HYDRA
Error Model
Presentation

Petros Zantis

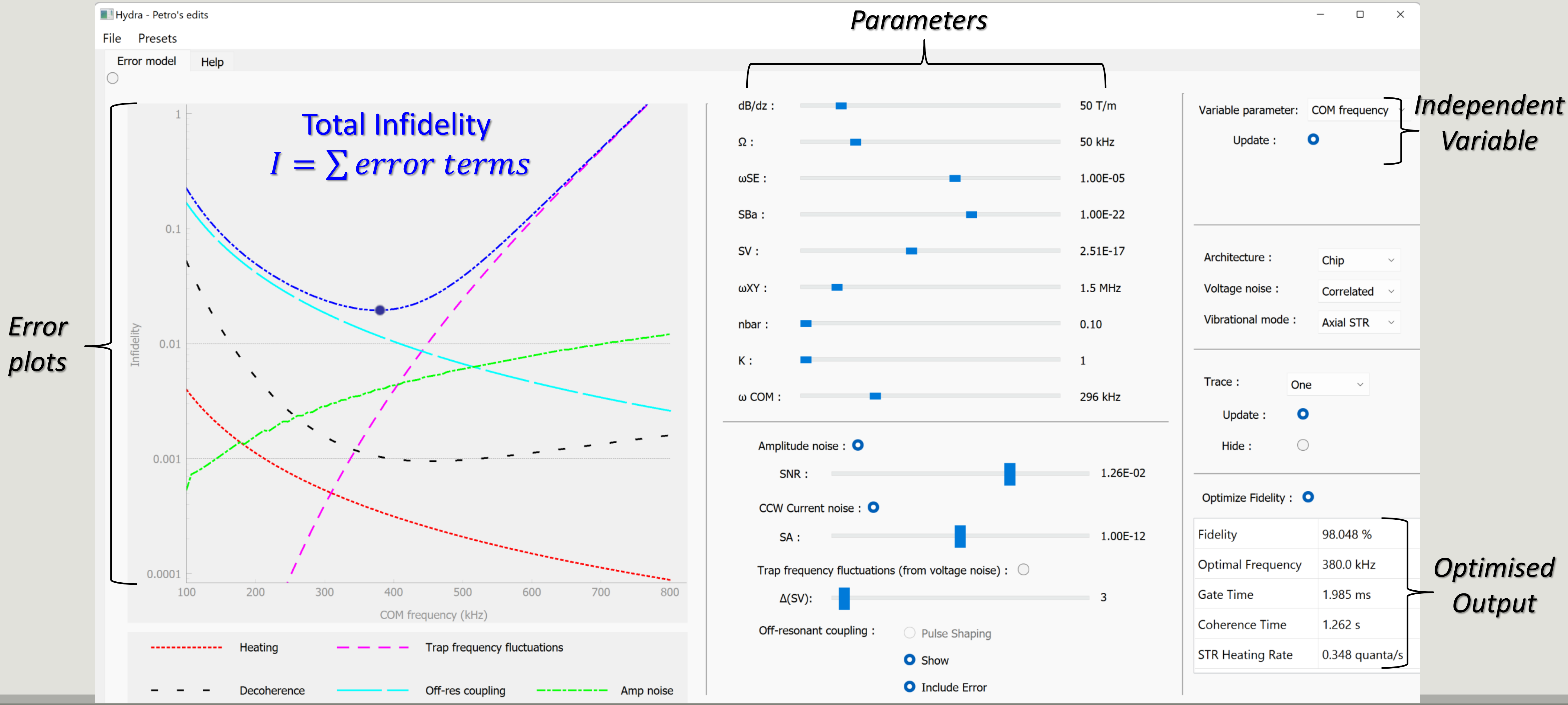
HYDRA

"For every head chopped off, the Hydra would regrow two" →
For every noise source eliminated, two more pop up.

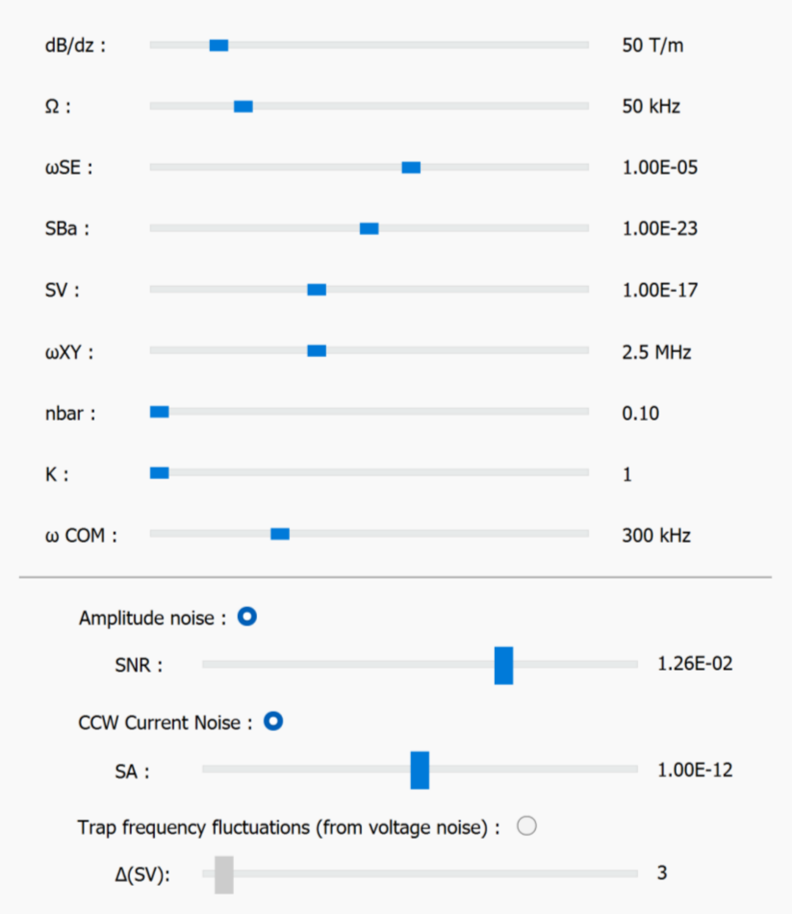
- A software which visualises the error models for two-qubit microwave gates using trapped ions in a static magnetic field gradient.
- Assumes perfect calibration of experimental parameters (no mis-set errors).
- Allows the option to optimise the variable parameter for minimising errors (maximising fidelity).
- Calculates maximum achievable fidelity due to inherent error/noise mechanisms.
- Also calculates expected heating rate, gate time and coherence time.

Credit: https://github.com/christophevalahu/hydra
My version: https://github.com/PetrosZantis13/Noise_IQT

User Interface



Parameters



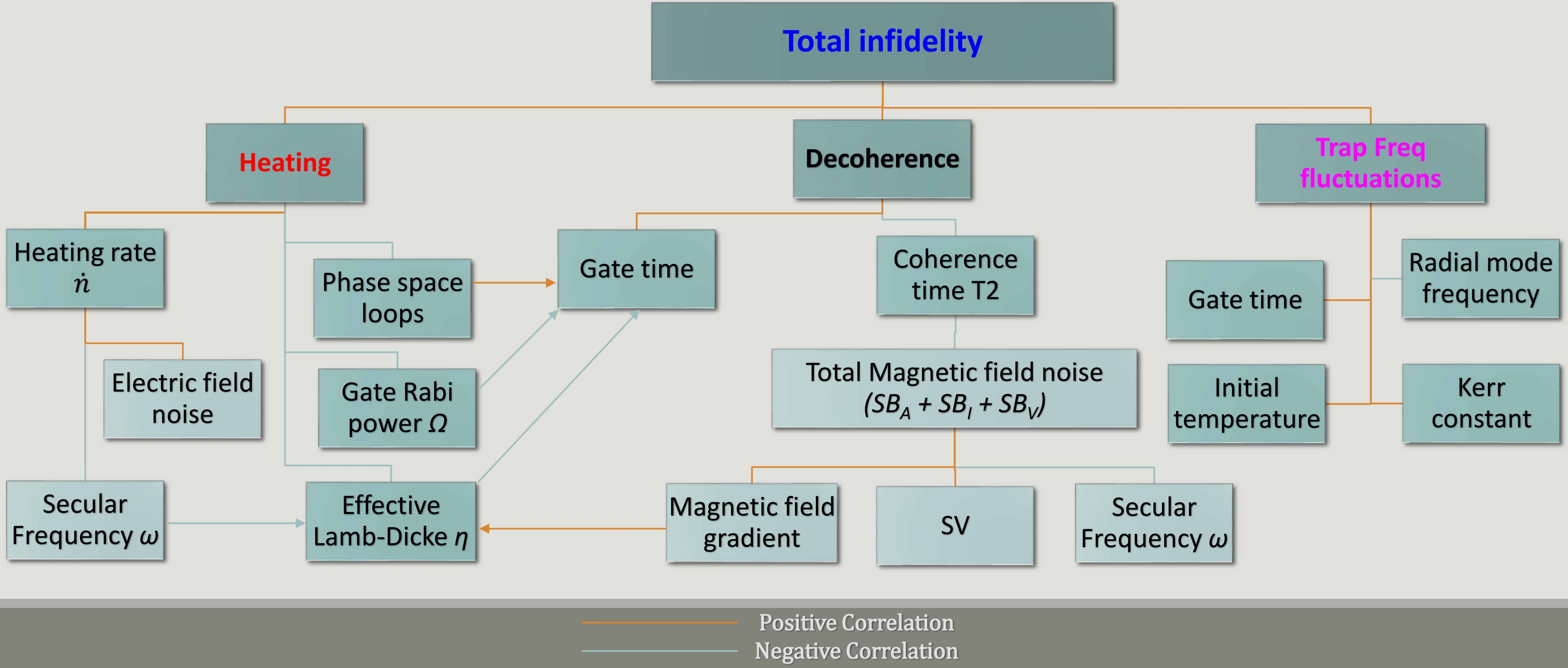
- Magnetic field gradient
- Gate Rabi frequency/power
- Scaled Electric field noise PSD $(V/m)^2$
- Ambient Magnetic field noise PSD (T^2/Hz)
- Voltage noise PSD (V^2/Hz)
- Radial mode frequency
- Initial temperature $\bar{n}$
- Loops in phase space
- Secular COM frequency

- Signal-to-Noise Ratio

- Current noise PSD (A^2/Hz)

- Variance from voltage noise

Dependencies



Heating errors

- Motional heating changes the populations of the Fock states $|n\rangle$
- Caused by stray electric fields matching the frequency of the ion's vibrational mode.
- The infidelity due to heating is derived analytically in the original MS paper:

$$\mathbf{err_{heating}}:\ 1-\frac{1}{8}\left(3+4e^{-\frac{\dot{n}\,\pi}{2\,\varphi\,\eta\,\Omega}}+e^{-\frac{2\,\dot{n}\,\pi}{\varphi\,\eta\,\Omega}}\right)$$

Where $\dot{n}$ is the heating rate, φ is the heating factor ($\sqrt{k}$ for a k-loop MS gate; tone-dependent for Multi-Tone MS gate), η is the effective Lamb-Dicke parameter and Ω is the gate Rabi frequency.

Heating errors

- For 2 ions, there are two vibrational modes: the centre-of-mass (COM) mode and the stretch (STR) mode.
- Heating the STR mode is more difficult since the two ions vibrate out of phase.
- Also, electric field noise PSD usually scales as 1/ω, and since $\omega_{STR} = \sqrt{3}\,\omega_{COM}$, the STR mode experiences less noise.

$$\dot{n}_{COM} = \frac{e^2}{4\,m\,\hbar\,\omega_z^2} * \omega SE \qquad or \qquad \dot{n}_{STR} = \frac{e^2}{4\,m\,\hbar\,\omega_z^2} * \omega SE * \left(\frac{\Delta z}{d}\right)^2$$

Where ω_z is the secular frequency, d is the ion-electrode distance,

and the ion-ion spacing $\Delta z = \frac{2}{2^{2/3}}\left(\frac{e^2}{4\pi\varepsilon_0\, m\omega_{com}^2}\right)^{1/3}$

D.F.V. James, "Quantum dynamics of cold trapped ions with application to quantum computation", Applied Physics B: Lasers and Optics 66, 181–190 (1998).

Decoherence errors

- Depolarisation and dephasing arise from magnetic field noise which fluctuates the energy levels of the qubit.
- Exact expression is dependent on the chosen gate scheme.
- For the dressed state scheme, the decoherence of the $|D\rangle$ state is analytically derived and numerically confirmed by simulating the full Hamiltonians.

$$\boldsymbol{err_{decoherence}} : \frac{\mathbf{1}}{\mathbf{3}}\left(1 - e^{-\frac{t_{gate}}{T_2}}\right)$$

$$\eta_{eff} = \frac{\mu_{\mathrm{B}}}{\sqrt{2}\,\hbar\,\omega_z} * \frac{\partial B}{\partial z} * \sqrt{\frac{\hbar}{2\,m\,\omega_z}}$$

$$\text{Coherence time } T_2 = \frac{\hbar^2}{\mu_{\mathrm{B}}^2\,SB}$$

$$t_{gate} = \frac{\pi}{\eta * \Omega} * Cost \longrightarrow$$

Trade-off for using MTMS or PM gates ($\sqrt{k}$ for a k-loop MS gate; tone-dependent for Multi-Tone MS gate)

Decoherence errors

Total PSD of B-field noise $SB = SB_A + SB_I + SB_V$

$SB_A \rightarrow$ *Input parameter*

$$SB_I = \left(\frac{\partial B}{\partial I}\right)^2 * \ SA$$

$$\frac{\partial B}{\partial I} = \frac{(1.6827 * 10^{-5}) * dx}{10 \ z_0^{1.78}}$$

$$dx = 4 \times 10^{-9}$$

(Ion's position resolution allowable with DAC)

$$SB_V = \left(\frac{\partial B}{\partial V}\right)^2 * \ SV$$

$$\frac{\partial B}{\partial V} = \frac{e \ g}{m \ \omega_z{}^2} * \frac{\partial B}{\partial z}$$

(g = Geometric factor for voltage noise found by numerically simulating chip geometries)

Geometric factor

$$g = \frac{\partial E}{\partial V}$$

1st nn: 25 m^{-1} , 2nd nn: 15 m^{-1} , 3rd nn: 8 m^{-1} , 4th nn: 5 m^{-1} , etc…

$$g \approx \sqrt{25^2 \times 4 + 15^2 \times 4 + 8^2 \times 4 + 5^2 \times 4} \approx 62\, m^{-1}$$

Incoherent sum of independent sources

Trap frequency fluctuation errors

- Non-linear coupling between the axial stretch mode (ω_{STR}) and the radial rocking modes (ω_r) causes a Kerr-type effect.
- The Kerr constant χ describes the coupling strength and is derived analytically.

$$err_{trap\_freq\_fluct} : \left(2 * K_{var} + (2\pi\, \Delta SV)^2\right) \quad * t_{gate}^2 * \left(\frac{1+2(1+2\,\bar{n})}{16}\right)$$

$$K_{var} = \chi^2\, (\overline{n_r})(2\,\overline{n_r} + 1)$$

$$where\ the\ Kerr\ coefficient\ \chi = -\omega_{STR}\left(\frac{1}{2} + \frac{\omega_{STR}^2}{2\,(4\omega_r^2 - \omega_{STR}^2)}\right)\left(\frac{\omega_{COM}}{\omega_r}\right)\left(\frac{2\,\hbar\,\omega_{COM}}{\alpha^2\, mc^2}\right)^{1/3}$$

$$\omega_r = \sqrt{\omega_{xy}^2 - \omega_{COM}^2} \quad and\ the\ Radial\ mode\ temperature\ \overline{n_r} = \frac{19.6}{2\,\omega_{xy}}$$

19.6 → *S→P linewidth in MHz*

ω_{xy} → *Radial mode frequency*

X. R. Nie, C. F. Roos, and D. F. James, "Theory of cross phase modulation for the vibrational modes of trapped ions", [Physics Letters A 373, 422–425 (2009)]

Amplitude noise errors

- Noise in the dynamical decoupling drive amplitude will eventually cause decoherence in the same way that magnetic field fluctuations lead to dephasing.
- The associated infidelity is derived by evaluating the errors due to amplitude noise during a Rabi oscillation, using the PSD of Rabi frequency noise.

$$\boldsymbol{err_{amp\_noise}} : 1 - e^{-\frac{t_{gate} * amp\_noise * \omega_{dd}^2}{2}}$$

$$amp\_noise = Amp\_PSD(f) = \frac{A}{f^2 + f_0^2} = \frac{2\, SNR^2\, f_0}{\pi\,(f^2 + f_0^2)}$$

$$\text{with sampling frequency } f_s = \frac{n_{rot}}{tgate} \text{ } and\ n_{rot} = int\left(\frac{tgate}{T_{rot}}\right) = int\left(\frac{tgate}{2\pi/\omega_{dd}}\right)$$

$$f_0 = 1000\ Hz,\ \text{Power of continuous DD drives } \omega_{dd} = 2\pi\ 30 \times 10^3$$

Off-resonant coupling errors

- The MS sideband fields can couple to the carrier transitions and induce errors.
- Not a priority since not dominant (at low powers) & can be eliminated by pulse shaping.
- Not accurately modelled in HYDRA, but can be included by numerically simulating the Hamiltonians.

$$\boldsymbol{err_{off-res}} : \; 2\left(\frac{\Omega}{\delta}\right)^2$$

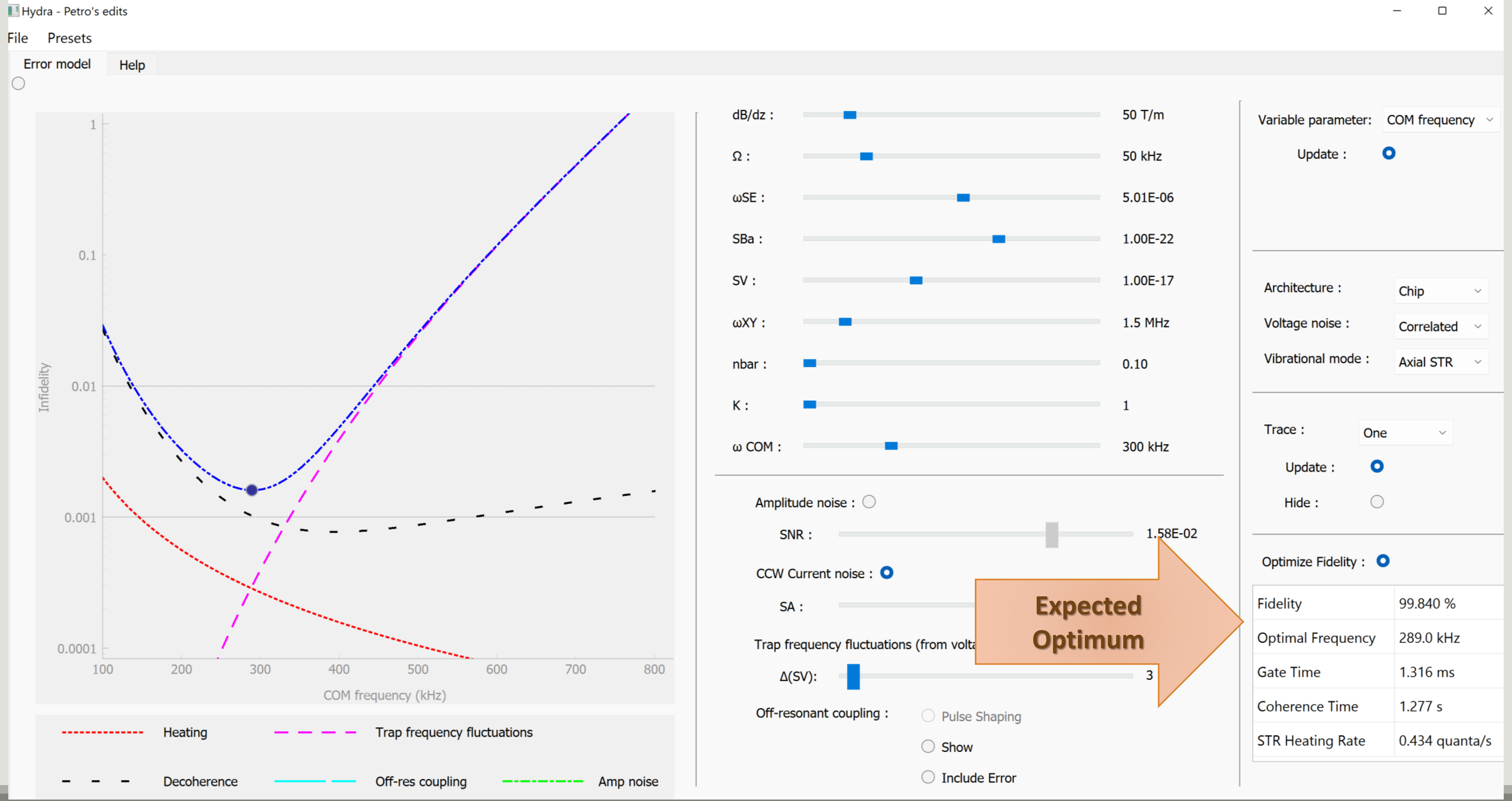
$$where\ the\ detuning\ \delta = \ \omega_{STR}$$

Preliminary Results

Setting the parameters

- B-field gradient : **50 T/m**
- Gate Rabi frequency : **50 kHz**
- Scaled E-noise : **5×10^{-6} V^2/m^2** *(estimated from literature using d=125 µm & T=70 K)*
- Ambient B-noise : **1×10^{-22} T^2/Hz** *(corresponds to ambient B-noise of 10 pT/$Hz^{1/2}$)*
- Voltage noise : **1×10^{-17} V^2/Hz** *(corresponds to electrode noise ~ 3 nV/$Hz^{1/2}$)*
- Radial frequency : **1.5 MHz**
- Initial temperature : **0.1**
- Phase space loops : **1** *(primitive MS gate)*
- Current noise : **1×10^{-12} A^2/Hz** *(corresponds to CCW noise of 1 µA/$Hz^{1/2}$)*

- **Using STR mode and assuming correlated noise in the electrodes**

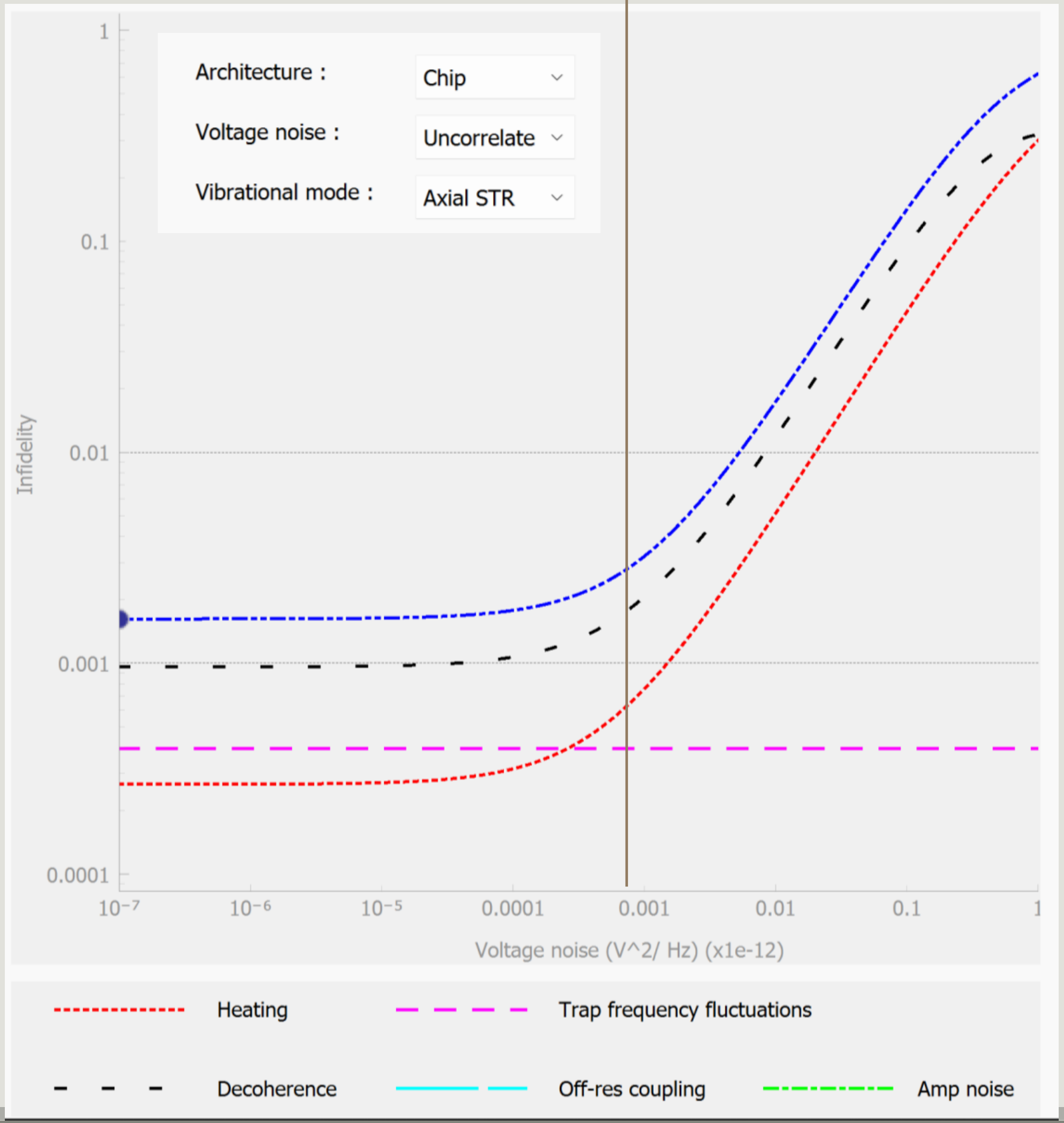


> 99% theoretically achievable!!!

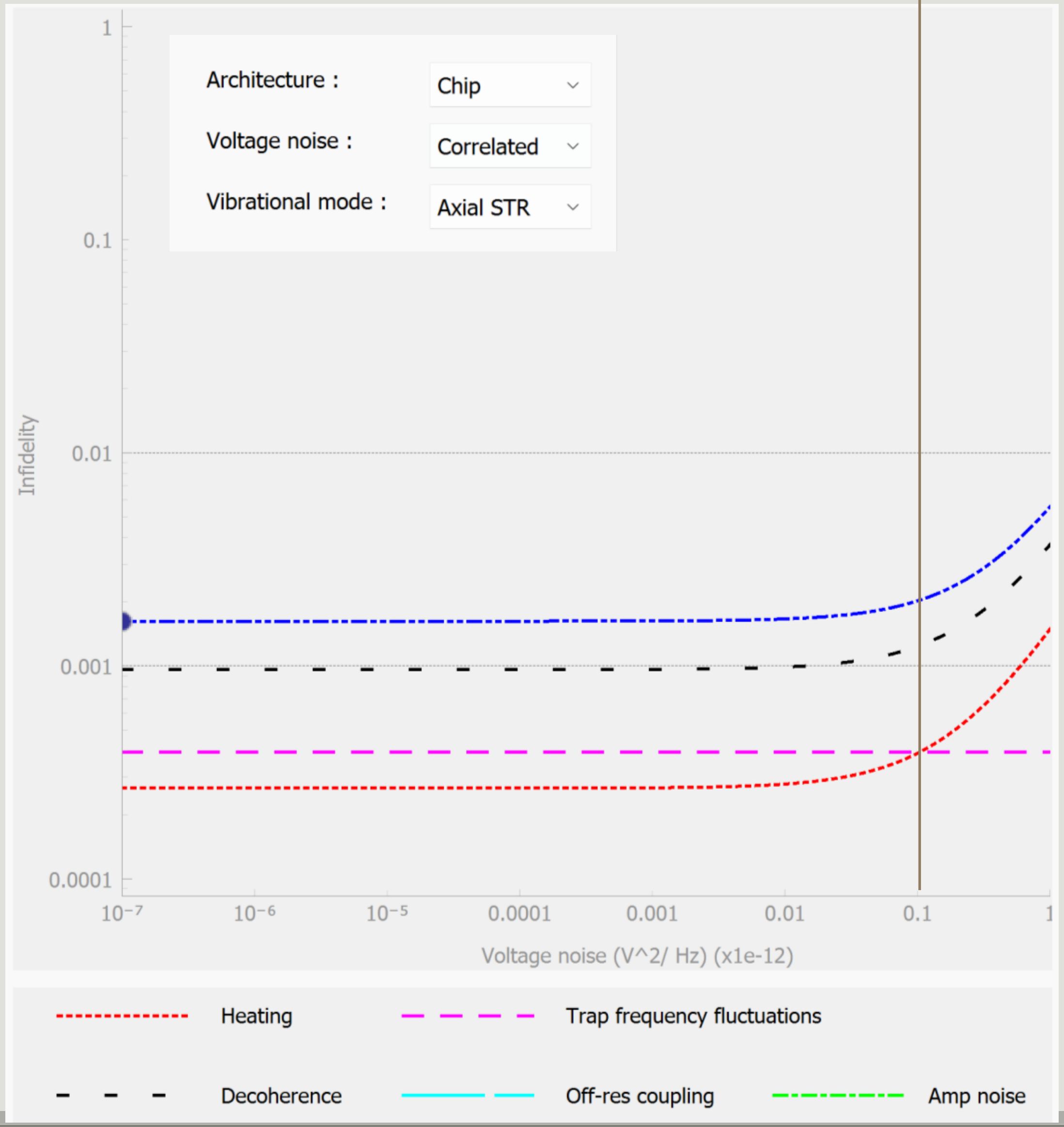
Electrode pairing

- In the case of uncorrelated noise between all electrodes, the E-field PSD is an incoherent sum of independent noise sources.
- In the case of correlated noise, the E-field PSD is derived from a coherent sum of the geometric factors for each electrode.
- Therefore, in the latter case, one can engineer the electrode geometry such that the geometric factor summation reduces to ~ 0.
- Numerical simulations show that this reduction is by a factor of ~ 20. *(Alex Owens)*
- The resulting effect is increased “resilience” to voltage noise.

- Voltage noise above 30 nV/Hz$^{1/2}$ starts causing decoherence.



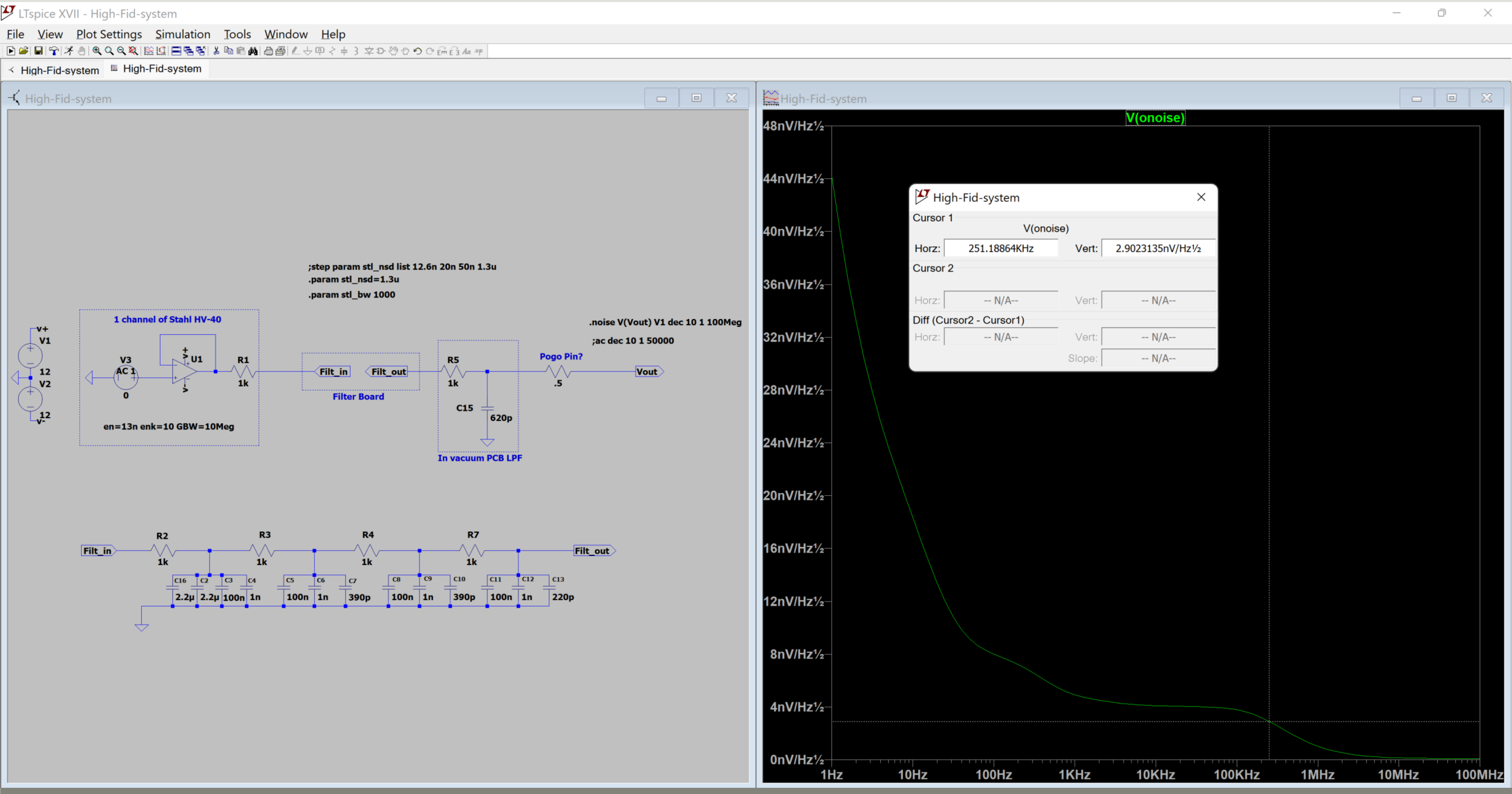
- Voltage noise above 300 nV/Hz$^{1/2}$ starts causing decoherence.



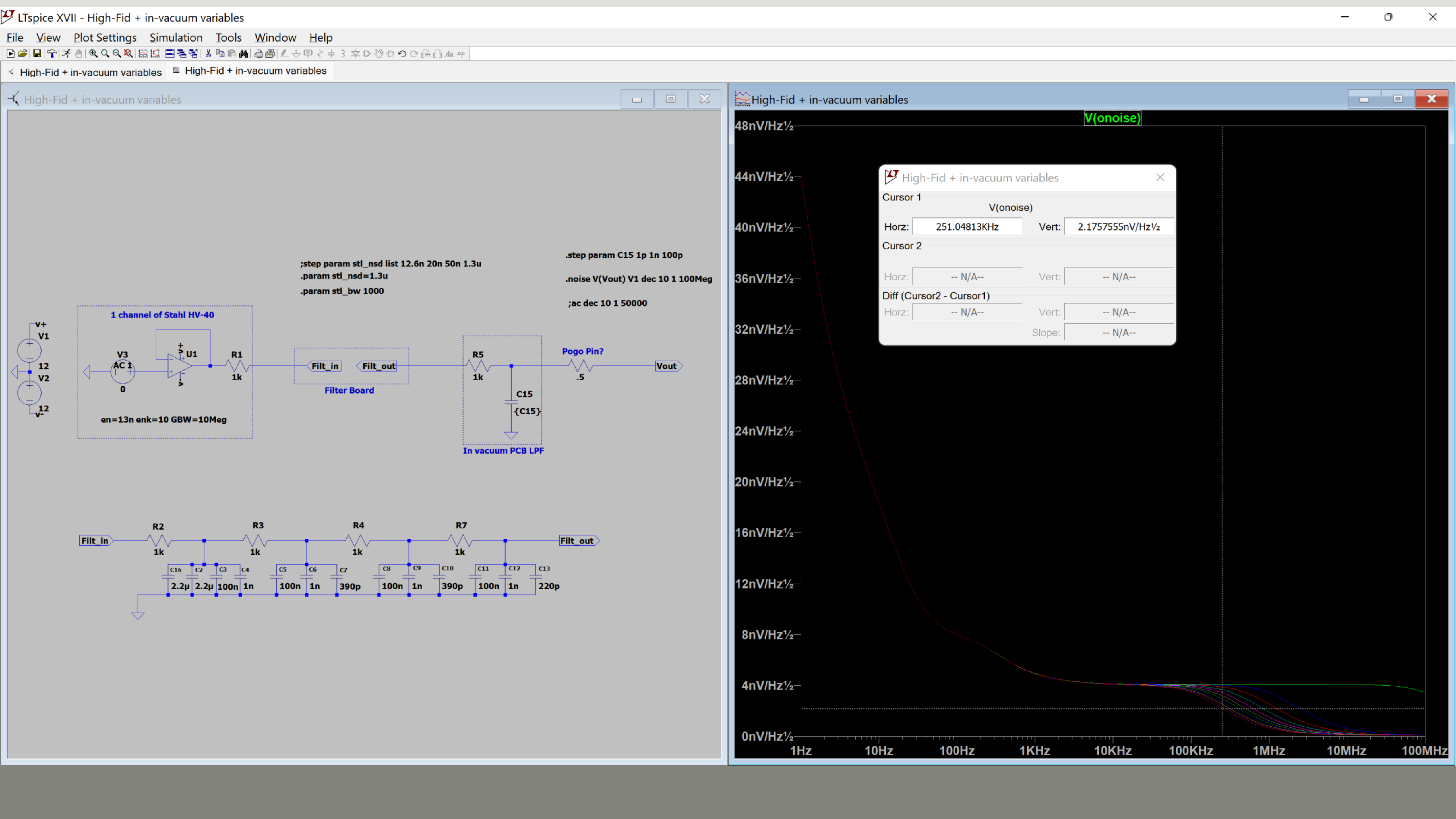
Filter redesign

- Voltage noise output from a channel of the *Stahl HV-40* is **~ 13 *nV/Hz$^{1/2}$*** ,which corresponds to setting the SV slider at **$1.7\text{x}10^{-16}$ V^2/Hz.** (relatively high noise)
- After passing through the filter box and the in-vacuum filters *(R = 1kΩ & C = 620pF),* the voltage noise at the chip electrodes is **~ 2.9 nV/Hz$^{1/2}$** (SV slider at **$8.4\text{x}10^{-18}$ V^2/Hz**).
- Assuming we redesign and change the in-vacuum capacitors with same size but *C = 1000pF,* the electrode noise drops to **~ 2.2 nV/Hz$^{1/2}$** (SV slider at **$4.8\text{x}10^{-18}$ V^2/Hz**).

  (LTspice simulation screenshots in the next slides)
- This leads to a negligible fidelity improvement due to the effect of electrode pairing.
- Thus, we have to focus on our dominant noise source : surface/anomalous heating.



LTspice XVII - High-Fid-system
File View Plot Settings Simulation Tools Window Help
High-Fid-system
High-Fid-system
;step param stl_nsd list 12.6n 20n 50n 1.3u
.param stl_nsd=1.3u
.param stl_bw 1000
1 channel of Stahl HV-40
.noise V(Vout) V1 dec 10 1 100Meg
;ac dec 10 1 50000
V+
V1
12
V2
12
V-
V3
AC 1
0
U1
R1
1k
Filt_in
Filt_out
Filter Board
R5
1k
C15
620p
In vacuum PCB LPF
Pogo Pin?
.5
Vout
en=13n enk=10 GBW=10Meg
R2
1k
R3
1k
R4
1k
R7
1k
Filt_in
Filt_out
C16 2.2µ
C2 2.2µ
C3 100n
C4 1n
C5 100n
C6 1n
C7 390p
C8 100n
C9 1n
C10 390p
C11 100n
C12 1n
C13 220p
V(onoise)
48nV/Hz½
44nV/Hz½
40nV/Hz½
36nV/Hz½
32nV/Hz½
28nV/Hz½
24nV/Hz½
20nV/Hz½
16nV/Hz½
12nV/Hz½
8nV/Hz½
4nV/Hz½
0nV/Hz½
1Hz 10Hz 100Hz 1KHz 10KHz 100KHz 1MHz 10MHz 100MHz
High-Fid-system
Cursor 1
V(onoise)
Horz: 251.18864KHz
Vert: 2.9023135nV/Hz½
Cursor 2
Horz: -- N/A--
Vert: -- N/A--
Diff (Cursor2 - Cursor1)
Horz: -- N/A--
Vert: -- N/A--
Slope: -- N/A--


LTspice XVII - High-Fid + in-vacuum variables
File View Plot Settings Simulation Tools Window Help
High-Fid + in-vacuum variables
High-Fid + in-vacuum variables
;step param stl_nsd list 12.6n 20n 50n 1.3u
.param stl_nsd=1.3u
.param stl_bw 1000
.step param C15 1p 1n 100p
.noise V(Vout) V1 dec 10 1 100Meg
;ac dec 10 1 50000
1 channel of Stahl HV-40
V+
V1
12
V2
12
V-
V3
AC 1
0
U1
R1
1k
en=13n enk=10 GBW=10Meg
Filt_in
Filt_out
Filter Board
R5
1k
C15
{C15}
In vacuum PCB LPF
Pogo Pin?
.5
Vout
Filt_in
R2
1k
R3
1k
R4
1k
R7
1k
Filt_out
C16 2.2µ
C2 2.2µ
C3 100n
C4 1n
C5 100n
C6 1n
C7 390p
C8 100n
C9 1n
C10 390p
C11 100n
C12 1n
C13 220p
V(onoise)
48nV/Hz½
44nV/Hz½
40nV/Hz½
36nV/Hz½
32nV/Hz½
28nV/Hz½
24nV/Hz½
20nV/Hz½
16nV/Hz½
12nV/Hz½
8nV/Hz½
4nV/Hz½
0nV/Hz½
1Hz
10Hz
100Hz
1KHz
10KHz
100KHz
1MHz
10MHz
100MHz
Cursor 1
V(onoise)
Horz: 251.04813KHz
Vert: 2.1757555nV/Hz½
Cursor 2
Horz: -- N/A--
Vert: -- N/A--
Diff (Cursor2 - Cursor1)
Horz: -- N/A--
Vert: -- N/A--
Slope: -- N/A--

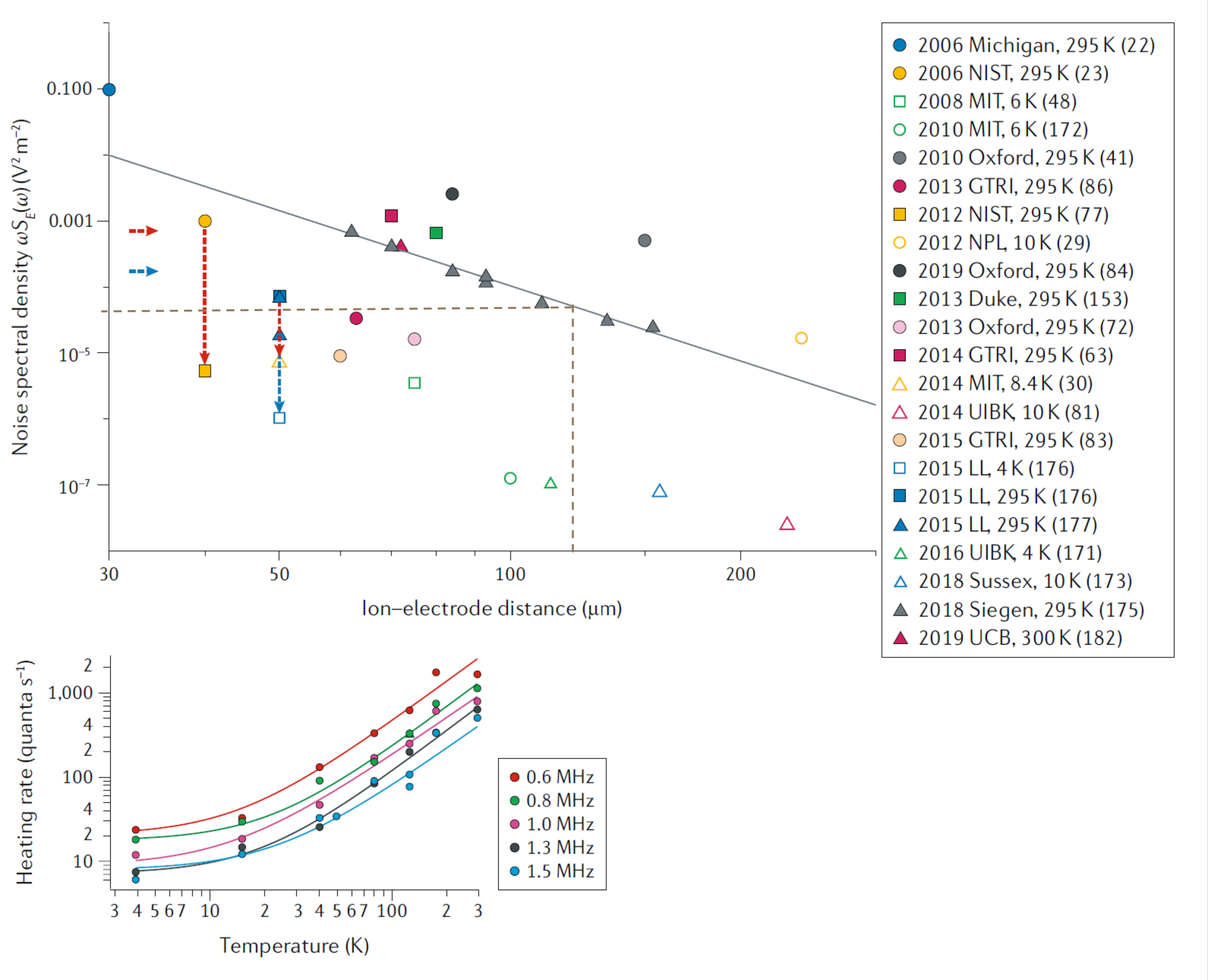
Anomalous Heating

- Thought to be induced by electric field noise from the surface of the chip.
- Modelled as random distribution of adsorbed atoms (fluctuating charges) producing $\mathrm{SE} \propto \omega^{-1}$.
- A good measure of anomalous heating is the ions' heating rates.
- Methods to reduce the heating rate:
  - Increase the distance *d* between the ion and the chip surface (since $\omega\mathrm{SE} \propto d^{-4}$).

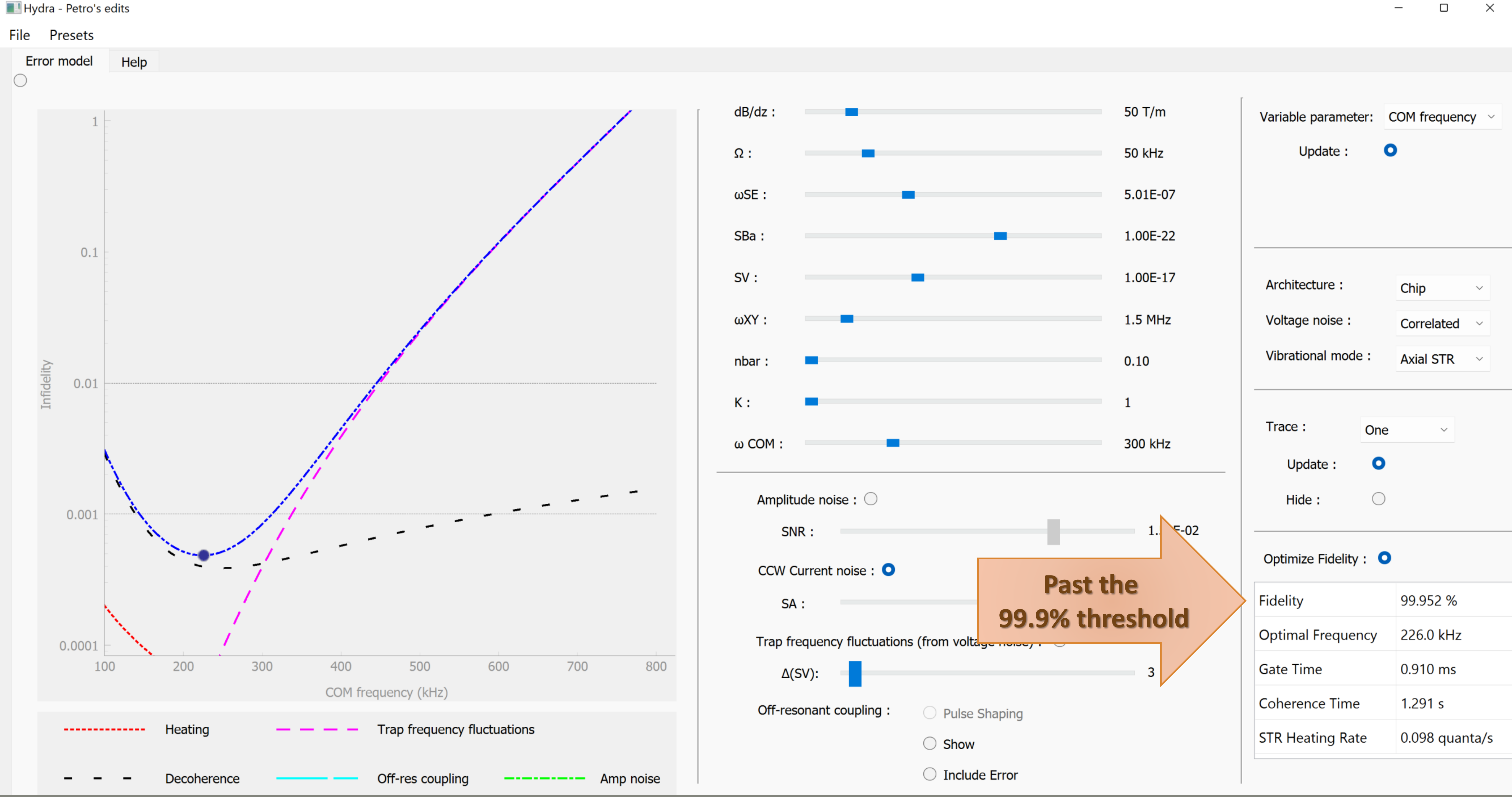
    The disadvantage of this is that it leads to a shallower trap depth due to hardware limitations.
  - Cooling the ion-trap to cryogenic temperatures.
  - In-situ cleaning of the chip's surface to remove adsorbed contaminants.

D. T. C. Allcock, L. Guidoni, T. P. Harty, C. J. Ballance, M. G. Blain, A. M. Steane, and D. M. Lucas,
"Reduction of heating rate in a microfabricated ion trap by pulsed-laser cleaning", New Journal of Physics 13, 123023 (2011)

- At 125 µm and 295 K, we should expect $\omega S_E \sim 5 \times 10^{-5}$.
  (brown dashed line)
- Following the plot below, by cooling the trap to 70 K, we should expect the heating rate (and thus ωS_E) to decrease by an order of magnitude.
  (also blue dashed line in top plot)
  (so $\omega S_E \sim 5 \times 10^{-6}$)
- We can also expect to drop by another order of magnitude with proper surface cleaning
  (red dashed line in top plot)
  (so best-case scenario $\omega S_E \sim 5 \times 10^{-7}$)

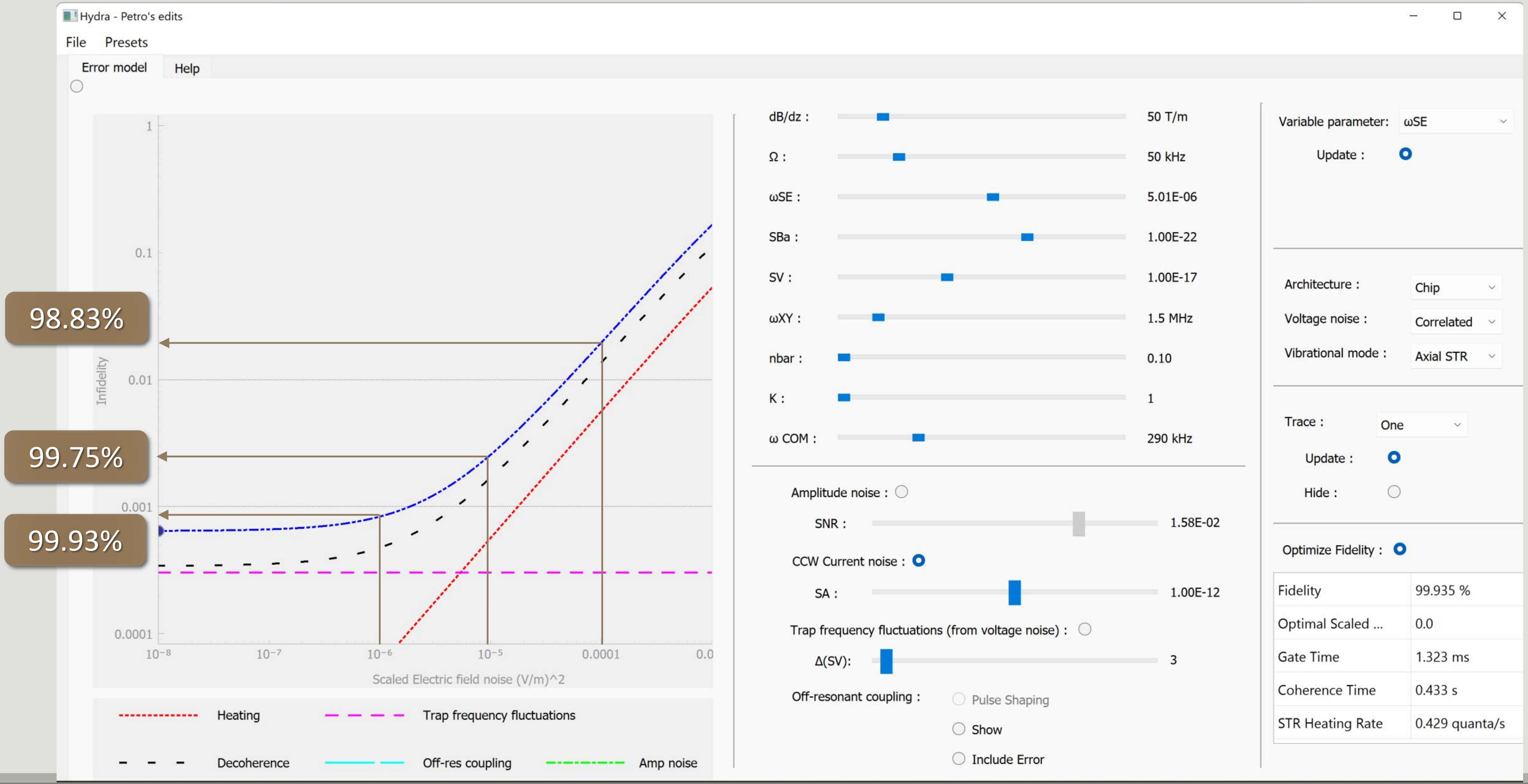


Z. D. Romaszko, S. Hong, M. Siegele, R. K. Puddy, F. R. Lebrun-Gallagher, S. Weidt, and W. K. Hensinger, “Engineering of microfabricated ion traps and integration of advanced on-chip features”, Nature Reviews Physics 2, 285–299 (2020).

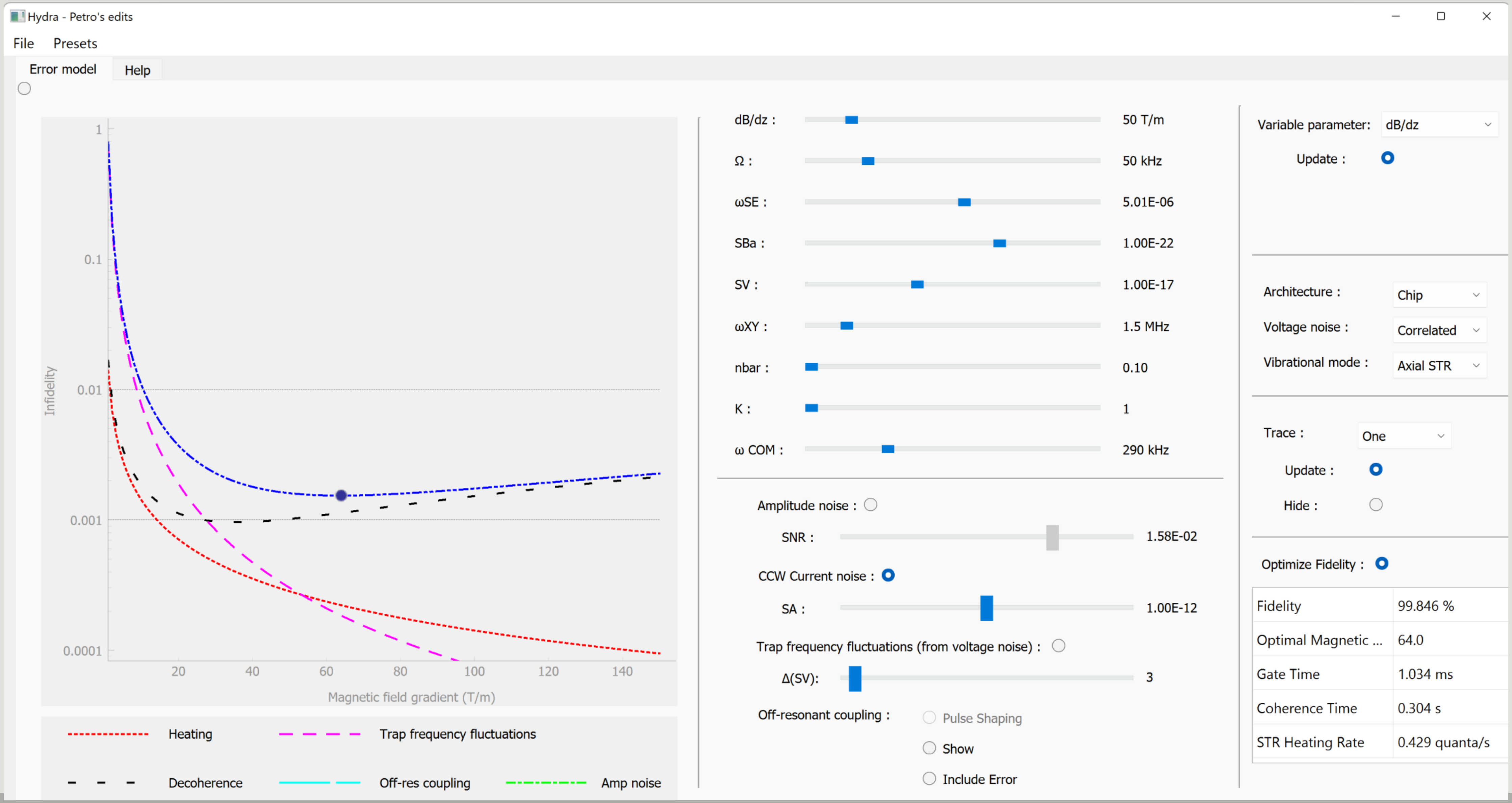

Hydra - Petro's edits
File
Presets
Error model
Help
Infidelity
1
0.1
0.01
0.001
0.0001
100
200
300
400
500
600
700
800
COM frequency (kHz)
Heating
Trap frequency fluctuations
Decoherence
Off-res coupling
Amp noise
dB/dz :
50 T/m
Ω :
50 kHz
ωSE :
5.01E-07
SBa :
1.00E-22
SV :
1.00E-17
ωXY :
1.5 MHz
nbar :
0.10
K :
1
ω COM :
300 kHz
Amplitude noise :
SNR :
CCW Current noise :
SA :
Trap frequency fluctuations (from volta
Δ(SV):
3
Off-resonant coupling :
Pulse Shaping
Show
Include Error
Past the
99.9% threshold
Variable parameter:
COM frequency
Update :
Architecture :
Chip
Voltage noise :
Correlated
Vibrational mode :
Axial STR
Trace :
One
Update :
Hide :
Optimize Fidelity :
Fidelity
99.952 %
Optimal Frequency
226.0 kHz
Gate Time
0.910 ms
Coherence Time
1.291 s
STR Heating Rate
0.098 quanta/s

Parameter Sensitivity

Scaled Electric field noise



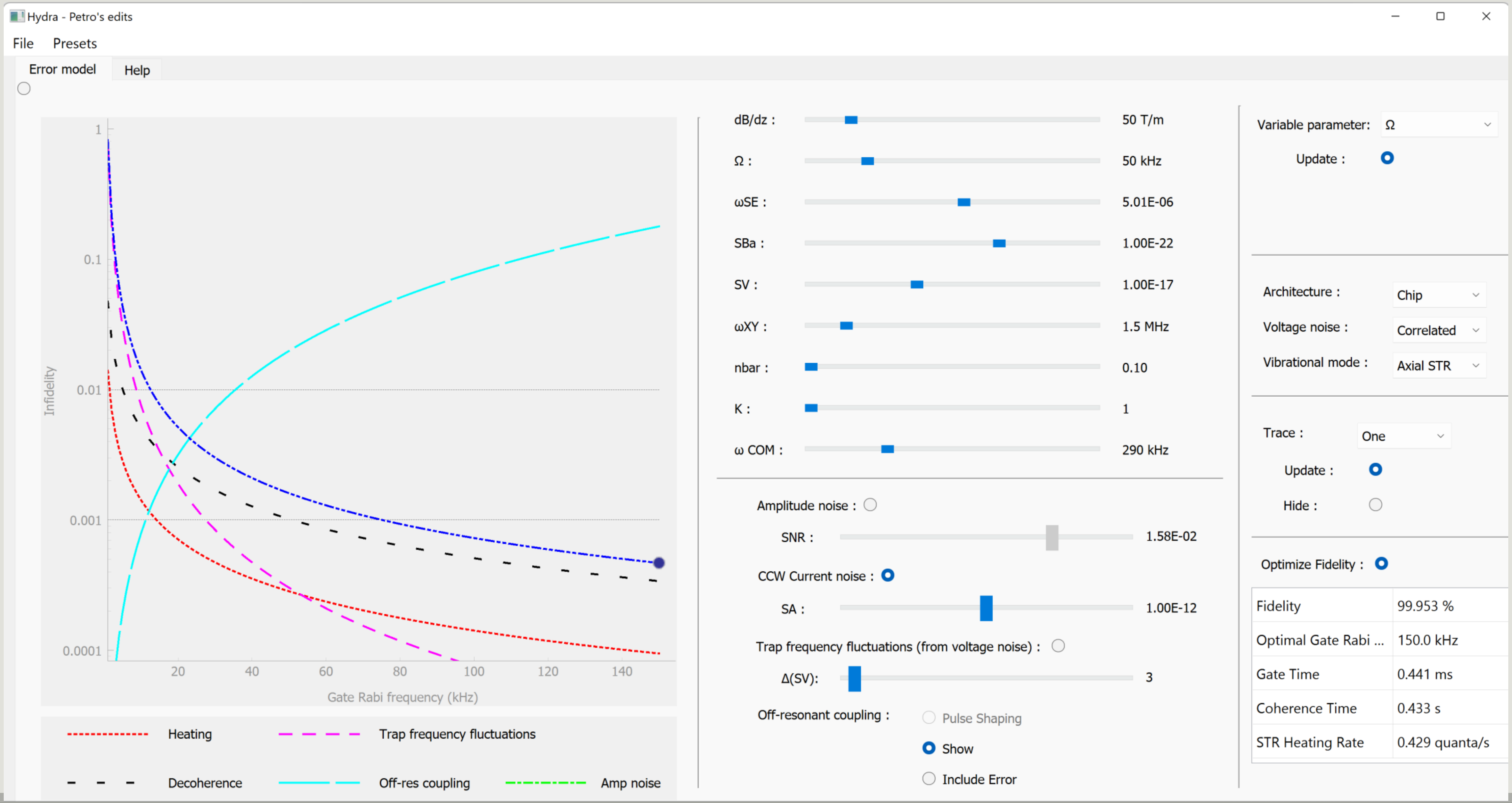
Magnetic field gradient



Higher gradients?

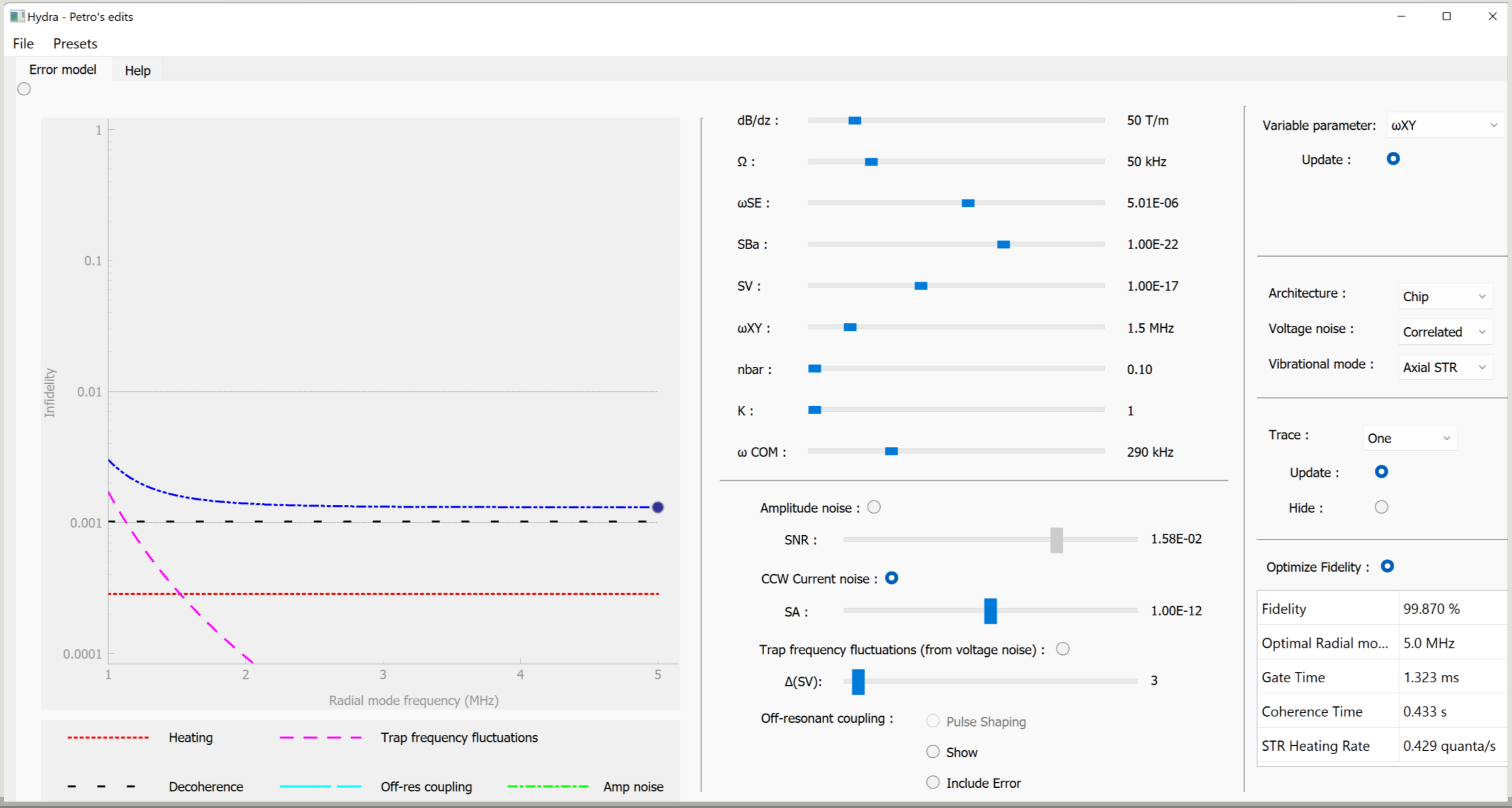
ONLY IF we mitigate the effects of anomalous heating!

Gate Rabi frequency (power)

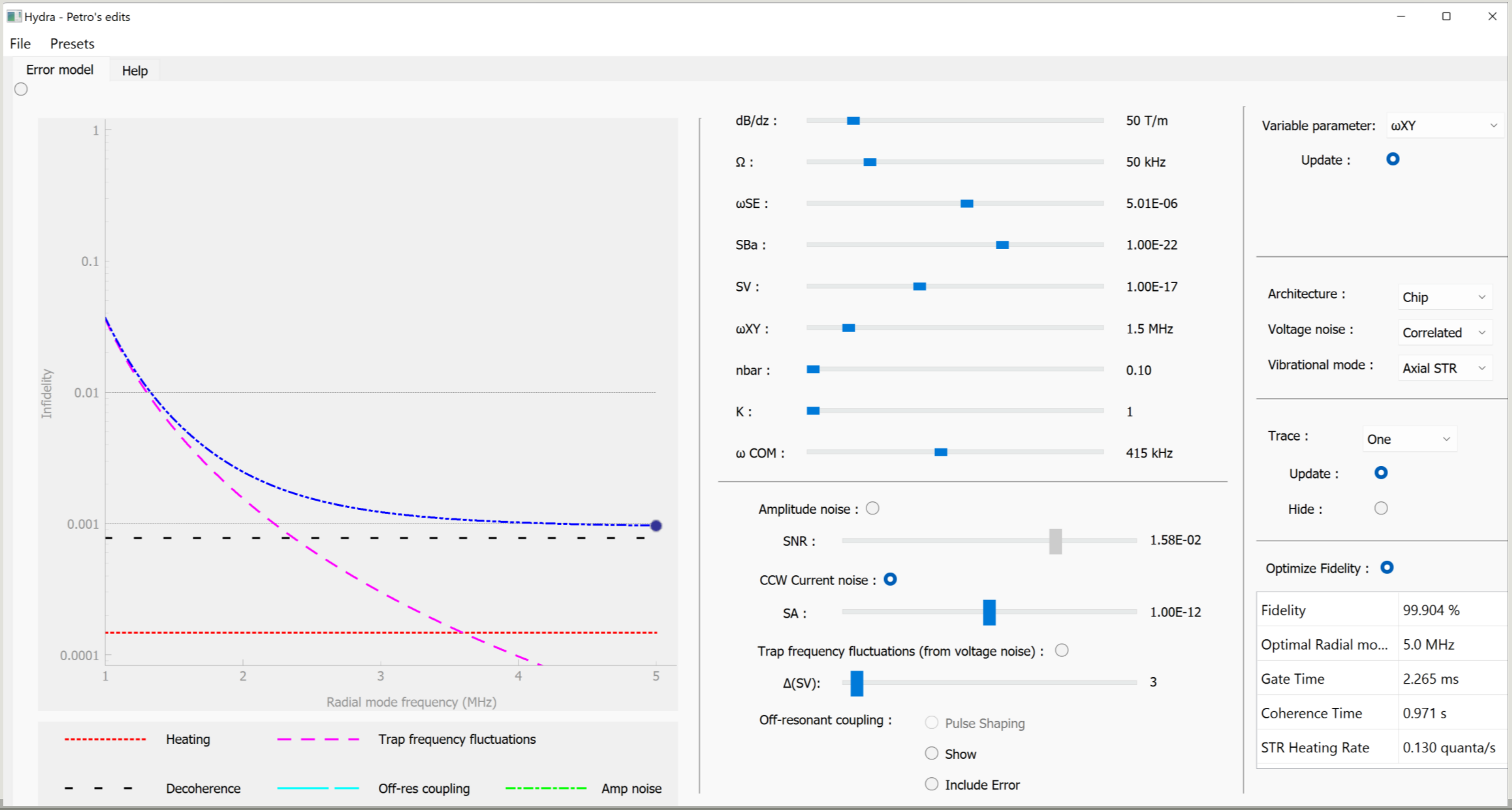


Investigate Power Broadening?

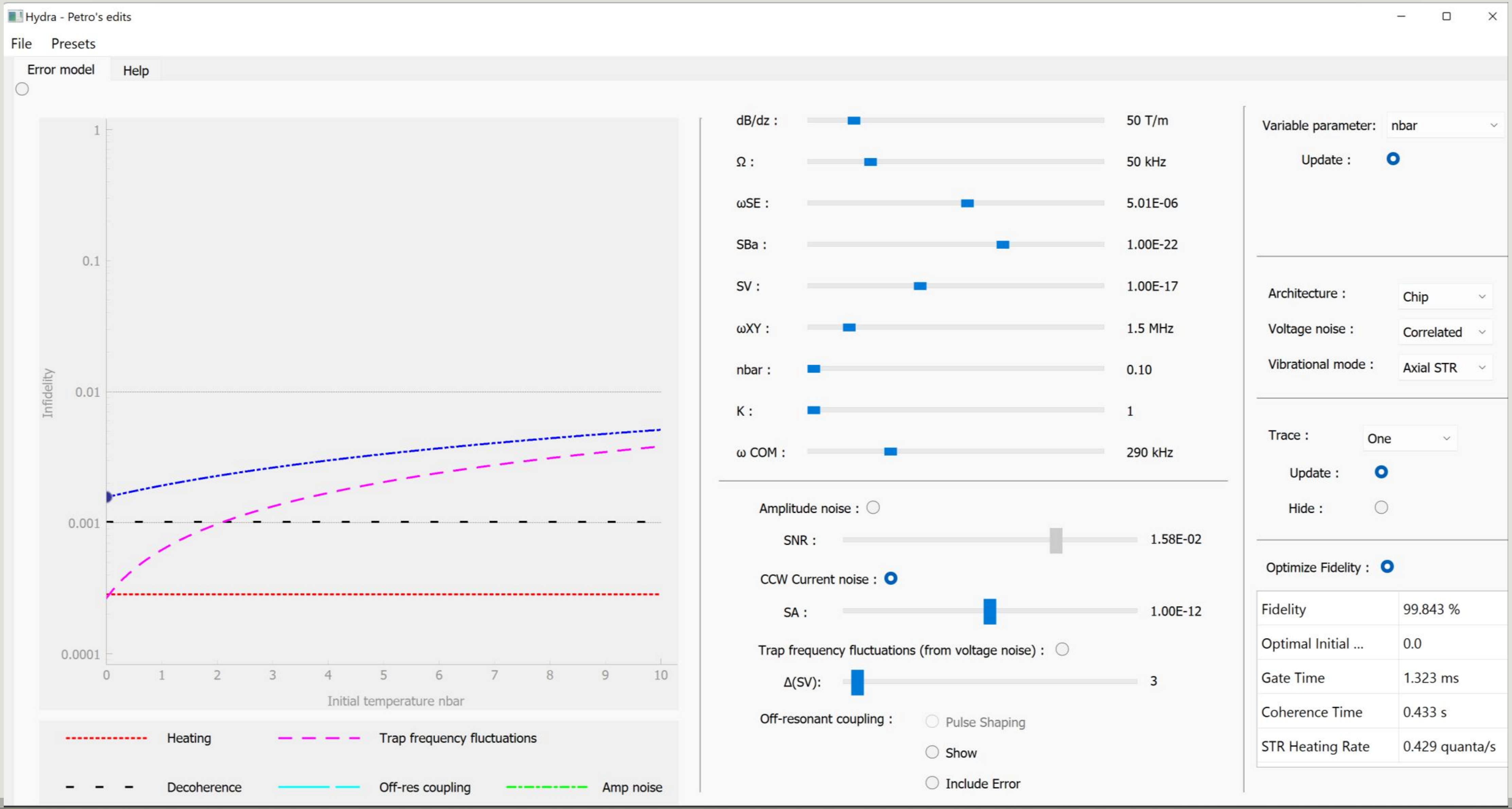
Radial frequency



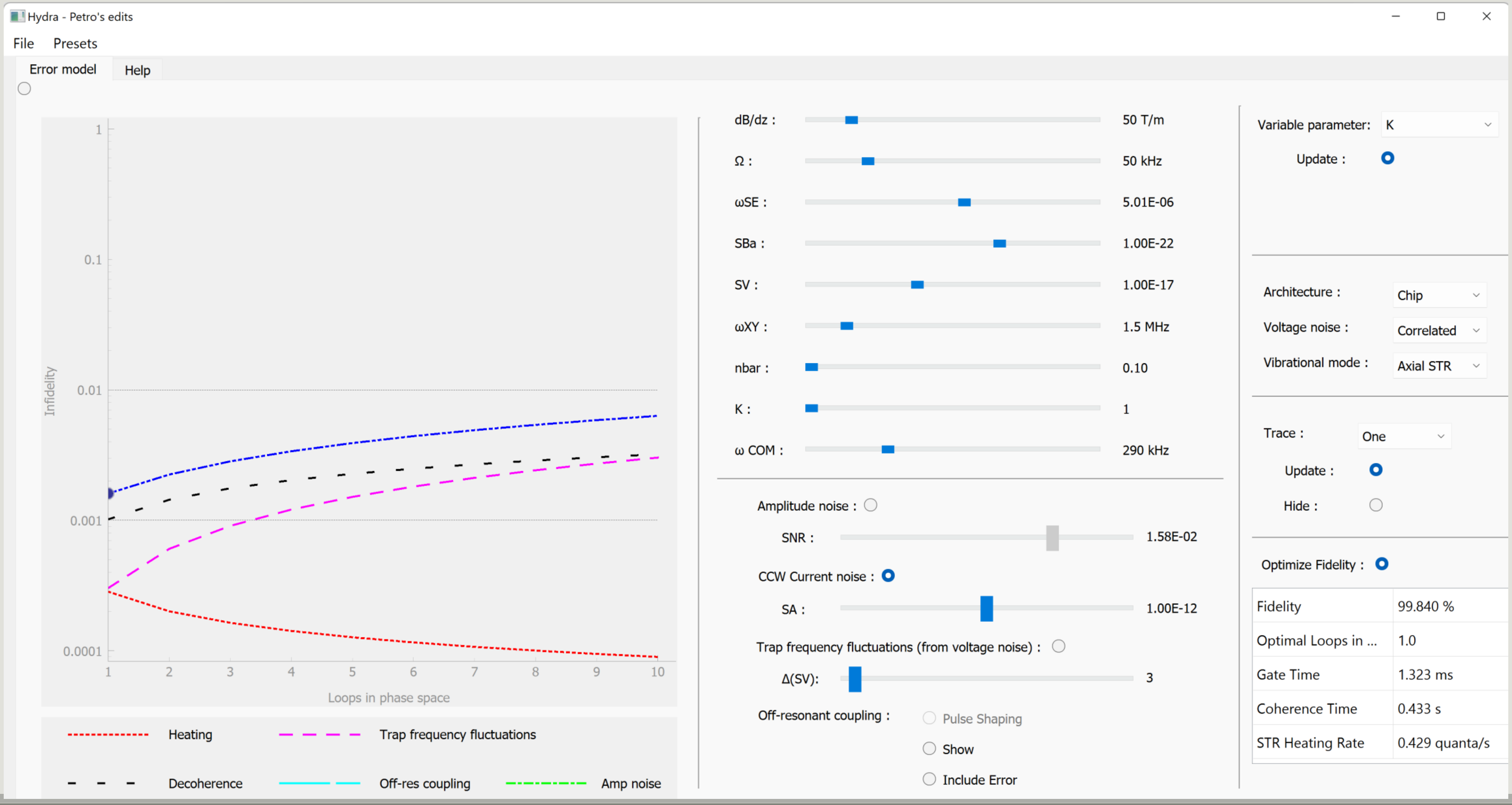
Higher radial + secular frequencies



Initial temperature



Phase space loops



Similar effect with more tones in MTMS

Future

- Investigate whether other terms from literature become dominant when changing parameters (e.g. increasing Ω)

- Make sure the input parameters (e.g. B-ambient, heating rates!) closely resemble our lab/experiment conditions. If not, measure them and modify accordingly .

- Extend to other MS gate schemes (currently Dressed States, but could include PDD & CDD along with phase, frequency and amplitude modulations: PM, FM & AM)

- Possibility to add LO noise, but found to be dominated by voltage noise and magnetic field noise.

- Include effects of mis-set parameters (e.g. incorrect symmetric/asymmetric detuning). This will enhance HYDRA to serve as a sanity checking tool when actually running experiments, and not just as a prediction tool.

PDD = Pulsed Dynamical Decoupling, CDD= Continuous Dynamical Decoupling