Design/Analysis Project

Frame Design for the Cooper Hyperloop

P. Sklavounos, K. Deolall, and J. Sutton

Cooper Union for The Advancement of Science and Art

May 14, 2022

ABSTRACT

. The frame designed has the objective of providing a stable structure that brings all the components of the hyperloop pod together. Our frame will be designed to withstand the loads and interferences between other sub-components of the pod, whilst also making it as light as possible, and simple to manufacture.
. To achieve this, various primary analysis techniques and research was conducted by comparing cross-sections with both hand calculations and FEA simulations.
.
.

1. Introduction

The major focus is in mounting the brake module, as seen in Figure 1, since the main force that the frame will have to withstand is the braking force. This is the main force as there is no friction due to the electromagnetic brakes. These forces will act axially through the four holes where each rod will go through. The magnitude of this force will be taken to be 7200N, which is 2018's braking force as measurements from the braking dynanometer are currently being recorded. That means that each hole will experience 1800N. The other design constraint is that the frame is to be comprised of flat surfaces to allow for L-bracket mounting and nuts and bolts. Before the material analysis took place, the cross-sectional analysis was conducted by using Aluminium 1060, with the area being kept consistent.

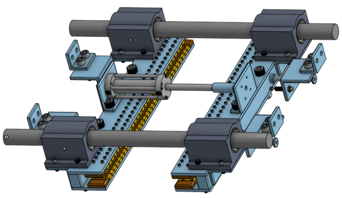


Fig. 1: Overview of braking module

The stages that will be outlined, along with their respective analysis will be: the multiple iterations through the frame's cross-section choice, whether it was a circular tube, I-beam, square beam, or C-bracket; how these would be organized in the assembly (mixtures of cross-sections for either the width or the length of the rectangular frame as they come in pairs) to minimize weight; and the material that will be used.

2. Process timeline & Decision making

The first assembled iteration, showcased in Figure 2 below, was a construction built entirely out of square beams. With our newfound knowledge on the rigidity of these structures, we were curious as to how such a design would behave. As observed, this square beam cross-section does not withstand axial forces through its walls well. Part of the reason for this is the wall thickness that is constrained, as well as the fact that it is the maximum distance from the neutral axis causing bending moments.

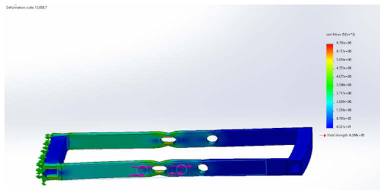


Fig. 2: The first assembly of the frame

Another issue that was noticed simply by observation was the wasted space - the dimensions were noticeably larger than they had to and the frame looked 'bulky'. The use of square beams all around meant a wider frame is then needed. To combat the issue of dimensions, another cross section was considered. This time around, an I-beam cross section was experimented with. The advantage of an I-beam cross section is a smaller area; therefore less mass, while keeping the design constraint and goal of flat surfaces for mounting intact.

2.1. Using I-beams cross sections

Using I-beams to carry the axial load is a more robust design as the line of action acts through the line of symmetry of the I beam, where there is mass to act through, while the hollow square beam was off-centered from the center line. Using a small section of I-beam, primary FEA analysis is conducted to see if there is failure with this geometry for the desired applied loads. The material used is Aluminium 1060 in the figure.

The FEA analysis on the stresses acting through a conventional I-beam, showcased in Figure 3 below, demonstrate the maximum and yield stress behaviors through this particular structure. From the stress chart, it is evident that the I-beam is considerably more rigid than the previous design we tested

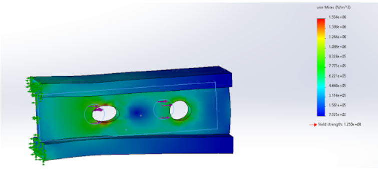


Fig. 3: Primary research on axial I-beam behavior

for the metal frame, causing us to focus our attention on designs with a similarly shaped structure.

2.2. New iteration with I-beam and cylinder connections

With the success and desirable behavior to axial forces applied to the I-beam, it was time to try out the I-beam as the main component for the members that would be receiving the axial forces. For aesthetics, cylindrical connections between each side of the I-beams would be used. In terms of manufacturing this, the exterior cylinder would be welded to the interior I-beam web, but to simplify the model, weld mates have been ignored and they are instead considered fixed on the ends. This iteration is shown in Figure 4.

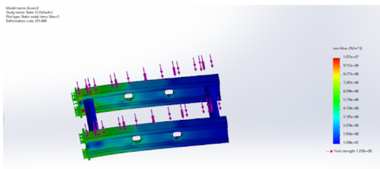
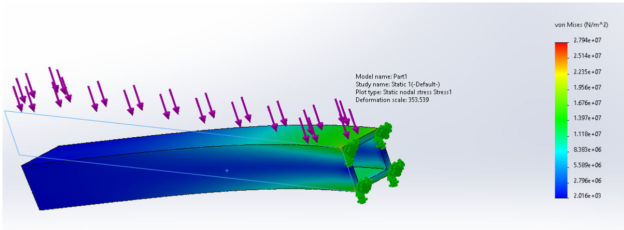


Fig. 4: I-beams connected by tubes

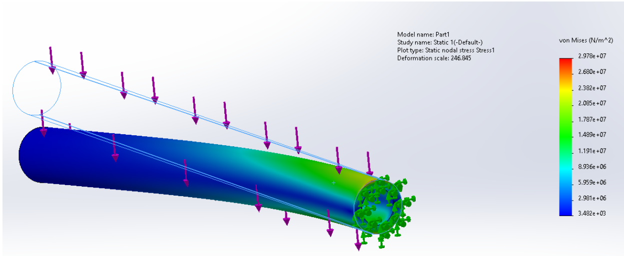
This FEA analysis revealed that even though the maximum stresses were a single order away from the yield stress, the design can be further optimized to minimize the deflection seen. The deflection scale is significant as the stress is not being distributed well. Despite the cylindrical supports that are placed at the ends, the constrained end is still showing areas of high concentrations which indicates that cylindrical tubes may not be an effective cross-sectional area.
Therefore, primary research was done on an isolated cylindrical tube of the same length and material and compared to a square beam. This comparison is necessary as the force was arbitrary as the weight of the components are unknown, as well as to see if a square-hollow beam tube could be used instead. Figure 5a and Figure 5b , along with hand calculations of the buckling load (Calculations 3.2) will give mechanical insight on the stability and strength of the cross section.

2.3. Final iteration with cross-sectional insight

Combining all of the findings so far, the I-beams will be connected with hollow square beams, as cylinders were proven to not provide enough stability. The FEA analysis of the frame in Figure 6 with a downward force (as components are to be mounted), finally indicate promising results!



(a)



(b)

Fig. 5: Compressive load using Von Mises, applied to (a) square beam (b) cylinder

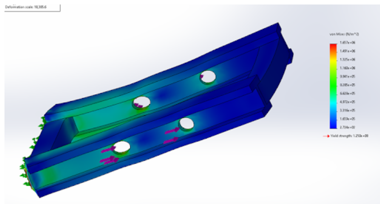


Fig. 6: Compiled cross-sections to form new frame

3. Calculations

3.1. Buckling symbolical calculation and comparison of cross-sections

In order to choose which shaped cross section to use, we looked for the one with a larger moment of inertia; the larger the moment of inertia, the less likely that the beam will buckle. To standardize the beams, the area and height were held constant. Through some algebra, we were able to show that the hollow square cross section had a larger moment of inertia with less material and therefore would be the most stable.

$$
\begin{aligned}
I_c &= \tfrac{A}{4}(r_o^2 - r_i^2) \\
I_s &= \tfrac{A}{12}(s_o^2 + s_i^2) \\
s_o &= 2r_o \\
A = \pi(r_o^2 - r_i^2) &= (4r_o^2 - r_i^2) \\
s_i &= \sqrt{(4-\pi)\cdot r_o^2 + \pi r_i^2} \\
I_s &= \tfrac{A(4r_o^2+4r_o^2-\pi r_o^2+\pi r_i^2)}{12} \\
I_s &= \tfrac{8r_o^2}{12} + \tfrac{\pi}{3}I - c \\
\sigma_x &= \tfrac{Mc}{I} \\
\sigma_s &\simeq \sigma_c + \tfrac{12M}{8r_o}
\end{aligned}
$$

3.2. Calculating the buckling loads of the various cross-sections

The buckling load will give us design insight as it represents the maximum load that the beam/cylinder can experience without buckling.The material of it will be taken to be aluminium 1060.

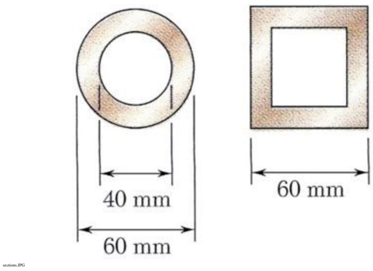


Fig. 7: Different cross-sections of same area to be analysed

$$
\begin{aligned}
P_{cr} &= \frac{\pi^2 EI}{L^2} \\
I_{circle} &= \frac{\pi}{4}(r_o^4 - r_i^4) \\
I_{square} &= \frac{1}{12}(a_o^4 - a_i^4) \\
\hookrightarrow I_{circle} &= \frac{\pi}{4}(60^4 mm - 40^4 mm) \\
I_{circle} &= \underline{5.105 \cdot 10^5 mm^4} \\
\hookrightarrow I_{square} &= \frac{1}{12}(60^4 mm - (60 - 2 \cdot 7.48mm)^4) \\
I_{square} &= \underline{7.371 \cdot 10^5 mm^4}
\end{aligned}
$$

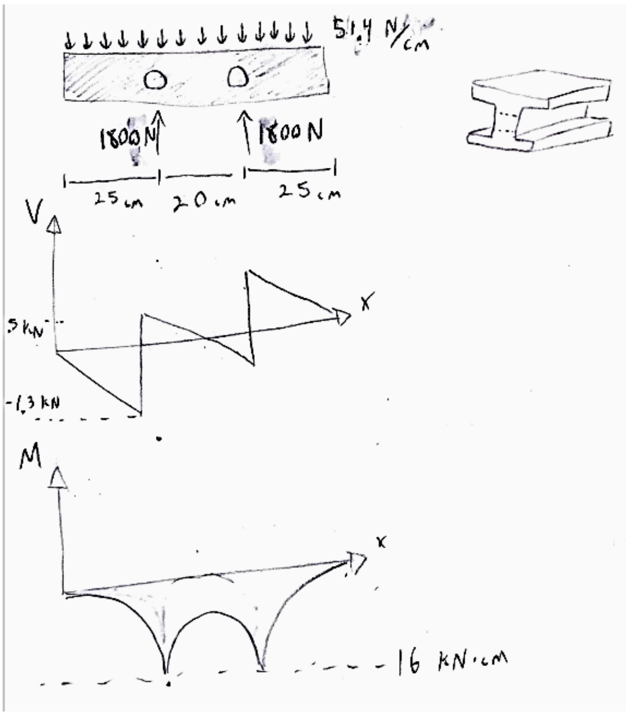
$$P_{cr_{circle}} = \underline{4.1987kN} \; By \; substituting \; above \; values$$
$$P_{cr_{square}} = \frac{7.371 \cdot 10^5 mm^4}{5.105 \cdot 10^5 mm^4} P_{cr_{circle}} = \underline{6.062kN}$$

∴ It is clear that the hollow square cross-section has a higher buckling load maximum and so is the better choice within this regard. While the main focus is on axial forces, there will be sub-components mounted, as well as the weight that will act downwards. This result is also verified by the FEA simulations seen in Figure 5b and Figure 5a that simulate how each cross-section reacts to a downward force. As it is not a brittle material, the Von Mises stress approach will be used. As seen, the cylinder experiences larger stresses in critical locations than the square beam, indicating that the square beam carries stress in a more conservative and safer way.

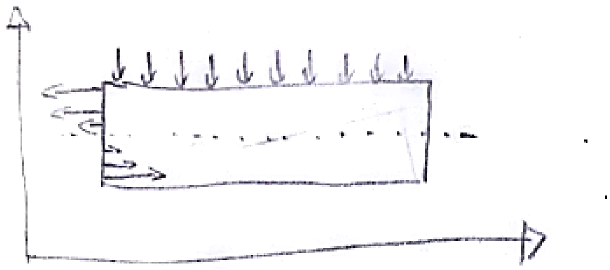
3.3. I-Beam shearing stresses

We conducted a bending analysis to demonstrate how the I beam can handle shear loads. First, a free body diagram was made with the 1800 Newton force acting through each hole and a distributed load balancing that out (Figure 8a). Using shear and moment diagrams, the maximum bending moment was found. With that, and the dimensions to the I-beam, we were able to determine the maximum normal stress. For the moment of inertia we had to subtract the area of the hole since the maximum moment of inertia was centered at each hole. With this information we could apply a factor of safety and decide what material could withstand that stress.

$$
\begin{gathered}
\text{Maximum Normal stress} = \frac{M_x c}{I}, c = 10.16cm \\
I = 2I_1 + I_2 - I_{hole} \\
I_1 = \frac{bh^3}{12} + bh(c - \frac{h}{2})^2 = 343.8cm^4 \\
I_2 = \frac{bh^3}{12} = 42.7cm^4 // \; I_{hole} = 13.1cm^4 \\
I = 717.2cm^4 \\
\text{Maximum normal stress} = \frac{16071 \cdot 10.16}{717.2} = 227.7Pa
\end{gathered}
$$



(a)



(b)

Fig. 8

4. Material analysis choice

With the information of the maximum buckling force,types and magnitudes of stresses that is felt by the frame, a more robust material choice can be made as the constraints are known.

Steel							
Material	Yield Stress(ksi)	Shear Modulus (ksi)	Modulus of Elasticity (ksi)	Density (lb/in^3)	Applicable Shape	Cost	Section used for cost
A36	36	11300	29000	0.28	Channel	$23.40	C channel 3/4 X 3/8 X 1/8 web - 4 ft
A572 Gr. 50	55	11600	29000	0.28	I-beam	$90.50	S 3 x 5.7# (3.00" x .170" x 2.33") - 5 ft
A513 Gr. 1010	42	11900	29000	0.28	Tube	$14.16	1 OD x .083 wall - 4 ft
A500	46	11600	29000	0.28	Tube	$14.56	1.05 OD X .113 wall - 4 ft
Aluminum							
Material	Yield Stress(ksi)	Shear Modulus (ksi)	Modulus of Elasticity (ksi)	Density (lb/in^3)	Applicable Shape	Cost	Section used for cost
6061-T6	40	3770	10000	0.098	Channel	$33.00	2 x 1 x .130 web - 4 ft
6061-T6	40	3770	10000	0.098	Tube	$21.82	1 OD x .125 wall - 4 ft
6063-T52	21	3740	10000	0.1	Tube	$16.79	1 OD x .125 wall - 4 ft

The above properties in Figure 4 were researched for different alloys of steel and aluminum. Material cost, weight, and yield stress were the most important factors considered. The aluminum alloys have a yield stress comparable to steel while having a much lower weight. However, this comes at a higher cost. A513 steel has a desirable yield stress and is the cheapest of the materials researched. A513 steel and 6063 aluminum appear to be the best options, and the onet is ultimately utilized depends on whether yield stress or weight is prioritized.

5. Conclusion

From the research we conducted using FEA analysis on differently shaped cross-sections, we were able to visually see the distribution and strength of shear forces acting on each beam with different moments of inertia under given loads, which would be experienced by a traveling hyperloop pod. We accounted for any sort of buckling that the frame might experience by calculating the maximum force values using critical dimensions of the cross-sections associated with each beam. We also conducted calculations for an overall shape of the frame to be used in the pod (the optimized shape being that of an I-beam) and respective maximum stresses the frame could withstand. To complete the overall design of the frame, we also conducted an analysis on various metal materials that are cost-effective and rigid enough to be used in the overall design.

6. Relevance to course

This research project found us primarily focusing on understanding the bending of beams under various loads. From what we learned in lecture over the semester, we were able to relate much of the design process to a real world scenario: the forces experienced by the metal frame of a hyperloop pod as it travels down a track. Using simulation software to consider the stresses of metal beams with varying moments of inertia, we were able to use calculations associated with shear forces and the bending of materials to optimize a design for a frame that was structurally sound.