Measuring the Young's Modulus of a Vibrating Beam

Determination of Young's Modulus for Varying Beam Materials at Different Free Lengths via Accelerometer Measurements

By Scott Losirisup[1], Petros Sklavounos[1], Angel Jimenez[1]

[1] *The Cooper Union*

Technical Report for ME360: Instructed by Professors G. Sidebotham, D. Wootton, K. Wright

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Abstract

The experimental determination of the Young's modulus for unknown beam materials remains mostly destructive and complicated, with limitations in shapes and sizes being tested (i.e., point bending test, torsion test, ultrasonic method). In this work, the resonant frequency method is used to measure the natural frequency of a vibrating cantilever beam for the calculation of the Young's modulus through mass-spring characterization and beam theory. By quantifying this elastic constant, the stiffness and the resistance to deformation when under load is known, crucial to most mechanical engineering applications. Numerical results indicate that the relationship between natural frequency and the free length is a monotonically decreasing function, with the power law fit best capturing the data. The Young's Modulus for the twelve different beams(aluminum, brass, steel, and carbon fiber of varying dimensions) was calculated with a 2.3 GPa uncertainty.

Introduction

The measurement of the Young's modulus of a material is crucial as it reflects the stiffness and elasticity of a material under stress. The resonant frequency method provides a simple, minimal-equipment approach that is non-destructive to the beams. A beam of the material is clamped at one end allowing a free oscillation of the beam by plucking it with a finger at the free length(Figure 1). By measuring the natural frequency of the beam using an accelerometer, and knowing its dimensions and mass, the Young's modulus can be calculated using a theoretical model.

In this study, twelve beams of varying materials and dimensions (aluminum, brass, steel, and carbon fiber) were subjected to an initial displacement by plucking, with the free vibration responses being captured using an accelerometer whose data is relayed to the data acquisition system (The paper describes the testing protocol used to measure the natural frequency of the cantilever beam(Experimental). It also discusses the uncertainty introduced by the instrumentation, provides a detailed step-by-step procedure for the experiment, and presents the results obtained (Results). The processed raw data is analyzed, and the theory of the experiment is discussed (Discussion). Finally, the paper concludes by summarizing the findings of the study(Conclusion) and outlining some recommendations for any future work.

Experimental) . The beam of interest has a free length that is varied by approximately 17 well-spaced lengths up until the minimum of 5" (Appendix 1: Lab Instructions). The acceleration signal from the excited beam is analyzed in the LabView software, with the signal using Fourier transforms to finally identify the natural frequency of the beam. Given that the mass of the accelerometer is negligible compared to the beams, and that a displacement rather than an impulse is used to excite the beam, the system can be characterized and modelled by the general equation for natural frequency of an undamped cantilever beam (Equation 1). The final form of the equation used to calculate the Young's Modulus is rearranged in Appendix 2: Young's Modulus Calculation, where a sample calculation is outlined as well. The polar moment of Inertia, I, in this equation varies depending on the geometry and cross-section of the beam, Appendix 2: Young's Modulus Calculation [3].

The paper describes the testing protocol used to measure the natural frequency of the cantilever beam(Experimental). It also discusses the uncertainty introduced by the instrumentation, provides a detailed step-by-step procedure for the experiment, and presents the results obtained (Results). The processed raw data is analyzed, and the theory of the experiment is discussed (Discussion). Finally, the paper concludes by summarizing the findings of the study(Conclusion) and outlining some recommendations for any future work.

Experimental

The testing protocol developed to measure the natural frequency of a variety of cantilevered beams is described in this section. The experimental procedure calls for a beam to be mounted parallel to a tabletop, as outlined in Figure 2. The experiment consists of manually initiating a free oscillation of the beam by plucking it with a finger at the free length, as shown in Figure 1. The data acquisition system measures the acceleration as a function of time once the beam tip is plucked, long enough to obtain several oscillations. The objective of the investigation is to calculate the Young's Modulus of beams of varying materials at different free lengths. Some of the tested beam materials include aluminum, brass, carbon fiber, and steel.

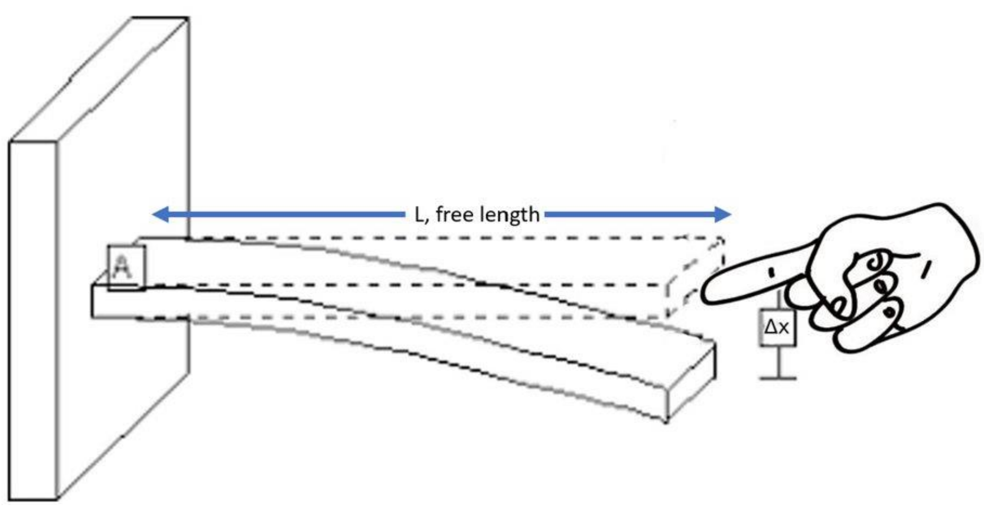

Figure 1: Simple graphic of cantilever beam being plucked to simulate a small displacement dx, fixed at A

Equipment

A schematic diagram of the test stand is shown in Figure 2, which consists of the apparatus needed to hold the cantilevered beam in place and the instrumentation required to measure the natural frequency of the plucked beam. All the components within these two are to be discussed separately in the sections to come, with the objective of a natural frequency vs. length curve of the beam for each beam sample. A copy of the lab instructions is included in Appendix 1: Lab Instructions.

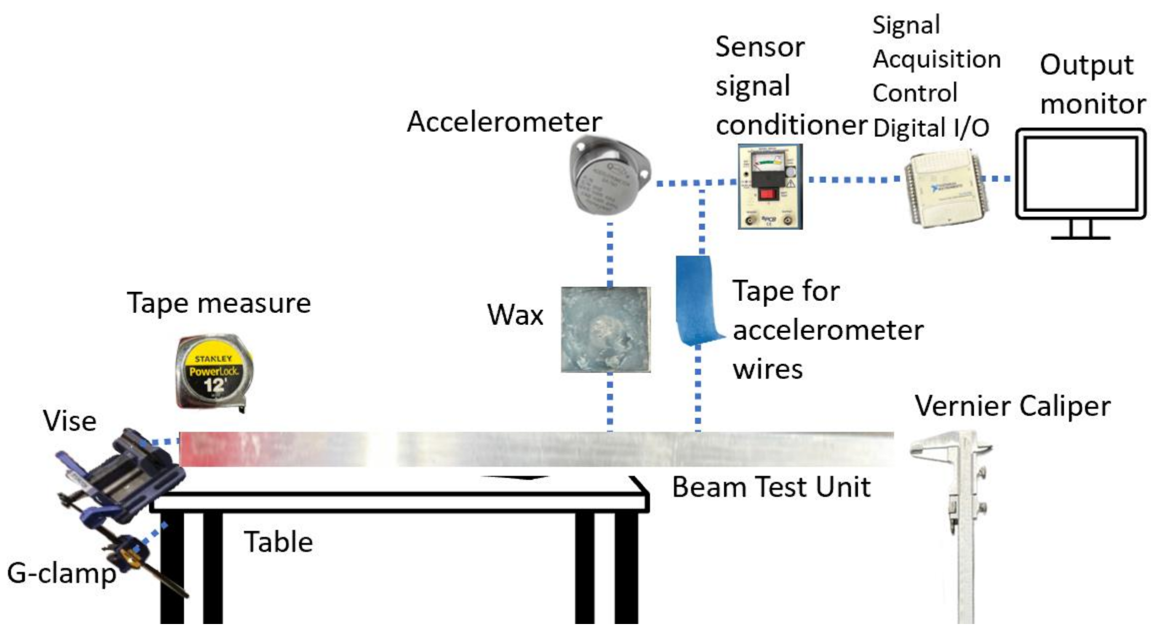

Figure 2: Schematic of Experimental Arrangement - outlining the clamped beam on a workbench, with the accelerometer transmitting data to the computer.

Apparatus

The apparatus for the beam vibration experiment is designed to hold the beam in place in a manner that allows it to be manually excited to initiate a free vibration. A combination of a clamp connected to a vice fixes the setup to a lab bench, with the vice allowing for the effective free length of the beam to be varied. Twelve student teams were each assigned a different beam with varying material and dimensions, detailed in Figure 3. An individual experiment is initiated when the beam is plucked manually and made to vibrate freely.

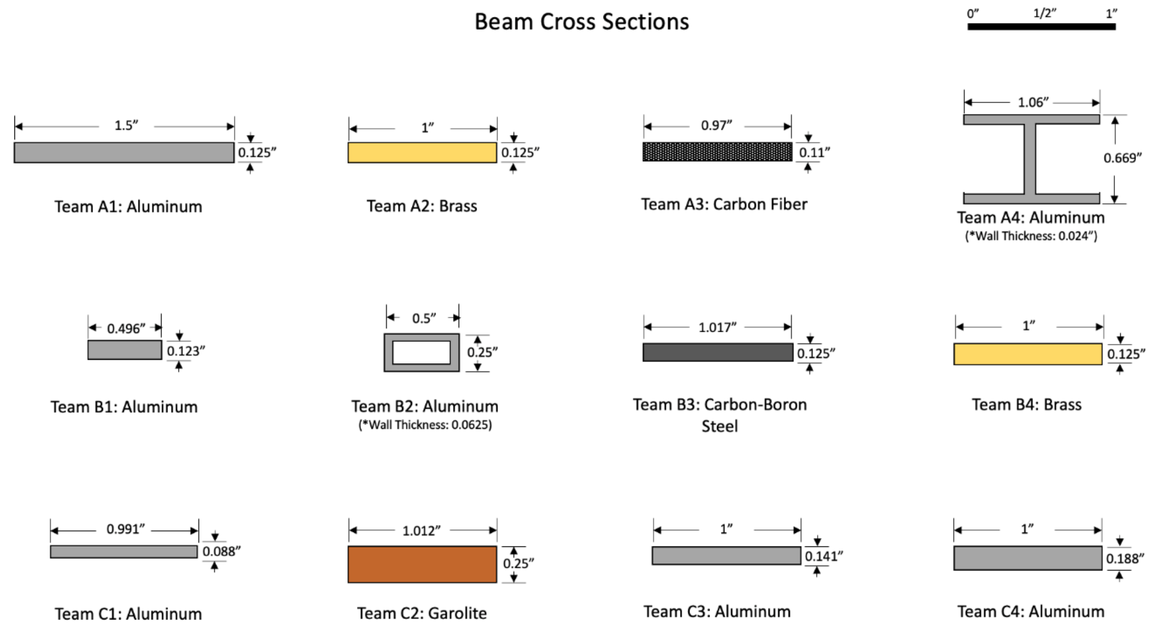

Figure 3:Beam Cross Section Samples

Instrumentation

The width and thickness of the beams is measured using a standard, analog vernier caliper (Mitutoyo Analog Vernier Caliper Series 530) that has graduations of 0.001 in, with a readability of 1/128 in and manufacturer reported uncertainty of ± 0.5/128 in (up to 6 in).

The beams' length is measured with a Stanley 12 feet measuring tape, with graduations of 1/16 inches, which is readable to ±1/32 inches. The systematic uncertainty in determining the free length is 0.123 inches (Appendix 3: Uncertainty Determination). The manufacturer reported uncertainty is ±1/32 inches over 12 feet. When using the tape measure for the beam samples, the same beam was used at a wide enough range as possible.

The mass of the beam and accelerometer was measured using a Royal ds3 digital scale. The random uncertainty is 1 gram, the smallest graduation of this instrument and readability, while the systematic uncertainty reported by the manufacturer is 0.05 grams, up to 100 grams. The scale should be tared and calibrated to remove any possible zero error. The mass of the accelerometer was measured to be 1.4% of the mass of the beam on average and so it was assumed negligible for this experiment.

A PCB Piezotronics 100.1mV/g accelerometer was mounted on the beam with wax to measure the acceleration of the beam near its free end. The masking tape is used to secure the wiring to the beam. Wires were used to transfer information from the accelerometer, through the National Instruments NI USB-6008 10 kS/s Multifunction I/O and PCB ICP Sensor Signal Conditioner (Model 480C02), to Labview Software on the computer. This measurement process is affected by systematic errors from the aforementioned instruments with the accelerometer introducing both systematic and random uncertainty due to its inherent noise and sensitivity to environmental conditions such as temperature and humidity. The signal conditioner adds to the uncertainty by its gain, linearity, and noise. Similarly, the signal acquisition control digital I/O contributes to the uncertainty in the frequency measurement due to factors such as sampling rate, quantization error, and noise. In The total random and systematic uncertainty that was calculated for the frequency was found to be ±5% due to the main contribution of the accelerometer’s reported uncertainty of ±5%.

Lastly, wax secured the accelerometer to the beam, with the wires connected from the accelerometer to the rest of the data acquisition taped down using masking tape to ensure that the wires would remain untangled and out of the way of the oscillating beam.

Procedure

Safety glasses were worn and the workspace was cleared. The clamps were then used to firmly hold down the vise from the benchtop, leaving enough space for the beam to be mounted.

The measurements of the beam itself were taken, ensuring that the scale was tared. After weighing and recording the value, the vernier caliper and tape measure were used to measure and record the length, width, and height of the beam. The beam was loaded parallel to the table in between the jaws of the vise, leaving 22” of it free and 2” of it clamped.

The computer was switched on to launch the LabView Software. Next, the accelerometer was connected to the data acquisition system with the wires coming out of the system. After connecting, the wires were checked to ensure there were no kinks present and that the USB A cable was connected to a free port on the computer. The accelerometer was rubbed in wax and stuck onto the beam near the free end. Masking tape was used to secure the accelerometer cable about 2” down the beam to make sure it does not obstruct the beam vibration.

The LabView VI file (BeamM1_V1_023.VI) was opened and the “front panel” automatically displayed two graphs. Afterwards, the block diagram was opened by hitting Ctrl+E and following the previous step, the DAQ Assistant block was launched. Within the DAQ Assistant block, the settings were checked to ensure they followed all systematic parameters:

- Signal Input Range: Max 5 V (NI 6008/9), Min -5 V (NI 6008/9)
- Scaled Units: Volts (NI 6008/9)
- Acquisition Mode: N Samples
- Samples to Read: 20000
- Rate (HZ): 2kHz
- Verify measurement does not time out before 10 seconds in the Advanced Timing tab

The setup was then shown to the supervisor to ensure all the connections were set properly, all the hardware was configured securely, and all system settings were correctly set. The accelerometer signal was then tested by gently plucking the beam, hitting the run button on the top right of the computer software, and observing the two displayed graphs to see whether the setup could provide reliable information. After confirming the setup was complete, the front panel window was opened again.

To begin collecting data, the free end of the beam was gently plucked, and the run button was pressed where the plots with the sinusoidal response were displayed through the data acquisition system (**Error! Reference source not found.**). A decaying vibration signal without saturation and without high frequency noise was displayed.

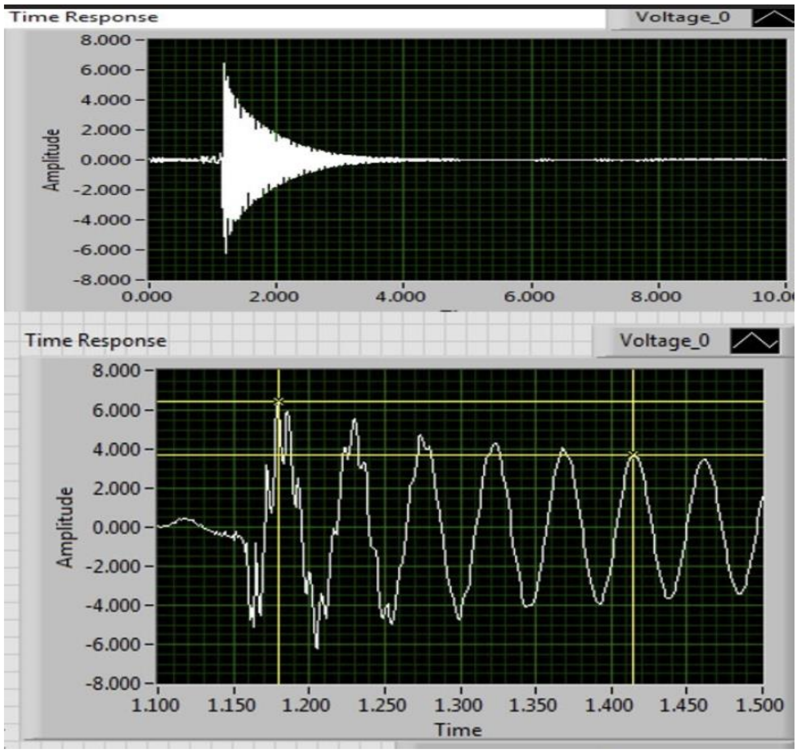

Figure 4:A screenshot of the scope showing data from accelerometer.

With a reliable frequency response, the Spectrum plot was opened. Another scope of a Fourier Transform being used by the software is displayed with the Frequency (Hz) vs. Time(s) with which the peak found and recorded as the natural frequency.

The jaws of the vise were opened, and the beam was adjusted to a new free length and clamped shut again. Recording the new free length, the beam was plucked and the new frequency response and spectrum plot frequency with its max amplitude were displayed and recorded. The measurement of natural frequencies was repeated for approximately 17 well-spaced lengths up until the minimum of 5", which is where data begins to be difficult to record. Once a sufficient breadth of data was collected and recorded onto the data sheet, the natural frequency vs. beam free length was plotted by hand, and then ultimately by Excel.

Results

Figure 5 shows the data taken from all 12 groups who have conducted the experiment on a singular scatter plot. This allows for the raw data to be assessed and any possible insight to be revealed from the basic trend. Each group had a different dimension, material and in some cases, cross-section beam, all of which when plotted in this way reveal monotonically decreasing, functional trends and relationships to each other.

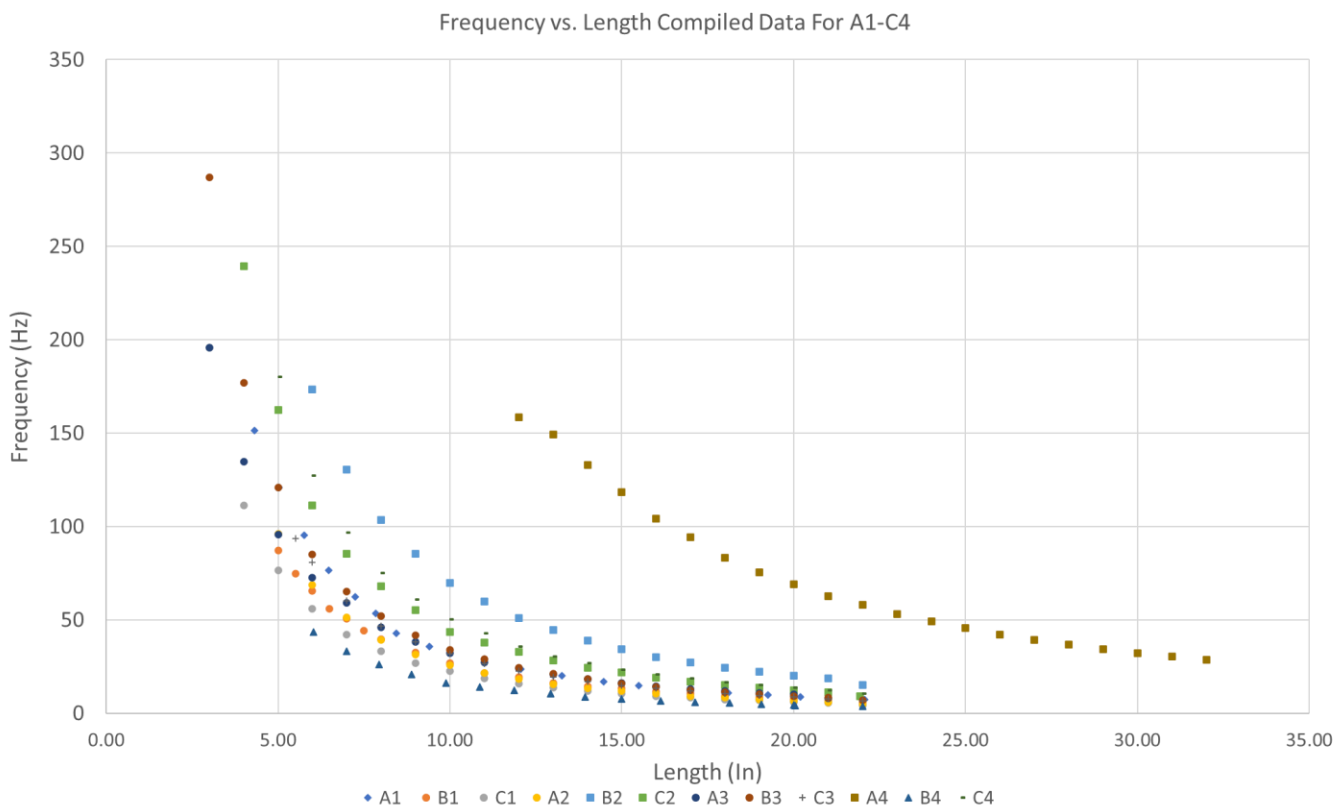

Figure 5: Frequency vs. Free Length for all raw data.

The rightmost A4 data appears consistent in shape as the rest of the groups but shows a significantly higher natural frequency. This beam was the Aluminum I-beam whose offset is due to the cross-sectional difference, further discussed in Discussion. Data before 5 inches for the groups and 10 inches for the I-beam was instructed to not be recorded given the limitations of the current setup and accelerometer. It proved difficult to excite and record accurate data. The frequency response graphs that were displayed appeared unstable and did not follow the desired naturally damped sinusoidal response shown in Figure 4.

Since the general trends of all beams are similar, a case study of group C4 (the aluminum 1"x0.188") is chosen to consider experimental uncertainty and the underlying functional dependence by applying various curve fitting functions. Using the Grubbs Test (Appendix 4: Outlier tests: Grubbs Test), an outlier for the 18th trial was identified and excluded due to its assessed impact.

Initially, a linear fit was applied to the frequency vs. length graph, Figure 6 . It is evident that the line was not adequate as the linear fit missed many of the data points. The R^2 value which measures the trendline's reliability was seen to be 0.7838.

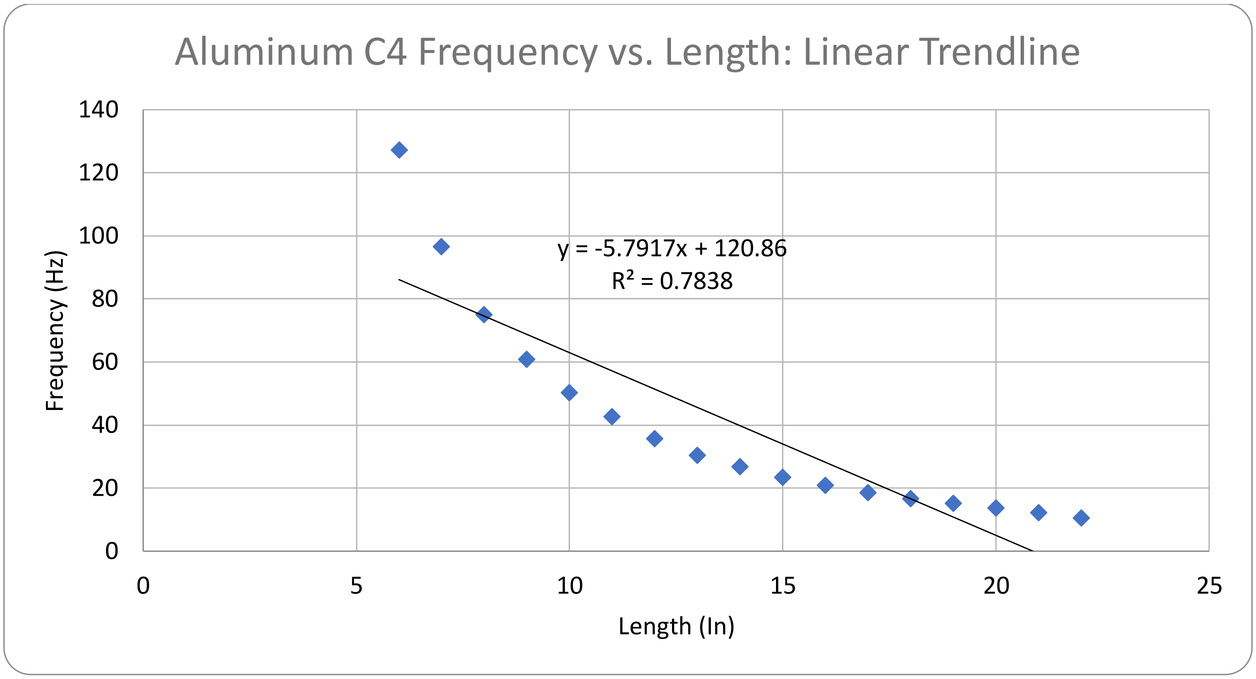

Figure 6: Frequency vs Free Length data for group C4 with Linear Trendline.

A similar matching of the data points to the fit line was done with both the exponential (Figure 7) and second-order polynomial (Figure 8) trendlines. These best line fits visually matched up with the raw data better than a linear fit but was still not able to capture the true shape. The R^2 value of the exponential and polynomial lines which were 0.9598 and 0.962, respectively, were closer to one, but still not able to capture the shape over the whole range.

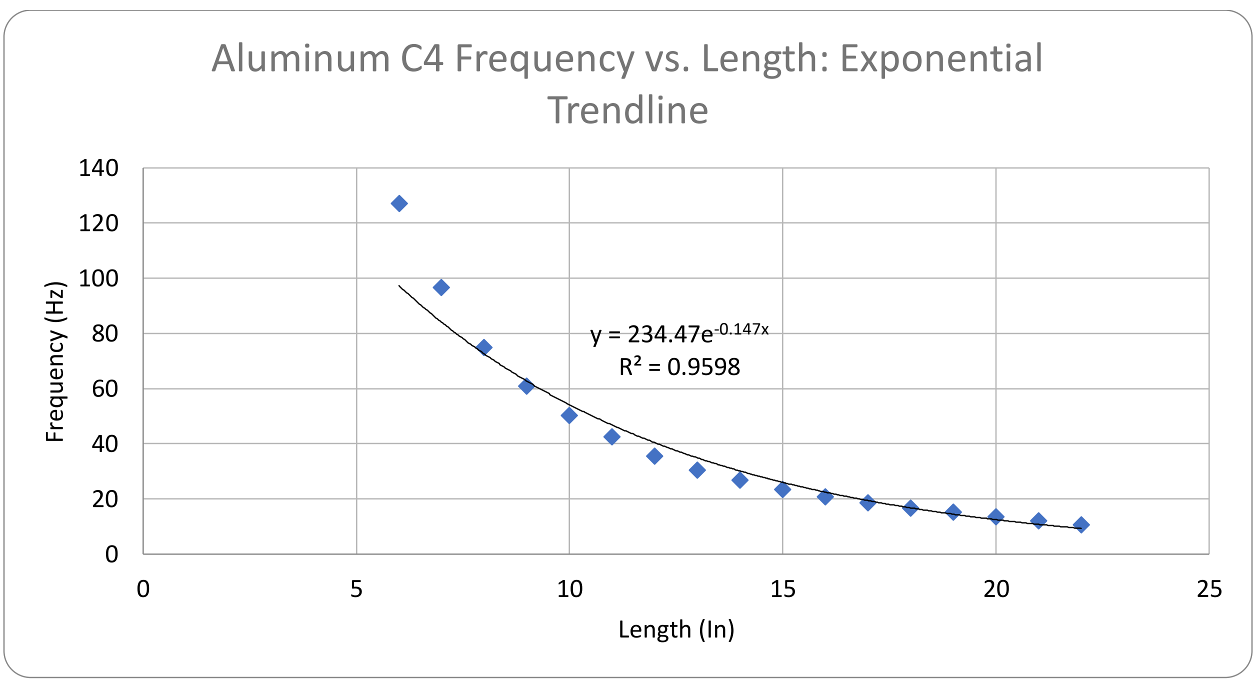

Figure 7: Frequency vs Free Length data for group C4 with Exponential Trendline.

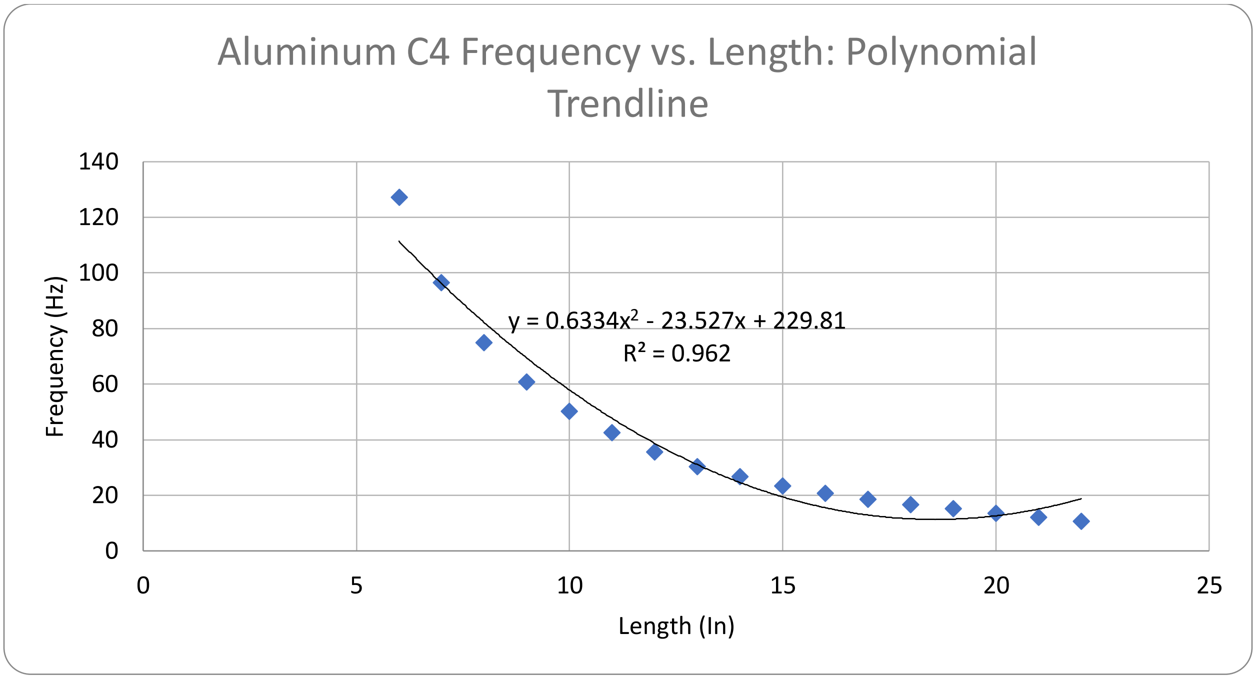

Figure 8: Frequency vs Free Length data for group C4 with Polynomial Trendline.

Finally, a power trendline applied to the frequency vs. length graph shown in Figure 9 displayed a line-of-best-fit that matched ideally with the data points of Group C4. The R^2 value of the line being 0.9997 further proved the assertion and reliability of the trend. A residual plot displaying the deviation of the raw data to the power trendline can be found in Figure 14 in Appendix 7, whereby the 6-inch free length is also seen to be an outlier given its large deviation. This brings the exponent closer to 2, with a value of 1.89.

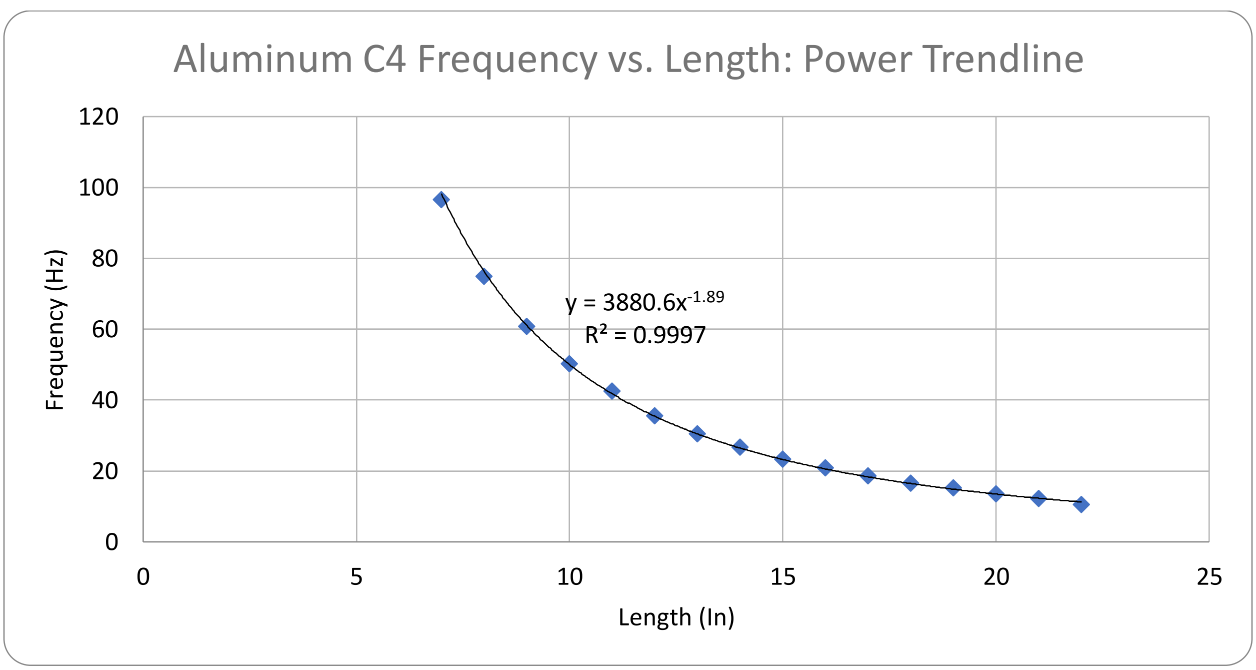

Figure 9: Frequency vs Free Length data for group C4 with Power Trendline.

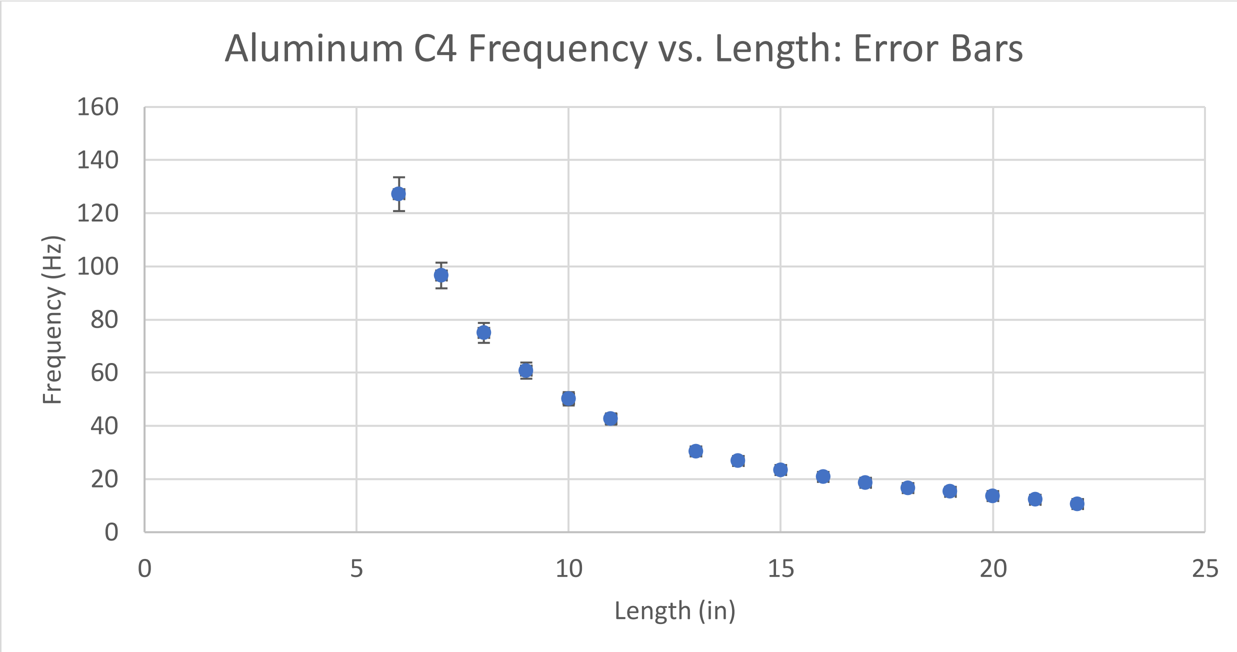

Figure 10: Frequency vs Free Length error bars for group C4.

Alternate views for better ease of pattern and trend visibility of Figure 11Figure 5 are shown below in a log-log scale, and against $1/l^2$ as the y-axis. A plot portraying the coefficient x against $\sqrt{I/A\rho}$ in Figure 12 was also created to further show the trend and consistency in the material across the different beams. The coefficient x contains α_n^2 and $\sqrt{EI/A\rho}$. For different materials, the plot would be linear but with different slopes.

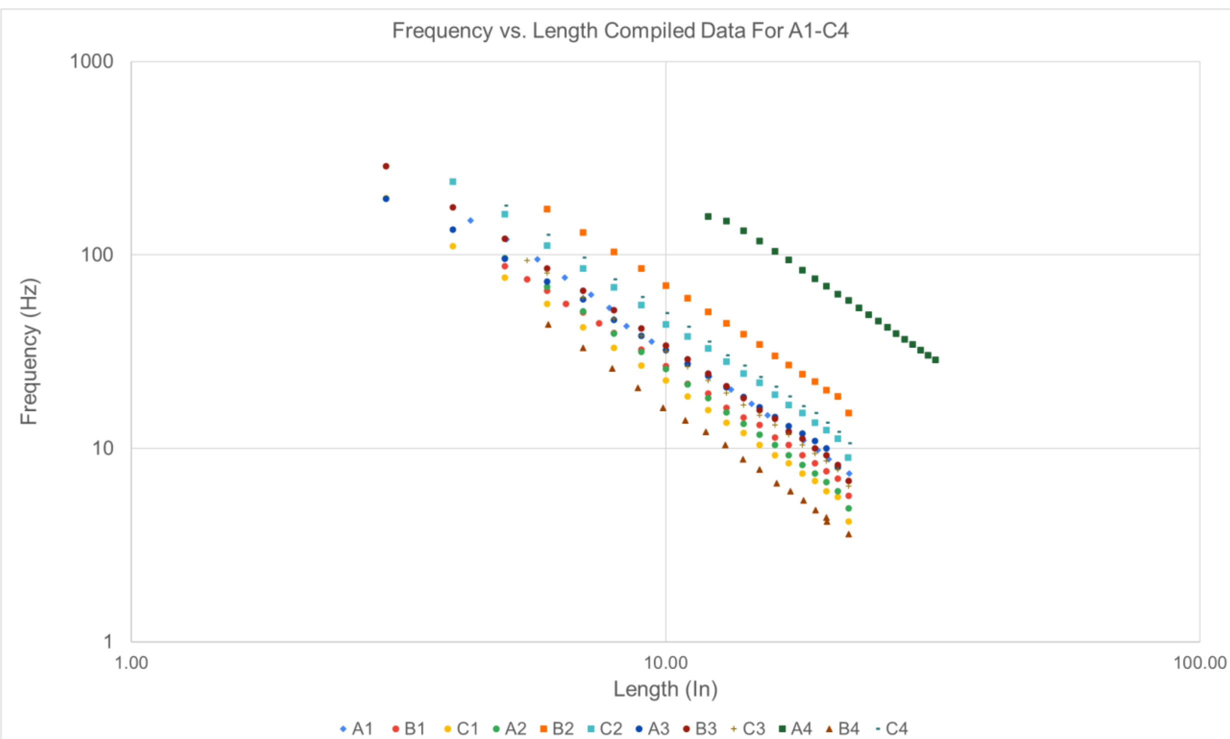

Figure 11: Raw Data plotted on Log-Log scale.

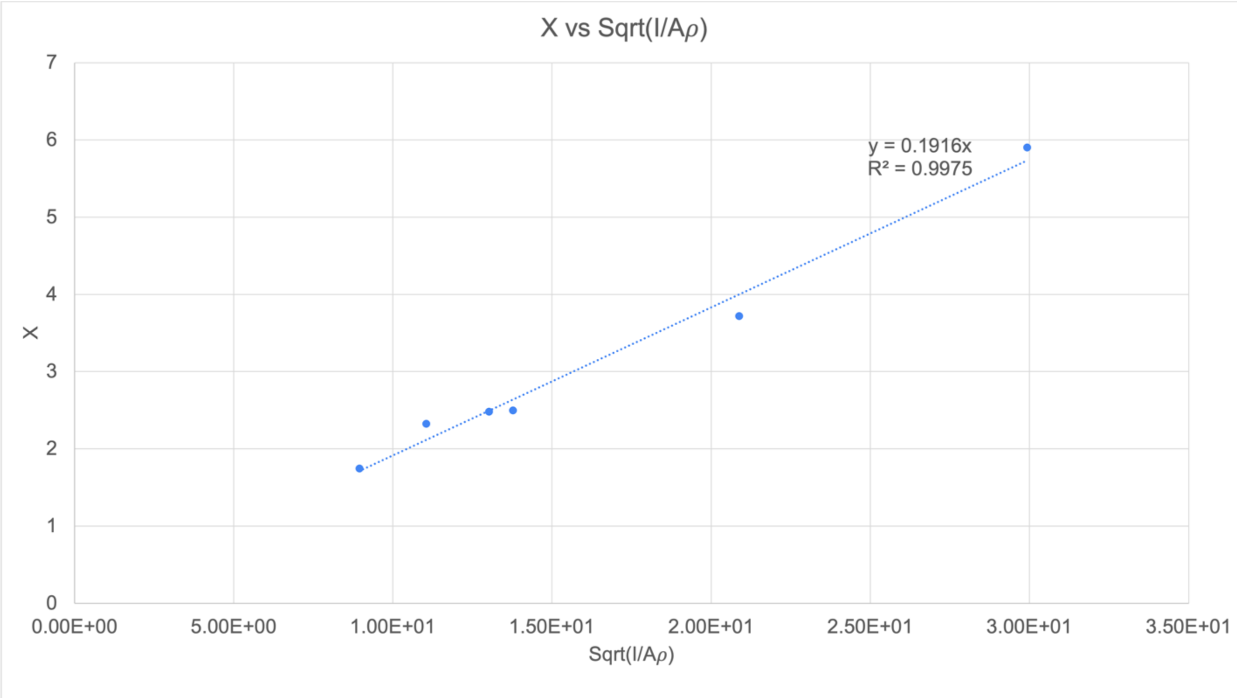

Figure 12: Plot of X vs. Sqrt(I/Aρ).

Safety

Materials with sharp edges are present that must be handled with care to avoid injury or equipment damage. Electrical equipment is also used, so electrical safety measures must be followed, including proper grounding and insulation. Although the experiment does not involve fire hazards, the location of fire extinguishers and emergency exits should be familiar. Quick moving objects and small objects are prone to projecting unpredictably. Safety glasses should be used to prevent impact from the accelerometer that is only held by wax to the beam.

Discussion

The experimental data will be compared to the theoretical model of a cantilevered beam that is undamped and given a small displacement. As the beam had many periods before decay, it is known to be lightly damped. Despite this, it was negligible and not included in the mass-spring system characterization of the beam since a small displacement was given to the beam rather than an impulse. The general equation for natural frequency of an undamped cantilever beam is simplified to Equation 1 (Appendix 2: Young's Modulus Calculation).

$$\omega_n = \alpha_n^2 \cdot \sqrt{\frac{EI}{\rho A L^4}}$$

Equation 1: Natural frequency as a function of the free end

Where ω_n is the natural frequency, α_n is the mode constant, E is the Young's Modulus of the beam, A is the cross-sectional area, L is the length of the beam, and ρ the density of the beam's material. The mode constant is taken to be 1.875 as this is the first mode of vibration [1].

By manipulating the equation, the relationship of proportionality is clearer between ω_n and L^{-2}. This is done by substituting, α_n^2 and $\sqrt{\frac{EI}{\rho A}}$ into a coefficient "x" , where $\omega_n = xL^{-2}$. In comparing the experimental to this theoretical model, the data was best fit with a power law. The values of the exponential for the groups by using the power law fit equations are as given:

Teams	Exponent value from Power Law Fit
A1	-1.877
B1	-1.8
C1	-1.844
A2	-1.963
B2	-1.952
C2	-1.867
A3	-1.622
B3	-1.853
C3	-1.875
A4	-1.796
B4	-1.921
C4	-1.89

The average was -1.855, which is 7% off the theoretical model of an undamped cantilever beam.

A case study was then conducted to determine the Young's Modulus value of the 6061 aluminum beams experimentally using the beam parameters and the acquired frequency. In the equation for natural frequency $\omega_n = \alpha_n^2 \cdot \sqrt{\frac{EI}{\rho A L^4}}$ all values are known except the Young's Modulus. Given that literature values for natural frequency as a function of free length are not as readily available as the Young's Modulus for the different material beams, the paper shifted its focus to determining $E = \frac{\rho A L^4 \omega_n^2}{I \alpha_n^4}$. It should be noted that all beams in the study were rectangular with the exception of Team B2 who had a box beam.

CASE STUDY FOR 6061 ALUMINUM BEAMS

Beam	X ($\alpha^2 \sqrt{\frac{EI}{\rho A}}$)	ρ (Density)	A (Cross-Sectional Area)	I (Moment of Inertia)	E (Young's Modulus)
Team A1: Aluminum	$13.8\ m^2/s$	$2725\ kg/m^3$	$1.21\times10^{-4}\ m^2$	$1.02\times10^{-10}\ m^4$	49.1 GPa
Team B1: Aluminum	$11\ m^2/s$	$2581\ kg/m^3$	$3.94\times10^{-5}\ m^2$	$3.20\times10^{-11}\ m^4$	31 GPa
Team C1: Aluminum	$8.94\ m^2/s$	$2711\ kg/m^3$	$5.63\times10^{-5}\ m^2$	$2.34\times10^{-11}\ m^4$	41.5 GPa
Team B2: Aluminum (*Wall Thickness: 0.0625)	$29.9\ m^2/s$	$2676\ kg/m^3$	$5.04\times10^{-5}\ m^2$	$2.46\times10^{-10}\ m^4$	40 GPa
Team C3: Aluminum	$13\ m^2/s$	$2409\ kg/m^3$	$9.07\times10^{-5}\ m^2$	$9.65\times10^{-11}\ m^4$	31.2 GPa
Team C4: Aluminum	$20.9\ m^2/s$	$2699\ kg/m^3$	$1.22\times10^{-4}\ m^2$	$2.31\times10^{-10}\ m^4$	49.5 GPa

Average E: 40.9 GPa Actual E: 70 GPa

Figure 13: Case Study Results of 6061 Aluminum Beam's Young's Modulus Determination

As can be seen in figure 13, the six different aluminum beams had a Young's Modulus value ranging from 31 – 49.5 GPa with an average of 40.9 GPa. The actual value of the 6061-aluminum beam, as noted in the Engineering Toolbox, is 70 GPa [10]. Although the value attained was off by a factor of 1.71 to the actual, there is some truth with the results attained. The uniformity in all six values shows that the beams are in fact made up of the same aluminum material despite not being able to narrow it down to one. Beyond this, the attained Young's Modulus average value is also within the same order of magnitude of pressure of gigapascal. As a check, the calculation was conducted several times by multiple individuals and the same conclusion was reached. Different approaches were also taken to solve for the Young's Modulus which yielded in the same general result of 40.9 ± 2.3 GPa (See Appendix 2 for sample error calculation). Due to the trends seen in the plots of the data, it is believed that the equation used was not to blame for the discrepancy and the error lies in the calculation of the Young's Modulus. Another reason as to why it was thought that the result may have varied from the actual value could be due to the attached accelerometer mass. The mass despite being thought of as negligible, may not be and could be lowering the resulting natural frequency. The beams themselves could also be advertised as being 6061 aluminum but not exactly be that specific identity. To be sure, the Young's Modulus values of two other beams were calculated. The value of the Team A2 brass beam was determined to 92.5 GPa which is closer to the lower end of the actual Young's Modulus value of 113.5 GPa [10]. However, the result if still off by a factor of 1.23. Similarly, the value for the Team B3 Carbon Boron Steel beam was calculated and found to be 129 GPa. This was off by a factor of 1.47 to the actual Young's Modulus Value of 190 GPa [11] but still within the same order of magnitude.

Conclusion

The line of best fit for the aluminum, brass, garolite, carbon fiber, and carbon-boron steel beams was a power law fit with an R^2 value of 0.999. The beams can therefore be modelled as an undamped cantilever, where the natural frequency of the beams decreased as the free length was increased. The log-log graph of the Frequency Vs. Free length plot further supports the use of a power law fit given that linear relationships were shown on these axes. In comparing their quotients to the theoretical negative-squared value, there was a 93% agreement to this model. The calculated average Young's Modulus value for the case study of the 6061 aluminum beams was 40.9 ± 2.3 GPa. Despite not attaining the actual value of 70 GPa [10], there was some truth to the results. The value attained was within the same order of magnitude as the actual aluminum value and there was some uniformity in all of our beam results. A reason as to why it was believed that the Young's Modulus could have differed was due to attached accelerometer mass. The discrepancy may be also have been due to limitations of the experimental setup or the exact identity of the beam not being Aluminum 6061. Overall, it is believed that with more time, using the length and the natural frequency of the beam could prove to be a viable method for determination of the Young's Modulus of a beam.

Recommendations

A greater breadth of materials can be used as the majority were metals, such as composites or plastics. Also, different plucking methods, such as plucking the center of the beam or using different plucking strengths, could provide valuable insights. The experiment setup could be modified to automate the plucking as human error is introduced with inconsistent plucking strength and location.

The main source of experimental uncertainty lies in the accelerometer used. Multiple accelerometers along the beam or a higher accuracy one would improve accuracy of the data recorded.

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Appendices

Appendix 1: Lab Instructions

BROAD GOAL: to experimentally determine the natural frequency of a cantilevered beam.

SPECIFIC OBJECTIVE: to generate a natural frequency vs length curve of a cantilevered beam (with at least 17 well-spaced points), along with geometry and mass measurements sufficient to determine density, area moment of inertia, and mass distribution.

DESCRIPTION OF TEST STAND
The test stand consists of a vise into which a beam sample can be mounted as a cantilever and displaced at its free end. The test stand is instrumented with a piezoelectric accelerometer and digital data acquisition system that allows for a set of measurements of the beam acceleration at one specific location near the beam free end. Calipers and a standard tape measure are used for measurement of beam and frame dimensions. A standard laboratory scale is used for mass measurements.

PROCEDURE
- Clamp vise base firmly to the benchtop in a space, allowing for mounting a 24" beam parallel to the benchtop.

- Measure and record in the table all beam dimensions required, using calipers (width, thickness) and tape measure (length)
- Weigh the beam and record it.
- Load the beam into the frame, parallel to the benchtop and with narrow edges up/down, with 22" free length (2" clamped)
- Start and log in to the computer. Start Lab View Software (National Instruments).
- Locate/check the accelerometer, amplifier, and data acquisition system components:
  - Accelerometer (PCB 333B3), mounting wax, and cable (BNC female to small threaded female coaxial)
  - Signal conditioner (PCB 480C02). Turn on and test battery charge by pressing "test" end of toggle switch.
  - BNC female to 3 wire cable with bias resistor, screwed into first three ports of A/D module.
  - Analog to digital converter (NI 6008/6009), and USB A to USB B cable.
- Weight the accelerometer (detached from the cable) and record it.
- Connect the accelerometer and data acquisition system as indicated in the diagram, taking care not to kink the accelerometer cable or drop the accelerometer. Connect the USB A cable to a free port on the computer.
- Using a small amount of wax on the accelerometer base, attach the accelerometer to the beam near the free end. Use blue masking tape to secure the accelerometer cable about 2" down from the beam.
- Find and open the LabView VI file BeamM1 V1 2023.VI). The "front panel" will open automatically and display two graphs.
- Open the block diagram (Window > Show Block Diagram, or CTRL+E), then open the DAQ Assistant block.
  - Check the DAQ Assistant settings:

    Signal Input Range
    Max 5 V (NI 6008/9)
    Max -5 V (NI 6008/9)
    Scaled Units Volts (NI 6008/9)
    Acquisition Mode N Samples
    Samples to Read 20000
    Rate (Hz) 2kHz

Verify the Advanced Timing tab to ensure the measurement does not time out before 10 seconds.

- Show your setup to the supervisor to check at this point, prior to taking data.
- Test the accelerometer signal. GENTLY pluck the beam tip and hit the Run button. After 10 seconds you should get a decaying vibration signal around OV average. Debug, if necessary, with the supervisor.
- After verifying that the system is working correctly, return to the Front panel (Window > Front Panel or CTRL+E).
- Measure the beam natural frequency.
  - GENTLY pluck or displace and release the beam free end.
  - Press the Run button (large arrow at the top of the Front Panel, or Ctrl+R). Wait 10 seconds for data acquisition.

  - Page up to the histogram graph to check the signal quality. You should see a decaying vibration signal without saturation (constant amplitude or "clipped" signal) and without too much high frequency noise. If saturation occurs, you are plucking the beam too forcefully; pluck more gently and run again.
  - With a verified good histogram, move in the GUI to display the Spectrum plot, cursor, and cursor control.
  - The spectrum shows Amplitude (in dB) vs. Frequency (in Hz), and there are often several local peaks in the magnitude. Find the peak at the lowest frequency. It should also have the highest amplitude. Record frequency of the lowest peak as the natural frequency, using the cursor display to maximize the resolution.
- Loosen the clamp, taking care not to drop the beam/accelerometer, slide the beam to a new free length, and reclasp.
- Measure and record the free length. Gently pluck beam, run, check acceleration histogram, and measure the natural frequency.
- Repeat measurement of natural frequency for approximately 17 well-spaced lengths, from 22" to about 6".
- Return experiment components to their original locations.
- Plot natural frequency vs. beam free length on the provided (semilog) grid.

Appendix 2: Young's Modulus Calculation

$$\omega_n = \alpha_n^2 \cdot \sqrt{\frac{EI}{\rho A L^4}}$$

$$\left(\frac{\omega_n}{\alpha_n^2}\right)^2 = \frac{EI}{\rho A L^4}$$

$$\frac{\rho A L^4}{I} \cdot \left(\frac{\omega_n}{\alpha_n^2}\right)^2 = E$$

$$\therefore E = \frac{\rho A L^4 \omega_n^2}{I \alpha_n^4}$$

<u>Sample calculation: Team A1</u>

Variables

$$\alpha = 1.875; \mathrm{I} = 1.016 \times 10^{-10}\, m^4\,;\ \rho = 2725\frac{kg}{m^3};\ \omega_n = 7.4\ Hz; A = 1.2 \times 10^{-4} m^2; L = 0.6096\ m$$

$$E = \frac{\rho A L^4 \omega_n^2}{I \alpha_n^4}$$

$$E = 5.60 \times 10^{10}\ or\ 56\ GPa$$

*The Young's Modulus value was calculated in excel using the listed variables and equation.

<u>Sample Error Calculation of Young's Modulus: Team A1</u>

Variables

$$\alpha = 1.875; \mathrm{I} = 1.016 \times 10^{-10}\ m^4;\ \rho = 2725 \frac{kg}{m^3};\ \omega_n = 7.4\ Hz; A = 1.2 \times 10^{-4} m^2; L = 0.6096\ m$$

$$\sigma_\rho = 13.56\ \frac{kg}{m^3};\ \sigma_A = 9.7 \times 10^{-6} m^2;\ \sigma_L = \ 7.9 \times 10^{-4} m;\ \sigma_{\omega_n} = \ 0.05\ Hz;\ \sigma_I = 2.9 \times 10^{-10} m^4$$

Equations

$$\omega_n = \alpha^2 \sqrt{\frac{EI}{\rho A L^4}} \quad \rightarrow \quad E = \frac{\rho A L^4 \omega_{n^2}}{I \alpha^4}$$

$$\sigma^2 = \left[\left(\frac{\partial f}{\partial \rho}\right)\sigma_\rho\right]^2 + \left[\left(\frac{\partial f}{\partial A}\right)\sigma_A\right]^2 + \left[\left(\frac{\partial f}{\partial L}\right)\sigma_L\right]^2 + \left[\left(\frac{\partial f}{\partial w_n}\right)\sigma_{w_n}\right]^2 + \left[\left(\frac{\partial f}{\partial I}\right)\sigma_I\right]^2$$

$$\sigma^2 = \frac{A^2 L^8 {\omega_n}^4 {\sigma_\rho}^2}{I^2 \alpha^8} + \frac{\rho^2 L^8 {\omega_n}^4 {\sigma_A}^2}{I^2 \alpha^8} + \frac{16 A^2 \rho^2 L^6 {\omega_n}^4 {\sigma_L}^2}{I^2 \alpha^8} + \frac{4 A^2 \rho^2 L^8 {\sigma_{\omega_n}}^2}{I^2 \alpha^8} + \frac{A^2 \rho^2 L^8 {\omega_n}^4 {\sigma_I}^2}{I^4 \alpha^8}$$

$$\text{Simplified Equation: } \frac{\Delta E}{E} = \frac{2\Delta X}{X} + \frac{\Delta \rho}{\rho} + \frac{\Delta A}{A} + \frac{\Delta I}{I}$$

$$\sigma = 5.7 \times 10^9 \text{ or } 5.7 \text{ GPa}$$

*The propagation of error value was calculated in excel using the listed variables and equations. It should be noted that the average of all the propagation of error values of the 6061 aluminum beams was used for the margin of error.

Appendix 3: Uncertainty Determination

Appendix 3a: Uncertainty in Mounting Beam

An experiment was conducted where 36 students measured the free length of the beam to determine the readability of a tape measure when the setup is simulated. The determined readability was found to be 0.106115 in, supporting the belief and tendency for readability to be greater than the smallest graduation. When clamping the free length, the uncertainty is 1/16" minimum given that two measurements of the beginning alignment and free length are made.

Source of uncertainty	**Uncertainty (in)**
Clamping the recorded amount	1/16
Measuring the free length	0.106115

As this process is repeated , the total uncertainty of the measurement of the free length can be found by adding these in quadrature as these variables are independent of one another.

Systematic uncertainty in length of Free Length: $\sqrt{\left(\frac{1}{16}\right)^2 + (0.106115)^2} = 0.123\ in$

Appendix 3b: Uncertainty in Measuring Natural Frequency

To measure natural frequency from the data received from the accelerometer that is attached at the free end of the beam, there is systematic error introduced from the accelerometer, sensor signal conditioner, and signal acquisition control digital I/O that is to be added using the root-sum-square method. The human error in selecting the highest peak using the mouse cursor is included for accuracy purposes, but can be considered negligible as the scale of the scope was zoomed in and was estimated to be 0.04 Hz, which is the same as the readability of the grid on the monitor.

Source of uncertainty	**Manufacturer Uncertainty (%)**
Accelerometer	5
Signal Conditioner	2
USB 6008 Multifunction I/O	0.4

Systematic uncertainty in natural frequency readings: $\sqrt{(0.05)^2 + (0.002)^2 + (0.004)^2}$

$= \ 0.050 \ or \ 5\%$

Appendix 4: Outlier tests: Grubbs Test

<u>Sample calculation: B1 Aluminum</u>

For a data set of 21 samples, the critical value is found to be 2.860 for a significance level of 0.05.

$$Grubbs\ test\ statistic = \frac{maximum\ deviation\ from\ mean}{sample\ standard\ deviation}$$

Mean = 43.9667, standard deviation = 29.5714, standard deviation =24.5811

$$\therefore \frac{|5.7 \ - \ 29.5714|}{24.5811} = 0.97$$

We compare the critical value to the test statistic G, 0.6< 2.860. This indicates that the first datum is not an outlier. Doing this iteratively for each datum, we find that Trial 18 is furthest from the rest, and while not a significant outlier, it is still and can be excluded.

Appendix 5: Raw Data Tables

Table 1: Measuring Beam Student Experiment

Student	Length (in)
1	19.9
2	19.84375
3	19.875
4	19.875
5	19.875
6	19.90625
7	19.90625
8	19.83333
9	19.75
10	19.875
11	19.9375

12	19.8125
13	19.9375
14	19.875
15	19.8125
16	19.875
17	19.8125
18	19.9375
19	19.90625
20	19.875
21	19.75
22	19.75
23	19.875
24	19.875
25	19.875
26	19.875
27	19.8125
28	19.90625
29	19.875
30	19.875
31	19.875
32	20
33	19.875
34	19.875
35	19.875
36	19.8125

Appendix 6: Raw Data Plots

Table 2: Brass Beam Raw Data

Length (In)	Frequency (Hz)
22	4.9
21	6
20	6.7
19	7.4
18	8.2
17	9.2
16	10.4
15	11.8
14	13.4
13	15.4
12	18.2
11	21.4
10	25.8
9	31.5
8	39.2
7	51.2
6	68.5
5	96

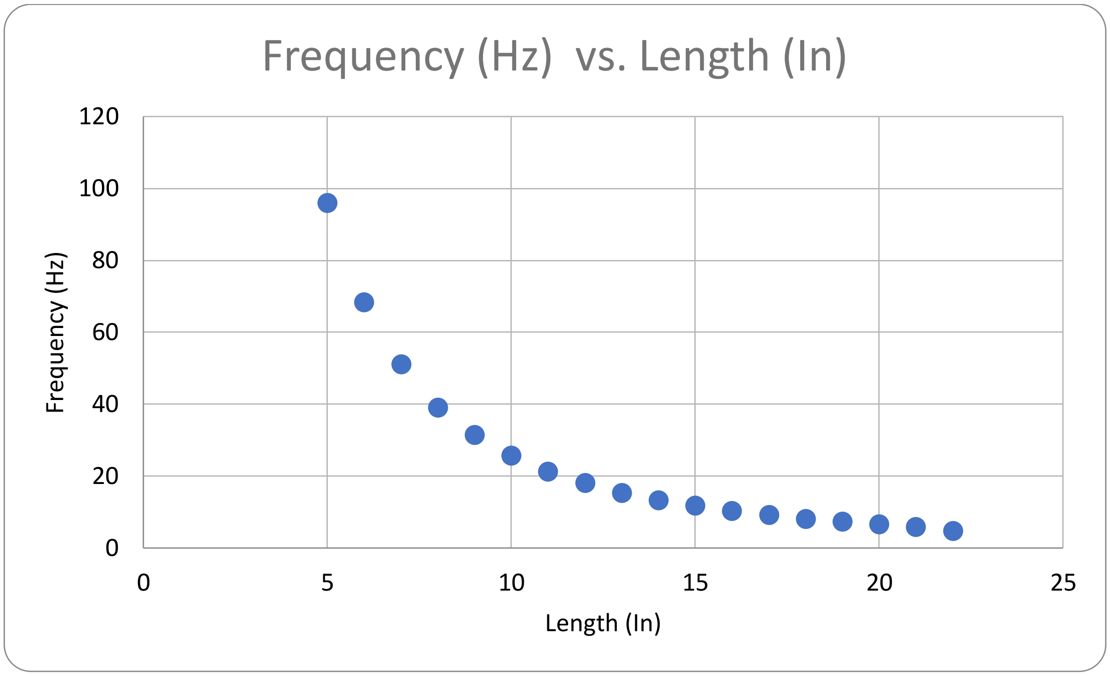


Table 3: Garolite Beam Raw Data

Length (In)	Frequency (Hz)
22	9
21	11.2
20	12.4
19	13.6
18	15.2
17	16.8
16	19
15	21.8
14	24.4
13	28.2
12	32.8
11	37.8
10	43.6
9	55.2
8	68
7	85.4
6	111.4
5	162.4

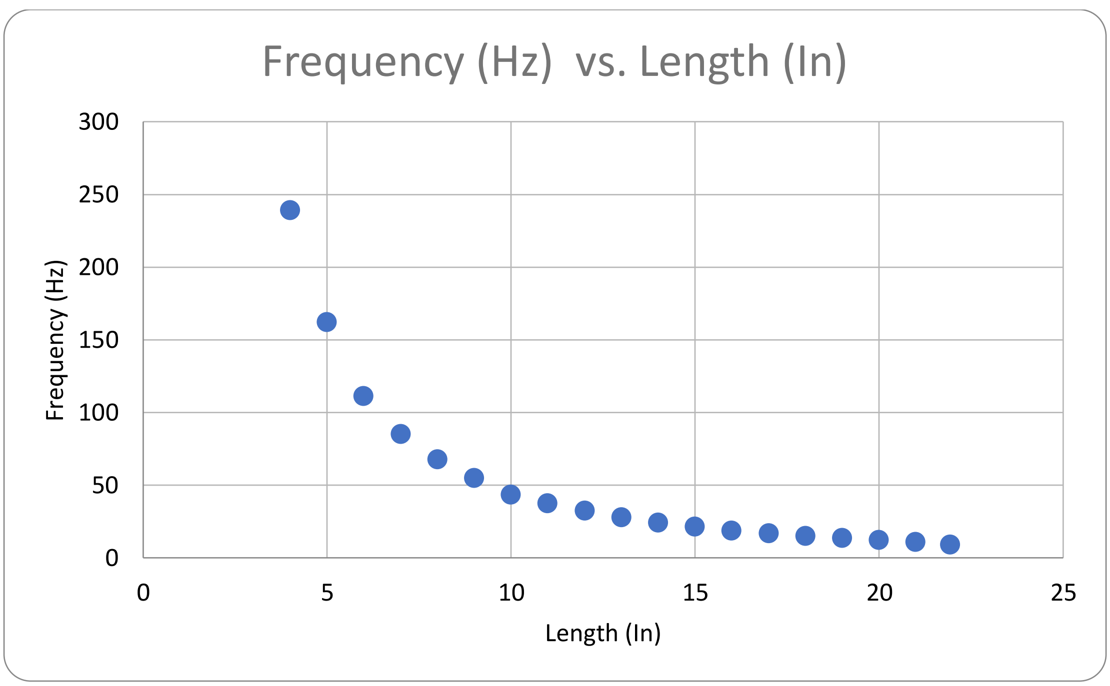


Table 4: Aluminum Beam Raw Data

Length (In)	Frequency (Hz)
22	15.2
21	18.6
20	20
19	22.2
18	24.2
17	27
16	30
15	34.4
14	39
13	44.4
12	50.8
11	59.8
10	69.6
9	85.4
8	103.4
7	130.6
6	173.2

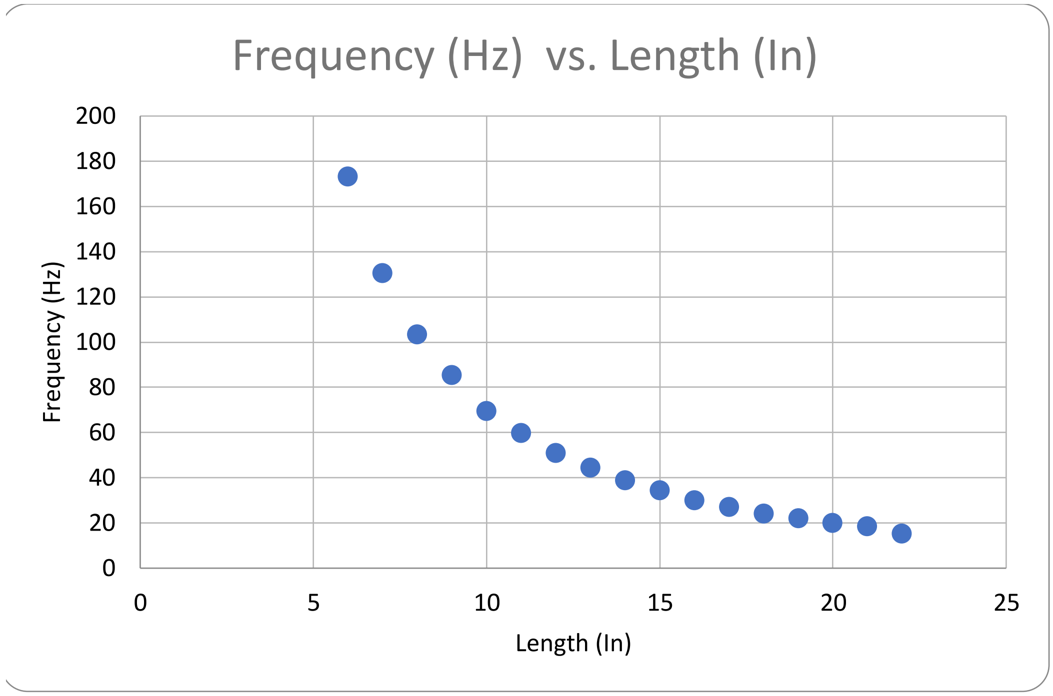


Table 5:Carbon Fiber Team A3 Beam Raw Dat

Length (In)	Frequency (Hz)
21	8
20	10
19	10.9
18	11.9
17	13
16	14.5
15	16.3
14	18.4
13	20.7
12	23.8
11	27.3
10	32.3
9	38.2
8	46.1
7	59
6	72.6
5	95.8
4	134.8
3	195.6

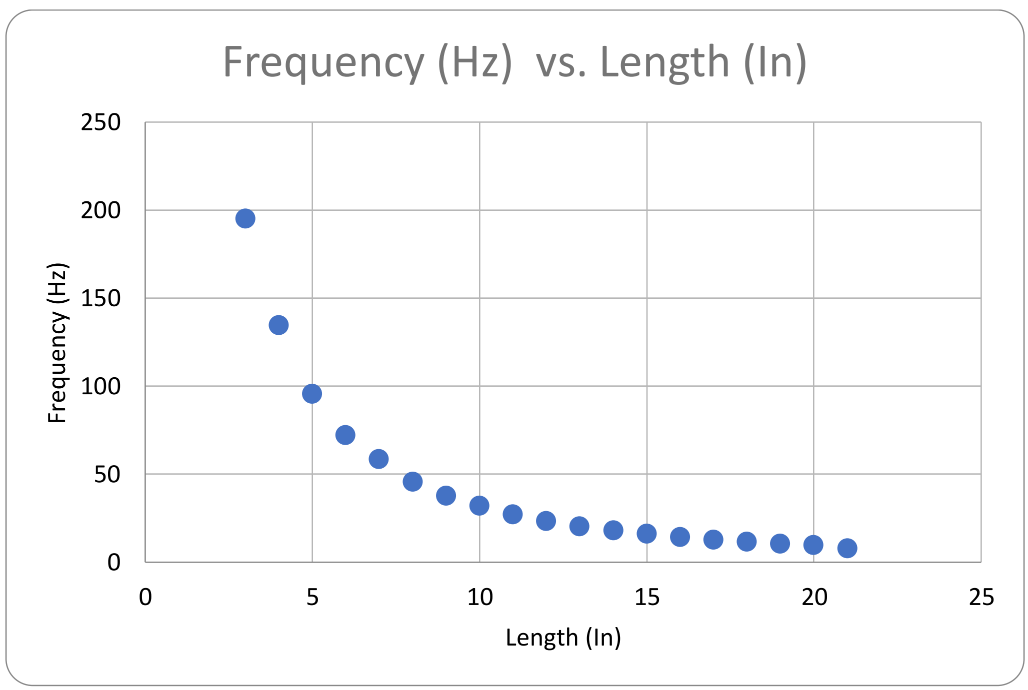


Table 6 :Steel Team B3 Raw Data

Length (In)	Frequency (Hz)
22.0	6.8
21.0	8.2
20.0	9.2
19.0	10.0
18.0	11.2
17.0	12.2
16.0	14.2
15.0	15.8
14.0	18.2
13.0	21.0
12.0	24.4
11.0	28.8
10.0	34.0
9.0	41.8
8.0	52.0
7.0	65.2
6.0	85.0
5.0	121.0
4.0	176.8
3.0	287.0

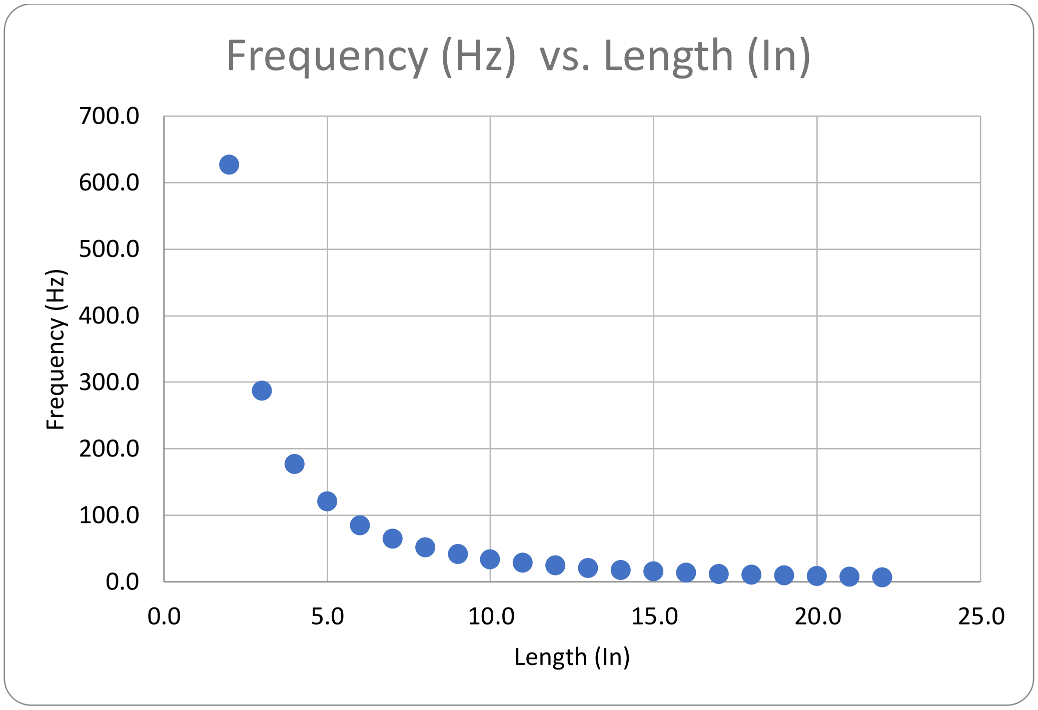


Table 7: Aluminum Beam C3 Raw Data

Length (In)	Frequency (Hz)
22	6.4
21	7.8
20	8.6
19	9.4
18	10.4
17	11.8
16	13.2
15	14.8
14	16.8
13	19.4
12	22.4
11	26.4
10	31.8
9	38.2
8	46.8
7	60.0
6	80.6
5.5	93.4

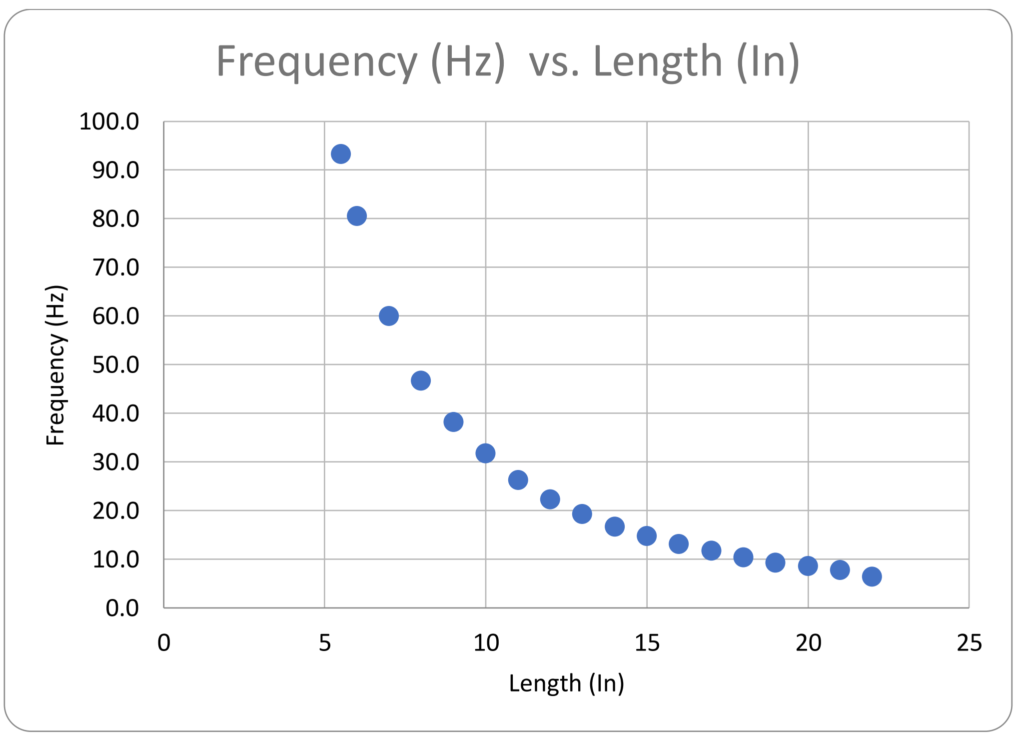


Table 8: Aluminum Beam A4 Raw Data

Length (In)	Frequency (Hz)
32	28.6
31	30.2
30	32.2
29	34.4
28	36.7
27	39.2
26	42.2
25	45.6
24	49.0
23	53.2
22	58.0
21	62.8
20	69.0
19	75.4
18	83.3
17	74.4
16	104.3
15	118.2
14	133.0
13	149.2
12	158.3

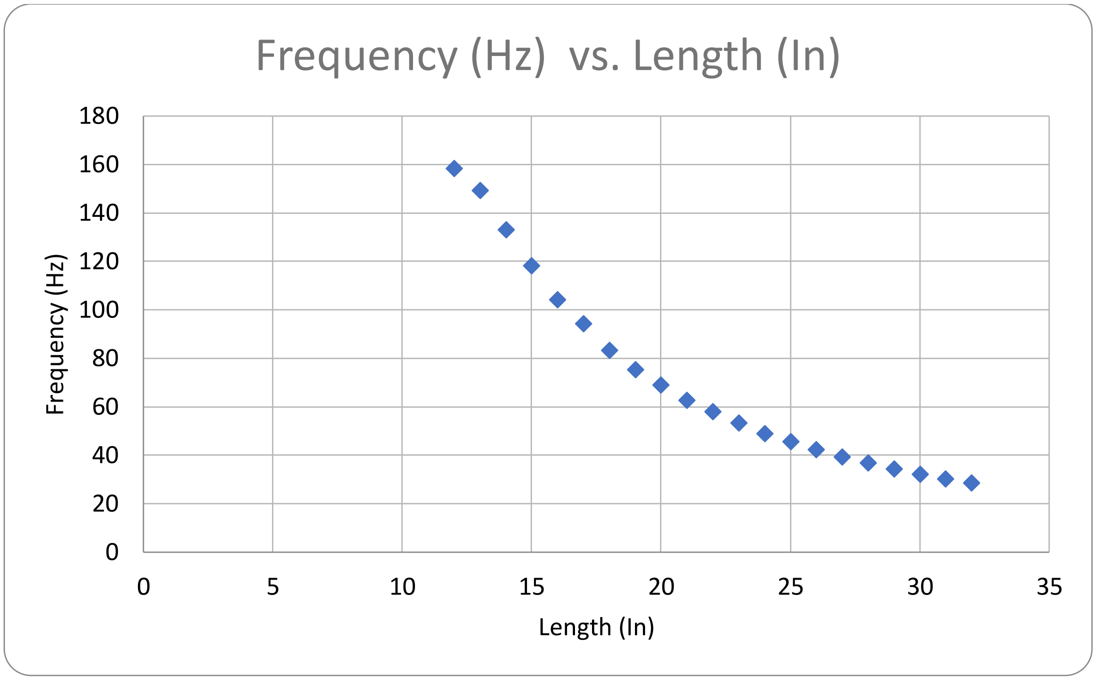


Table 8: UHMWPE Beam Raw Data

Length (In)	Frequency (Hz)
22.0	3.6
20.0	4.4
19.1	4.8
18.1	5.4
17.3	6.0
16.1	6.6
15.0	7.8
13.9	8.3
12.9	10.4
11.5	12.2
10.5	14.0
9.5	16.2
8.5	20.6
7.9	26.0
7.0	33.2
6.0	43.6

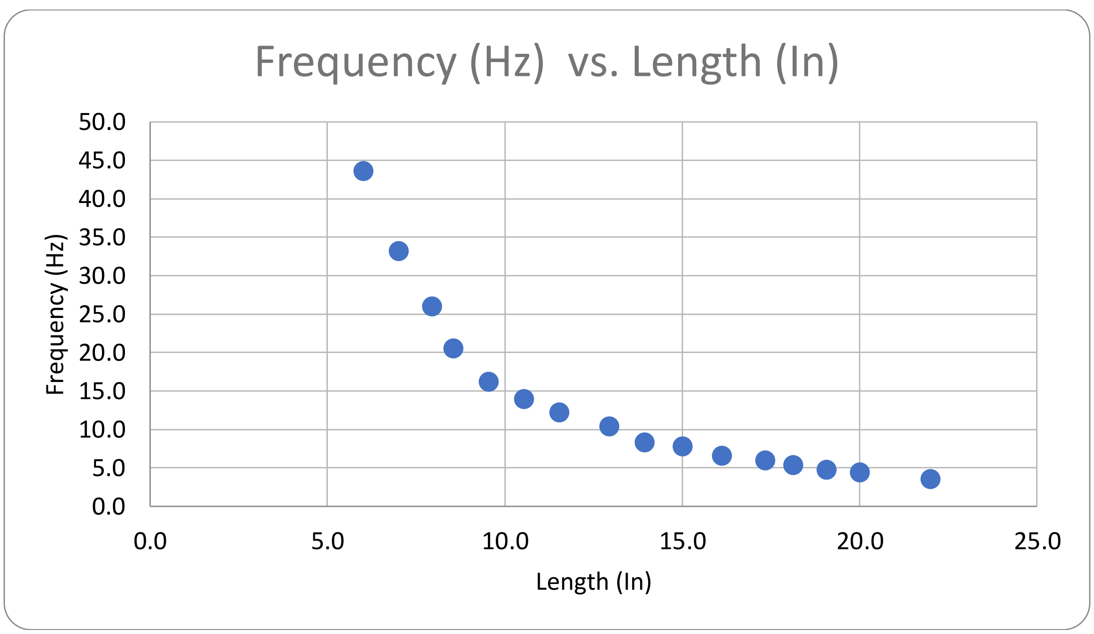
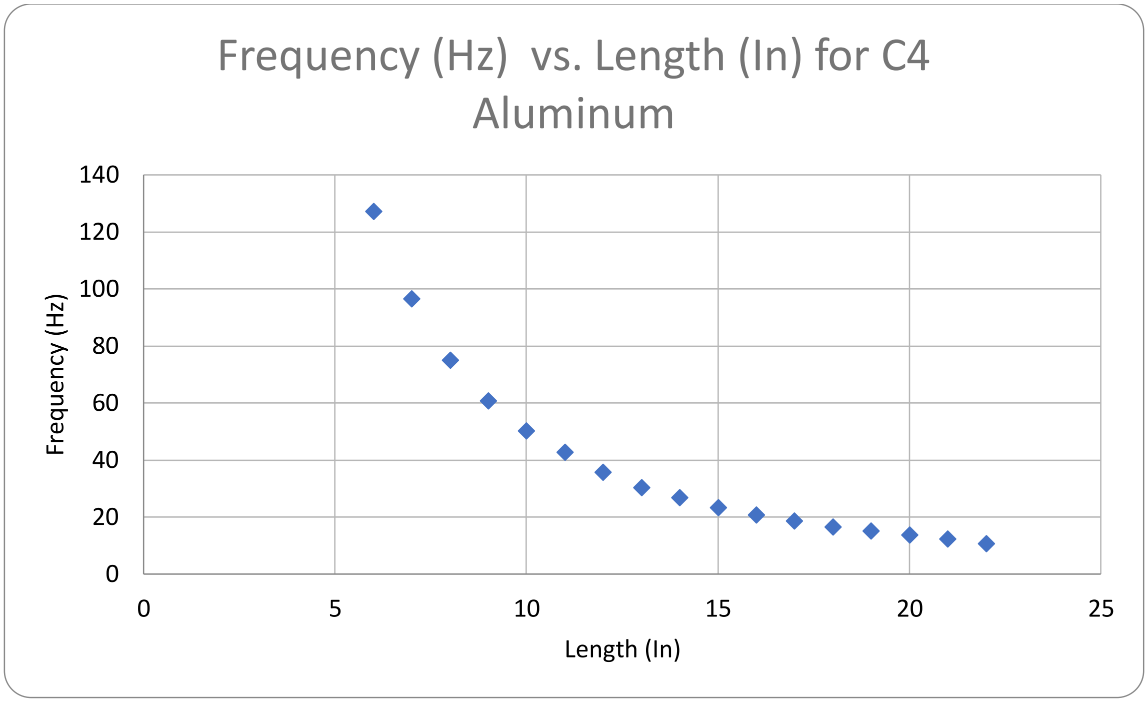


Table 9: Aluminum C4 Beam Raw Data

Length (In)	Frequency (Hz)
22	10.6
21	12.2
20	13.6
19	15.2
18	16.6
17	18.6
16	20.8
15	23.4
14	26.8
13	30.4
12	35.6
11	42.6
10	50.2
9	60.8
8	75.0
7	96.6
6	127.2



Appendix 7: Deviation Plot

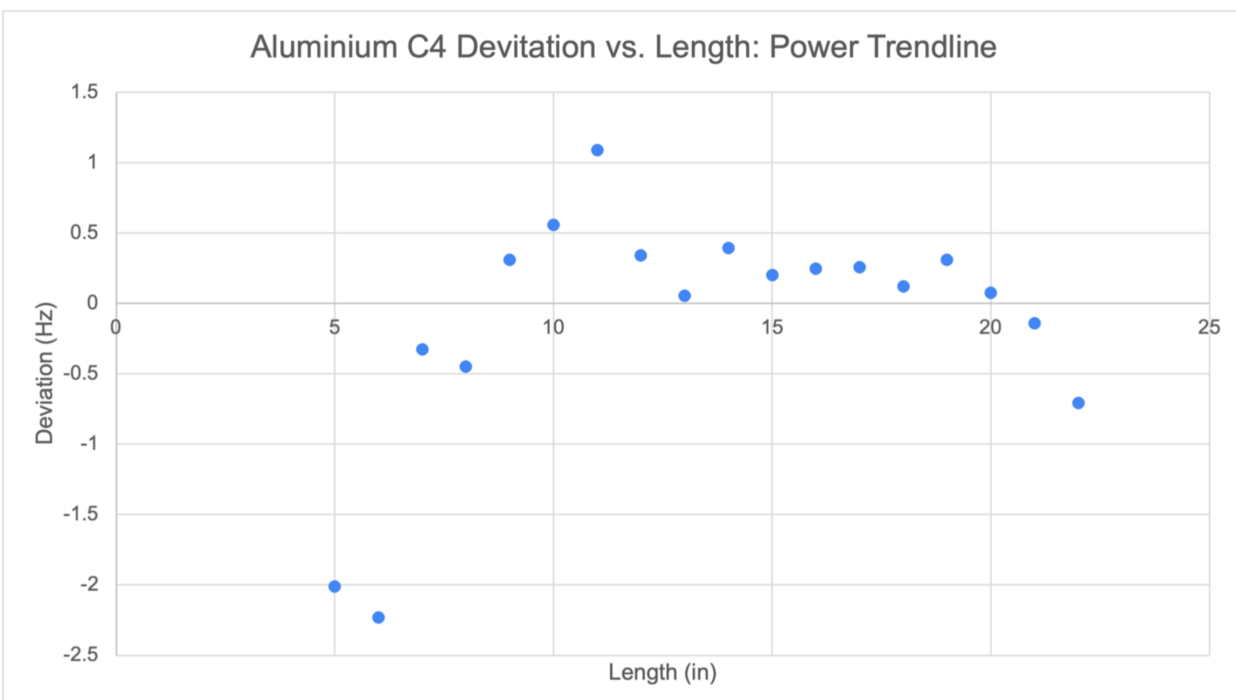


Figure 14: Residual plot of the power line of best fit