

Skewer Design for Delicious Turducken Cooking & Transient Thermal Analysis using Altair Hypermesh and Ansys Workbench

A Areya, S Khan, R Lam, P Sklavounos

The Cooper Union for the Advancement of Sciences and Art

Abstract: A set of heated skewers is to be designed to speed up the cooking of a turducken, a turkey stuffed with a mixture of chicken and duck, followed by stuffing. The design goal is to reduce the amount of cooking time, while minimizing the amount of the turducken that is burnt. This will be achieved through a transient thermal analysis of the system using a combination of manual calculations and Computer Aided Engineering (CAE) software.

Design Statement: A wealthy captain of industry who owns a turkey farm has put out a competition to help him design and then market a new concept in cooking his brand of turducken. The concept is to cook a turducken in a conventional oven, with the addition of electric resistance skewers which may be placed through the turkey. The design of the skewers and the temperature of the oven will be designed to optimize for the fastest cook time, with the least amount of turducken burnt.

Design Constraints & Assumptions:

- The ducken is a homogeneous mixture of duck and chicken.
- The turkey begins the cooking process as being fully refrigerated at 42 °F.
- The turducken is considered fully cooked when all parts of the turducken exceed 165 °F
- A part of the turducken is considered burned when its temperature reaches 280°F.
- The skewers may be turned on or off once during the cooking process at a time specified by the customer
- The skewers must be able to be inserted in the turducken in a conventional home kitchen.
- The skewers may be no bigger than 0.5" in any one dimension of the cross section and the cross section area must be less than 0.2 in at every point along its length.
- The oven in which the turducken is cooked may operate between 325°F and 525°F.
- The oven is purely convective, there is no radiative heating of the turkey

I. TURDUCKEN DIMENSIONS

As shown in Figure 1, the turducken is composed of a turkey exterior, stuffed with an inner layer of ducken, an even mixture of chicken and duck, followed by a core of stuffing, consisting of 60% potato, 20% carrots, and 20% onions. A tube is present at the butt of the turkey to provide easy access to stuff the turkey.

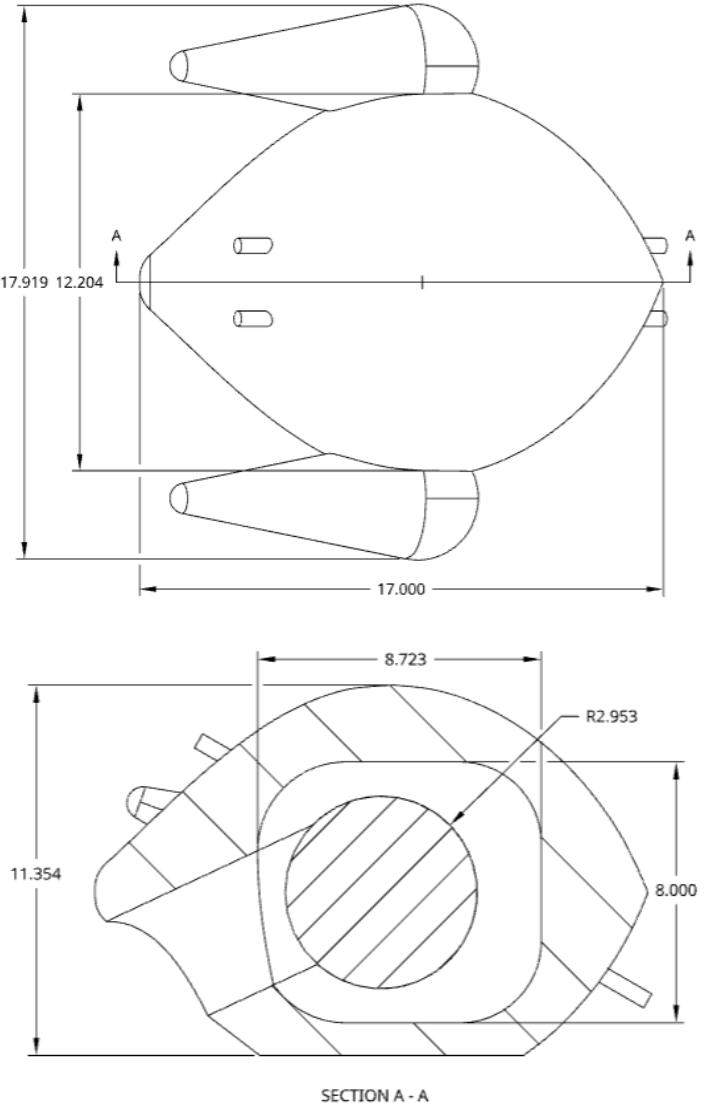


Figure 1: Top view and cross sectional view of turducken with dimensions in inches.

II. SKEWER DESIGN

SIMULATION ASSUMPTIONS

- Assume skewer emits a constant rate of heat uniformly across the heating surface.

- Assume that the skewer is adiabatic across its volume (no conduction through the rod)
- Assume no energy losses in the skewer. $W_{in} = Q_{..}$

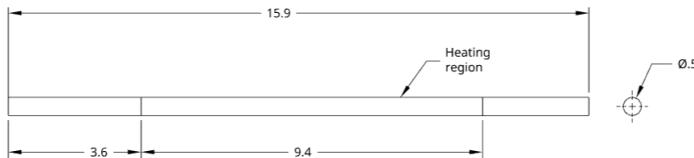


Figure 2: Dimensioned drawing of one of the skewers in inches.

The material of the rods were chosen to be Nichrome 60-15(60Ni-20Fe-18Cr-1Mn-1Si). The nichrome contains a high iron content, giving it a high thermal conductivity. Nichrome 60-15 also has a high electrical resistivity, making it optimal for electric resistance heating. Moreover, it has one of the highest thermal emissivity amongst other high temperature wiring and materials with a value of 0.65 - 0.79. While we reject heat loss and inefficiency of the appliance in our analysis, a higher emissivity means a greater thermal output from the same input. Given its availability in stores in that it is a common material, this was a good choice for our rods. The cylindrical shape allows for easy insertion and heating, as well as even heating across its surface as the direction of heat flux do not intersect, and instead emit radially outwards.

The heating region of the skewer is designed to coincide with the length of the ducken. Current injected into the skewers will inject heat into the turkey along the heating region of the skewer using I^2R heating, and will heat the core of the turkey why convection in the oven.

III. HAND CALCULATIONS

To begin thermal analysis of the Turducken, the thermal properties of each food product present with respect to temperature was determined. The density, specific heat, and thermal conductivity coefficient with respect to temperature for all the food products in the turducken were calculated. For this paper, we calculated the properties at critical temperatures during the cooking of the turducken: 42F (initial temperature), 165F(cooked), 210F(boling point of water),and 280F(burned).

A. Approximations

Referring to ASHRAE, Thermal Properties of Foods [8] handbook, the percentage compositions of each food product was found. Food products constitute of moisture (water), protein, fat, carbohydrates, and ash, all of which have different thermal properties. In order to simplify our calculations, we incorporated the following assumptions:

- The ducken is a homogenous mixture of equal parts chicken and duck
- No carbohydrates are present in poultry products
- All components have the same initial temperature of 42F.
- At 212F, all water in the food products evaporates (0% moisture) and the proportions of protein, fat, carbohydrates, and ash in the remaining material remain constant.

- At 280F, all other components are proportionally 0% while ash is at 100%.
- The material properties are isotropic and are calculated under isobaric conditions.
- ***The material properties of each food product is a proportional combination of the material properties of its constituents***
- Forced convection for heating is constant, laminar and steady state flow.

B. Thermal Properties of Food Components

The composition of each component of turkey, ducken, and stuffing were needed to proportionally analyze thermal properties

| Composition above freezing | | | | | |
|----------------------------|--------------|--------------|--------------|----------------|---------------|
| | Moisture | Protein | Fat | Ash | Carbohydrates |
| Turkey | 0.704 | 0.204 | 0.0802 | 0.00880 | 0 |
| Chicken | 0.660 | 0.186 | 0.151 | 0.00790 | 0 |
| Duck | 0.485 | 0.115 | 0.393 | 0.00680 | 0 |
| DUCKEN | 0.572 | 0.150 | 0.272 | 0.00735 | 0 |
| Carrots (20%) | 0.878 | 0.0103 | 0.00190 | 0.00870 | 0.101 |
| Potato (60%) | 0.790 | 0.0207 | 0.00100 | 0.00890 | 0.180 |
| Onion (20%) | 0.897 | 0.0115 | 0.00160 | 0.00370 | 0.0863 |
| Stuffing | 0.829 | 0.0168 | 0.00130 | 0.00782 | 0.145 |

Table 1: Food composition of all food products at temperatures 42F<T<212F(boling).

| Composition at Boiling Point (212F) | | | | | |
|-------------------------------------|----------|--------------|--------------|---------------|--------------|
| | Moisture | Protein | Fat | Ash | Carbohydrate |
| Turkey | 0 | 0.696 | 0.274 | 0.0300 | 0 |
| Chicken | 0 | 0.540 | 0.437 | 0.0229 | 0 |
| Duck | 0 | 0.223 | 0.764 | 0.0132 | 0 |
| DUCKEN | 0 | 0.350 | 0.633 | 0.0171 | 0 |
| Stuffing | 0 | 0.0979 | 0.00759 | 0.0456 | 0.849 |

Table 2: Food composition at boiling point, no moisture present within any food product of turducken.

| Composition at Burned Temp. (280F) | | | | | |
|------------------------------------|----------|----------|----------|-------------|--------------|
| | Moisture | Protein | Fat | Ash | Carbohydrate |
| Turkey | 0 | 0 | 0 | 1.00 | 0 |
| Chicken | 0 | 0 | 0 | 1.00 | 0 |
| Duck | 0 | 0 | 0 | 1.00 | 0 |
| DUCKEN | 0 | 0 | 0 | 1.00 | 0 |
| Stuffing | 0 | 0 | 0 | 1.00 | 0 |

Table 3: Composition of components at 280F, all food components burns off while only ash remains.

The composition of the food properties are used to proportionally find thermal properties that are necessary for transient thermal analysis. The composition changes at different temperatures, which is why the values are calculated at 42F, 165F, 212F, and 280F to include within a tabulated data for thermal simulations. To calculate the thermal properties of each component, all equations were provided as a function of temperature as stated in the ASHRAE handbook [8].

All equations for the various thermal properties are given between the temperature range of -40F<T<300F

| Thermal Property | Thermal Property Model |
|---|--|
| Thermal conductivity, Btu/(h ft °F) | $k_w = 3.1064 \times 10^{-1} + 6.4226 \times 10^{-4}t - 1.1955 \times 10^{-6}t^2$ |
| Thermal diffusivity, ft²/h | $\alpha_w = 4.6428 \times 10^{-3} + 1.5289 \times 10^{-5}t - 2.8730 \times 10^{-8}t^2$ |
| Water Density, lb/ft³ | $\rho_w = 6.2174 \times 10^{-1} + 4.7425 \times 10^{-3}t - 7.2397 \times 10^{-7}t^2$ |
| Specific heat, Btu/(lb °F) (For temperature range of -40 to 32°F) | $c_w = 1.0725 - 5.3992 \times 10^{-3}t + 7.3361 \times 10^{-7}t^2$ |
| Specific heat, Btu/(lb °F) (For temperature range of 32 to 300°F) | $c_w = 9.9827 \times 10^{-1} - 3.7879 \times 10^{-3}t + 4.0347 \times 10^{-7}t^2$ |

Table 4: Thermal properties for water

| Thermal Property | Food Component | Thermal Property Model |
|-------------------------------------|----------------|---|
| Thermal conductivity, Btu/(h ft °F) | Protein | $k = 9.0535 \times 10^{-2} + 4.1486 \times 10^{-4}t - 4.8467$ |
| | Fat | $k = 1.0722 \times 10^{-1} - 8.6581 \times 10^{-5}t - 3.1652$ |
| | Carbohydrate | $k = 1.0133 \times 10^{-1} + 4.9478 \times 10^{-4}t - 7.7238$ |
| | Fiber | $k = 9.2499 \times 10^{-2} + 4.3731 \times 10^{-4}t - 5.6500$ |
| | Ash | $k = 1.7553 \times 10^{-1} + 4.8292 \times 10^{-4}t - 5.1839$ |

Table 5: Conductivity of food components

| | | |
|-----------------------------|--------------|---|
| Density, lb/ft ³ | Protein | $\rho = 8.3599 \times 10^1 - 1.7979 \times 10^{-2}t$ |
| | Fat | $\rho = 5.8246 \times 10^1 - 1.4482 \times 10^{-2}t$ |
| | Carbohydrate | $\rho = 1.0017 \times 10^2 - 1.0767 \times 10^{-2}t$ |
| | Fiber | $\rho = 8.2280 \times 10^1 - 1.2690 \times 10^{-2}t$ |
| | Ash | $\rho = 1.5162 \times 10^2 - 9.7329 \times 10^{-3}t$ |
| Specific heat, Btu/(lb °F) | Protein | $c_p = 4.7442 \times 10^{-1} + 1.6661 \times 10^{-4}t - 9.6784 \times 10^{-8}t^2$ |
| | Fat | $c_p = 4.6730 \times 10^{-1} + 2.1815 \times 10^{-4}t - 3.5391 \times 10^{-7}t^2$ |
| | Carbohydrate | $c_p = 3.6114 \times 10^{-1} + 2.8843 \times 10^{-4}t - 4.3788 \times 10^{-7}t^2$ |
| | Fiber | $c_p = 4.3276 \times 10^{-1} + 2.6485 \times 10^{-4}t - 3.4285 \times 10^{-7}t^2$ |
| | Ash | $c_p = 2.5266 \times 10^{-1} + 2.6810 \times 10^{-4}t - 2.7141 \times 10^{-7}t^2$ |

Table 6: Density and Specific Heat

| To | Tf | Tb | Boiling point | |
|-----------------------------|-------|-------|---------------|-------|
| | 42 | 165 | 280 | 212 |
| Specific Heat of components | | | | |
| water | 0.997 | 1.00 | 1.02 | 1.01 |
| protein | 0.481 | 0.499 | 0.513 | 0.505 |
| fat | 0.476 | 0.494 | 0.501 | 0.498 |
| carbohydrates | 0.372 | 0.397 | 0.408 | 0.403 |
| Ash | 0.263 | 0.290 | 0.306 | 0.297 |

Table 7: Specific heat calculation for all food components at critical temperatures.

Using the equations in *Table 4*, all properties of moisture were calculated at the critical temperatures. Likewise, using the equations provided in *Table 5* and *Table 6*, the properties of the other food components were determined. An example calculation is presented in *Table 7*. The values for ‘t’ as shown in the equations are substituted with the critical temperature values in Fahrenheit. The equations are simplified by the handbook within the given -40F to 300F range, therefore, the units all represent the respective property.

Specific Heat

To determine the total specific heat of each element, the sum of specific heat for each food component multiplied by the fraction present in each element of the turducken yielded the net specific heat.

$$C_{total} = \sum C_i x_i$$

C_{total} = Specific heat of element (BTU/lb°F)

C_i = Specific heat of each food component(BTU/lb°F)

x_i = fraction of each food component

| SPECIFIC HEAT (BTU/lb°F) | | | | |
|--------------------------|-------|-------|-------|-------|
| | 42F | 165F | 280F | 212F |
| TURKEY | 0.841 | 0.850 | 0.306 | 0.497 |
| DUCKEN | 0.775 | 0.786 | 0.306 | 0.497 |
| STUFFING | 0.891 | 0.900 | 0.916 | 0.906 |

Table 8: Specific heat of each component of the turducken.

Density

The total density was calculated using the following equation provided by the handbook. Since porosity is only significant for granular foods, which is absent from the current model, therefore, it was assumed to be 0. The denominator represents the sum of the ratio of the fractional composition of the food component with its respective density.

$$\rho = \frac{(1-\varepsilon)}{\sum x_i / \rho_i}$$

ρ_i = Density of each component

ε = Porosity of food component

x_i = fraction of each food component

| DENSITY (lb/ft ³) | | | | |
|-------------------------------|------|------|------|------|
| | 42F | 165F | 280F | 212F |
| TURKEY | 65.8 | 65.8 | 149 | 72.0 |
| DUCKEN | 63.4 | 63.0 | 149 | 62.6 |
| STUFFING | 66.6 | 67.0 | 149 | 96.7 |

Table 9: Density of each component

$$k = \sum x_i v k_i$$

k = Total thermal conductivity

k_i = Thermal conductivity of each food component(continuous phase)

x_i = volume percentage, proportional to percent composition

Since the majority of the components are continuous volumes, we simplified the model to that of a parallel model, assuming that all components are uniformly distributed and aligned parallel to each other.

| THERMAL CONDUCTIVITY (BTU/h*ft°F) | | | | |
|-----------------------------------|-------|-------|-------|-------|
| | 42F | 165F | 280F | 212F |
| TURKEY | 0.268 | 0.310 | 0.270 | 0.141 |
| DUCKEN | 0.238 | 0.268 | 0.270 | 0.115 |
| STUFFING | 0.299 | 0.346 | 0.255 | 0.173 |

Table 10: Thermal conductivity of each component

The values in *Tables 9-11* were used to define the materials in ‘Engineering Data’ after the meshed geometry was imported to Ansys Workbench for thermal analysis.

C. Film Coefficient for Convection and Conductive Heat Flow

In order to simulate the cooking process for the turkey, a set of thermal loads needed to be determined and consequently applied in Ansys Workbench to simulate transient thermal analysis.

To simulate a convection oven, a series of coefficients and experimental values were needed to approximate a film coefficient of forced convection. In a commercial convection oven, a fan is used to evenly distribute heat. Usually the type of convection emits forced air over a surface. Since air is an incompressible fluid a series of non-dimensional quantities are used to aid in calculating the film coefficient.

To get an approximated film coefficient, Nusselt-Reynold-Prandtl relationship is used for convective heat transfer over a spherical body[2][3][4].

$$Nu = 2 + [0.4Re^{1/2} + 0.06Re^{2/3}]Pr^{0.4}$$

Nu = Nusselt number

Re = Reynolds number

Pr = Prandtl number

Relationship with Nusselt number with film coefficient:

$$Nu = hD/k$$

h = film coefficient of air (BTU/s*ft^2*F)

D = max vertical diameter of turkey (ft)

k = thermal conductivity of air (BTU/s*ft*F)

Reynolds Number formula:

$$Re = \rho V D / \mu$$

ρ = density of air (lb/ft^3)

V = velocity (ft/s)

μ = dynamic viscosity of air (lb/ft*s)

Since our model is limited to a cooking temperature between 350F and 525F, we only determined the film coefficient at temperatures of 350F and 450F. Properties of dry air at those temperatures were obtained from experimental data.

According to an experimental data conducted by authors from University of Canterbury, for a commercial oven, the maximum velocity of air in a cavity was recorded to be 1.8 m/s or 5.91 ft/s. Since the turkey is assumed to be a sphere and flow is assumed to be uniform throughout, the velocity will be consistent with the experimental data. [5]

Table 12 and 13 documents the properties needed for calculating the film coefficient. Assuming that the system is isobaric, the experimental data is consistent at all temperatures. Furthermore, the coefficients have a negligible difference within the given range(350F < T < 525F), therefore inputting an approximate value at any given temperature will suffice. This approximation allowed us to run simulations at various temperatures between 350F and 450F, anything above would part parts quickly.

Using the calculated values, a load input of convection and heat flow was applied to the surfaces and every face of the solid inside, where the rods will make contact within the body. A convection load required an ambient temperature input, the temperature of the oven(350F or 450F), and a film coefficient of the two temperatures, which is represented in Table 12 and 13, respectively. The convection load was applied to every exposed surface, including an outer face of the stuffing along with the cut leading to it. The heat flow was applied to the faces making contact with the rods. The estimated heat in is determined by optimizing the parameters using the Circuit model as presented in the following section.

| | Air Properties | 350F |
|--------------------------|----------------|----------------------------|
| density | 0.0489684923 | lbm/ft^3 |
| velocity | 5.90551 | ft/s |
| Diameter | 0.9464 | ft |
| Dynamic viscosity | 0.00001669444 | lb/ft*s |
| Re | 16393.69694 | |
| Prandtl | 0.683609434 | |
| k (thermal conductivity) | 0.00000595842 | BTU/s*ft*F |
| Nu | 79.24189061 | |
| Film Coefficient | h | 1.796031932 BTU/h*ft^2*F |
| | | 0.00049889776 BTU/s*ft^2*F |

Table 11: Properties of air at 350F.

| Air Properties | 450F |
|-------------------|-----------------------------|
| density | 0.04407414 lb/ft^3 |
| velocity | 5.9055 ft/s |
| Diameter | 0.9464 ft |
| Dynamic viscosity | 0.0000179 lb/ft-s |
| Re | 13729.5011 |
| Pr | 0.68 |
| k | 0.000006490025 BTU/s*ft*F |
| Nu | 71.65332589 |
| h | 0.000491369295 BTU/s*ft^2*F |

Table 12: Properties of air at 450F

D. Heat Transfer Circuit Model

A simple heat transfer model is used to approximate the time required to fully cook the turkey for different oven temperatures and different heat fluxes injected by the two skewers. The system is modeled as three concentric spheres of turkey, ducken and stuffing with diameters 12.2in, 8in and 5.9in, respectively (corresponding to the dimensions in Figure 1). The system is modeled as an electric circuit by making temperature T analogous with voltage V, and heat flow q analogous with current i. Transfer of heat through the system is thus modeled as charge flowing through an electric circuit of resistors and capacitors representing their thermal counterparts. The conductance, density and capacitance of each food component is assumed to be constant in this model, with its magnitude equal to the average from T=42F to T=212F:

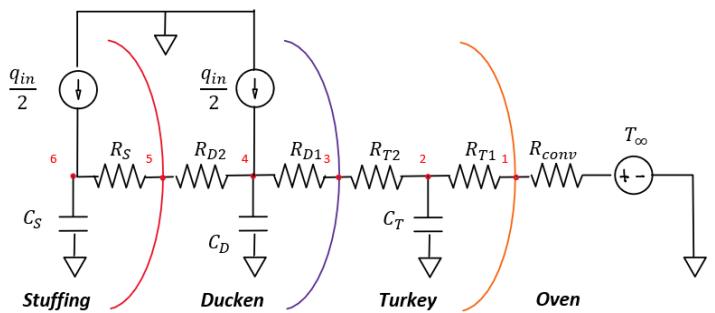


Figure 3: Thermal Circuit Model

Convection between the ambient temperature of the oven T_{∞} and node 1 is given by the convective heat transfer equation:

$$q = hA\Delta T$$

converting this to its electrical counterpart yields the convective resistance:

$$i = hA\Delta V$$

$$\Delta V/i = R_{conv} = 1/hA$$

where h is the film coefficient of the turkey and A is the surface area of the turkey sphere, given by $A = 4\pi r^2$

Conduction between consecutive components of the system is given by the thermal conduction equation in spherical coordinates:

$$q = -kA \frac{dT}{dr}$$

where k is the thermal conductivity of each component.
Simplifying the equation gives:

$$q \int_{r_1}^{r_2} \frac{1}{R} dR = -4\pi \int_{T_1}^{T_2} dT$$

$$q = \frac{4\pi k(T_1 - T_2)}{1/r_1 - 1/r_2} = \frac{4\pi k}{1/r_1 - 1/r_2} \Delta T$$

therefore:

$$I = \frac{4\pi k}{1/r_1 - 1/r_2} \Delta V$$

$$\Delta V/i = R_{cond} = \frac{1/r_1 - 1/r_2}{4\pi k}$$

To model the transient thermal behavior of each food component, each consecutive layer of the turkey is given a heat capacitance calculated using the density, volume and specific heat of each component, calculated in the previous section.

$$C_T = \rho_T V_T c_T$$

$$C_D = \rho_D V_D c_D$$

$$C_S = \rho_S V_S c_S$$

where $V = 4/3\pi(R_o^3 - R_i^3)$. To simplify the hand calculation, the density and specific heat is treated as a constant with respect to temperature, taken as the average of the values below 280F. The equations describing the system are derived by combining resistors in series between nodes before extrapolating the temperature at the in-between nodes by voltage division.

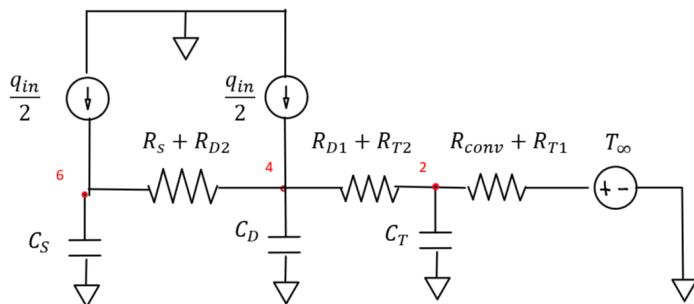


Figure 4: Simplified thermal circuit
solve the following differential equations for T2,T4,T6:

$$@node 2: \frac{T_\infty - T_2}{R_{T1} + R_{conv}} - \frac{T_2 - T_4}{R_{D1} + R_{T2}} - C_T \dot{T}_2 = 0$$

$$@node 4: \frac{T_2 - T_4}{R_{D1} + R_{T2}} - \frac{T_4 - T_6}{R_s + R_{D2}} - C_D \dot{T}_4 + \frac{q_{in}}{2} = 0$$

$$@node 6: \frac{T_4 - T_6}{R_s + R_{D2}} - C_S \dot{T}_6 + \frac{q_{in}}{2} = 0$$

using voltage division to solve for T3 and T5, (T1 is not interesting since temperature at the surface is nearly constant due to the constant ambient temperature).

$$@node 3: T_3 = T_2 - \frac{R_{T2}}{R_{T2} + R_{D1}} (T_2 - T_4)$$

$$@node 5: T_5 = T_4 - \frac{R_{D2}}{R_s + R_{D2}} (T_4 - T_6)$$

The code used to solve these differential equations can be found in the appendix.

When $q_{in} = 0$ Btu/hr, and $T_\infty = 400$ F. The temperature of each component over time is shown below:

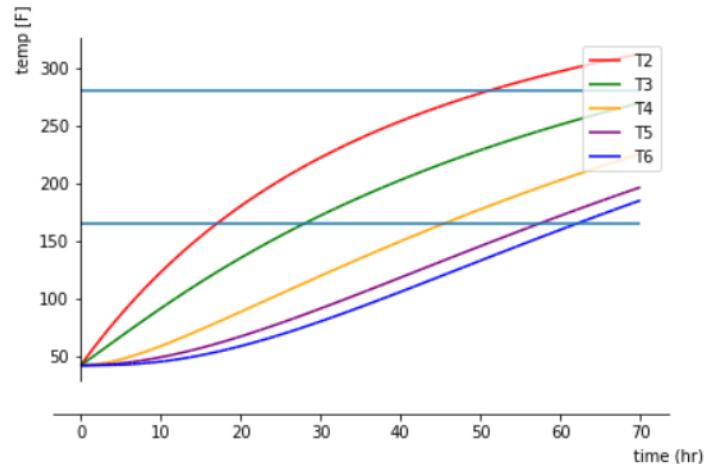


Figure 5: Temperature over time of nodes in the thermal circuit model when $q_{in}=0$ Btu/hr, $T=400$ F
With only convection heating, it takes 15 hrs for the exterior of the turkey to be cooked, while it takes 60 hours for the stuffing to be cooked (at which point the turkey is burnt). This is too long for a computer simulation to run.

With a $q_{in} = 120$ Btu/hr and $T_\infty = 450$ F the graph looks like:

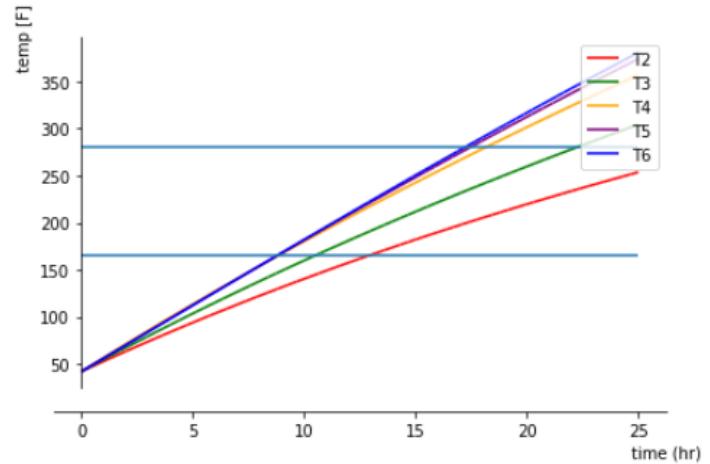


Figure 6: Temperature over time of nodes in the thermal circuit model when $q_{in}=120$ Btu/hr, $T=400$ F
As shown above, it only takes 8 hours to cook the stuffing and 13 hours to cook the turkey. Since T6 and T5 are below the 280F line, the stuffing is below the burning temperature while the turkey is fully cooked. This is an optimal starting point for the simulations.

If we consider turning off the rod once during the cooking, we can have a greater Qin such that the interior of the turkey cooks faster, and turn off the rods before it reaches burnt temperature.

With $Q_{in} = 180 \text{ Btu/hr}$ and $T_{\infty} = 450$, the graph looks like:

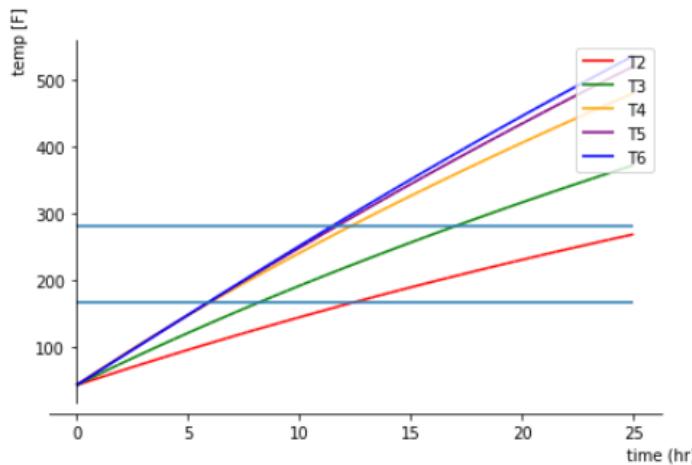


Figure 7: Temperature over time of nodes in the thermal circuit model when $q_{in}=180 \text{ Btu/hr}$, $T=450\text{F}$

In the graph above, the stuffing is burnt slightly before the turkey is fully cooked, but the rods can be turned off at the 10 hr mark to minimize burning.

However, a limitation of this simulation is that it conglomerates the heat capacitance of each food component to a single heat capacitance, ignoring local heating behavior. This means that the material adjacent to the rods and the surface of the turkey would likely burn much earlier than predicted in this model. Therefore, while hand calculated models predict increasing the oven to its maximum temperature of 525F, and having a high q_{in} is ideal to heat the turducken quickly, this is not the best practice in reality. Additionally, the thermal properties of materials change drastically at 212F when water evaporates. By averaging the resistance capacitance and densities, the model neglects such effects. Nonetheless, the circuit model gives a good approximation of magnitude, and a starting point for simulations.

IV. SIMULATIONS

A. Meshing Method [6]

To perform the meshing of the turducken, the various components were isolated and meshed separately. As the turducken is symmetrical, the thermal behavior will follow the symmetry. By only analyzing half of the turducken, the number of nodes is halved, thus reducing simulation time. To reject any loads at exposed interior, adiabatic conditions will be used across the cut surface. Additionally, splitting the circular holes and spherical bodies down the middle ensures the mesh at these areas of interest generate properly.

To begin, a surface automesh using mixed tetra and hexa elements on the surfaces of each component was applied. The toggle line tool (F11) was used to hide any edges that caused the element density to be too high or overdefined. For example, at the back end of the turkey, the circular geometry at the pointed end causes the very high element density at that point. The line tool was used to toggle the circle to reduce such effects. A surface mesh of element size 0.01m (0.03ft) was used. Using the element

refinement tool, the element density across certain areas was manually edited to ensure a fine enough mesh with an aspect ratio greater than 1:5 for all components, but not too fine to the point where it would overextend the computer's simulation resources. By tweaking density manually with Altair Hypermesh, The surface mesh better quadrilaterals and no collapsed shapes, which is optimal as it is not rigid in the physics that simulations allow for.

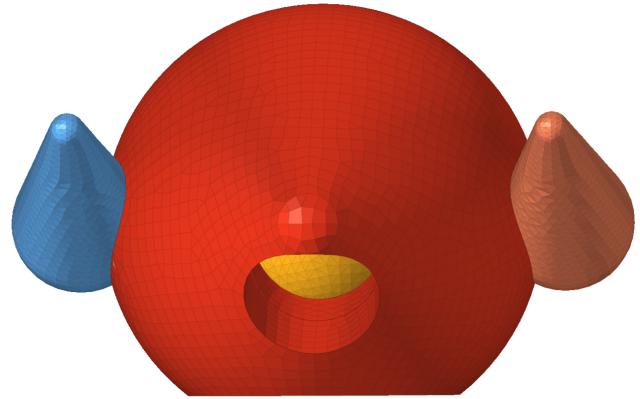


Figure 8: Surface mesh of Turducken

A CFD tetramesh of 3 boundary layers, first layer thickness of 0.002m (0.0066 ft), BL growth rate of 1.2 was generated. Delaunay tetrameshing, with a growth rate of 1.3, pyramid transition ratio of 0.8, and a tetrahedral collapse of 0.1 was applied to all bodies. The turkey was unable to be meshed using a fixed boundary layer, so it was specified that the surfaces on the turkey were floating in the cfd tetramesh tab.

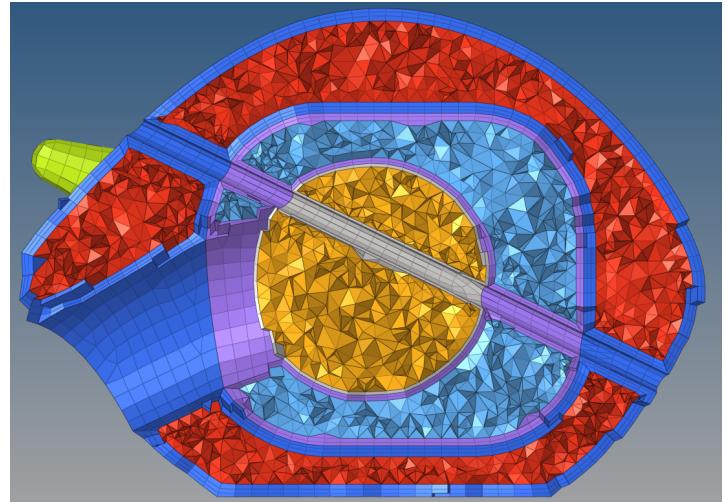


Figure 9: Full turducken mesh including boundary layers
To achieve closed volumes even for the layers that are seemingly open, like the chicken/duck mixture and the turkey, the inner layer surfaces were chosen as highlighted in the figure below.

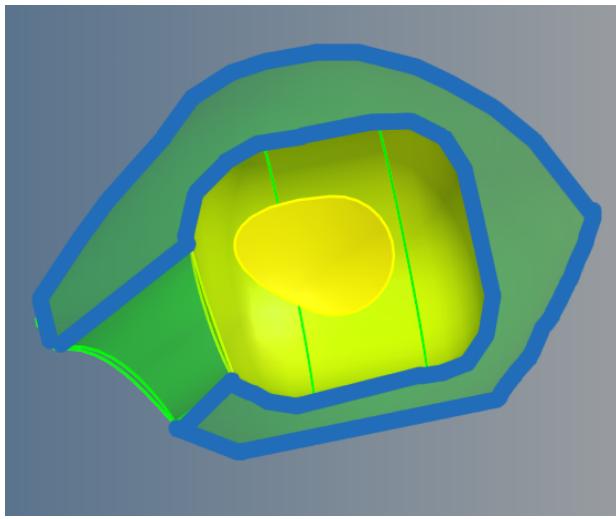


Figure 10: An example of how layers were chosen to ensure enclosed volumes for the volume tetrafill.

The reason for why CFD tetramesh volume filling is chosen is because there would be too many elements and nodes to simulate, so the areas of interest are given higher resolution, and then an exponential inflation of tetras takes place that grows to the center of the component.

Most importantly, in exporting to APDL, it is important to define the properties of each component of the turducken with sensors and properties according to their element, otherwise only nodes are imported without any elements.

Workbench has a useful python script hidden in its directory that helps convert invalid blocked code databases (.CDB) to valid ones, and so that was used on the file with the specified assemblies of each of the subcomponents. These subcomponents are then made into named selection for material property assignment and ease of thermal analysis.

The rods themselves, for simulation purposes, don't have to be meshed or tetra-filled since the faces of the holes created by the rods can be used for heat flow.

V. TRANSIENT THERMAL ANALYSIS

After completing meshing in Altair Hypermesh, all components of the turducken were exported as separate .bdf files using NASTRAN as the preferred output. The files were then imported separately into the workbench 'export model' component system into the setup, which was connected to 'model' under 'transient thermal'. The turducken model was imported as three separate geometries. The 'engineering data' was added by defining the property of each component using the numbers shown under *Hand Calculations*. Once updated, material properties for each element were assigned. After adding in loads and boundary conditions, the simulations were executed under different load conditions and cases. Refer to figure 6 for set up.

A. Boundary Conditions used

- convection heat on all outer surfaces of the turkey (labeled A and B)
- Heat flow from rod applied to the surface of the hole (labeled C)

- No heat flow on the ducken component; Adiabatic (labeled D)

How analysis is performed: max, avg and min temp over time exported, distributions when first part burnt and when fully cooked shown, and time until that happens found.. Temperature of every node when turkey is fully cooked extracted to find the percentage burnt. A simulation was run with the parameters shown in figure 7.

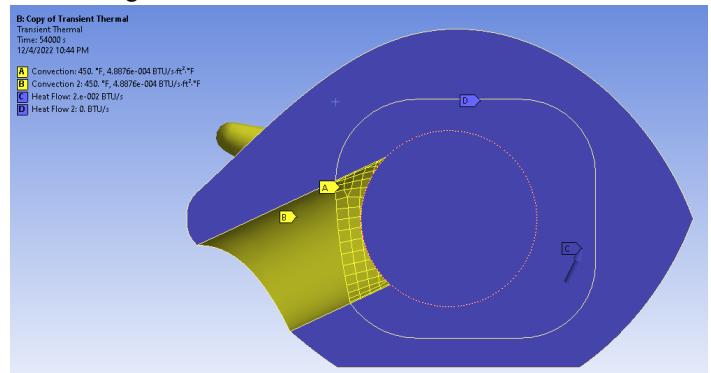


Figure 11: Boundary conditions and simulation setup

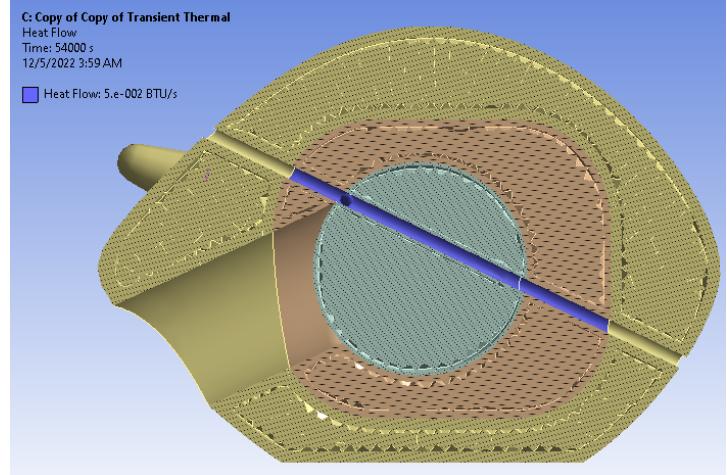


Figure 12: (Base case) Oven temperature = 350F, $q_{in} = 0$

Analysis Settings

| Properties | Step 1 |
|---------------------------|--------------------|
| Step Controls | |
| Step End Time | 54000 |
| Auto Time Stepping | Off |
| Define By | Time |
| Time Step | 100. |
| Time Integration | On |
| Nonlinear Controls | |
| Heat Convergence | Program Controlled |
| Temperature Convergence | Program Controlled |
| Line Search | Program Controlled |
| Output Controls | |
| Contact Data | Yes |
| Nodal Forces | No |
| Contact Miscellaneous | No |
| General Miscellaneous | No |
| Euler Angles | Yes |
| Volume and Energy | Yes |
| Calculate Thermal Flux | Yes |
| Store Results At | All Time Points |
| Advanced | |
| Contact Split (DMP) | Off |

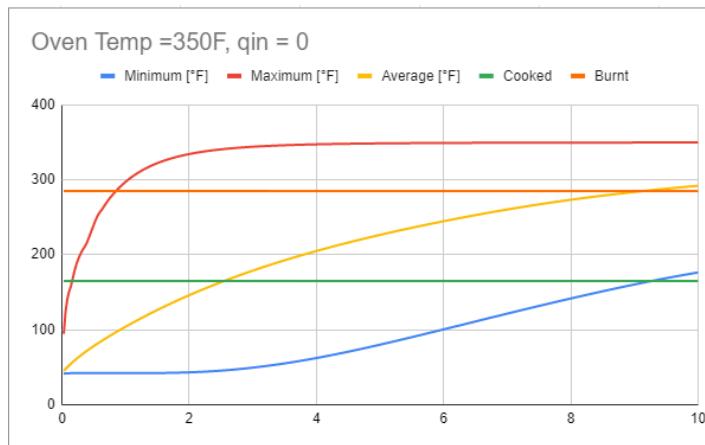


Figure 13: Graph of first case simulation. Cooked at $t = 9.3\text{hr}$. First signs of burning when $t = 0.806\text{hr}$

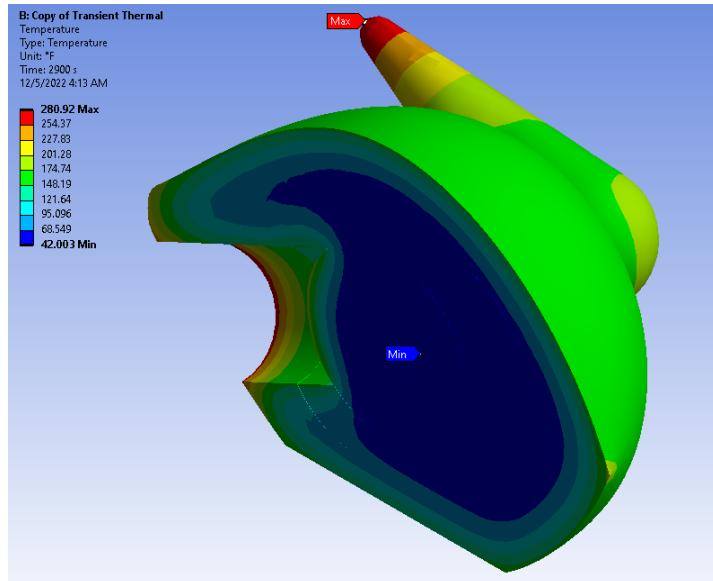


Figure 14: Temperature distribution when $t=0.806\text{hr}$, (first node burnt)

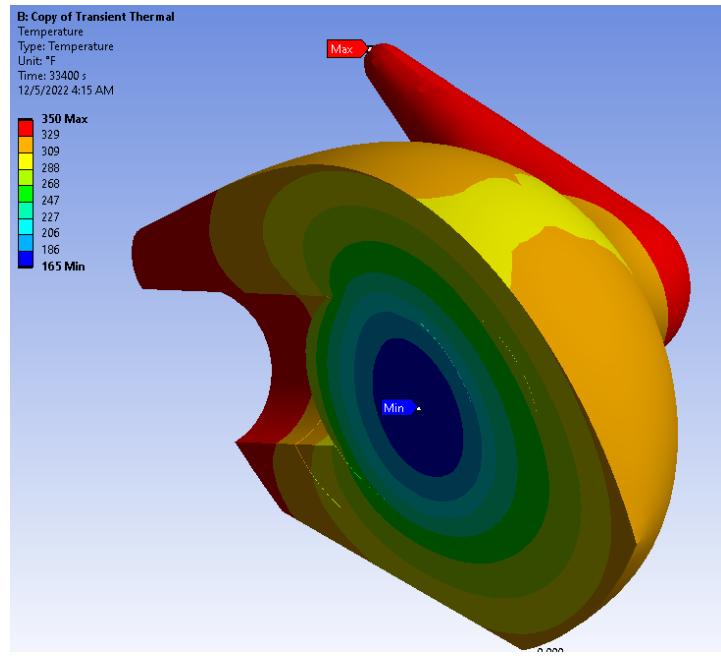


Figure 15: Temperature distribution when $t=9.3\text{hr}$. Fully cooked.

| | |
|--------------|-------|
| #Nodes Burnt | 26174 |
| Total #Nodes | 41234 |
| %Burnt | 63% |

Figure 16: Percentage burnt for base case

We observe from this base case that the Turkducken cooked too slow and majority of the turducken was burnt.

B. Test Case Simulations

Experiment 1: $q_{in} = 120\text{Btu/hr}$ (0.03Btu/s), $T = 450\text{F}$

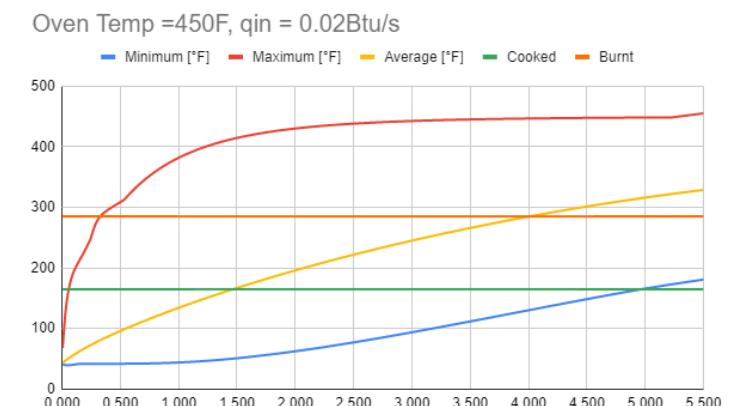


Figure 17: Fully cooked at 4.97hr , first burnt at 0.3 hr .

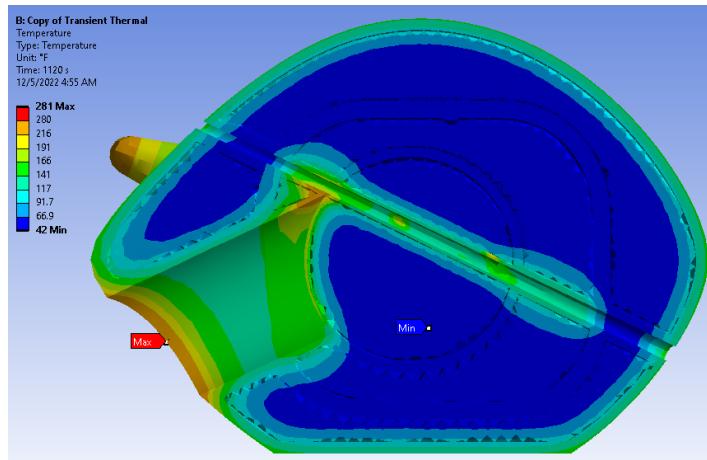
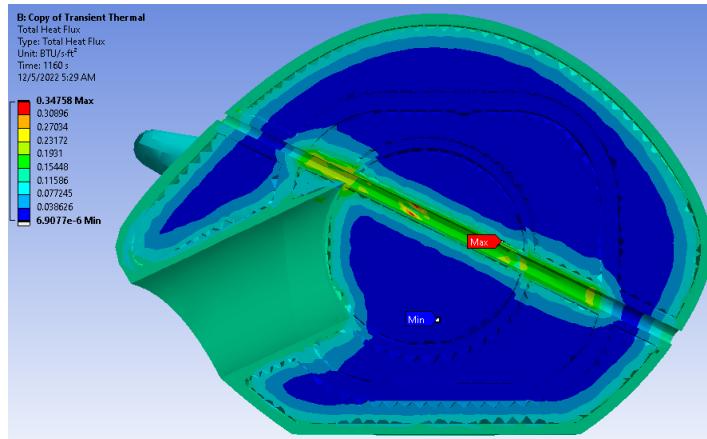
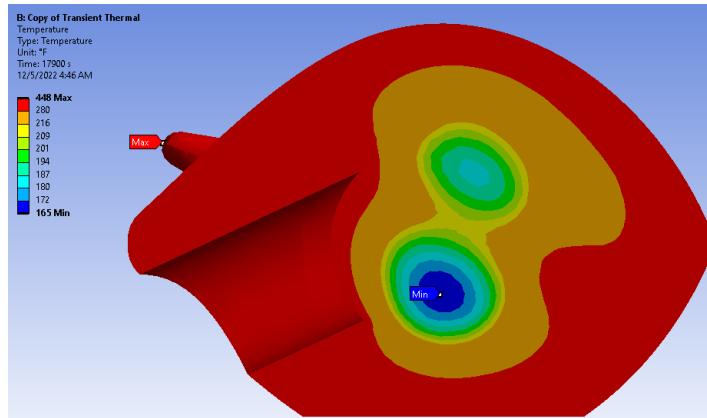
Figure 18: Temperature distribution when $t=0.31\text{hr}$, (first node burnt)

Figure 19: Heat flux when first node is burnt

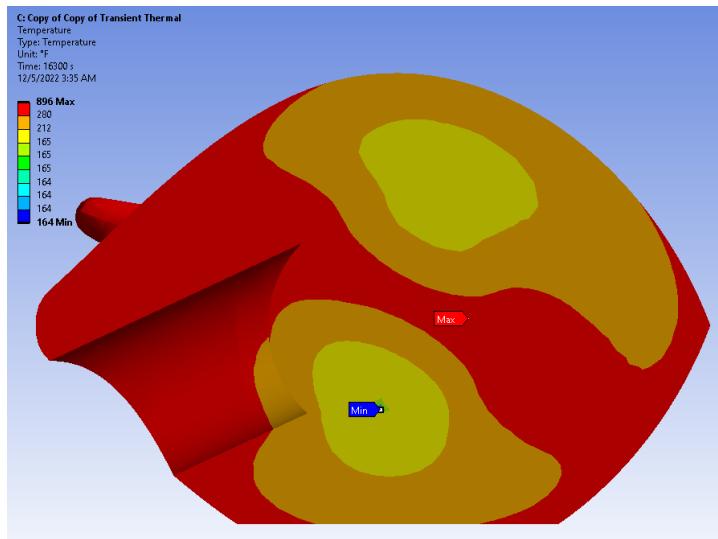
Figure 20: Temperature distribution at $t=4.97\text{hr}$ (fully cooked)

| | |
|---------------|-------|
| #Nodes Burnt | 30004 |
| Total # Nodes | 41234 |
| % Burnt | 73% |

Figure 20: Percentage burnt for this case

From this set of parameters, it seems like there is not enough q_{in} .

Experiment 2: $q_{in}=0.05\text{Btu/s}$, $T=350\text{F}$

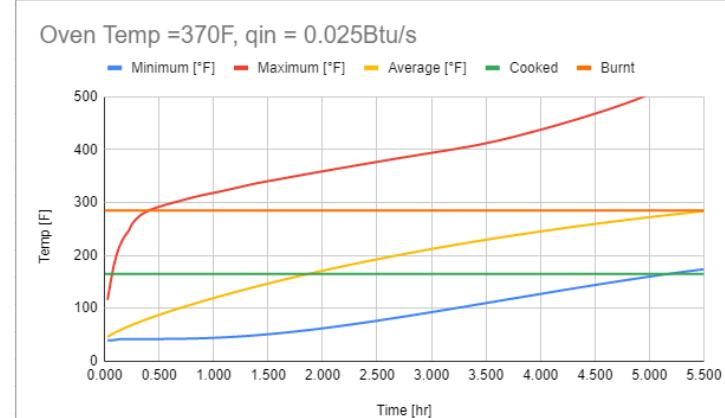
Figure 21: Temperature distribution at $t=4.5\text{hrs}$ (fully cooked)

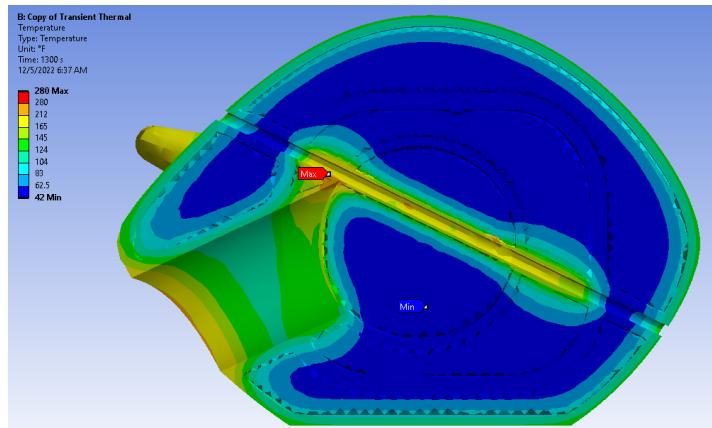
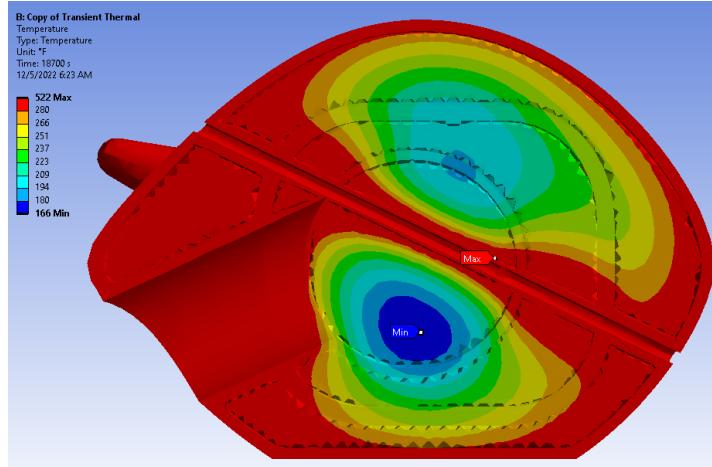
| | |
|------------------|--------|
| # of Nodes burnt | 41222 |
| Total # nodes | 41234 |
| % Burnt | 99.97% |

Figure 22: Percentage burnt for this case

At this point 99.97% of turkey is burnt. We can presume that there is too much energy being supplied by the rods.

Experiment 3: $q_{in} = 0.025$, $T=370\text{F}$

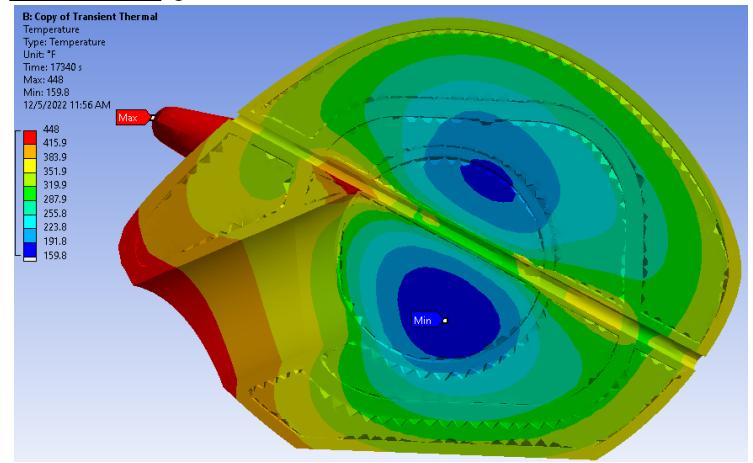
Figure 23: Fully cooked at $t= 5.2\text{hr}$. First burnt at $t= 0.36 \text{ hr}$.

Figure 24: Temperature distribution at $t=0.36$ (first burnt)Figure 25: Temperature distribution at $t=5.2$ hrs (fully cooked)

| | |
|--------------|-------|
| #Nodes Burnt | 21508 |
| Total #Nodes | 41234 |
| %Burnt | 52% |

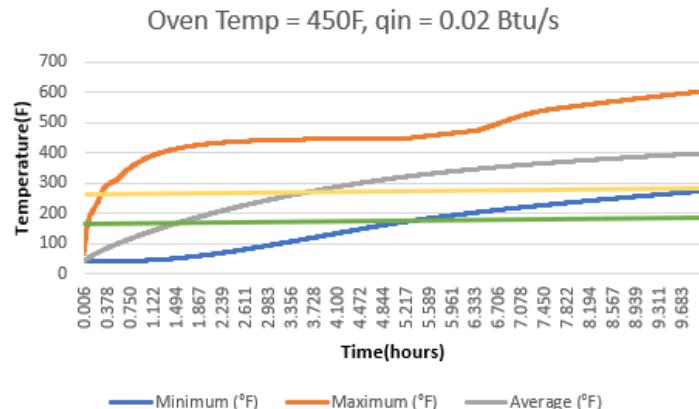
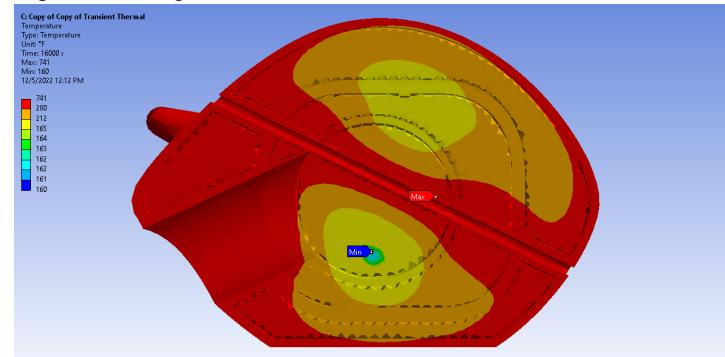
Figure 26: Percentage burnt for this case

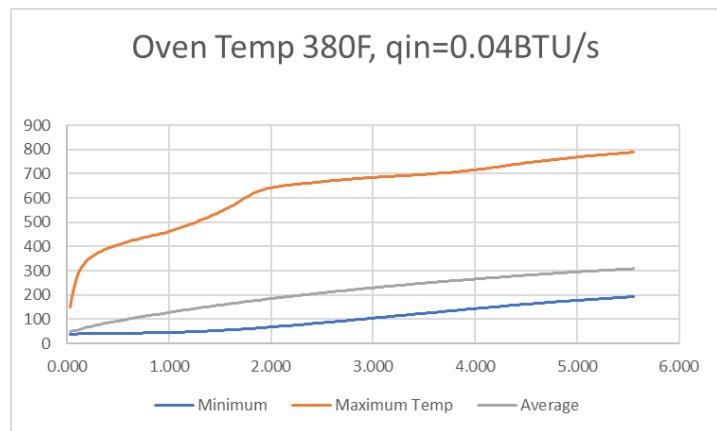
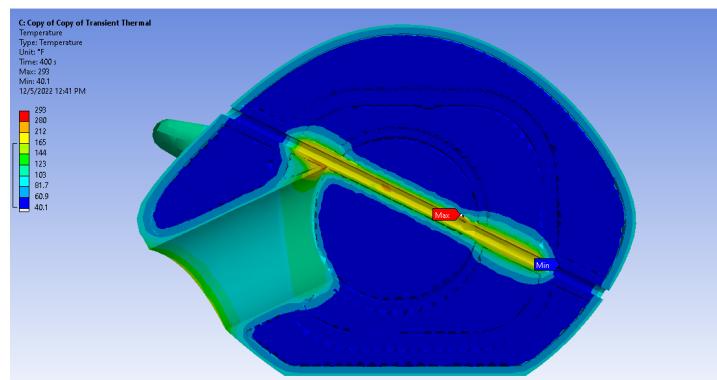
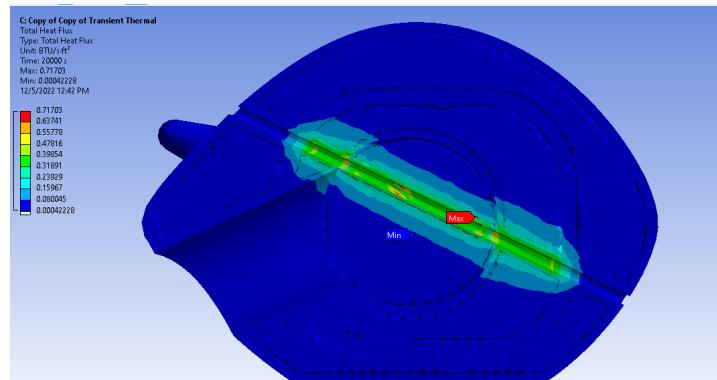
This case only has 52% being burnt, so we tried to change the heat in value in order to see how it affects the turkey.

Experiment 4: $q_{in} = 0.02 \text{ Btu/s}$ 450FFigure 27: Temperature distribution at $t=4.82$ hrs (fully cooked)

| | |
|---------------|-------|
| #Nodes Burnt | 30004 |
| Total # Nodes | 41234 |
| % Burnt | 73% |

Figure 28 : Percentage burnt for this case

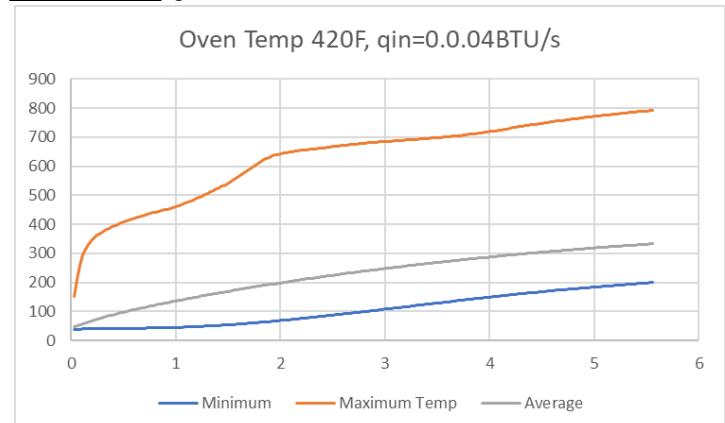
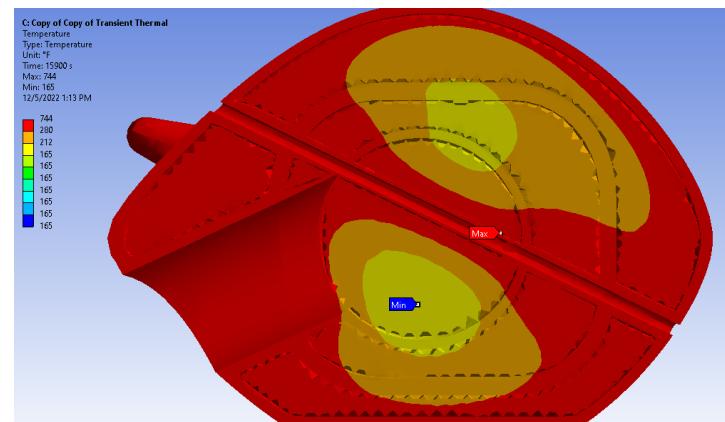
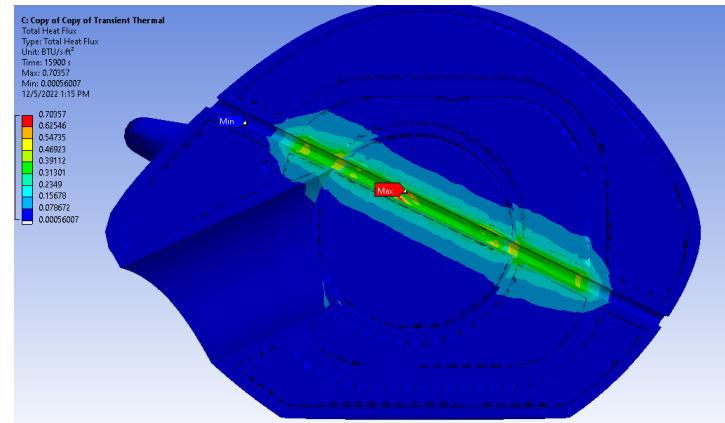
Figure 29: Fully cooked at $t=4.82$ hr. First burnt at $t=0.3$ hr.Experiment 5: $q_{in} = 0.04 \text{ Btu/s}$ 380FFigure 30: Temperature distribution at $t=4.4$ hrs (fully cooked)

Figure 31: Fully cooked at $t=4.4\text{hr}$. First burnt at $t=0.11\text{ hr}$.Figure 32: Temperature distribution at $t=0.11\text{hr}$ (first burnt)Figure 33: Heat flux at $t=0.11\text{hr}$ (first burnt)

| | |
|---------------|-------|
| nodes burnt | 21319 |
| total # nodes | 41235 |
| percent burnt | 52% |

Figure 34: Percentage burnt for this case

This case produced the same results as the previous test, so we decided to test with the same heat flow and different temperatures.

Experiment 6: $q_{in} = 0.04 \text{ Btu/s}$ 420FFigure 35: Fully cooked at $t=4.41\text{hr}$. First burnt at $t=0.11\text{ hr}$.Figure 36: Temperature distribution at $t=4.41\text{hr}$ (fully cooked)Figure 37: Heat flux at $t=4.41\text{hr}$ (fully cooked)

| | |
|---------------|-------|
| #nodes burnt | 27210 |
| total # nodes | 41235 |
| percent burnt | 66% |

Figure 38: Percentage burnt for this case

This case burnt more and we observed a pattern between our 0.04 BTU/s cases regarding the temperature changes. So, finally we decided to test with a lower temperature.

Experiment 7: $q_{in} = 0.04 \text{ Btu/s}$ 350F

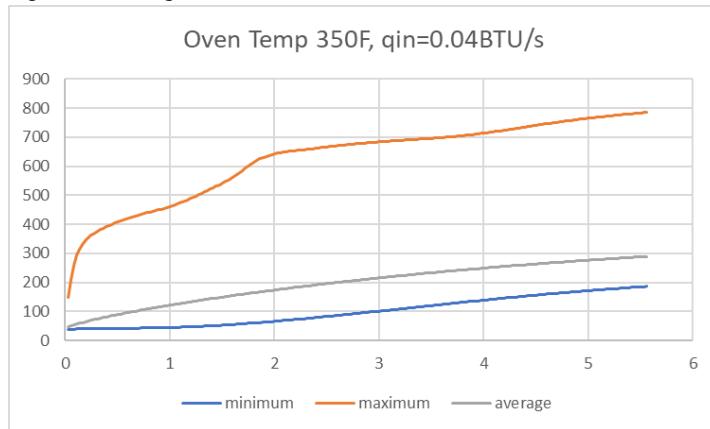


Figure 39: Fully cooked at $t=4.8\text{hr}$. First burnt at $t=0.11\text{ hr}$.

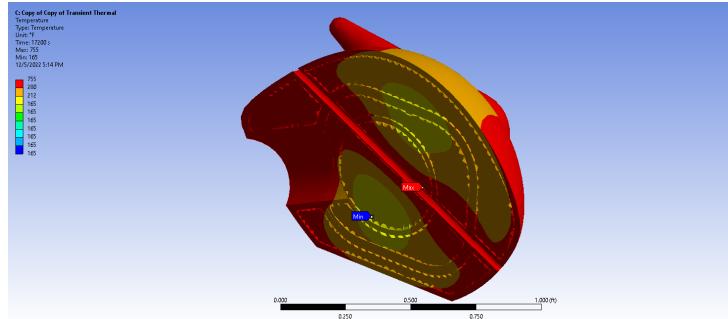


Figure 40: Temperature distribution at $t=4.8\text{hr}$ (fully cooked)

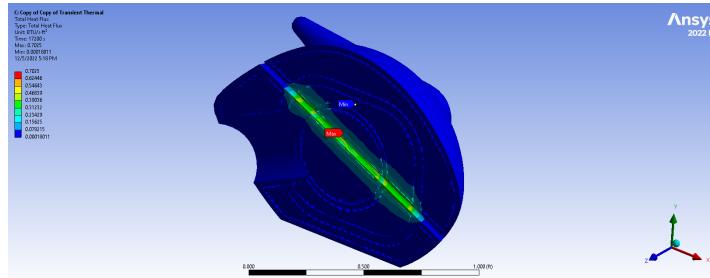


Figure 41: Heat Flux at $t=4.8\text{hr}$ (fully cooked)

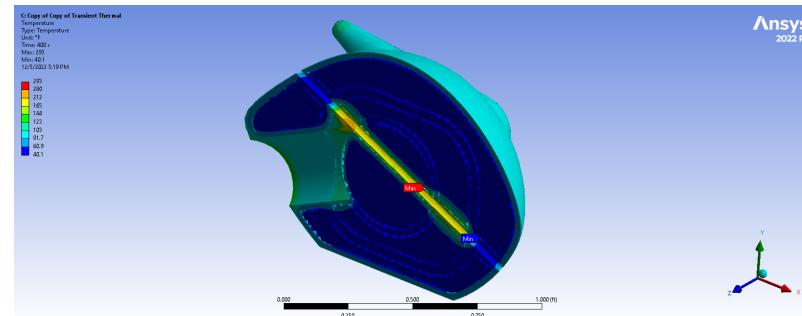


Figure 42: Temperature distribution at $t=0.11\text{hr}$ (first burnt)

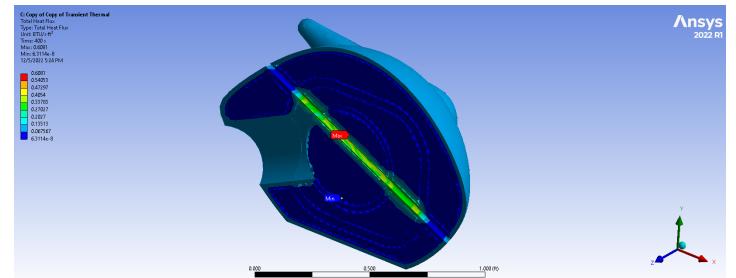


Figure 43: Heat Flux at $t=0.11\text{hr}$ (first burnt)

| | |
|--------------------|-------|
| # nodes burnt | 17051 |
| total number nodes | 41235 |
| percentage burnt | 41% |

Figure 44: Percentage burnt for this case

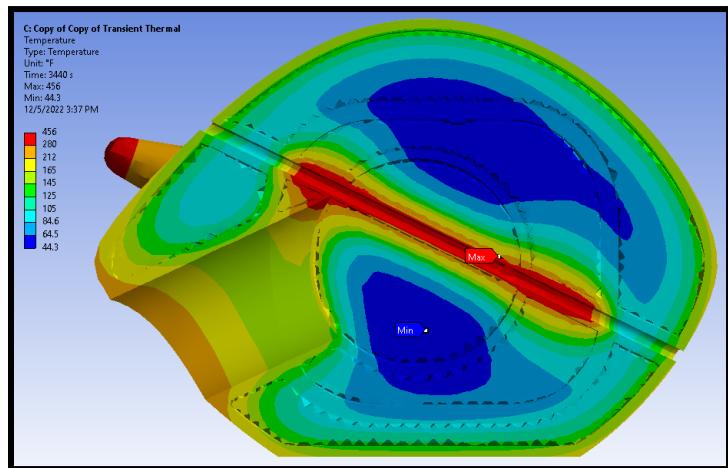
This test case yielded the best data, so this is the experiment we had chosen as our final model.

C. Concise Summary of Test Cases

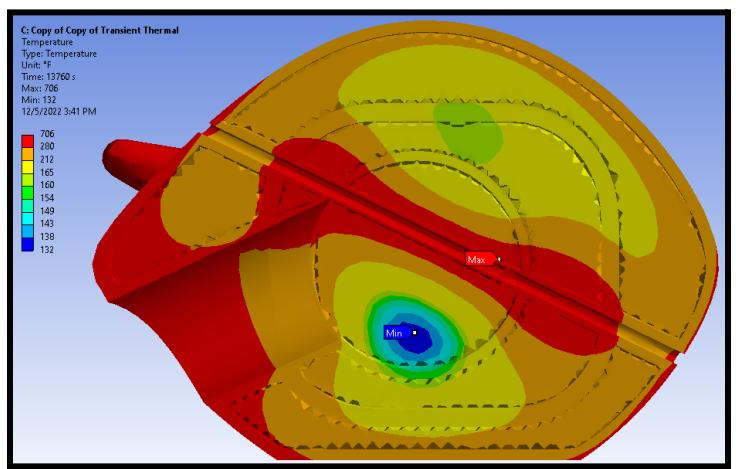
| Ambient Temperature (F) | Qin (BTU/s) | Cooking time (hours) | Percentage burnt (%) |
|-------------------------|-------------|----------------------|----------------------|
| 350 | 0 | 9.3 | 63 |
| 450 | 0.02 | 5.0 | 73 |
| 350 | 0.05 | 4.5 | 99.97 |
| 370 | 0.025 | 5.2 | 52 |
| 450 | 0.02 | 4.8 | 73 |
| 380 | 0.04 | 4.4 | 52 |
| 420 | 0.04 | 4.4 | 66 |
| 350 | 0.04 | 4.8 | 41 |

D. Cooking Timeline

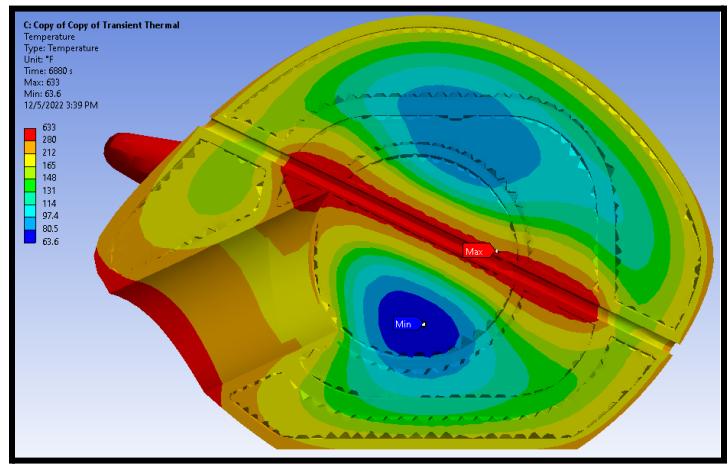
$t=0.96\text{hr}$



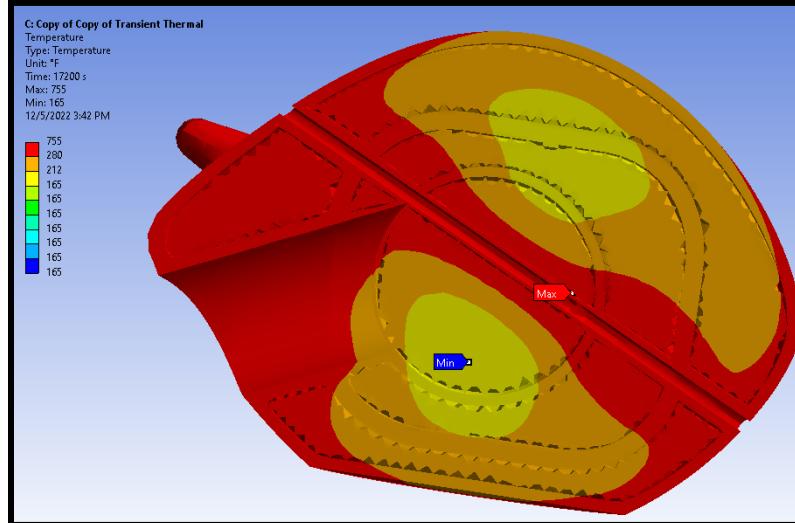
$t=3.8\text{hrs}$



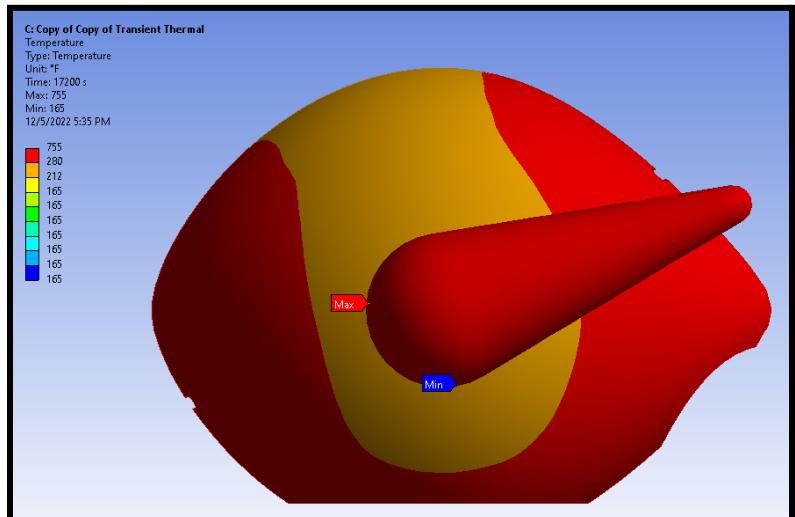
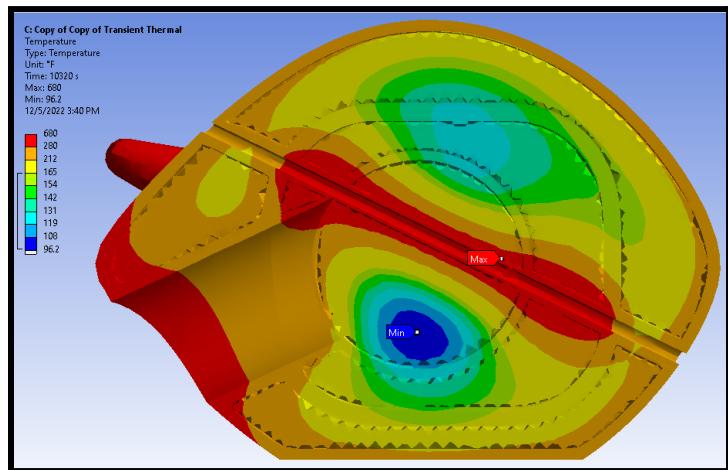
$t=1.91\text{hrs}$



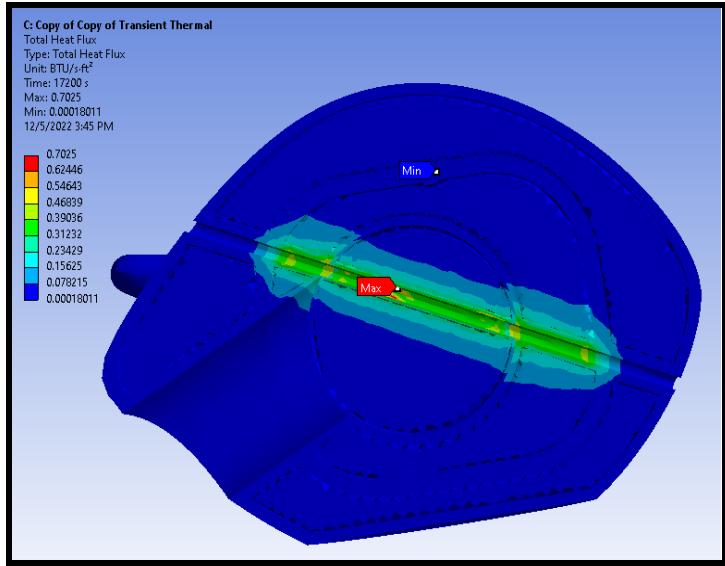
$t=4.8\text{hrs}$ (temp. gradient, below 2 views)



$t=2.9\text{hrs}$



t=4.8hrs (heat flux)



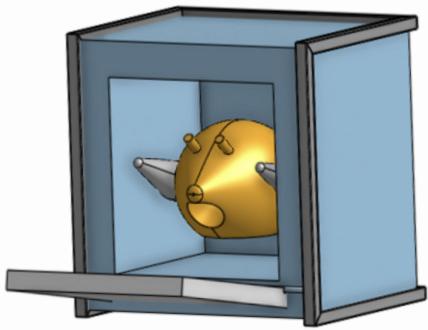
E. Final Cooking Time

| | |
|--------------------|------------------------------|
| Total Cooking time | 4.8 hrs = 4 hours 48 minutes |
| Mass/volume burnt | 41% |

VI. DESIGN TIME ESTIMATE

Time spent designing the model was approximately 300 hours between running simulations, iterating our design , and documenting work. We tested different heat fluxes and oven temperatures in order to find the most optimal values. Given more time, our next steps forward would be to further optimize the position of the insertion of the rods, as well as test rectangular rods. This would require meshing all the components differently for each iteration.

VII. INSTRUCTION MANUAL



Place your turducken as shown in the oven, with the rods placed at 30 degrees through the turkey at the specified location. The turducken is placed on the middle rack of the oven to ensure convection of the bottom of the turducken.

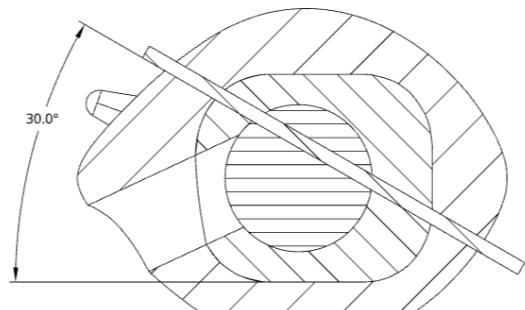
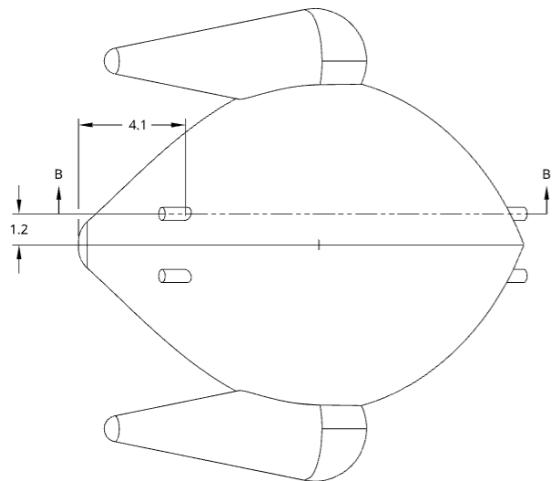
Step 1: Preheat the oven to 350F



Step 2: Fill the cavity with 20% carrot, 60 % potato, and 20% onion. Ensure that the cavity is fully stuffed.



Step 3: Insert rods approximately 120 degrees from the base of the turkey above the opening and ensure they are approximately 3.6 inches apart (center of rod to the other center of rod). Refer to image of the scaled cross-sections.



SECTION B - B

Step 4: Place the turkey on a 17.8inch wide by 17 inch long aluminum tray, on the middle height rack.



Step 5: Turn skewers on and let the turducken cook for 4.8 hours before switching off the oven and skewers.



VIII. APPENDIX

Hand Calculations code:

```

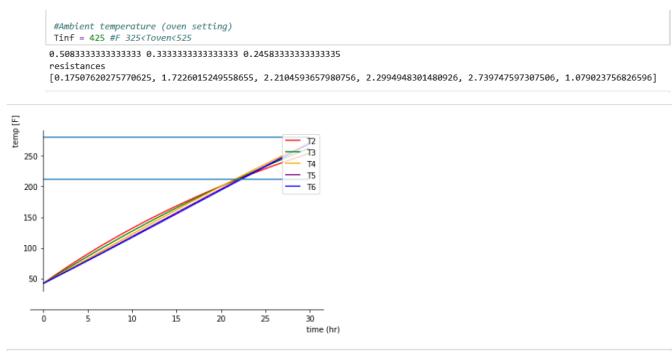
#!/usr/bin/env python
# coding: utf-8
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
m = 3.2808 #ft
#archive
# t = sp.symbols("t")
# x = sp.Function("x")(t)
# y = sp.Function("y")(t)
# eqns = []
# eqns.append(sp.Eq(x, x.diff(t)))
# eqns.append(sp.Eq(x, 2 * y.diff(t, t)))
# sols = sp.dsolve(eqns, ics={x.subs(t, 0): 2})
# display(sols)
# sp.plot(sols[0].rhs, (t, 0, 5))
# display(eqns)
#adding equations ***BROKEN VERS***
# eqns.append(sp.Eq((T1-Tinf)/res[0],(T2-T1)/res[1]))
# eqns.append(sp.Eq(-capT*T2.diff(t)+(T2-T1)/res[1],(T2-T3)/res[2]))
# eqns.append(sp.Eq((T2-T3)/res[2],(T3-T4)/res[3]))
# eqns.append(sp.Eq((T3-T4)/res[3]-capD*T4.diff(t)+qin/2,(T4-T5)/res[4]))
# eqns.append(sp.Eq((T4-T5)/res[4],(T5-T6)/res[5])) #not showing up for some reason
# eqns.append(sp.Eq((T5-T6)/res[5]+qin/2, capS*T6.diff(t)))
# # sols = dsolve_system(eqns, funcs=[T1,T2,T3,T4,T5,T6], t=t, doit=False)
## display(eqns[6].atoms(sp.Function))
# sols = sp.dsolve(tuple(eq for eq in eqns))
# display(sols)
sp.init_printing(use_latex="mathjax")
#Variables
rT_o = 3.224e-1/2 *m #outer radius of turkey
rD_o = 3.192e-1 /np.pi *m #radii of ducken
rs = 2.356e-1/np.pi *m #radii of the stuffing
radii = [rT_o, (rT_o+rD_o)/2, rD_o, (rD_o+rs)/2, rs] #outer radii of each section
kT = 0.2395 #conductivity of turkey
kD = 0.20692 #conductivity of ducken
kS = 0.299998 #conductivity of stuffing
h = 79
resConv = 1/(h*4*np.pi*rT_o**2) #convective resistance
#building the resistances of each section
k = [kT,kT,kD,kD]
res = [resConv]
for i in range(len(radii)-1):
    res.append(1/radii[i+1]-1/radii[i]/(4*np.pi*k[i]))
res.append(1/rs/(4*np.pi*kS)) #resistance of stuffing
print('resistances')
print(res)
densT = 67.85 #density of turkey

```

```

densD = 63.013 #density of ducken
densS = 76.757 #density of stuffing
cT = 0.7293 #specific heat of turkey
cD = 0.6858 #' ducken
cS = 0.8991 #' stuffing
#capacitance
capT = 4/3*np.pi*(rT_o**3-rD_o**3) * densT *cT
capD = 4/3*np.pi*(rD_o**3-rs**3) * densD *cD
capS = 4/3*np.pi*rs**3 * densS *cS
#rod heat in
qin = 0 #Btu
#Ambient temperature (oven setting)
Tinf = 525 #F 325<Toven<525
from sympy.solvers.ode.systems import dsolve_system
t = sp.symbols("t")
T2 = sp.Function("T2")(t)
T4 = sp.Function("T4")(t)
T6 = sp.Function("T6")(t)
eqns = []
#equations
eqns.append(sp.Eq(T2.diff(t),1/capT * ((Tinf-T2)/(res[0]+res[1]) - (T2-T4)/(res[2]+res[3]))))
eqns.append(sp.Eq(T4.diff(t),1/capD*((T2-T4)/(res[2]+res[3]) - (T4-T6)/(res[4]+res[5])+qin/2)))
eqns.append(sp.Eq(T6.diff(t),1/capS*((T4-T6)/(res[4]+res[5])+qin/2)))
sols = sp.dsolve(eqns, ics={T2.subs(t,0):42,T4.subs(t,0):42,T6.subs(t,0):42})
# In[7]:
T2s = sols[0].rhs
T4s = sols[1].args[1]
T6s = sols[2].args[1]
T1s = (Tinf - T2s)*res[0]/(res[0]+res[1])+Tinf
T3s = -(T2s-T4s)*res[2]/(res[2]+res[3])+T2s
T5s = (T4s-T6s)*res[4]/(res[4]+res[5])+T4s
line280 = 280
p = sp.plot(T2s,T3s,T4s,T5s,T6s,line280, (t, 0, 30), show = False, axis_center = [0,0], xlabel = 'time (hr)', ylabel = 'temp [F]', legend = 'True')
p[0].line_color = 'r'
p[0].label = 'T2'
p[1].line_color = 'g'
p[1].label = 'T3'
p[2].line_color = 'orange'
p[2].label = 'T4'
p[3].line_color = 'purple'
p[3].label = 'T5'
p[4].line_color = 'blue'
p[4].label = 'T6'
p[5].label =
p.show()
# sp.plot(T1, (t,0,100))

```



- [4] ENSC 388 engineering thermodynamics and Heat transfer Majid Bahrami - Teaching - ENSC 388. Available at: <https://www.sfu.ca/~mbahrami/ensc388.html>.
- [5] FINLAYSON, B.R.U.C.E.A. and OLSON, J.O.H.N.W. (1987) "Heat transfer to spheres at low to intermediate Reynolds Numbers," *Chemical Engineering Communications*, 58(1-6), pp. 431–447. Available at: <https://doi.org/10.1080/00986448708911980>.
- [6] Mechanical APDL Element Reference (2017) Mechanical APDL element reference. Available at: https://www.mmm.bme.hu/~gyebro/files/ans_help_v182/ans_elem/Hlp_E_ElemTOC.html.
- [7] UQ Espace - University of Queensland (no date). Available at: <https://www.espace.library.uq.edu.au/>
- [8] 1998 Ashrae handbook: Refrigeration (1998). Atlanta, GA: American Society of Heating, Refrigeration and Air-Conditioning Engineers.

REFERENCES

- [1] Air - dynamic and kinematic viscosity (no date) Engineering ToolBox. Available at: https://www.engineeringtoolbox.com/air-absolute-kinematic-viscosity-d_601.html.
- [2] CARDARELLI, F.R.A.N.O.J.S. (2019) Materials handbook: A concise desktop reference. SPRINGER.
- [3] Dry air - thermodynamic and physical properties (no date) Engineering ToolBox. Available at: https://www.engineeringtoolbox.com/dry-air-properties-d_973.html.