Analysis of Heat Transfer in Rods of Various Cross-Sections

Determining the Largest Heat Transfer Rate for Various Cross-Sectional Rods under Different Cooling Conditions

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Introduction

Heat transfer is the transfer of thermal energy through spatial temperature difference. It depends on many factors including the shape and flow characteristics. The shapes that will be primarily investigated include: a circular cross section, a square cross section, and a star cross section. When the rod is cooling down, there is convection, conduction, and radiation heat transfer occurring. The remaining seven cross-sections that are to be used for further analysis are the trapezoid, the semi-circle, the hexagon, oval, octagon , dodecagon, and triangle.

Givens:

- Initial temperature of rod: 100°C
- Final temperature of rod equal to ambient: 20°C
- Rods have same length of 6 inches and fixed surface area of 7.289 in^2
- Quiescent air

Assumptions:

- Steady state and one-dimensional heat transfer
- Uniform cooling
- Materials are isotropic
- The effects of radiation can be neglected for most of the analysis
- Rods suspended in air without contact to mounting

Conduction

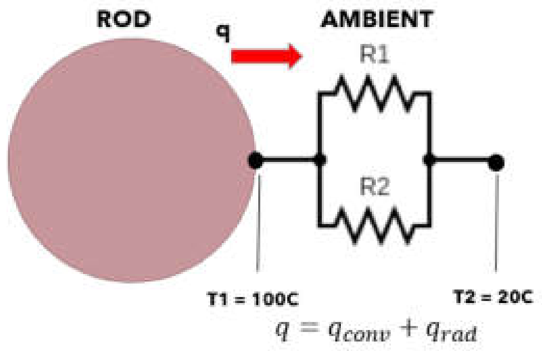
Material Properties:

Table 1: Thermal properties from Bergman Table A.1 pg. 923 :

Material A: Copper	Material B: Aluminum	Material C: Platinum
K= 401 W/mK	K= 237 W/mK	K=71.6 W/mK
α =117 x 10^6 m^2/s	α = 97.1 x 10^6 m^2/s	α =25.1 x 10^6 m^2/s
ρ =8933 kg/m^3	ρ = 2702 kg/m^3	ρ =21450 kg/m^3
c_ρ =385 J/kgK	c_ρ =903 J/kgK	c_ρ=133 J/kgK
ϵ =0.03	ϵ= 0.4375	ϵ = 0.054

Since there is static air, there is free convection occurring. According to Bergman Table 1.1, the range of convection coefficient is from a range of 2 to 25 W/m^2 K. Therefore, our chosen convection coefficient will be 14 W/m^2 K.

Circuit analogy:



Where R1 = thermal convective resistance, R2 = thermal radiative resistance

Figure 1: Thermal Circuit Analogy, showing the hotter rod in the cooler ambient temperature, and the direction of the heat transfer. Rod taken as a point mass.

Cross-Section calculations

Cross-Sections	Surface Area Equation	Total Surface Area(in^2)
Cylinder	$2\pi rh + 2\pi r^2$	7.289
Square	$2s \cdot (s + 2h)$	7.289
8-sided Star	$2 \cdot b + 6 \cdot e + 12 \cdot f$	7.289

Design dimensions

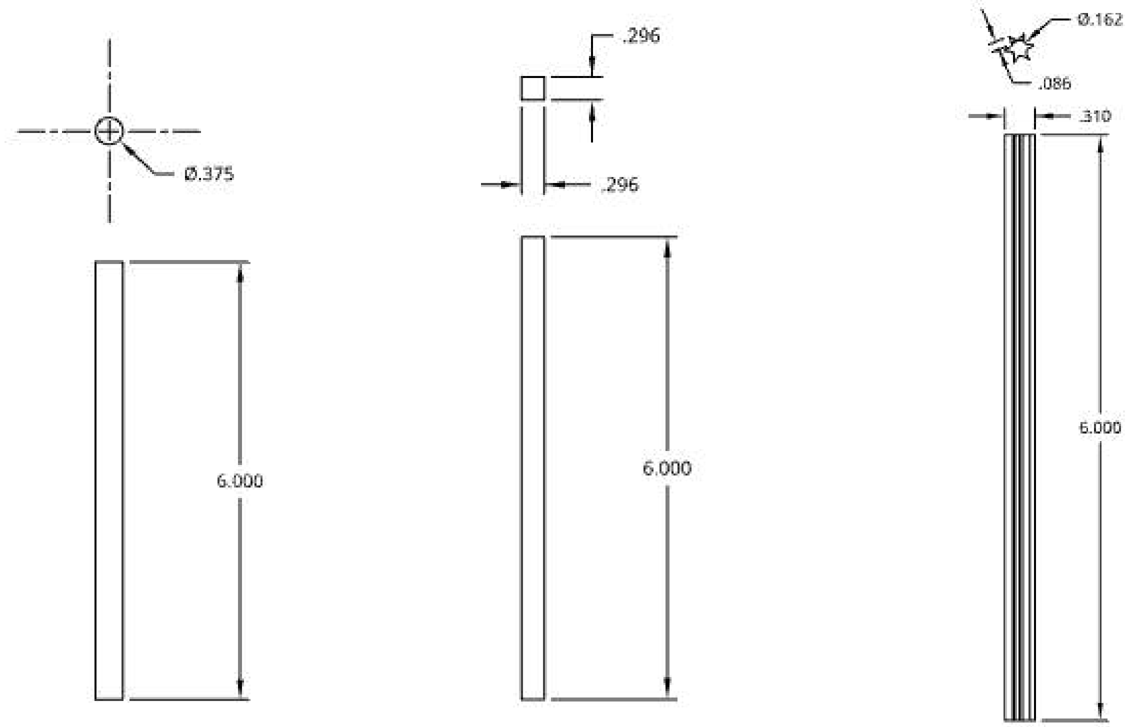


Figure 2: Cross-section dimensions of a Cylinder, Square and Star, accordingly (units of inches)

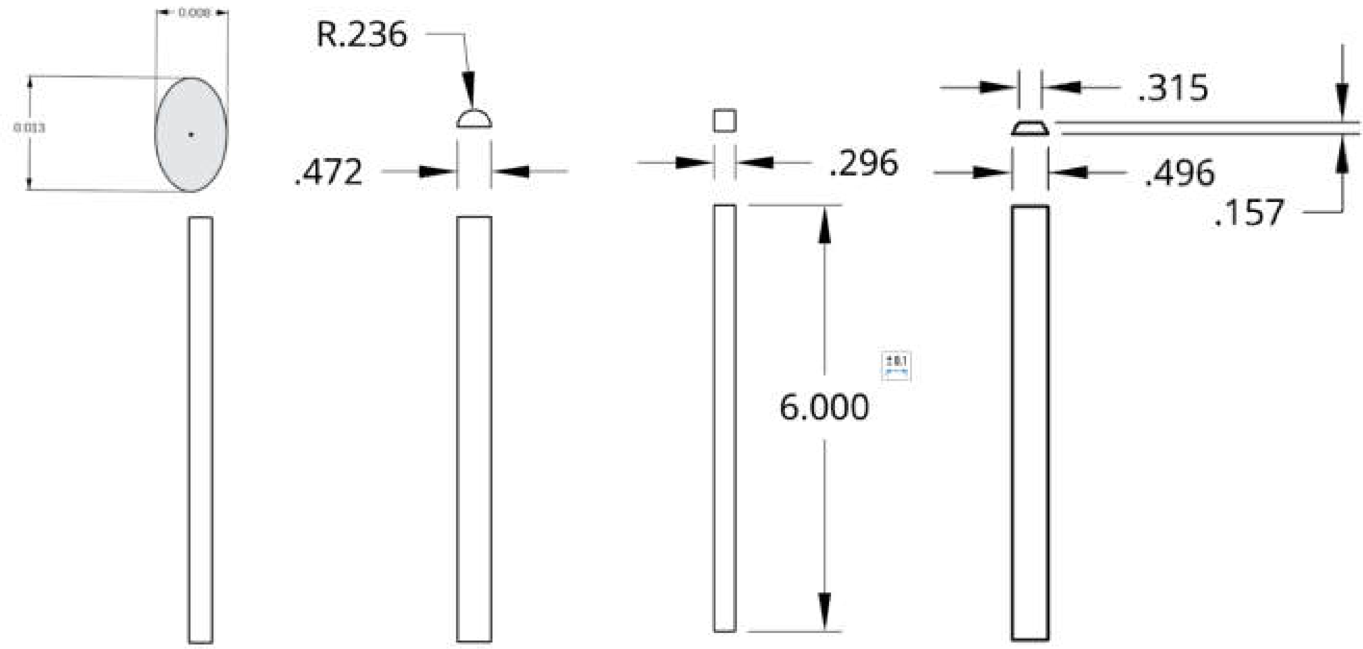


Figure 3: Cross-section dimensions of an Oval, Semi-circle, square and trapezium, accordingly (units of inches)

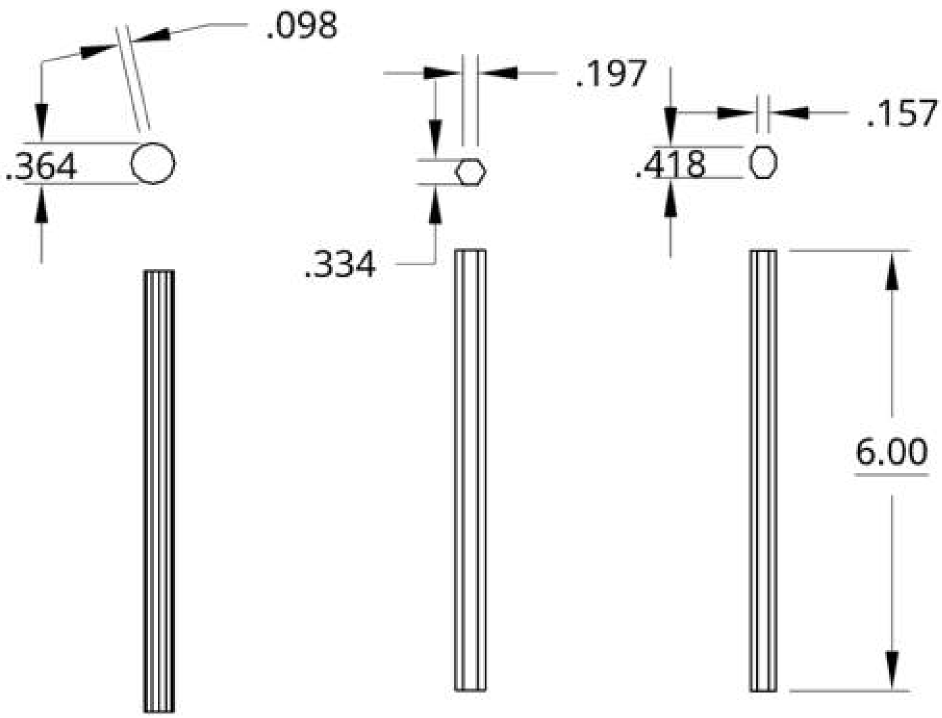


Figure 4:Cross-section dimensions of a Dodecagon, Hexagon, and Octagon, accordingly (units of inches)

Calculations

1D, Steady State Conduction Analysis

Rods in quiescient air and left to cooldown

$$q = q_{conv} + q_{rad}$$

$$q_{conv} = h \cdot A \cdot \Delta T \ , \ \ q_{rad} = \sigma \epsilon A \Delta T$$

$$q = h \cdot A \cdot \Delta T + \sigma \epsilon A \Delta T$$

where σ is the Stefan- Boltzmann's constant and ϵ is the emissivity of the material

Table 2: Using h = 14 W/Km^2

Material A: Copper	Cylinder (W)	Rectangular (W)	Star (W)
Convective	10.84	10.84	10.84
Radiation	0.20	0.20	0.20
Total heat transfer	**11.04**	**11.04**	**11.04**
Material B - Aluminum			
Convective	10.84	10.84	10.84
Radiation	2.89	2.89	2.89
Total heat transfer	**13.73**	**13.73**	**13.73**
Material C - Platinum			
Convective	10.84	10.84	10.84
Radiation	0.36	0.36	0.3567599
Total heat transfer	**11.20**	**11.20**	**11.20**

Table 3: Using h = 10 W/Km^2

Material A - Copper	Cylinder	Rectangular	Star
Convective	7.74	7.74	10.84
Radiation	0.20	0.20	0.20
Total heat transfer	**7.94**	**7.94**	**11.04**
Material B - Aluminum			
Convective	7.74	7.74	10.84
Radiation	2.89	2.89	2.89
Total heat transfer	**10.63**	**10.63**	**13.73**
Material C - Platinum			
Convective	7.74	7.74	10.84
Radiation	0.36	0.36	0.36
Total heat transfer	**8.10**	**8.10**	**11.20**

2D, Steady State Conduction Analysis

Governing Equations

Laplace equation in rectangular coordinates for square cross-section

$$\boldsymbol{\nabla^2 T = 0}$$

$$\boldsymbol{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0}$$

where T is the temperature and x and y are coordinates of the x-y reference frame for the square cross-sectional rod.

The Laplace equation can be re-written using separation of variables, under the assumptions the temperature distribution is indeed separable:

$$\boldsymbol{T(x,y) = X(x)Y(y)} \rightarrow \boldsymbol{X}''/\mathrm{X} + \mathrm{Y}''/\boldsymbol{Y} = \boldsymbol{0}$$

where X” and Y” are the second derivatives with respect to their according variable dependencies.

Taking orthogonal functions to satisfy this set of equations, we can substitute trial solutions:

$$\boldsymbol{X(x) = A\sin\left(\frac{n\pi x}{a}\right)}$$

$$\boldsymbol{Y(y) = Bsin(\frac{m\pi y}{b})}$$

where A and B are constant coefficients, and n and m are integers.

Thus, the temperature distribution can be expressed in the form of:

$$\boldsymbol{T(x,y) = \Sigma[A_n in\left(\frac{n\pi x}{a}\right) \cdot B_n sin(\frac{m\pi y}{b})]}$$

where $\boldsymbol{A_n}$ and $\boldsymbol{B_n}$ are constant coefficients determined from the boundary conditions.

Boundary conditions

- At x=0 (for cylindrical rods) or x=±L/2 (for rectangular rods):
  - Cylindrical rods: T(r, 0) = 100°C or T(r, L) = 100°C
  - Rectangular rods: T(±L/2, y) = 100°C or T(x, ±L/2) = 100°C

- At x=± ∞:
  - Cylindrical rods: T(∞, θ) = 20°C or T(0, θ) = 20°C
  - Rectangular rods: T(±∞, y) = 20°C or T(x, ±∞) = 20°C

- The heat transfer coefficient h between the rod and the ambient air is constant and uniform:
  - Cylindrical rods: -k(∂T/∂r) = h(T - T∞) at r = R
  - Rectangular rods: -k(∂T/∂y) = h(T - T∞) at x = L/2 or x = -L/2

- The thermal conductivity k of the rod is constant and uniform:
  - Cylindrical rods: (1/r)(∂/∂r)(r∂T/∂r) = 0
  - Rectangular rods: (∂^2T/∂x^2) + (∂^2T/∂y^2) = 0

Cylindrical Governing equations

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

The Laplace equation can be re-written using separation of variables, under the assumptions the temperature distribution is indeed separable:

$$T(r,\theta) = R(r)\Theta(\theta)$$

$$r^2R'' + rR' + k^2r^2R = 0$$

where R' and R" are the first and second derivates of R. To solve this, Bessel functions could be an approach:

$$R(r) = AJn(kr) + BYn(kr)$$

where Jn and Yn are the first and second kind of Bessel functions, and the constant coefficients A and B are found by the boundary conditions imposed.

Substituting this expression, the temperature distribution takes the form of:

$$T(r.\theta) = \Sigma[A_nJn(kr) + B_nYn(kr)][C_m\, cos(m\theta) + D_m\, sin(m\theta)]]$$

Where An, Bn, Cm, and Dm are constants that depend on the boundary conditions.

The temperature distributions could then be solved with the boundary conditions and with numerical methods since a system of linear algebraic equations arises.

Shape factors do not apply to the current setup of the problem of cooling down as the mode of heat transfer consists of convection and not conduction. This is because the shape factor relates to conductive heat transfer and measures how efficiently heat is transferred between two surfaces through the heat transfer mode of conduction.

Solving the ordinary partial differential governing equations is challenging and so the Lumped Capacitance method simplifies the analysis allowing for quicker estimations of temperature changes over time, without having to account or the temperature distribution within the object of interest.

Before using the Lumped Capacitance , its validity needs to be tested. To do so, the Biot number must be less than 0.1 indicating that the internal thermal resistance of the object is much smaller than the external thermal resistance at the surface of the object. This suggests that the internal temperature of the object is nearly uniform and that heat transfer is mainly one-dimensional and perpendicular in direction to the surface, which aligns with the assumptions made by the Lumped Capacitance Method.

Biot number $Bi = \frac{hL_{ch}}{k}$

where, h is the convection coefficient, L_{ch} is the characteristic length of the geometry, and k is the conduction coefficient of the solid.

<u>Free convection in air and water</u>

The Biot number was found to be valid for all three shapes, with them all being a couple order of magnitudes lower than the requirement. These data can be found in the Appendix.

Since the Lumped Capacitance Method is valid, the time constant can simply be found using:

$$\tau = \frac{1}{hA_s}(\rho V c) = R_t C_t$$

where τ is the time constant, h is the convective heat transfer coefficient, A_s is the surface area of the rod, ρ is the density of the rod , V is the volume of the rod, and c is the specific heat capacity of the rod.

Table 4: Star, Free convection in air/water

Star			
	Aluminum	Copper	Platinum
Time constant in air(s)	161.05	114.26	133.59
Time constant in water (s)	22.35	16.00	18.70

Table 5: Rectangle, Free convection in air/water

Square			
	Aluminum	Copper	Platinum
Time constant in air(s)	434.34	308.14	360.29
Time constant in water (s)	60.81	43.14	50.44

Table 6: Cylinder, Free Convection in air/water

Cylinder			
	Aluminum	Copper	Platinum
Time constant in air(s)	533.57	378.53	442.60

Time constant in water (s)	74.70	52.99	61.96

As seen from the tables above, the time constant for heat dissipation of the rectangular, star, and cylindrical shapes made of aluminum, copper, and platinum is consistently higher in air than in water. This indicates that heat dissipates more slowly in air than in water for these shapes and materials. Among the three materials, copper has the lowest time constant in both air and water, indicating that it dissipates heat more quickly than aluminum and platinum, with the star shape being the most optimal for heat dissipation.

External Flow

External Flow analysis was conducted to determine the effects of forced convection at different mediums, water and air, with a total of ten different shapes. The shapes we chose to analyze included our three primary shapes from the preceding analysis as well as trapezoids, oval, semi-circle, and polygons as shown in *table 7*. Calculating the heat transfer rate with varying mediums and shapes offered more parameters to compare, and this narrowed down which shape is the most flexible and effective provided various constraints. The properties of the aforementioned fluids was determined at the film temperature for analysis, where the final temperature is 100C and initial temperature is 20C.

Film Temperature:

$$T_{film} = \frac{T_{final} + T_{initial}}{2} \quad [=] \, K$$

Forced convection of air at V = 3m/s

Before delving into the analysis, the flow needs to be characterized as either turbulent or laminar for each of the cross-sections. To do this, the Reynolds number is found and compared to the Reynold’s critical number for internal flow, for flow over a cylinder and external flow where applicable. As shown, the Reynolds number is not distinguished by material since the equation does not depend on material properties, the value was observed to be the same regardless of material.

Reynolds Equation for air:

$$Re = \frac{VD}{\nu}$$

$$V = Velocity \left[\frac{m}{s}\right]$$
$$D = diameter\ of\ the\ cross\ section$$
$$\nu = kinematic\ viscosity\ of\ air\ at\ film\ temperature$$

Table 7: Reynold's Numbers for forced convection of air of varying rods

Cross-sectional shape	Reynolds Number	Characteristic flow
Star	1304	Laminar
Square	1304	Laminar
Circular	1630	Laminar
Trapezoid	652	Laminar
Oval	2119	Laminar
Semi-circle	978	Laminar
Octagon	1793	Laminar
Hexagon	1304	Laminar
Dodecagon	1467	Laminar
Equilateral triangle	1467	Laminar

Calculations and data on how these numbers and flows were arrived at can be found in Appendix 1b.

Generally, the Reynolds number was calculated differently using the diameter of the cross-sections. While for the square and the cylinder, the diameter/length was used, for the star shaped, we had used the longest distance between two endpoints, therefore, we account for the largest shape possible. For the rest of the shapes, since they were an iteration of the existing geometry, the diameter would be the vertical distance, and they were also mostly symmetrical.

To determine the heat transfer rate, Reynolds-Nusselt relationship was applied as follows:

$$Re \rightarrow Nu \rightarrow h \rightarrow q$$

Following this relationship allowed us to determine the resulting q for each shape.

To calculate the Nusselt number, the Prandtl number is also needed. The property is determined at the film temperature. Properties of air is found in Table A.4 provided in the textbook (*Fundamentals of Heat Transfer, 8th Edition written by Bergman and Levine).* The determined values was then used in the Churchill-Bernstein equation to determine the Nusselt number. The Churchill-Bernstein equation is favorable provided our conditions, since the Reynolds and the Prandtl number and the temperature constraints all apply for this equation. The following equation, equation 7.54 from the textbook, is used to obtain a Nusselt value:

Nusselt Number: Churchill-Bernstein

$$Nu_D = 0.3 + \left[0.62 * Re_D^{\frac{1}{2}} * \mathrm{Pra}^{\frac{1}{3}} * \left[1 + \left(\frac{0.4}{Pra} \right)^{\frac{2}{3}} \right]^{-\frac{1}{4}} \right] * \left[1 + \left(\frac{Re_D}{282{,}000} \right)^{\frac{5}{8}} \right]^{\frac{4}{5}}$$

$$Re_D = Reynolds$$

$$Pra = Prandtl$$

This equation was applied with the given assumption that all the shapes we had chosen would follow a similar trend as to a Cylinder given the Reynolds number calculated, since the conditions for this equation is maintained by all the shapes under analysis, the equation was applied to determine the Nusselt number for all the shapes. The convection coefficient was then determined using the Nusselt relations:

Nusselt-Convective Coefficient:

$$Nu = \frac{hD}{k_{fluid}}$$

$$h = convection\ coefficient$$

$$D = diameter\ of\ cross - section$$

$$k_{fluid} = thermal\ conductivity\ of\ the\ fluid$$

After determining the convectioncoefficient, it was correlated with the forced convection limits that were determined experimentally and presented in Table 1.1 of the textbook. All the shapes fall within the range of forced convection (25-250 $\frac{W}{m^2 * K}$). Using the convection coefficient, the resulting heat rate was determined for all the shapes by applying the following equation:

Heat Rate due to Convection:

$$q = h * A_s * \left(T_f - T_o\right)$$

$$A_s = Surface\ area\ of\ cross\ section\ subtracting\ the\ ends$$

The surface area usecd for the calculation neglected the bases of the shapes, since convection was considered to be cross-flow, which translates to fluid flow over the length of the rod. The resulting values are shown in *table 8*.

Table 8: Forced Convective Coefficients and Heat Rate at Air for all the shapes

Cross-sectional shape	**Convection Coefficient: h [W/m^2*K]**	**Heat rate: q [W]**
Star	64.24	25.48
Square	64.24	25.11
Circular	57.59	22.38
Trapezoid	90.62	35.66
Oval	50.69	19.60
Semi-circle	74.04	28.95
Octagon	54.97	21.32
Hexagon	64.24	25.03
Dodecagon	60.63	23.58
Equilateral triangle	60.63	23.83

Sample Calculations are shown in the attached excel spreadsheet

Based on the results in *table 8*, the highest heat rate is shown by the trapezoid and the lowest is shown by oval. Among the three primary shape that we selected (star, square, circular), the star cross-section generated the highest heat rate. The variations in the other shapes are also a result of the assumptions made due to the limited experimental data available. The Nusselt-Reynolds relations that is assumed is mainly applicable for Cylinders, however, not much experimental data is available for non-circular cross-sections, and assumptions are necessary for external flow calculations to be made possible. The main conclusions that can be drawn from the calculation is that the star cross-section has potential for delivering the fastest heat transfer rate when compared to square and circular cross-section, however, the fastest transfer occurs with a trapezoidal cross-section as shown by the results.

Forced Convection of Water at V = 3 m/s

Much of the calculations that was performed for air is similar to that of water. The primary difference is within the Reynolds calculations and the properties of water at the film temperature. The Reynolds formula for water is as follows:

Reynolds Number for Water:

$$Re = \frac{\rho V D}{\mu}$$

$$\rho = density\ of\ saturated\ water$$

$$\mu = dynamic\ viscosity$$

Furthermore, the Prandtl number and the thermal conductivity were all determined from Table A.6 from the textbook. Water was assumed to be fully saturated liquid. After calculating the Reynolds number, the same process of obtaining heat rate using Reynolds-Nusselt relations was used. After running the calculations. The final results for the heat rate in water is shown in *table 9*.

Table 9: Forced Convective Coefficients and Heat Rate at water for all the shapes

Cross-sectional shape	**Convection Coefficient: h [W/m^2*K]**	**Heat rate: q [W]**
Star	19,748	7,832
Square	19,748	7,720
Circular	18,199	7,072
Trapezoid	25,919	10,198
Oval	16,599	6,417
Semi-circle	22,038	8,616
Octagon	17,591	6,824
Hexagon	19,748	7,694
Dodecagon	18,907	7,352
Equilateral triangle	18,907	7,432

Calculations are presented in the attached excel sheet.

The highest heat rate was again observed to be with the trapezoid, however, as opposed to the air, the range for forced convection for liquids in 100-20,000 $\frac{W}{m^2K}$. As observed, shapes like Trapezoid and semi-circle does not fall within such limits, therefore, there is a flaw within the assumptions that was made to obtain the results. Therefore, after exempting the two shapes from analysis, the star shape yields the highest heat rate among the remaining shapes. By comparing with the three primary shapes, the star still seems to outperform the reaming two. Therefore, after considering the flaw in the assumptions, the star outperforms. This analysis is not ideal, since the non-circular shapes have limited experimental methods to determine external flow. The only remaining method that can be applied is combining existing geometry and their associated Nusselt-Reynold correlation to determine the heat rate. However, there aren't any existing experimental methods to prove the approach. Different methods should be considered to justify the performance of star cross section.

Internal flow

In order to begin the internal flow analysis, a few of the parameters had to be re-characterized. Internal flow involving the cooling down of a fluid was occurring in a hollow tube. Some assumptions made about this system included assuming that the flow inside each of the hollow tubes was a mean flow, assuming a constant surface temperature for tubes, and assuming steady state incompressible flow. The fluid flowing inside the hollow tubes was also assumed to be water flowing at 3m/s with properties at the film temperature. Three rods were compared for this analysis: the star, the square, and the cylinder.
Initially, the hydraulic diameter was calculated for Reynold's number and the surface area was re-calculated for the convection heat formula. The surface area is different for internal flow in comparison to external flow due to the change in the interaction of the flow of the fluid with the shape. With internal flow, the flow surface area will only be inside the tube. The diameter was also adjusted for Reynold's formula since hydraulic diameter is different for a solid tube compared to a hollow tube. Next, the below procedure was followed for each shape:

$$Re \rightarrow Nu \rightarrow h \rightarrow q$$

While this procedure implemented the same format generally, the Nusselt formula used for each shape was different. For the star, a superposition method was used. Once the Reynold's number was calculated, it was seen that it was within the turbulent range. However, assuming that Table 8.1 (*Fundamentals of Heat Transfer, 8th Edition written by Bergman and Levine)* was relevant and most applicable for non-circular shaped hollow tubes, the Nusselt number was calculated. The Nusselt for the triangle was multiplied by 6 (representing how many spikes the star had) and added the Nusselt number given for the circle from the applicable column. This was used as an approximate Nusselt value for the star shape and then further implemented for the convective coefficient formula. Thus, allowing us to calculate the heat rate. For the square, Table 8.1 (*Fundamentals of Heat Transfer, 8th Edition written by Bergman and Levine)* was implemented once again to find the Nusselt value. Finally, for the cylinder equation 8.60 (F*undamentals of Heat Transfer, 8th Edition written by Bergman and Levine)* was used to calculate Nusselt number as it accurately aligned with the Reynold's number range, shape, and flow region. Table of the values for each shape can be seen below.

Table 10: Calculation of Internal Flow

Star:

	Aluminum	Copper	Platinum
Reynolds	10028.35	10028.35	10028.35
Nusselt	18.60	18.60	18.60
h ($\frac{W}{m^2K}$)	7714.31	7714.31	7714.31
q (W)	1851.43	1851.43	1851.43

Square:

	Aluminum	Copper	Platinum
Reynolds	8810.036343	8810.04	8810.04
Nusselt	2.98	2.98	2.98
h ($\frac{W}{m^2K}$)	1406.86	1406.86	1406.863923
q (W)	471.23	471.23	471.23

Cylinder:

	Aluminum	Copper	Platinum
Reynolds	56464.97	56464.97	56464.97
Nusselt	202.74	202.74	202.74
h ($\frac{W}{m^2K}$)	96722.21	96722.21	96722.21
q (W)	68595.39	68595.39	68595.39

Based on these results, it can be seen that the heat rate for the Square is the lowest and the heat rate for the Cylinder is the highest. This indicates that the Square is the best for cooling down fluid as it has the least heat transfer.

Free Convection

To end the analysis, free convection equations are applied. The flow over the rods are once again external and the rods are solid. Initially, the Grashof number can be calculated. Using the Grashof number, the Rayleigh number can be found.This allows for the characterization of the flow. Then, the same steps can be taken to find the heat rate: calculate Nusselt number, use Nusselt to calculate the convective coefficient, and finally find the heat rate.

Table 11: Free Convection Calculations

Star:

	Aluminum	Copper	Platinum
Gr	1.96041078	1.96041078	1.96041078
Ra	1.38	1.38	1.38
Nusselt	1.07	1.07	1.07
h	45.93	45.93	45.93
q	18.22	18.22	18.22

Square:

	Aluminum	Copper	Platinum
Gr	3562.157176	3562.157176	3562.157176
Ra	2505.98	2505.98	2505.98
Nusselt	3.25	3.25	3.25
h	11.43	11.43	11.43
q	4.47	4.47	4.47

Cylinder:

	Aluminum	Copper	Platinum
Gr	6957.338234	6957.338234	6957.338234
Ra	4894.49	4894.49	4894.49
Nusselt	3.59	3.59	3.59
h	10.10	10.10	10.10
q	3.92	3.92	3.92

Based on the results, it is clear that the Star has the highest rate. This makes this shape the best for heat transfer in comparison to the Square and Cylinder. A sample calculation for the star case is displayed in the Appendix: Calculations: Sample Heat Transfer Rate Calculation for Free Convection of a Star

Heat Exchangers

One potential application for the rods could be in the field of heat exchangers. Heat exchangers are devices used to transfer heat between two or more fluids, and they are commonly used in a wide range of applications, including air conditioning systems, refrigeration systems, and power plants.

The shape of the rods could play a role in improving the efficiency of heat exchangers. For example, the six-pointed star and square rods could increase the surface area available for heat transfer, potentially improving the overall heat transfer rate. The cylindrical rod could also be used to create turbulence in the fluid flow, which can also improve heat transfer.

The reasoning behind why cylindrical rods are commonly used in heat exchangers as opposed to other is due to their ease of manufacturing, but of maintenance. When retubing Scotch Marine boilers, firetubes are plugged until changed, and with the pre-existing supply of tubes ready to be used, it has become the industry's standard.

Another potential application for these rods could be in the field of fluid mechanics, where they could be used to study the effects of different shapes on fluid flow patterns. This could provide valuable insights into the design of more efficient fluid flow systems.

Appendix

I. Calculations

a. Surface Area Sample Calculation

The displayed total surface area values from Onshape when modelling the rods 1:1 were confirmed by simplifying complex shapes into a combination of simpler ones.

For example, the six-sided star's surface area was made up of triangles and rectangles, which when simplified yielded the equation of :

$$\boldsymbol{SA = 12 * rectangle_1 + 6 * rectangle_2 + 2 * (6 * triangle + circle)}$$

$$\boldsymbol{SA = 12 * 0.5614 + 6 * 0.081 + 2 * 0.033}$$

$\boldsymbol{SA = 7.289}$ in^2 = **0.005 m^2**

SA neglecting front and back: SA=6.737 in^2 = **0.00496 m^2**

To find the equivalent side lengths for each different cross-section, the length was fixed at 6 inches, with manual calculations for the simpler shapes being carried out, and Excel's Goal Seek feature for the more complex rods. The same approach was followed for the volume of the rods.

b. Sample Reynolds number calculation

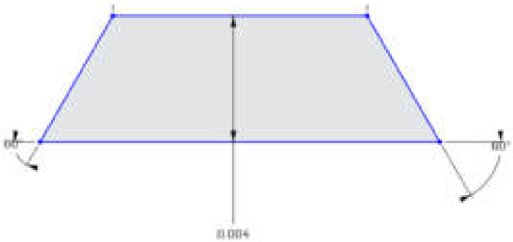
$$\boldsymbol{Re = \frac{\rho v D}{\mu} = \frac{vD}{\nu}}$$

Properties of air at 333K film temperature used from Table A.4, Thermophysical Properties of Gases at Atmospheric Pressure, Bergman.

Film temperature,T	333 °K
Kinematic viscosity, $\boldsymbol{\nu}$	$\boldsymbol{1.84 \cdot 10^{-5} \frac{m^2}{s}}$
Conduction coefficient, k	0.02815 $\frac{W}{m \cdot k}$
Prandtl number, Pr	0.7035

For a complex shape like the trapezoid, the equivalent value for D in Reynolds number equation is taken to be the height of the shape as that makes most sense in crossflow in terms of dimensions to use.

Aluminum Trapezoid in forced convection, air at 3 m/s



$$\boldsymbol{Re = \frac{\rho v D}{\mu} = \frac{vD}{\nu}}$$

$$\boldsymbol{Re} = \frac{3\left[\frac{m}{s}\right] \cdot 0.004[m]}{1.84 \cdot 10^{-5}\left[\frac{m^2}{s}\right]} = 652[unitless]$$

c. Sample Heat Transfer Rate Calculation for Free Convection of a Star

Aluminium star

$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{v^2} = \frac{g(\frac{1}{T_f})(T_s - T_\infty)L^3}{v^2} \quad (Grashof\ number)$$

$$\frac{(9.81\left[\frac{m}{s^2}\right])(\frac{1}{333}[K^{-1}])(100-20)(6.56\cdot10^{-4}[m])}{1.84\cdot10^{-5}[\frac{m^2}{s}]} = 1.96$$

$$Ra_L = Gr_L Pr \ (Rayleigh\ number)$$
$$Ra_L = 1.96 \cdot 0.7035 = 1.38$$

$$\overline{Nu_d} = \frac{\bar{h}D}{k} = CRa_D^n \ (Nusselt\ number)$$
$$= 1.02 * 1.38 = 1.07$$

Rearranging the previous equation in terms of average heat convection coefficient :

$$\frac{1.07 \cdot 0.02815\left[\frac{W}{m \cdot K}\right]}{6.54 \cdot 10^{-4}} = 45.93[\frac{W}{m^2K}]$$

Using Newton's Law of Cooling:

$$q = hA(T_s - T_\infty)$$
$$q = (45.93)\left[\frac{W}{m^2K}\right](100 - 20) = 18.22[W]$$

A limitation of this method is that material properties aren't accounted for resulting in the same q value for all Platinum, Copper, and Aluminum.

II. Tables & Data

Table 11:Star Biot Number results, less than 0.01

Star			
Volume ($10^{-6}\ m^3$)	3.25		
Surface Area ($10^{-3}\ m^2$)	5.00		
Characteristic Length (10^{-4} m)	6.56		
	Aluminum	**Copper**	**Platinum**
Biot number @ h =14 (air)	0.00004	0.00002	0.00013
Biot number @ h =100 (water)	0.00028	0.00016	0.00092

Table 8:Rectangle Biot Number results, less than 0.01

Rectangle	
Volume ($10^{-6}\ m^3$)	8.64
Surface Area ($10^{-3}\ m^2$)	5.0

Characteristic Length (10^{-3} m)	1.77		
	Aluminum	**Copper**	**Platinum**
Biot number @ h =14 (air)	0.0001	0.0001	0.0003
Biot number @ h =100 (water)	0.0007	0.0004	0.0025

Table 9: Cylinder Biot Number Results, less than 0.01

Cylinder			
Volume ($10^{-5}\ m^3$)	1.09		
Surface Area ($10^{-3}\ m^2$)	5.00		
Characteristic Length (10^{-3} m)	2.24		
	Aluminum	**Copper**	**Platinum**
Biot number @ h =14 (air)	0.00013	0.00008	0.0004
Biot number @ h =100 (water)	0.0009	0.0006	0.0031

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