Atlas, God of Angled Beam Shelves, Carrier of 200 lbs

Deflection and Stress Analysis of a Shelf system for edge beam Design Optimization using hand calculations and FEA.

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ABSTRACT

Context. The purpose of this report is to comprehensively summarize the final shelf design parameters and measurements,for the shelf to carry 200 lb, have a maximum deflection of 1/2 in relative to the corners, and a minimum factor of safety of 3.
Methods. Governing equations of stress and deflection derived from stress/strain relations with the help of boundary conditions will be expressed, to help relate the properties of deflection, types of stresses, and twist to parameters like length and other physical properties of the shelf itself. These provide the reader with a better understanding of internal and external behavior and reactions of the shelf and supporting beams when under load.
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1. Introduction

The rectangular shelf is of length L, width D, and thickness ts. Numerically, these will initially be taken to be 48", 18" and 0.125" respectively for a baseline of deflection, stresses, and factor of safety. These parameters are displayed below in Figure 1. The materials of these components will be chosen to be plywood for the shelf, and stainless steel for the edge beam. The properties for the shelf (plywood), according to Matweb [1], are a Young's Modulus of 1190 ksi, Poisson ratio of 0.29 and a yield strengh of 4 ksi. The edge supports are wooden.
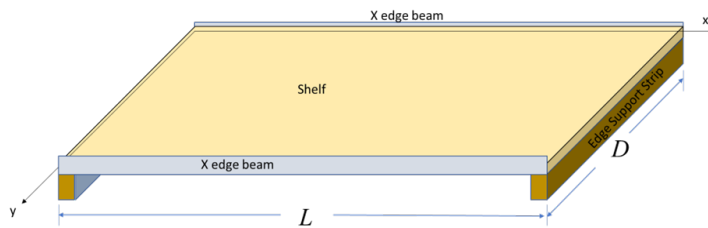
Fig. 1: Shelf design graphic

The beam is supported at each end by a ¾" wide x 1.5" deep wooden edge support strip, length D, attached securely to a wall. On the length L edges, the beam is supported by two angle beams, oriented as shown in Figure 2. The deflection in the shelf and edge beams can be modelled by simplifying the beams as a simply supported beam with a uniform load, and consulting the pre-existing, derived deflection curve found in Beer and Johnston's Mechanics of Materials Appendix. As for the shelf, the governing equations in the form of partial differentials and the flexural rigidity of the plate is used, alongside the boundary conditions for a meaningful expression. Its distribution can be derived by beginning with a general load configuration based off of the boundary conditions, that are then integrated over the length and width. The amplitudes are solved for and expanded

2D Fourier sine series is finally substituted.Given the plywood's fiber orientation, it fails in 'rolling shear', which is the shear stress acting on the radial-tangential plane perpendicular to the grain of the wood. Therefore, the factor of safety will use the lower rolling shear strength as the ultimate stress of the shelf.
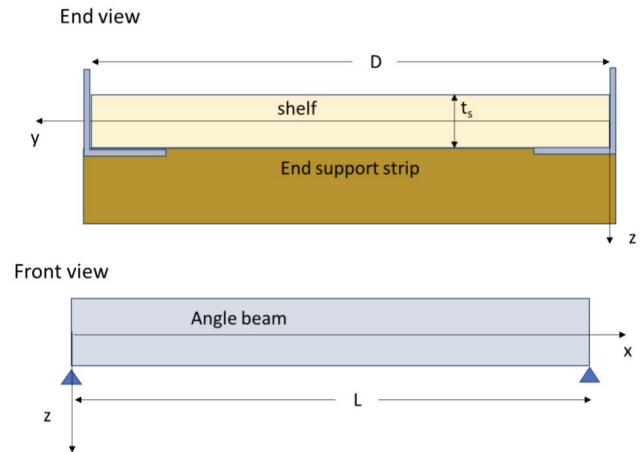
Fig. 2: Shelf design graphic

2. Shelf deflection and stress analysis equations

Plate deflection distribution
A partial differential equation of the fourth order enforces static equilibrium of the shelf, and is expressed in terms of the applied pressure onto the shelf over the flexural rigidity, D. Since the thickness is much less than the width and length of the shelf, we model this to be a thin plate. Here are some forms of this PDE for the plate deflection:
$\nabla^4 w = \frac{p}{D}$, *where w is deflection, p is pressure applied to the normal face of the shelf, and D is the flexural rigidity of this*

plate.

Or when expanded: $\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D}$

$D = \frac{Eh^3}{12(1-\nu^2)}$, *Flexural Rigidity*

To get this we use the fact that we know lateral strain in y is zero for continuity purposes in :

$$M = \int_{-a/2}^{a/2} \sigma_x z \, dz, \text{ where } \sigma_x = \frac{E\epsilon_x}{1-\nu^2} = -\frac{Ez^2}{1-\nu^2}\frac{d^2w}{dx^2}$$

$$-\int_{-a/2}^{a/2} -\frac{Ez^2}{1-\nu^2}\frac{d^2w}{dx^2} dz = \boxed{\frac{Eh^2}{12(1-\nu^2)}}$$

Another form which could prove useful is deflection expressed in terms of an amplitude and trigonometric expressions:
w(x,y) = $w_o \sin(\frac{\pi x}{a})\sin\frac{\pi y}{b}$

$\hookrightarrow \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{w_o}{D}\sin(\frac{\pi x}{a})\sin\frac{\pi y}{b}$

The assumptions and boundary conditions that are kept into account throughout and used particularly for the deflection distribution of the shelf are:
w = 0, $M_x = 0$ at x = 0, x=a
w = 0, $M_y = 0$ at y=0, y=a
Displacement in z-direction « thickness, so displacement is neglected

We see that expressing in terms of x and y and trigonometric expressions will be the route for our distribution solution. By U&F, deflection and uniform load can be represented as a 2D Fourier sine series. The coefficients m, n are numbers of half wavelengths of the defining shape in x, y. Utilizing the above boundary conditions of deflection and moments at the corners of the plate, the amplitude A_{mn} and B_{mn} will be solved for:

$$W(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\sin(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b}), \text{ where a, b = D, L}$$
respectively.
$$Q(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\sin(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})$$

To solve, substitute Fourier expansion of these and set them to zero:

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}[A_m[(\frac{m\pi}{a})^4 + 2(\frac{m\pi}{a})^2(\frac{n\pi}{b})^2 + (\frac{n\pi}{b})^4] - \frac{B_{mn}}{D}]\sin(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b}) = 0$$

$$\hookrightarrow A_{mn}\pi^4(\frac{m^2}{a^2} + \frac{n^2}{b^2}) - \frac{B_{mn}}{D} = 0$$

$$\boxed{A_{mn} = \frac{\pi^4 B_{mn}}{D(\frac{m^2}{a^2} + \frac{n^2}{b^2})}}$$

To get B_{mn}, $B_{mn} = \frac{4}{ab}\int_0^a\int_0^b B(x,y)\sin(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b})dxdy$

Finally, combine these to get the maximum deflection distribution and setting the sine terms to 1.

$W_{max} = \frac{4Pa^2}{\pi^4 D}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{m^2+n^2}$

This derivation doesn't account for only the odd numbers being used for the sum of the sines, the adjusted equations from above can be re-written:

$W(x,y) = \sum_{m=1,odd}^{\infty}\sum_{n=1,odd}^{\infty}\frac{16p_o}{D\pi^6 mn[(\frac{m}{L})^2)+(\frac{n}{D})^2]}\sin(\frac{\pi m x}{L})\sin(\frac{\pi n y}{D})$

$w_{max} = \frac{16p_o}{D_{fr}\pi^6}\sum_{m=1,odd}^{\infty}\sum_{n=1,odd}^{\infty}\frac{-(\frac{m+n}{2}+1)}{mn[(\frac{m}{L})^2+(\frac{n}{D})^2]}$ *where D_{fr} is the flexural rigidity and not D, the parameter length of the plate. Any previous occurances have been the flexural rigidity, but notation will be updated from here on out.*

Normal stresses to the plate

$\overline{\sigma_{xx}(x,y,z) = -\frac{M_z(x)y}{I_{zz}(x)}}$ no z dependence since we are analyzing planar stress. We can take this to be equal to $\sigma_{zz}(x,y)$
Moment taken about z axis. And y is distance from netural axis, where $I_{yy} = \int y^2 dA$ over area A(x)

$\sigma_{xx}max = \frac{M_{max}(h/2)}{I} = \left|\frac{6M_{xmax}}{t_s^2}\right|$

$\sigma_{yy}max = \frac{M_{max}(h/2)}{I} = \left|\frac{6M_{ymax}}{t_s^2}\right|$

Shearing stresses

$\overline{\tau_{xy} = -B_{mn} \cdot D_{fr} \cdot (1-\nu)\pi^2 \cdot (\frac{m}{L})(\frac{n}{D})}$ *Twisting moment per length/shear on xy*

$\tau_{xz} = B_{mn} \cdot D_{fr} \cdot \pi^3 \cdot \frac{m}{L}((\frac{m}{L})^2 + (\frac{n}{D})^2)$ *Shearing stress per unit length on the x directed face*

$\tau_{yz} = B_{mn} \cdot D_{fr} \cdot \pi^3 \cdot \frac{m}{L}((\frac{m}{L})^2 + (\frac{n}{D})^2) \cdot (\frac{n}{D})(\frac{L}{n})$ *Shearing stress per unit length on the y directed face*

Also written as:
$M_x = -D_{fr}(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial l^2}{\partial y^2})$

$M_y = -D_{fr}(\frac{2w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2})$

$M_{xy} = -D_{fr}(1-\nu)\frac{\partial^2 w}{\partial x \partial y}$

$\tau_{xy}max = 6 \cdot \left|\frac{\tau_{xy}}{t_s^2}\right|$

$\tau_{xz}max = 1.5\left|\frac{\tau_{xz}}{t_s^2}\right|$

$\tau_{yz}max = 1.5\left|\frac{\tau_{yz}}{t_s^2}\right|$

Applied distributed load by shelf to edge beams.

Using the fact that the physical parameters *L, D* of the shelf are of differing sizes, we know that the shorter size will experience the greatest shear, with the location being at the edges due to the supports. In our case, the width D is the short side, so it will be the y-directed face experiencing this. With some slight changes to the deflection equation, the distributed force that the shelf applies to the edge angle beam can similarly be expressed as:

$Q_y = D_{fr}\frac{\partial}{\partial y}(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2})$

$Q_y = \frac{16p_o}{D_{fr}\pi^6}\sum_{m=1,odd}^{\infty}\sum_{n=1,odd}^{\infty}\frac{1}{m[(\frac{m}{L})^2+(\frac{n}{D})^2]}\sin(\frac{\pi m x}{L})\cos(\frac{\pi n y}{D})$

Applying boundary conditions of the location of intererst being the edge of the plate at y = 0, the cosine term is set to 1. This expression tells us that the shear distribution is directly proportional to the average distributed load and affected by the plate aspect ratio. This tells us that in iterating, if we are trying to reduce this shear force, there is no point in targeting plate stiffness (flexural rigidity), the Young's modulus or the poisson ratio.

2.1. Shelf analysis results

$$M_z(x) = \left(\tfrac{wL}{4}\right)\left(\tfrac{x^2}{L^2} - \tfrac{x}{L}\right)$$

| Parameter	Symbol	Units	Initial Value	Final Value
Shelf thickness	t_s	in	0.75	0.60
Shelf Modulus	E_s	ksi	1.19E+06	1.19E+06
Shelf Poisson Ratio	v_s	-	0.29	0.29
Shelf yield strength	σ_Y	ksi	4	4
Shelf edge face shear strength	$\tau_{xy}U$	ksi	0.8	0.8
Shelf rolling shear strength	$\tau_r U$	ksi	0.25	0.2
Max shelf deflection	w_{max}	in	0.0063	0.0047
Max shelf normal stresses	$\sigma_{xx}max$	ksi	0.035	0.052
Max shelf normal stresses	$\sigma_{yy}max$	ksi	0.092	0.15
Max shelf shear stresses	τ_{xy}	ksi	0.038	0.058
Max shelf shear stresses	τ_{xz}	ksi	0.003	0.005
Max shelf shear stresses	τ_{yz}	ksi	0.004	0.007

3. Edge beam deflection and stress analysis equations

Edge Beam Governing Equations

As Figure 2 shows, the angle beam can be viewed in 2D, with it being further simplified to a 1D beam that is simply supported. This allows for the distributed load, internal shear load and moment distribution to be easily derived using our previous Mechanics of Materials beam theory :

$W(x) = \frac{W}{2L}$ *Distributed load,* w_0

$V(x) = \frac{W}{4} - w_0 \cdot x \rightarrow \frac{W}{4}\left(\frac{1-2x}{L}\right)$*The internal shear distribution, with the sign convention being displayed below*

Beam Section with loading
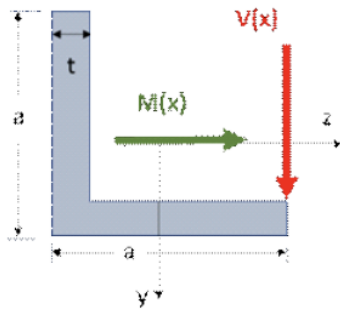
Fig. 3: The cross-section view of the angle beam with the axes displayed.

To find the moment distribution, we take the integral of the negative of the internal shear distribution. For the integrating limits we recycle the boundary conditions expressed in the previous section, where the bending moments at either ends are zero due to the supports opposing this motion.Hence the moment about the z-axis is given by:

$M_z(x) = \int_0^x -V(x)dx = \int_0^x \left(\frac{w}{4}\right)\left(\frac{2x}{L} - 1\right)dx$

$M_z max(x) = \frac{WL}{16}$*Maximum bending moment at the center of the beam.*

For the deflection distribution, we found the expression to be:

$w(x) = \frac{w_0 \cdot (x^4 - 2Lx^3 + L^3 x)}{24EI_z cos\phi - I_{yz} sin\phi}$

Moment of Inertia

The location of the centroid is alongside the line of symmetry, but outside of the physical beam. Its exact coordinates are (0.018809, 0.018809) inches.

$I_z = \frac{1}{12}a^4 + a^2\left(\frac{a}{2} - \bar{y}\right)^2 - \frac{1}{12}(a - t)^4 - (a - t)^2\left(\frac{a}{2} + \frac{t}{2} - \bar{y}\right)^2$

Where the centroid distance $\bar{y} = \bar{z}$ at:

$\bar{y} = \frac{a^2 + at - t^2}{2(2a - t)}$

Product of Inertia

$I_{zy} = at\left(\bar{y} - \frac{t}{2}\right)\left(\frac{a}{2} - \bar{y}\right) + (a - t)\left(\bar{y} - \frac{t}{2} - \frac{a}{2}\right)\left(\frac{t}{2} - \bar{y}\right)$

Maximum Deflection

Begin by computing the deflection curve using our simply supported beam model with uniform distributed force along it.To get the maximum deflection, the maximum of sinusoidal functions are set giving the following:

$w_{max}(x) = \frac{5w_0 L^4}{384EI_z cos\phi - I_{yz} sin\phi}$

Maximum Twist

According to Shigley's, the increment in twist can be expressed in terms of the change in teh length of a beam Δx from an applied torque T:

$\Delta\psi = \frac{T\Delta x}{2\beta ab^3 G}$

Another form of beam twist through a derivation that is based on the integration from x = 0 and x=L/2 since it would can be thought of the sum of these increments.

$\psi = \frac{WL(a - \frac{t}{2})}{32\beta ab^3 G}$

Maximum Axial Normal Stress

Navier Stokes' formula allows for stress to be calculated at any point of a cross-section due tot bending and axial forces.

$$\sigma = \frac{P}{A} + \frac{M}{I_y} \cdot z$$

3.1. Beam analysis results

Parameter	Symbol	Units	Initial Value	Final Value
Beam Modulus	E_B	ksi	29000	29000
Beam Poisson Ratio	ν_B	-	0.3	0.3
Moment of Inertia	I_{zz}	in^4	0.02172	0.01964-
Product of Inertia	I_{YZ}	in^4	0.01276	0.001111
Polar Moment of Inertia	J	in^4	0.001199	0.00323
Shear Center Offset	Z_{shear}	in	0.0775	0.06723
Max Beam z deflection	Z_{max}	in	0.00089	0.00084
Max Beam y deflection	y_{max}	in	0.00028	9.5e-05
Max Beam axial stress	$\sigma_{xx}max$	ksi	69.8	55.94
Max Beam Twist	θ_{max}	degrees	2.4	1.6

4. Recommended Design

The iterated design came out to be approximately 28% lighter, at 15.23 lb. The factor of safety was lowered to 5.2 from 6.5, which is still significantly greater than the desired minimum safety of 3. The changes made to achieve this were the cross-sectional height and width, a, being changed to 0.5 in; the thickness of the plate changed to 0.6 in;the thickness of the l-shaped cross section reduced to 0.1 in; the length L of the shelf changed to 40 in; and finally the width D at a value of 10 in. All of these changes are illustrated in the scaled figures below. The
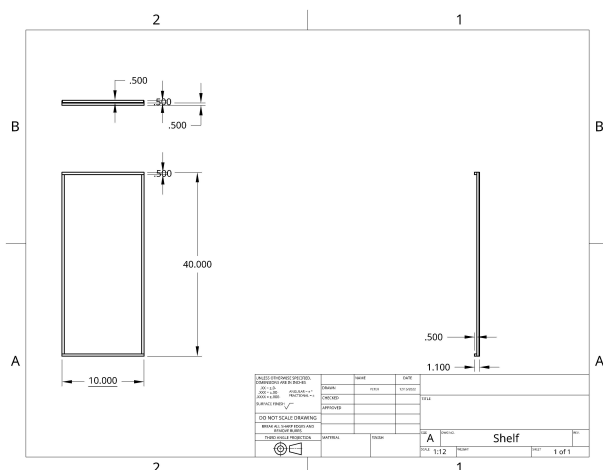

Fig. 4: Iterations made to the overall assembly

cross-sections are compared in an unorthodox, but direct way in the scaled drawing side by side to each other in Figure 5.
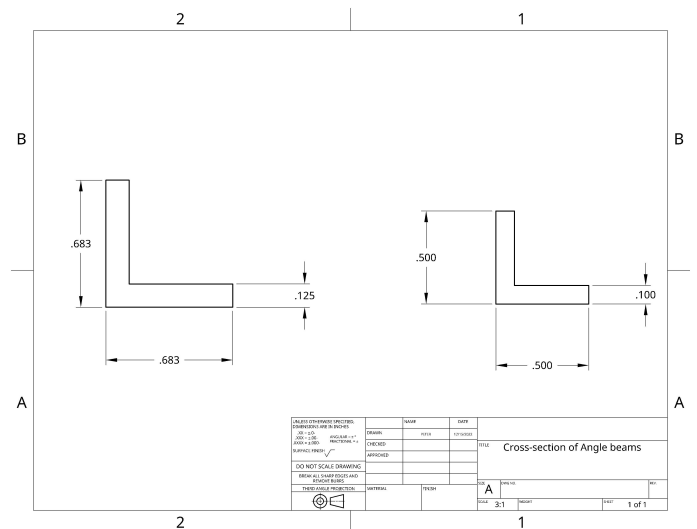

Fig. 5: On the left is the original angle beam cross-section, while on the right is the iteration.

4.1. Benefits of Design

The iterated design allows for less material to be used which contributed to lighter shelves. The initial design was over-engineered and so parameters like cross-sectional widths were shrunk. Despite this, the changes made caused an observable increase in stability where there often wasn't a tendency for there to be a moment along given axes. This was shown in the greater value of the polar moment of inertia from the initial value - an increase of 71%. A greater polar moment of inertia signifies a

greater resistance to torsional deformation, something that was also seen in the lowered axial stress and beam twist. Lastly, the design requirement of deflection being less than 1/2" was met. There is still potential for further improvement given the room between design requirements, with the limiting factor being the factor of safety since a deflection that high is hard to reach with stainless steel beams.

5. Discussion

The overall deflection with the uniform loading of the 200lb is similar in the isolated plate deflection contour plot in that there are concentric rings, with the greatest deflection being in the centre.
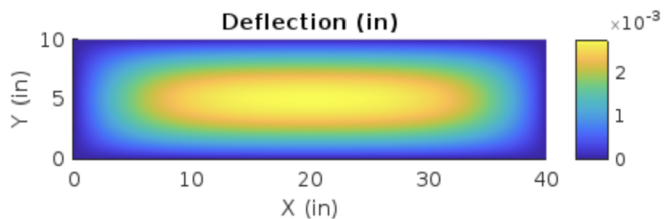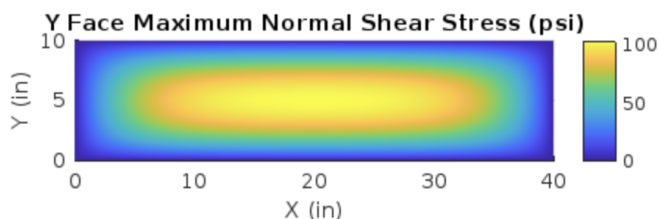

Fig. 6: A contour plot of just the plate experiencing deflection, with the greatest value at (20,5)
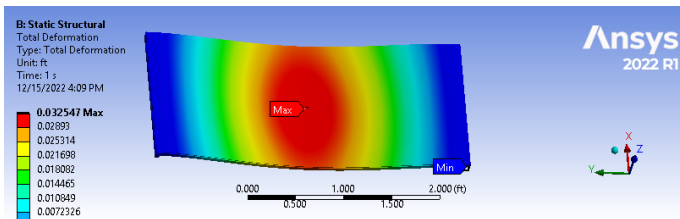

Fig. 7: A simulation of the entire system,with the greatest value of deflection at the center of the shelf.

As for the maximum stresses, the contour plot reveals that the greatest stress value is the y-directed normal stress, which also acts at the centre of the plate and has a similar contour plot to the deflection plot.

Fig. 8: A contour plot of just the plate experiencing maximum normal stress in the y-direction, with the greatest value at (20,5)

To consider the entire system, the same numbers are run in FEA software that generate the following distribution.
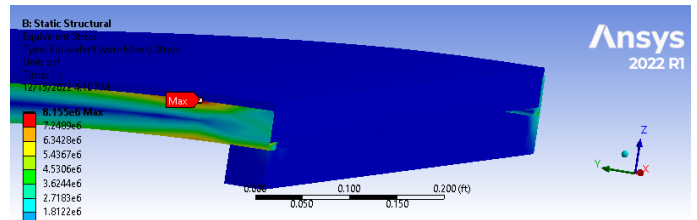

Fig. 9: This reveals to us that the greatest stress when taking the entire system as a whole is in the angled beam support where the probe is. The units in this are psf, but the maximum value is 56.6 ksi.

Factor of Safety

The factor of safety plot is extremely useful as it highlights areas of interest that dip below the set threshold. In this case Ansys highlighted such areas that in fact did dip below the required FOS of 3.While this may not necessarily be the case in reality since the stress value used may not be the correct one and some digging into the results file could locate this. Future iterations could enforce these weaker bands to make the design more robust.
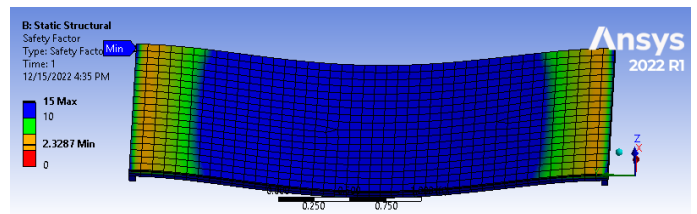
Fig. 10: A top view of the FOS , with a strip near the 'ends' where it dips significantly. The value of the number isn't as significant given that we have not looked at what stress values it used.
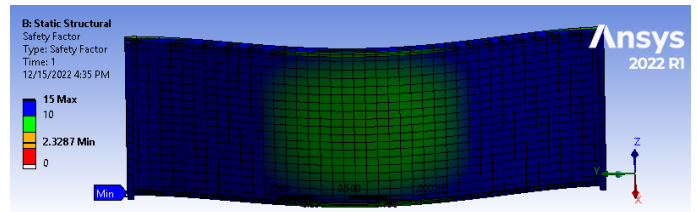
Fig. 11: A bottom view of the FOS since there is an area of interest that is concentric around the centre.

Therefore it is clear this design is more strength-orientated since it fluctuates in factor of safety values and even dips below the required 3 near the supports.To further improve, I would refine the mesh even further, and do a couple more iterations for the FOS specification to be confidently achieved throughout.Refining the mesh will allow for higher precision; the current mesh size was 0.01 ft.

References

[1] The Online Materials Information Resource: Plywood, MatWeb. Available at: https://www.matweb.com/search/datasheet.aspx?matguid=bd6620450973496ea2578c28

[2] Ugural, A.C. and Fenster, S.K. (2012) Advanced strength and applied elasticity. Upper Saddle River, NJ: Prentice Hall.

6. Appendix

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% Simply supported rectangular plate model under uniform load.
% Uses 2D Fourier sine series solution. Prepared for ME300 Applied Elasticity
% Parameters
E = 1.19E+06; % Young's modulus, psi (Plywood, worst case)
nu = 0.29; % Poisson ratio (worst case)
L = 40; % Plate length, in
D = 10; % Plate depth, in
ts = 0.6; % Plate thickness, in
W = 200; % total weight carried, lb
p0 = W/(L*D); % Distributed weight, psi
K = E*ts^3/(12*(1-nu^2)); % Plate rigidity per unit width, lb-in
% Simulation parameters
imax = 101; jmax = 101; % maximum indices of 2D Fourier series
ngrid = 100; % # of plot points -1 in x and y directions
% Set up X and Y grid for plotting
NDedge = [0:1/ngrid:1]; % make an equally-spaced vector from 0 to 1
NDX = NDedge'*ones(ngrid+1,1)'; % Expand into matrix using outer product
NDY = NDX';
X = L*NDX; Y = D*NDY; % Scale up to dimensional mesh
% Initialize Fourier series coefficient matrices
pij = zeros(imax,jmax); % pressure distribution
Bij = zeros(imax,jmax); % deflection
Mxij = zeros(imax,jmax); % bending moment/length on x-directed face
Myij = zeros(imax,jmax); % bending moment/length on y-directed face
Mxyij = zeros(imax,jmax); % twisting moment/length
Qxij = zeros(imax,jmax); % shear/length on x directed face
Qyij = zeros(imax,jmax); % shear/length on y directed face
% Compute coefficients
for i = 1:2:imax
 for j = 1:2:jmax
 pij(i,j) = 16*p0/(i*j*pi^2);
 Bij(i,j) = pij(i,j)/(K*pi^4*((i/L)^2 + (j/D)^2)^2);
 Myij(i,j) = -Bij(i,j)*K*pi^2*((j/D)^2 + nu*(i/L)^2);
 Mxij(i,j) = -Bij(i,j)*K*pi^2*((i/L)^2 + nu*(j/D)^2);
 Mxyij(i,j) = -Bij(i,j)*K*(1-nu)*pi^2*(i/L)*(j/D);
 Qxij(i,j) = -Bij(i,j)*K*pi^3*(i/L)*((i/L)^2 + (j/D)^2);
 Qyij(i,j) = Qxij(i,j)*(j/D)*(L/i);
 end
end
% Initialize fields
w = 0*X; p=w; Mx=w; My=w; Mxy=w; Qx=w; Qy=w; Sxxmax=w; Syymax=w;
Txymax=w;Txzmax=w; Tyzmax=w;
% Compute Fourier series
for i = 1:2:imax
 for j = 1:2:jmax
 sinxsiny = sin(pi*i*X/L).*sin(pi*j*Y/D);
 cosxsiny = cos(pi*i*X/L).*sin(pi*j*Y/D);
 sinxcosy = sin(pi*i*X/L).*cos(pi*j*Y/D);
 cosxcosy = cos(pi*i*X/L).*cos(pi*j*Y/D);
 p = p + pij(i,j)*sinxsiny;
 w = w + Bij(i,j)*sinxsiny;
 Mx = Mx + Mxij(i,j)*sinxsiny;
 My = My + Myij(i,j)*sinxsiny;
 Mxy = Mxy + Mxyij(i,j)*cosxcosy;
 Qx = Qx + Qxij(i,j)*cosxsiny;
 Qy = Qy + Qyij(i,j)*sinxcosy;
 end
end
figure;
surf(X,Y,p); shading interp, axis equal tight, colorbar, view(2)
xlabel('X (in)'), ylabel('Y (in)'), title('Pressure Load (psi)')
figure;
```

```
wmax = max(max(w))
surf(X,Y,w); shading interp, axis equal tight, colorbar, view(2)
xlabel('X (in)'), ylabel('Y (in)'), title('Deflection (in)')
figure
surf(X,Y,Mx); shading interp, axis equal tight, colorbar, view(2)
xlabel('X (in)'), ylabel('Y (in)'), title('X Face Moment/length Mx(lb)')
figure
surf(X,Y,My); shading interp, axis equal tight, colorbar, view(2)
xlabel('X (in)'), ylabel('Y (in)'), title('Y Face Moment/length My(lb)')
figure
surf(X,Y,Mxy); shading interp, axis equal tight, colorbar, view(2)
xlabel('X (in)'), ylabel('Y (in)'), title('Twisting Moment/length Mxy(lb)')
figure
surf(X,Y,Qx); shading interp, axis equal tight, colorbar, view(2)
xlabel('X (in)'), ylabel('Y (in)'), title('X Face Shear/length Qx(lb/in)')
figure
surf(X,Y,Qy); shading interp, axis equal tight, colorbar, view(2)
xlabel('X (in)'), ylabel('Y (in)'), title('Y Face Shear/length Qy(lb/in)')
figure
plot(X(:,1),Qy(:,1));
xlabel('X (in)'), ylabel('Qy (lb/in)'), title('Y=0 Edge Shear Profile')
figure
plot(Qx(1,:),Y(1,:));
ylabel('Y (in)'), xlabel('Qx (lb/in)'), title('X=0 Edge Shear Profile')
Sxxmax=6*abs(Mx)/ts^2;
Syymax=6*abs(My)/ts^2;
Txymax=6*abs(Mxy)/ts^2;
Txzmax=1.5*abs(Qx)/ts; Tyzmax=1.5*abs(Qy)/ts;
Sxxmaxmax = max(max(Sxxmax))
Syymaxmax = max(max(Syymax))
Txymaxmax = max(max(Txymax))
Txzmaxmax = max(max(Txzmax))
Tyzmaxmax = max(max(Tyzmax))
figure
surf(X,Y,Syymax); shading interp, axis equal tight, colorbar, view(2)
xlabel('X (in)'), ylabel('Y (in)')
title('Y Face Maximum Normal Shear Stress (psi)')
```