Analysis of Cooper Hyperloop's Horizontal Suspension

A base excitation analogy, using Tracker software, a servo DC motor, and an Arduino Microcontroller

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Introduction

The efficient and safe operation of a Hyperloop pod relies heavily on the performance of the stability system as it encounters various bumps along the I-beam during its high speed travel. To minimize the disruption and vibration that the unmanned pod will experience, and to better understand the vibrations and transmitted force , our team built a simplified, smaller scale testing rig to perform analysis. This will inform our design process with regards to springs, critical lengths and ratios of linkage arms, as well as empirical data that will be gathered and processed.

The experimental setup, shown in Figure 1c, will model an uneven I-beam track for an abstracted horizontal stability system, which is a 2-degree of freedom system. The experimental setup models a trailing arm suspension with the wheel moving in unison with the track . The experimental rig uses a stepper motor and Arduino microcontroller to move the bottom point of the linkage arm in a oscillatin motion. This was accomplished by using a Scotch yoke linkage, which consists of a 3D printed horizontal slot in which a pin on the motor hub is connected to, Therefore, allowing horizontal motion and translating it to only vertical motion from the circular motion. This motion, in turn, drives a linkage arm connected to the top part of the system, creating compression and decompression movements due to the spring that is attached in triangulation. The stepper motors frequency will be altered throughout the experiment to provide various sinusoidal forcing functions while using Tracker to observe the displacement and performance of the mass spring system.

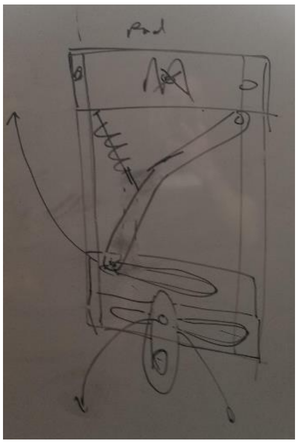


Figure 1a:Free hand drawing

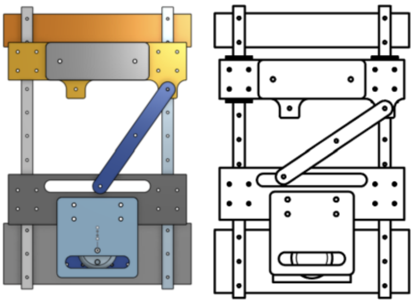


Figure 1b: CAD of build

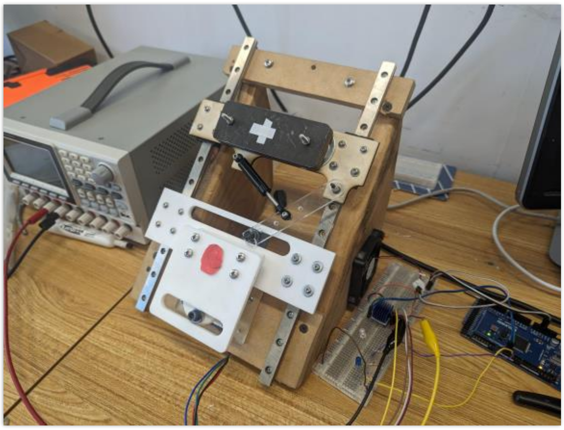


Figure 1c: Experimental Setup

Experimental:

The objective of this experiment is to model an even surface for a hyperloop wheel of a stability using a 2-degree of freedom system, which involves a servo motor, an Arduino microcontroller, a 3D printed component, and a linkage arm, allowing for a representation of vertical displacement of the wheel. The servo motor is responsible for rotating the 3D printed component, which in turn moves the linkage arm up and down. This motion causes the top part of the system to compress and decompress, representing the vertical displacement of the wheel.

To perform the experiment, the Arduino was connected to the step and direction pins of the stepper motor driver and The Arduino was programmed to send pulses to the step pin at the required frequency to cause the desired angler frequency in the motor. A camera was propped on a stand to point normal to the experimental setup, and a recording was done using a slow motion camera. Power supply for the 12 volt signal to the motor was turned on and I recording of about 10 seconds was done. This was repeated for angular frequencies between 100 and 400 RPM.

The recording of each frequency in rpm, were imported into Tracker software, where a defined point, indicated by a white piece of tape, was set as the reference point to track the displacement of the masses, called Mass A in Figure 2. Excel was then used to post-process the raw data of angular frequency, vertical displacement, and time.

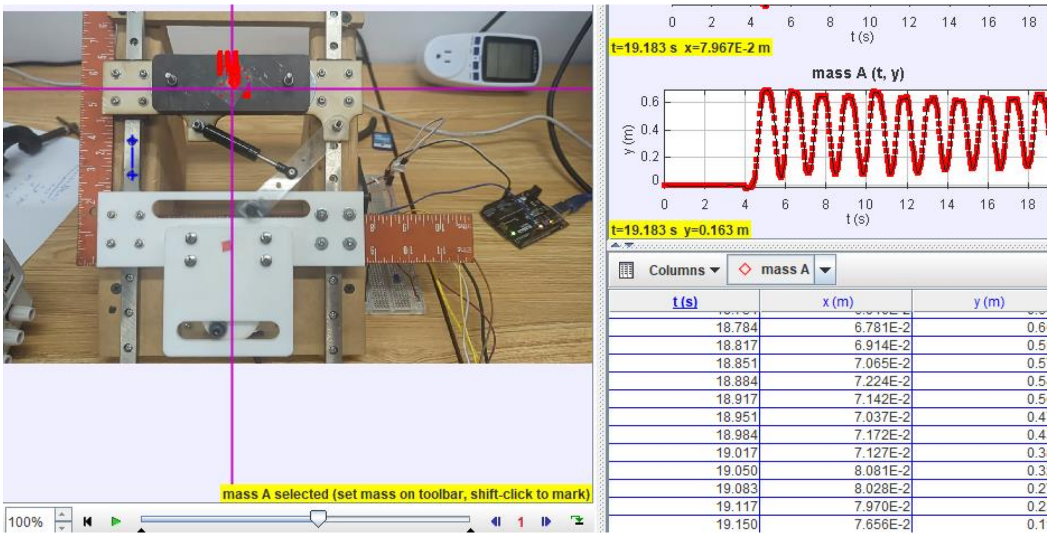


Figure 2: Tracker interface displaying the set origin, reference point and calibration stick.

After, the displacement data vs the time data was exported to an excel sheet, a raw data analysis was begun. Initially, the displacement for each rpm was plotted against its respective time data set. The displacement plots can be seen in Figure 3.

Results:

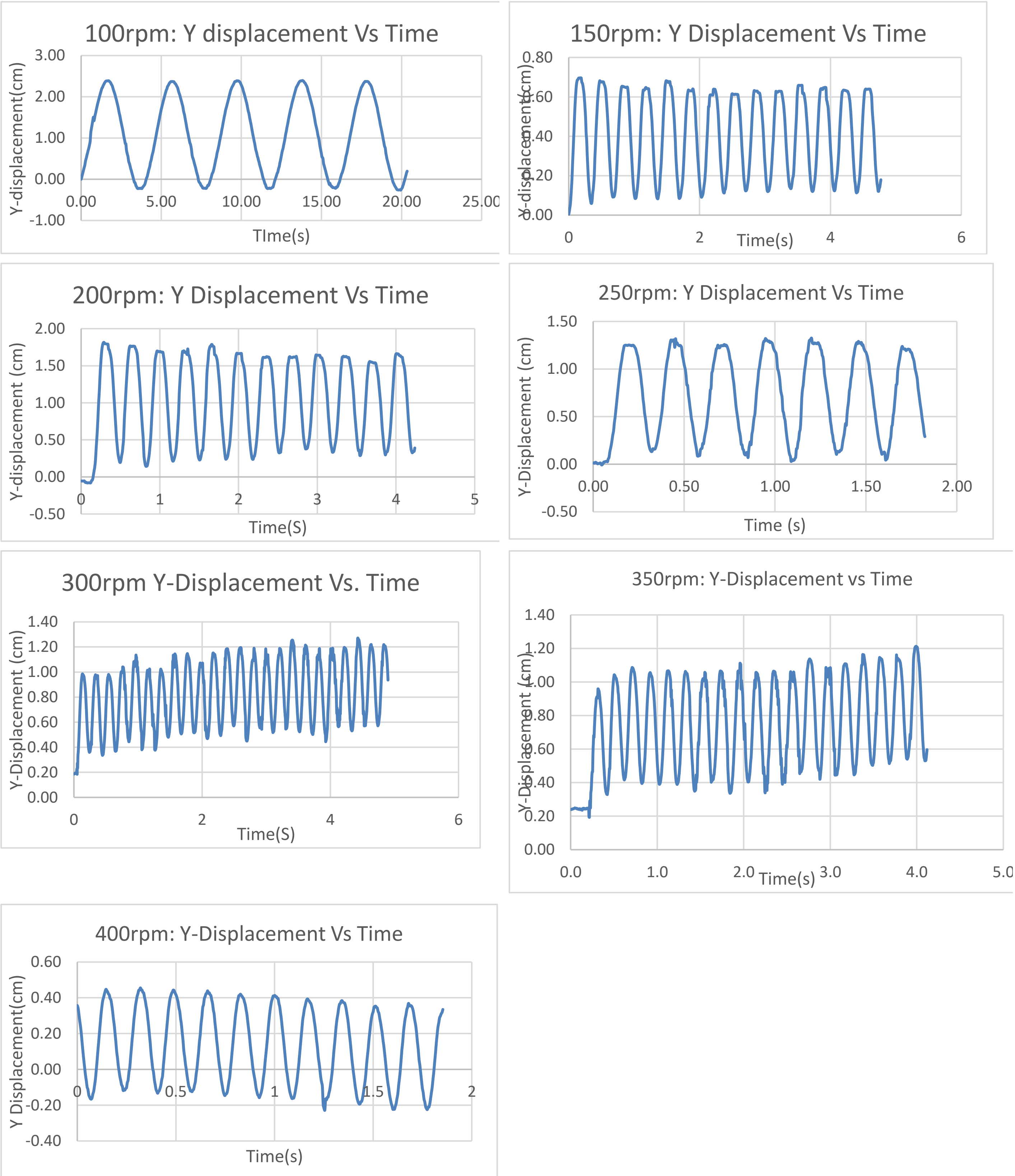
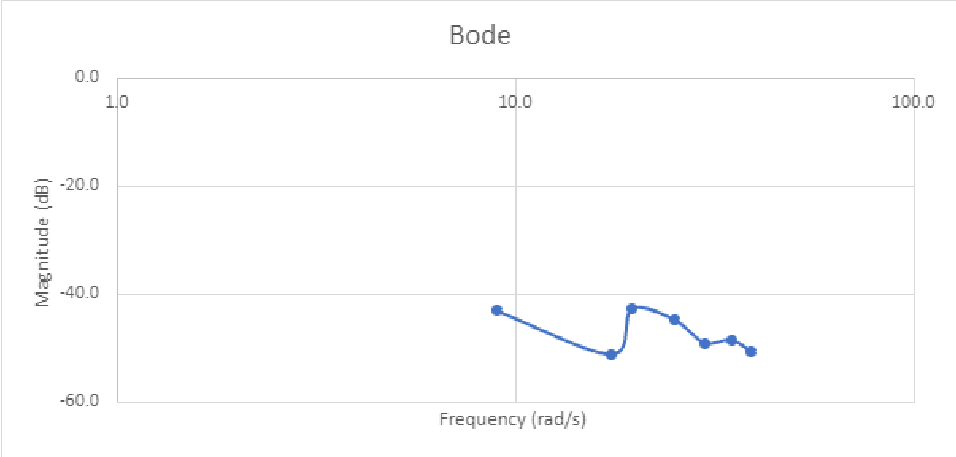


Figure 3: Vertical Displacement Vs. Time graphs of the Mass at different frequencies

To receive higher fidelity and less noisy data than originally, the tape on the masses used for tracker was shrunk to a single point as it would search for the color within the frame and pick the wrong points. This data of displacement against time is displayed in Figure 3, where there are smooth sinusoidal responses due to the sinusoidal input. Using this, the amplitude of a sine wave was calculated, the frequency was calculated, and the phase angle was calculated. The amplitude was found by subtracting the average highest peak from the average lowest peaks on the graphs and taking the average between them. To find the frequency, we implemented the formula: $\omega = \frac{2\pi}{T}$. Finally, to find the phase angle, we multiplied the time that it takes to get to the amplitude by the frequency. The amplitude was given in centimeters and converted to meters. For the magnitude plot, the amplitude was converted to dB by using the formula: $dB = 20\log_{10}(amplitude)$. The frequency was in radians/s and the phase angle was in radians. For the phase plot, the phase angle was converted to degrees. After all these conversions were made for each of the data sets, a bode plot was formed. The magnitude and phase angle was plotted against the set of frequencies acquired. The x-axis (frequency axis) was formatted logarithmically, the Magnitude axis was formatted in 20dB increments, and the Phase angle axis was formatted in 45 degree increments. This can be seen in Figure 4.



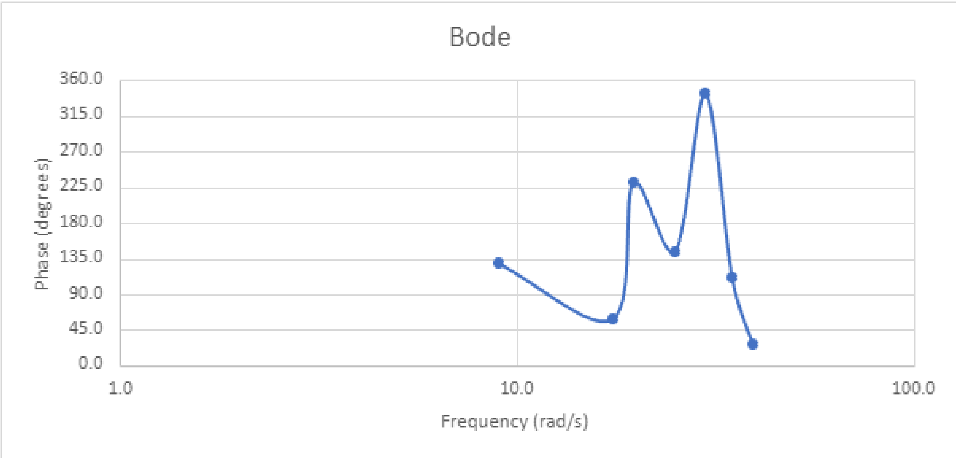


Figure 4: Bode Magnitude and Phase plots

The Bode magnitude and phase plots shown in Figure 4 are graphical representations of the frequency response of a system. They provide valuable insights into the behavior of a system in terms of its amplitude and phase shift response at different frequencies.

In the Bode magnitude plot, the vertical axis represents the magnitude of the system's response, usually measured in decibels (dB), while the horizontal axis represents the frequency in logarithmic scale. The magnitude plot shows how the system's response varies with different frequencies. It provides insight on the amplification or attenuation of the input signal at different frequencies. Peaks in the magnitude plot indicate resonance frequencies, where the system amplifies the input signal, while valleys represent frequencies at which the system attenuates the signal. The Bode plot magnitude provides crucial information about the system's frequency-dependent behavior, allowing us to identify resonant frequencies and potential stability issues.

The Bode phase plot, on the other hand, represents the phase shift between the input and output signals of the system. It is plotted against the same logarithmic frequency axis as the magnitude plot. The phase plot shows how the output signal is delayed or advanced relative to the input signal at different frequencies. By analyzing the phase plot, we can determine the time delay introduced by the system at different frequencies. This information is particularly useful in understanding the system's transient response, stability, and phase relationship between input and output signals.(Inman,2007)[1]

In the context of the mass-spring system suspension experiment, Bode plot magnitude and phase plots are invaluable tools. They allow for the dynamic response of the system to be evaluated at different excitation frequencies. By measuring and plotting the magnitude and phase response of the system, one can determine the system's natural frequencies, resonances, damping characteristics, and stability margins. These insights are crucial for designing and optimizing the suspension system to ensure proper vibration isolation, minimize resonance effects, and enhance the overall performance and safety of the experiment. More analysis specific to the data is outlined in Discussion, with a succinct summary found in Table 1.

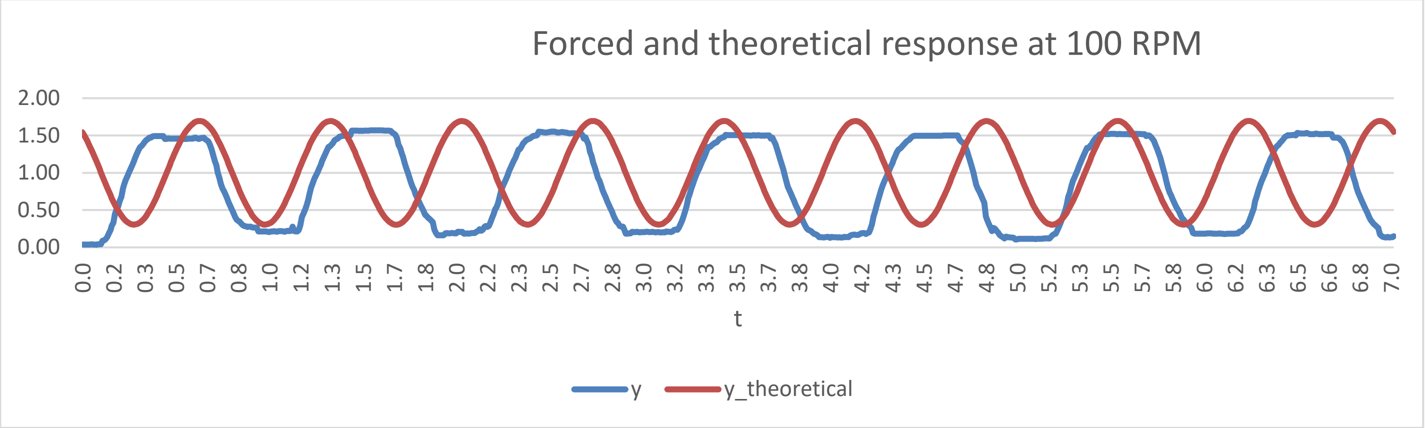


Figure 5: forced and theoretical response at 100 RPM

Generating a theoretical model from the given data used the generated graphs as a point of reference. The amplitude, the phase, and the natural frequency was calculated from the generated graph. Using the derived parameters, it was applied to the forced response, $y(t) = Asin(wt + \phi)$, such that the observed amplitude, natural frequency, and the phase would yield the theoretical response, as shown in Figure 5, the theoretical is represented in red. In a given period the forced response generates a transient and steady state response at max amplitude. The ideal curve shows a desired peak and has a continuous flow. In contrast, the forced response has inconsistent reading, since there are noise present when recording with tracker. Such uncertainties would be accounted for due to instrumentation error. Furthermore, there seems to be a greater phase lag between the two curves as more time passes.

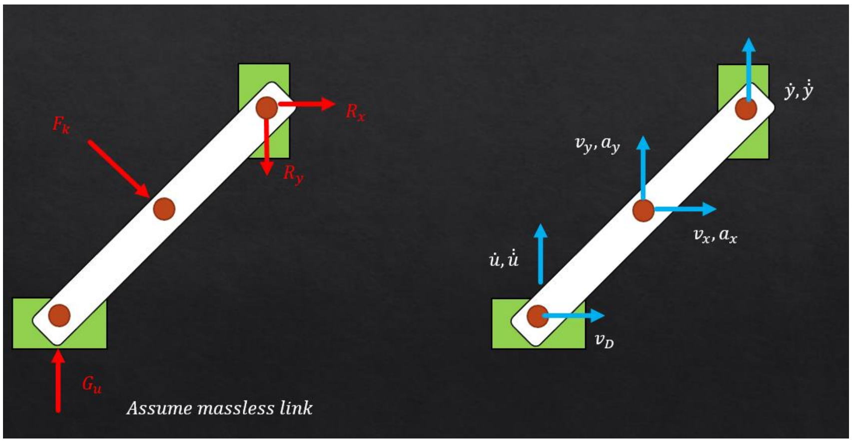


Figure 6: FBD and kinematic model of the linkage arm

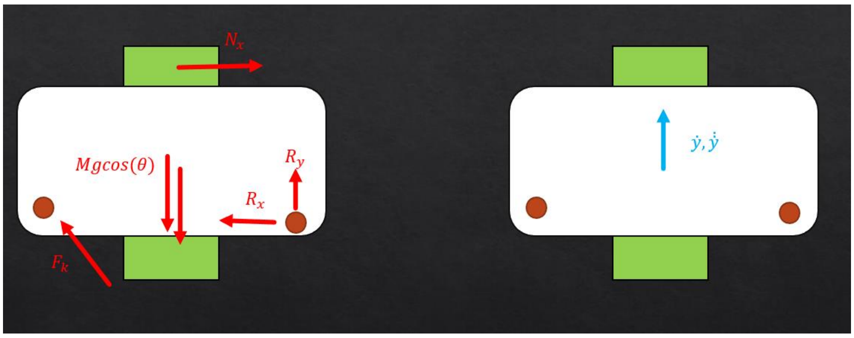


Figure 7: FBD and kinematic model of the mass

In relation to the theoretical model that was developed graphically, a series of calculations was done to model base excitation. The base excited by the stepper motor is connected by the linkage arm to the mass. Therefore, the force generated by the excited base transfers to the mass through the mass with reaction forces connecting the two bases. As shown in figure 6, the force *Gu* is the applied force, and this is related to the reaction force at the top right, as well as the spring force. Therefore, combining the two elements yields a general equation of motion, $y'' - 2k/m * (1.94/3.8) = (F/m) - 5\sqrt{2}$. This is the governing equation for the system.

Table 1: Summary of the Motor rpm along with the phase and output amplitude as the system responds to input

Motor rpm	Empirical omega(rpm)	Frequency(rad/s)	Phase (degrees)	Amplitude(m)	Magnitude (dB)
100	85.7	9.0	128.6	0.0070	-43.2
150	166.7	17.5	58.0	0.0028	-51.2
200	187.5	19.6	230.6	0.0073	-42.8
250	240.0	25.1	144.0	0.0058	-44.8
300	285.7	29.9	342.9	0.0035	-49.1
350	333.3	34.9	110	0.0038	-48.5
400	375.0	39.3	27	0.003	-50.8

Discussion

The Bode plot analysis reveals important features about the system's behavior. The phase plot points (9, 128.6), (17.5, 58), (29.9, 342.9), and (34.9, 110) indicate peaks and troughs, representing significant phase shifts in the output signal compared to the input signal at their respective frequencies. Notably, the point (25.1, 144) signifies the phase crossover frequency where the phase response crosses 180 degrees. This frequency holds significance in determining stability and system behavior, with positive phase shifts indicating a lead response above the crossover frequency and negative phase shifts indicating a lag response below it.

The phase values ranging from 58 degrees to 342.9 degrees demonstrate substantial variation in the system's phase shift across the analyzed frequency range, indicating frequency-dependent time delays or phase shifts in the output signal. A summary of the discussed values can be found in Table 1.

It is important to note that a comprehensive analysis requires additional data points covering a wider frequency range to accurately characterize the system's behavior. Furthermore, the interpretation of insights from the Bode plot magnitude and phase data relies on the specific characteristics and context of the system under study. Given the limited data provided, drawing definitive conclusions about the system's natural frequencies, damping, or stability is challenging. To gain a more thorough understanding, it is recommended to collect more data points across a broader frequency range and employ additional analysis techniques such as curve fitting, system identification, or modal analysis. These approaches can provide deeper insights into the system's behavior, resonant frequencies, damping ratios, and stability properties.

In terms of construction and motion, the model design was inconvenient and did not fully replicate the motion we expected. The linear rails had considerable friction due to the balls on the carriage escaping from each side. This caused the motion of the system to be stiffer than we expected and a larger lag in the translation of motion from the motor to the mass at the top. For a future design, the linear rails could be swapped out for something with less friction that is more convenient. However, the linkage arm design was an effective way to translate motion and help compress the shock absorber.

Conclusions

The experimental model was able to closely model the behavior of a trailing arm on a track. With a sinusoidal input into the vertical displacement of the bottom of the trailing arm, the experiment showed a amplitude and phase difference in the mass at the top of the model against the input force. They the data that was acquired was subject to noise because the linear rails were not tightened completely into the plate holding the top mass, allowing for some play in the rotation of the top plate creating noise in our system. However, the the experiment was able to demonstrate some attenuation in the motion of the mass on top, and the design of the experimental setup can be tweaked and fixed to become a highly iterable and modular model of the hyperloop suspension system. In the future, different linkage lengths, spring constants, dampening constants, and linkage designs will be tested on the experimental setup to see the base excitation response of the pod at the anticipated speeds that the hyperloop vehicle will travel

References

[1] Inman, D.J. (2007) *Engineering vibration*. 3rd edn. Englewood Cliffs, NJ: Prentice Hall.