

The Effect of Temperature on the Flame's Velocity and Thrust for an Isopropyl Bottle Rocket

Determination of flame velocity and thrust of an isopropyl rocket with varying temperature.

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Table of Contents

Abstract	1
Executive Summary	2
Introduction	2
Experimental	3
Equipment	4
Apparatus	5
Instrumentation	5
Procedure	6
Safety	8
Discussion	8
Recommendations	12
References	13
Appendices	13
Notes	17

Abstract

The whoosh bottle can be a valuable experiment given the multitude of fundamental principles that it demonstrates. An experimental setup to improve the quantitative analysis and controllability of the demonstration is proposed. A glass bottle with isopropyl vapor is heated to specific temperatures with a sous-vide till steady state before being ignited in a fume hood to observe the temperature's effect on flame velocity and thrust generated. To get the thrust generated, displacement data from videos captured using an iPhone's camera application is analyzed using Newton's Second Law. Impulse can also be attained in a similar manner. The data collected from the capped and uncapped tests conducted help

to show a direct dependence between temperature and flame speed, thrust, and impulse. Thus, the stand created proves to be a viable method of analyzing the effect of temperature as well captures all the qualities in a stand required from by the sponsoring client.

Executive Summary

This report explores the scientific and educational potential of the whoosh bottle demonstration, which involves igniting a bottle filled with isopropyl vapor and observing the resulting flame and thrust. The client is a Cooper Union professor, more specifically of the Mechanical Engineering department that specializes in classes relating to force characterization and fluid/rocket theory. The client's motivation is to create a setup that is safer, more repeatable, and more useful in terms of data collection than how whoosh bottle experiments are typically carried out.

Research questions

The primary research questions considered were: 1) What is the effect of temperature variation on the flame velocity and thrust generated by the whoosh bottle experiment? 2) Can a seesaw-type setup provide safer and more controllable measurements of these parameters? 3) What is the effect of nozzle diameter on flame velocity?

To address these questions, a setup was designed that consists of a see-saw setup, where the glass bottle is placed on one end and the thrust produced from ignition pushes down the plank of wood causing movement. The recordings of the wooden plank and of the flame allow for the exploration of how temperature affects the flame speed and thrust generated by the whoosh bottle with the use of software and mathematical models. Due to the fumes produced from the combustion, the experiment will take place in the fume hood.

Recommendations

Based on the key findings, the following are recommended actions:

1. Conduct further investigation with a different setup consisting of strain gauges or sensors that are attached directly to the beam to remove the need for cameras.
2. Vary cap sizes further to find the optimal maximum diameter size as only the minimum diameter of 0.25 in was determined experimentally and theoretically.

Introduction

The whoosh bottle class activity is a valuable pedagogical tool that demonstrates fundamental engineering principles such as combustion kinetics, stoichiometry, and heat transfer. However, the experiment's qualitative nature limits its scientific and educational value, while also being hazardous. To address these limitations, this paper proposes an experimental setup that includes measuring the thrust and flame velocity at varying temperatures, enabling a more controllable, safe, and quantitative analysis of the experiment's results.

The experiment is fueled by the combustion of isopropyl alcohol vapor, which is ignited and reacts with oxygen in the air to produce carbon dioxide and water vapor. This exothermic reaction produces a large

amount of heat and gas that causes the temperature within the bottle to increase such that pressure builds up within the bottle, as predicted by the Ideal Gas Law. As the pressure builds, it creates a high-velocity jet of hot gases that is expelled from the bottle's neck, producing thrust due to the pressure difference between the inside and outside of the bottle. This thrust is similar in nature to that of a rocket engine, where because of the conservation of momentum, the action of the expulsion of hot gases has an equal and opposite reaction force of the thrust. The strength of this thrust is dependent on several factors, such as the outlet diameter. By iterating the size of the exit diameter, it in essence acts like a nozzle which accelerates the working fluid to a high exit velocity jet, giving it a propulsive force known as the thrust (Sidebotham, 2022) [1].

The proposed experimental setup involves heating the glass bottle containing isopropyl alcohol to specific temperatures before igniting it to produce the combustion reaction. This thermal treatment alters the vapor's density, vapor richness, and temperature, which can affect the reaction rate and the amount of heat released. By measuring the thrust and flame velocity at varying temperatures, the proposed experimental setup enables a more quantitative analysis of the effect of thermal treatment on the combustion process and thrust generated.

In addition to the theoretical understanding that glass possesses a higher thermal mass compared to plastic, an empirical analysis was conducted to evaluate the retention of heat in these materials over time. As anticipated, the time constant for glass was found to be in the range of minutes, while for plastic, it was significantly shorter, spanning only a few seconds. This allows for moving the rocket to the vent hood without drastically altering the temperature of the vapors. Furthermore, the glass bottle is less likely to experience deformation due to its higher melting point and greater thermal stability, making it a more sustainable choice compared to using multiple bottles that could be compromised by heat exposure. Most notably, the transparency of glass, as opposed to plastic, plays a significant role in the experiment. Owing to its smooth and clear surface, glass allows for accurate tracking of the flame by a camera throughout the combustion process.

Experimental

The testing protocol developed to measure the flame velocity and thrust of an isopropyl glass bottle rocket is described in this section. The experimental procedure calls for a glass bottle with a specified amount of isopropanol alcohol to be heated uniformly by a sous-vide, as outlined in Figure 1. Once the heating of isopropyl vapors reaches a steady state, the glass bottle is quickly placed in the cupholder on the beam situated within the fume hood. Depending on the specific experiment, the bottle is either fully uncapped or the opening is exposed to a flame. Two cameras positioned level with the bottle and the beam end, initiate recording just before the combustion event. The video recording is subsequently imported into the Tracker Software for post-processing, with analysis focused solely on the impulse event. The tested opening sizes were 20 mm and 6.35 mm, while the temperature range of interest spanned from 20°C to 45°C.



Figure 1: Water treatment of Glass bottle in bucket with sous-vide and weight to keep it vertical.

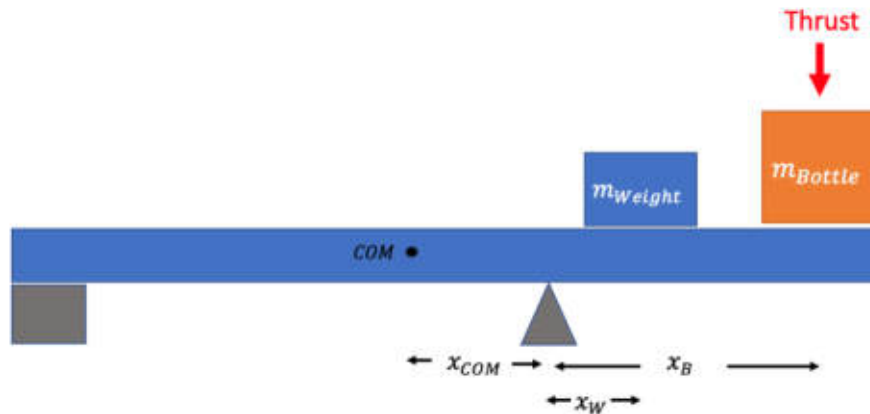


Figure 2: A simple schematic of the beam setup outlining the relative positions of the bottle and the added mass.

Equipment

A schematic diagram of the test stand is shown in Figure 3, which consists of the apparatus needed to constrain the glass bottle and the beam for motion in the vertical direction, as well as the instrumentation required to record the relative position of the bottle and beam for the duration of the event. All the components within these two are to be discussed separately in the sections to come, with their respective measurement errors and specifications, with the final objective of thrust data for varying temperatures.

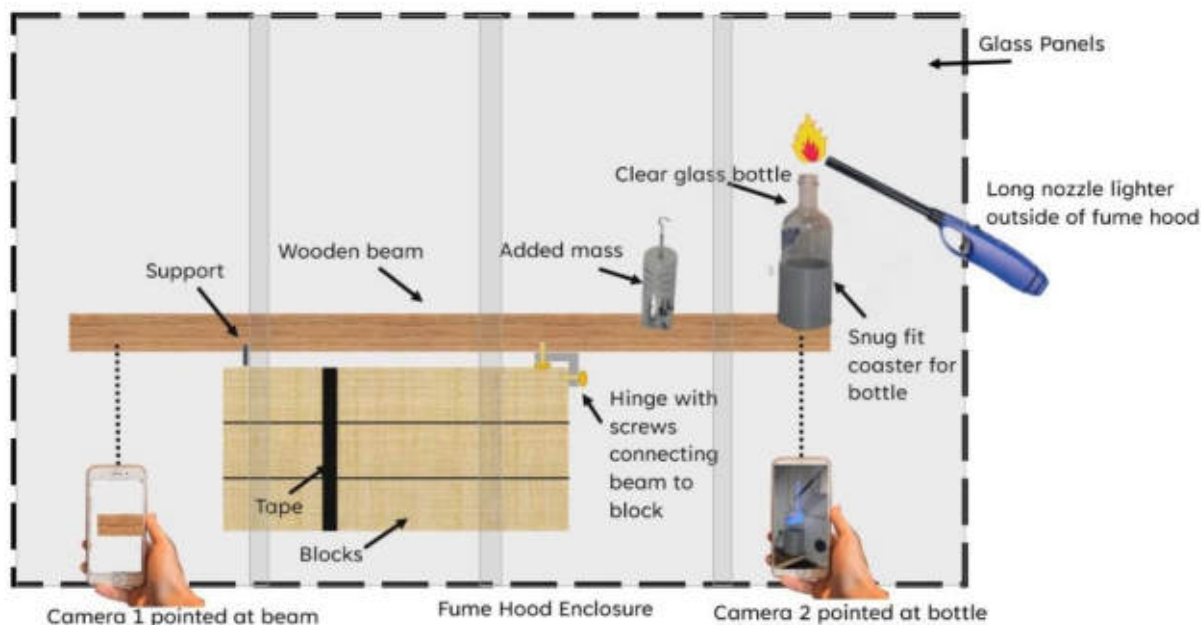


Figure 3: Schematic of Hinged beam setup for ignition, all within the fume hood.

Apparatus

The isopropanol rocket experiment apparatus is designed to safely measure flame velocity and thrust using a seesaw like setup. A wooden plank, nails, and a hinge form the structure, while safety glasses protect the experimenter. The Absolut Vodka Bottle, acting as a combustion chamber, is prepared with a drill for the various outlet diameters, held in place by a 3-D printed holder, and filled with isopropyl alcohol. A sous-vide is used to uniformly heat the glass bottle before placing it in the fume hood. A fume hood contains any hazardous fumes released from the combustion process. This apparatus ensures a controlled environment to study the isopropanol rocket's behavior. An individual experiment is initiated by igniting the isopropyl alcohol.

Instrumentation

The temperature of the water is measured using a hand-held infrared thermometer, namely Black & Decker's Thermal Leak Detector Model TLD100 to verify that the sous-vide had reached the desired temperature. Traditionally, this consumer-grade thermometer is designed primarily for temperature differences across larger areas to detect leaks. However, in the experiment, the focus was on localized temperature measurements at short distances so the spatial resolution may not be ideal. Additionally, a factor that may affect the accuracy of the measurements is the "point and shoot" functionality, which requires careful alignment of the IR beam to ensure it targets the water and avoids any potential interference from ice cubes or other objects in the bucket. The sensor's readability is $\pm 1.0^{\circ}\text{C}$, with a manufacturer-stated uncertainty of 2.2°C at 100°C and 0.3°C at 23°C .

A dial thermometer was used to further verify the temperature measurements, although it exhibited a time delay due to its non-instantaneous nature. This thermometer had a readability of 0.5°C and a manufacturer-reported uncertainty of $\pm 1.0^{\circ}\text{C}$. In total, three different instruments were used to determine the desired temperature in the bucket during the thermal treatment stage.

Lastly, when iterating the width of the bottle cap openings, a standard, analog vernier caliper (Mitutoyo Analog Vernier Caliper Series 530) was used that has graduations of 0.001 in, with a readability of 1/128 in and manufacturer reported uncertainty of $\pm 0.5/128$ in (up to 6 in).

Procedure

Safety glasses were worn, and the fume hood was cleared to ensure clear visibility during the experiment. The beam and hinge setup were placed in the fume hood, leveled with a leveler, and checked for adequate room for the bottle's rotation. Weights were attached to adjust for the offset of the hinge from the center of mass of the wooden plank to ensure that the beam was at the tipping point. Figure 2 can be used to calculate the theoretical center of mass. Two pieces of electrical tape, each about an inch long, were applied to a clear part of the glass bottle, three inches apart for tracking the flame's velocity. A bucket taller than the height of the bottle was filled with water to be heated. The sous-vide was plugged in, set to the desired temperature, and attached to the side of the bucket. A dial thermometer was attached to the side of the bucket using tape to further corroborate the sous-vide's temperature. When the sous-vide reached the target temperature that is indicated by a green light, 2.5 ml of 91% isopropyl alcohol was added to the bottle with a pipette, and the bottle was rotated to coat its inner surface. The bottle was submerged in the heated water, held down with a weight, and left to reach steady state for 15 minutes.

Upon completion of the thermal treatment, one person was stationed to record the flame speed, another to record the displacement of the beam, all while the third person carefully removed the bottle from the bucket and lightly dried any water from the surface of the bottle. The bottle was then swiftly placed in the 3-D printed bottle holder, and if needed, the weights were adjusted (only required for first run). The cap was removed, and the lighter was used to ignite the isopropyl alcohol. Following the test, the fume hood door was closed to allow any hazardous fumes to escape. A separate bucket with water, a beaker, and a funnel were brought to the fume hood to flush vapors from the bottle. The bottle was filled to the brim with water, which was allowed to overflow. The sous-vide temperature was adjusted for the next trial while another person emptied and dried the bottle using paper towels before repeating the steps of the run. For the capped experiments, a 0.25-inch hole was drilled into the cap and sealed with electrical tape. To ignite the contents, only the tape covering the hole was peeled back.

Results

The plot in Figure 4 shows the data taken from the capped ($D = 6.35$) and uncapped ($D = 20$ mm) experiments conducted on the final isopropyl rocket test stand. Results from both of the tests were combined into one scatter plot to help reveal any basic trends. Each data point represents a test conducted with 2.5 ml of 91% isopropyl alcohol and a bottle heated in a sous vide controlled water bath for 10-15 minutes.

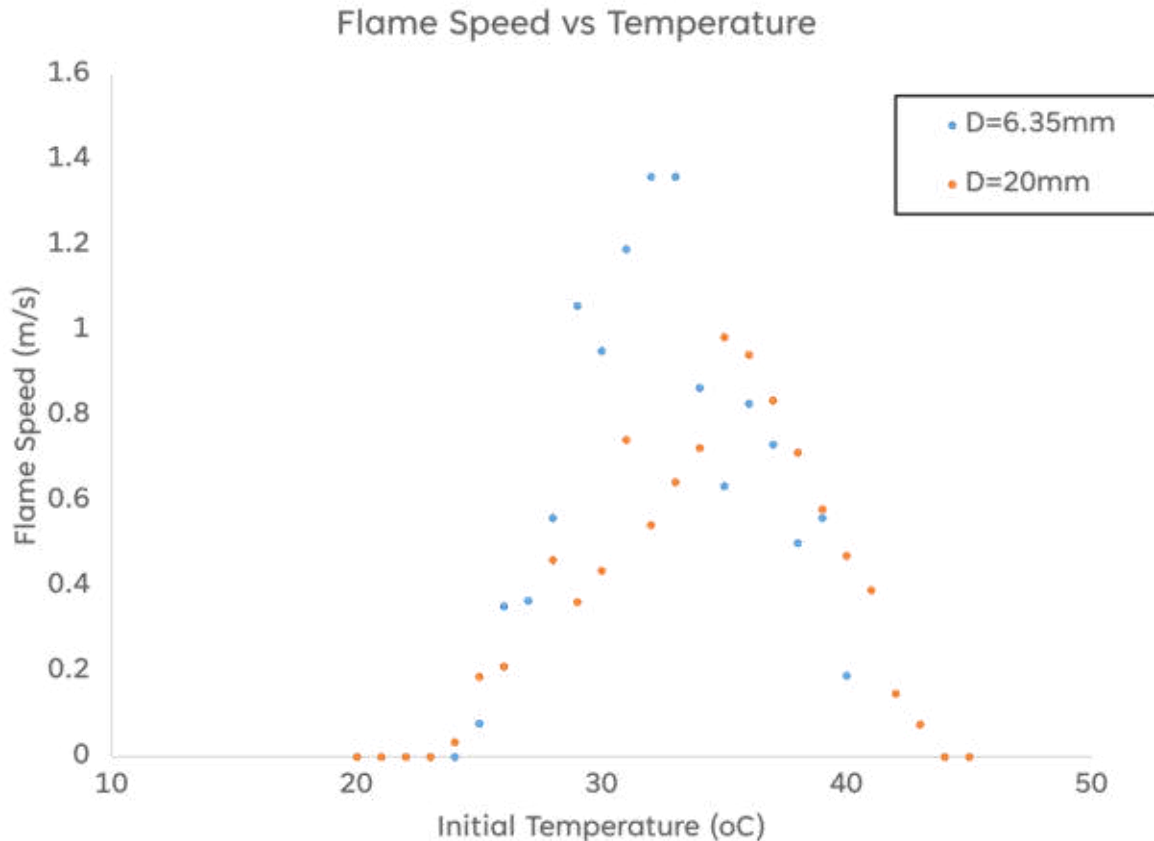


Figure 4: Flame Speed vs Temperature Plot for Uncapped and Capped Tests

From an initial inspection of the plot, the two experiments seem to have similar peaks around an initial bottle temperature of 33°C to 35°C. The two results also show similar flame propagation ranges. For the uncapped tests, a propagation temperature range of 24°C to 43°C was noticed, while the capped test had a range of 25°C to 40°C. Data for the tests conducted reveals that the flame speed typically increased with temperature up until it reached the peak around 33°C. After, the speed of the flame would decrease until it reached the upper temperature bound of the propagation range. It should be noted that outside of the temperature range, the combustion process produced a flame but did not last long enough for any data to be recorded.

Although the two data sets had a similar concave curvilinear shape, the data for the capped experiments performed seem to have higher flame speeds throughout. The peak of these tests also reached a max flame speed of 1.36 m/s at an initial bottle temperature of 33°C. This was not the case with the uncapped tests, which only had a max flame speed of 0.98 m/s at 35°C. It should be noted that the scatter plot data attained matches closely to data collected by previous groups who also performed a similar experiment. More tests should be conducted to be certain of the concave curvilinear shape seen and for a trendline to be applied. An error bar plot was also created to help see the uncertainty in the reported results (See Figure 5). The temperature uncertainty is $\pm 0.1^\circ\text{C}$ while the flame speed is ± 0.027 m/s (See Appendix 3 for the flame speed propagation of error calculation).

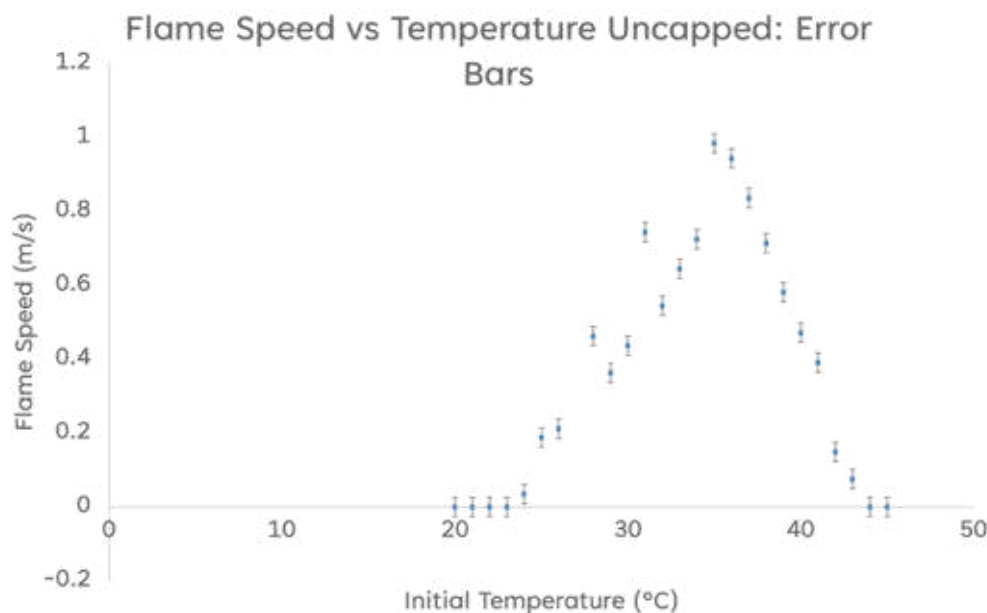


Figure 5: Error Bar Plot for Flame Speed vs Temperature of Capped Tests

Safety

In this experiment, flammable materials and hazardous gases are involved, requiring more thought and attentiveness. First, maintain a clean workspace and ensure clear visibility of all events. Keep the fume hood free of flammable items and operational throughout the experiment. Before igniting the fumes, close the fume hood as much as possible to minimize the release of gases. After ignition, properly flush out any hazardous fumes produced during combustion with the water and funnel method outlined in the Procedure. Take care to remove any excess alcohol on the exterior of the bottle and thoroughly clean it prior to ignition. Always wear safety goggles due to the presence of fire. When using the sous-vide, ensure all electrical cables and components are dry to prevent electrical shocks and fires. Finally, avoid placing the heated sous-vide in direct contact with the bucket's bottom to prevent melting the plastic and contaminating the water.

Discussion

In this study, the effect of temperature and varying cap sizes on flame velocity and thrust was investigated using a fulcrum-based setup, with a glass bottle as the primary vessel for the combustion process. The amount of alcohol used in each trial was determined based on calculations of the equivalence ratio of vapor and the volume of liquid added. This allowed for consistent conditions with each trial, ensuring that the observed effects were solely due to the manipulated variables.

The collected data provided valuable insights into the relationships between temperature, cap size, flame velocity, and thrust. However, there were several sources of error that could have impacted the accuracy of the findings. One potential source of error was the residual water inside the bottle between trials, which may have resulted in an alcohol-water mixture, altering the combustion process. To improve this, more time would be allocated to drying either naturally or with forced convection like a hair dryer.

Another potential source of error was the inconsistency in coating the bottle's inner surface area with alcohol. This could have led to varying fume distribution, impacting the combustion process and the resulting flame velocity. Additionally, the limitations of the equipment, such as the cameras, Tracker software, and the sous-vide, could have introduced inaccuracies in our measurements. For instance, the flame speed and Y-displacement of the fulcrum might not have been perfectly analyzed due to technological constraints. Thus, a camera with more frames per second and software with greater processing capability should be used in the future.

During the course of the experiment, there were instances when the isopropyl alcohol container was inadvertently left open for a period of time. This oversight could have led to inconsistencies in the experimental results. When isopropyl alcohol is exposed to the air, it evaporates more rapidly than water due to its higher volatility. This results in a decrease in the alcohol-to-water ratio within the remaining solution, which in turn affects the combustion properties of the isopropyl alcohol solution, such as flame speed and the amount of energy released. To maintain consistent experimental conditions and ensure the reliability of the results, it is crucial to keep the isopropyl alcohol container closed when not in use, thereby preserving a stable alcohol-to-water ratio throughout the experiment. (Brown et al., 2018)

Lastly, the sous-vide's temperature accuracy may have been an issue, as thermometer readings were off by ± 0.2 degrees Celsius at most. These minor temperature differences could have affected flame speeds and alcohol fume distributions, ultimately influencing the results.

After comparing the experimental data collected using the final isopropyl rocket test stand, it was noticed that capping the bottle had a direct effect on flame speed. The combustion process of the capped tests produced a flame that was slightly faster. This result was expected since the addition of a nozzle to the bottle would mean there would be an increase in the kinetic energy of the fluid as it passes through it. Despite getting data that seemed reliable and similar to data from previous groups, the experiment should be conducted more times. The thrust values of the each one of the tests was also calculated (See Appendix 1 for calculation). A general equation for the thrust produced by the rocket is shown in Equation 1.

$$F_{thrust} = \frac{I_{eff}\omega}{x_B\Delta t}$$

Equation 1: Thrust produced by Isopropyl Rocket

The equation consists of the effective moment of inertia of the stand (I_{eff}), the angular velocity of the beam (ω), the time of the event (Δt), and the bottle's distance from the fulcrum (x_B). Along with finding thrust, the impulse of the reaction was also calculated by multiplying the calculated force by the time of the event. The event time mentioned refers to the time it takes for the bottle to move from its equilibrium position to the ground. The values for thrust, impulse, and temperature for each of the tests were then plotted and shown in Figures 6-9. Despite all experiments being conducted in the rocket stand, thrust was not observed in every one of the tests. It is believed that not enough isopropyl 91% alcohol was added to the bottle to see thrust in all of the tests. If more tests are to be conducted with a greater amount of alcohol, more safety measures should be put in place to minimize the risks associated from the increased combustion process.

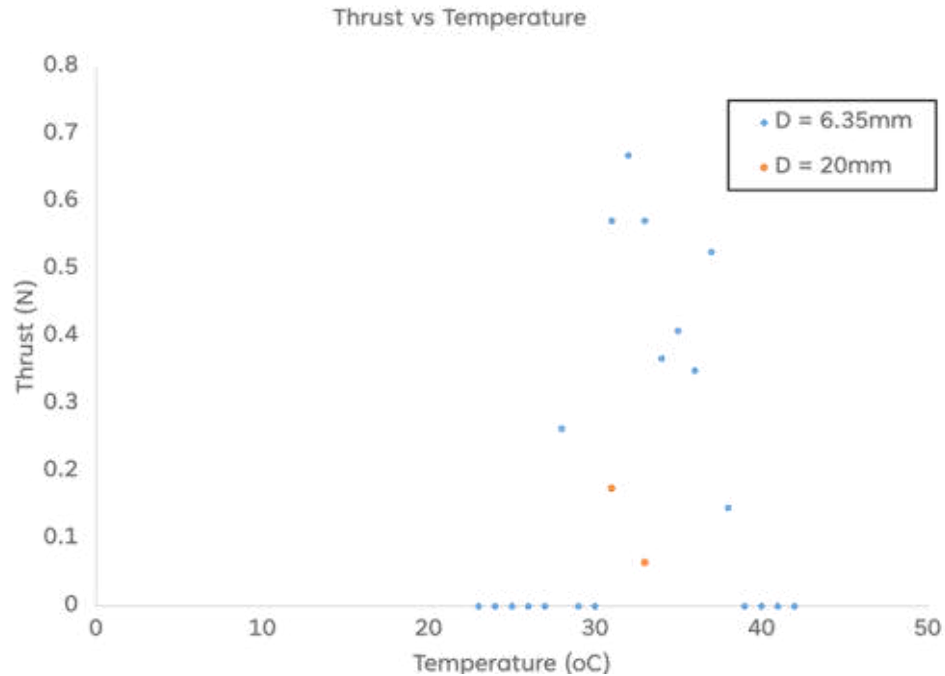


Figure 6: Thrust vs Temperature Plot for Uncapped and Capped Tests

The thrust versus temperature graph presented in Figure 6 shows that at the temperature range of 30°C to 35°C thrust was at its maximum. For the capped experiment the greatest thrust value seen was 0.67 N. Despite not getting many data points for uncapped tests, there was a greater success in achieving thrust for the capped experiments conducted due to the accelerated fluid. Thus, the thrust results shows some dependence on flame speed. A similar observation can be made for the impulse versus temperature plot shown in Figure 7.

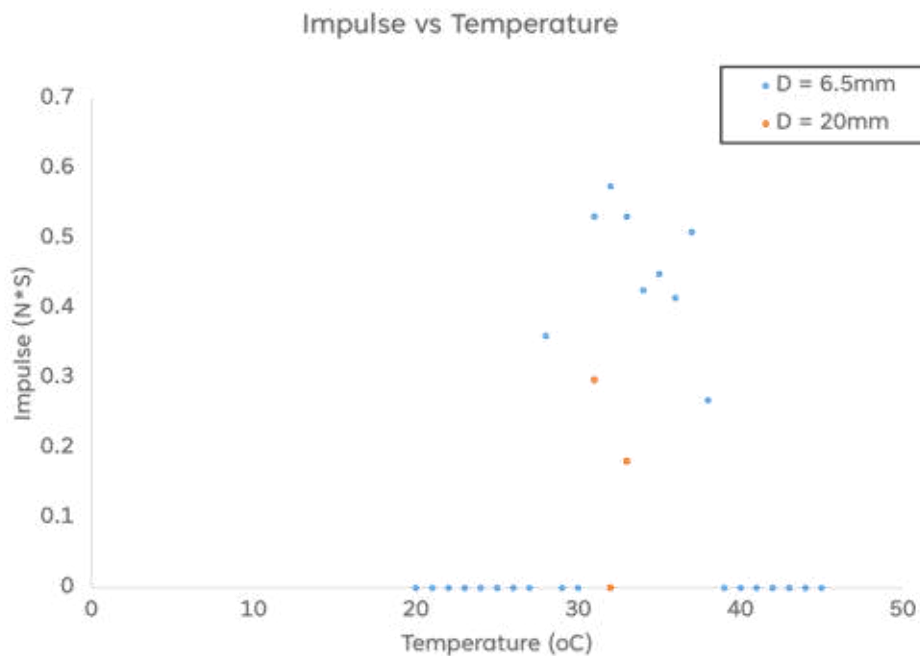


Figure 7: Impulse vs Temperature Plot for Uncapped and Capped Tests

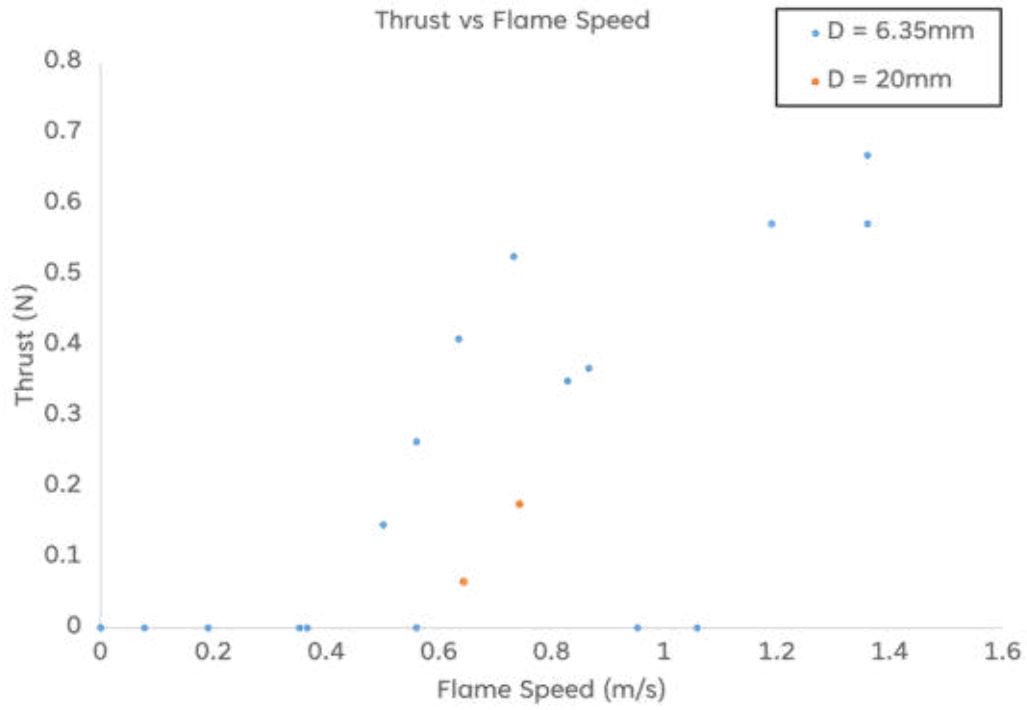


Figure 8: Thrust vs Flame Speed Plot for Uncapped and Capped Tests

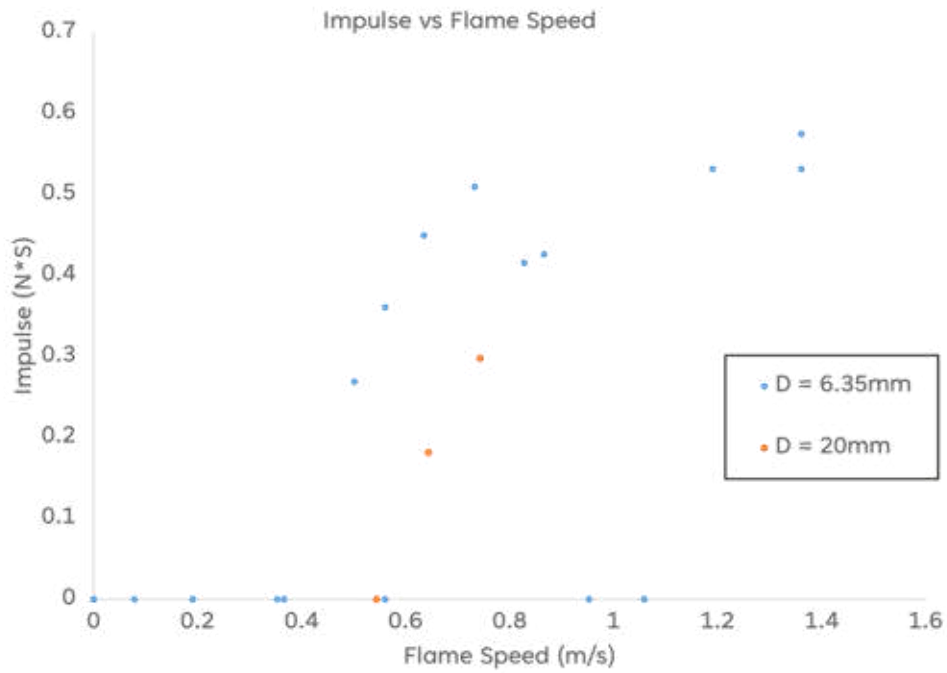


Figure 9: Impulse vs Flame Speed Plot for Uncapped and Capped Tests

The plots for thrust versus flame speed and impulse versus flame speed shown in Figures 8 and 9 revealed that impulse and thrust increase as flame speed increase. They increased rapidly between 0.4 to 0.8 m/s and then the thrust and impulse values began to plateau until it reached a temperature where the event no longer occurred. From inspection, it can be seen that the data in the plots has more of a logarithmic trend. Although, to be completely certain more tests should be conducted so as to fully establish the shape and for a trendline to be applied.

To better validate some of our assumptions and to create data plots with more reliable trends, more tests should be conducted. It should also be noted that the impulse value calculated was done with a more generalized approach and does not really dictate the exact impulse of the combustion process. To improve upon this calculation, an accelerometer can be introduced into the setup to find the exact angular acceleration of the event. Also, the real Δt of the impulse which pertains to the duration of time the thrust is affecting the beam can be found with the added piece of instrumentation. For a greater understanding of how the outer diameter affects the flame speed and thrust of the reaction, tests with a greater range of nozzle diameters should be conducted. Despite all the aforementioned errors and areas of improvement, there were still correlations between the specific test cases. Overall, the isopropyl rocket tests and trends were consistent with our predictions and the data from previous experiments. The final test stand also satisfied the requirements set by the client which were that it be safe, repeatable, and useful in terms of data collection.

Conclusion

The devised test stand allowed for the effect of temperature on flame speed, thrust, and impulse produced by the combustion process of an Isopropyl rocket to be analyzed. From the results collected for the capped (6.35 mm) and uncapped (20 mm) experiments, it was determined that as temperature increases, the flame speed also increases with it until it reaches a temperature where it peaks. Following the peak, the flame speed decreases gradually until it no longer produces a flame. The same plot showed that the capped tests had higher flame speeds. A concave curvilinear shape was noticed to represent the trend of results of the flame speed versus temperature plot. Thrust and impulse were then compared to temperature. Although, nothing concrete was attained from the comparison besides the fact that the capped experiments produced more thrust and impulse. The same comparison was then carried out with flame speed which showed that thrust and impulse are dependent on it. The plot showed logarithmic trend between the two. The results from the experiments carried matched those of our intuition as well the ones attained from groups from previous year. Despite getting data which seems reliable, further tests should be carried out to improve the shape of the plots and for further analysis to be conducted. Overall, the test stand produced helped to see the effect of temperature on flame speed, thrust, and impulse as well as satisfied the requirement of being safe, repeatable and, and useful for data collection.

Recommendations

Based on the results and analysis of this experiment, the following recommendations are made for future work:

First, the bottle should be immersed in water for at least 15 minutes after the sous vide device indicates the desired temperature, as glass has a high thermal mass and may take longer to heat up. Second, the

experiment should be performed quickly after taking the bottle out of the water to minimize changes in temperature.

Additionally, the water temperature should be verified with a thermometer to account for any error or variation in the sous-vide's reading.

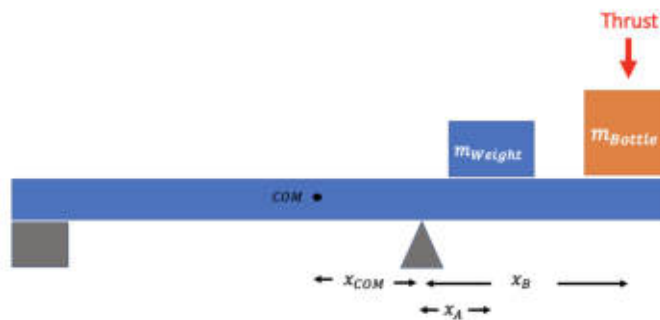
Lastly, the method and amount of alcohol used for each trial should be standardized, but it would be resourceful to empirically seek out the minimum volume of isopropyl alcohol required without causing a change.

References

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Appendices

Appendix 1: Calculation of Position of Added Mass and Thrust Produced by Isopropyl Rocket



Equations of Motion of Test Stand

$$\sum M_{Beam} = I_{Beam} \alpha = -N_B(x_B) - N_A(x_A) + M_{Beam} g x_{COM}$$

$$\sum F_y = M_B a_B = N_B - M_{Bottle} g - F_{Thrust}$$

$$\sum F_y = M_A a_A = N_A - M_W g$$

Determination of Position of Added Mass

$$0 = M_{Beam} g(x_{COM}) - M_W g(x_A) - M_{Bottle} g x_B$$

$$x_A = \frac{M_{Beam} g(x_{COM}) - M_{Bottle} g x_B}{M_W g}$$

*The position of the added mass on the test stand was found by setting the accelerations and thrust equal to zero.

Determination of Thrust Produced by Rocket

$$-(I_0 + M_{Beam} x_C^2) \alpha = -(-M_{Bottle}(x_B) \alpha + M_{Bottle} g + F_{thrust})(x_B) - (-M_W(x_A) \alpha + M_W g)(x_A) + M_{Beam} g x_{COM}$$

$$(-I_0 - M_{Beam} x_{COM}^2 - M_{Bottle}(x_B)^2 - M_W(x_A)^2) \alpha = g(-M_{Bottle}(x_B) - M_W(x_A) + M_{Beam} x_{COM}) - F_{thrust}(x_B)$$

$$\alpha = \frac{g(-M_{Bottle}(x_B) - M_W(x_A) + M_{Beam} x_{COM}) - F_{thrust}(x_B)}{-I_0 - M_{Beam} x_{COM}^2 - M_{Bottle}(x_B)^2 - M_W(x_A)^2}$$

$$* I_{eff} = (M_{Beam}(L^2/12 + x_{COM}^2) + M_{Bottle}(x_B)^2 + M_W(x_A)^2)$$

$$\alpha = \frac{F_{thrust}(x_B)}{I_{eff}}$$

$$F_{thrust} = \frac{I_{eff} \alpha}{x_B}$$

$$F_{thrust} = \frac{I_{eff} \omega}{x_B \Delta t}$$

*The equation for thrust derived is the previous. To achieve it, the beam is considered to be neutrally balanced and $a_R = -\alpha(x_B)$ and $a_A = -\alpha(x_A)$ must hold true. To derive impulse, the force due to thrust is multiplied by Δt .

Sample Thrust and Impulse Calculation: Capped Test at 33°C.

Variables

$$M_{Beam} = 1.215 \text{ kg}; L = 0.3937 \text{ m}; x_{COM} = 0.0254; M_{Bottle} = 0.536 \text{ kg}; M_W = 0.802 \text{ kg};$$

$$x_B = 0.3556 \text{ m}; x_A = 0.0445 \text{ m}$$

$$I_{eff} = (M_{Beam}(L^2/12 + x_{COM}^2) + M_{Bottle}(x_B)^2 + M_W(x_A)^2)$$

$$I_{eff} = 1.30 \text{ kg} \cdot \text{m}^2$$

Variables

$$I_{eff} = 1.30 \text{ kg} \cdot \text{m}^2; \omega = 0.1608 \frac{\text{rad}}{\text{s}}; x_B = 0.3556 \text{ m}$$

$$J = \frac{I_{eff}\omega}{x_B}$$

$$J = 0.5315 \text{ N} \cdot \text{s} \quad (\text{Impulse})$$

*The value for impulse was calculated in excel using the listed variables and equation. Omega was determined by dividing the unit of angle which the bottle travelled before it hit the ground divided by the time it took to travel. The angle for the tests conducted was $\theta = 0.149$ and time for this specific test was $\Delta t = 0.93$.

$$F_{thrust} = \frac{I_{eff}\omega}{x_B \Delta t}$$

$$F_{thrust} = 0.5716 \text{ N}$$

*In a similar manner to impulse, the force produced by the bottle rocket was calculated in excel. A similar calculation approach can be applied to other temperature cases.

Appendix 2: Raw Data Table

Table 1: Temperature and Flame Speed Results of Uncapped Test (20 mm Nozzle Diameter)

Initial Bottle Temperature (°C)	Flame Speed (m/s)
20	0
21	0
22	0
23	0
24	0.034
25	0.188
26	0.212
27	N/A
28	0.462
29	0.363
30	0.436

31	0.743
32	0.544
33	0.644
34	0.724
35	0.984
36	0.943
37	0.836
38	0.714
39	0.581
40	0.472
41	0.391
42	0.149
43	0.076
44	0
45	0

Table 2: Temperature and Flame Speed Results of Capped Test (6.35 mm Nozzle Diameter)

Initial Bottle Temperature (°C)	Flame Speed (m/s)
20	0
21	0
22	0
23	0
24	0
25	0.078
26	0.353
27	0.366
28	0.560
29	1.058
30	0.952
31	1.191
32	1.361
33	1.361
34	0.866
35	0.635
36	0.828
37	0.733
38	0.501
39	0.560
40	0.191
41	0
42	0
43	0
44	0
45	0

Appendix 3: Flame Speed Calculation of Propagation of Error

$$V_f = \frac{L}{t}$$

$$\left| \frac{\Delta V_f}{V_f} \right| = \left| \frac{\Delta L}{L} \right| + \left| \frac{\Delta t}{t} \right|$$

$$\frac{\Delta V_f}{V_f} = \sqrt{\left(\frac{\Delta L}{L} \right)^2 + \left(\frac{\Delta t}{t} \right)^2}$$

$$v = 0.027 \frac{m}{s} \text{ (Flame Speed Propagation of Error)}$$

*The propagation of error value was calculated in excel using a variation of the listed equations. The uncertainties used for the calculation were $\pm 0.1^\circ\text{C}$ for the thermometer and $\pm 0.0016 \text{ m}$ for the measuring tape.