

# HYDRA Error Model Presentation

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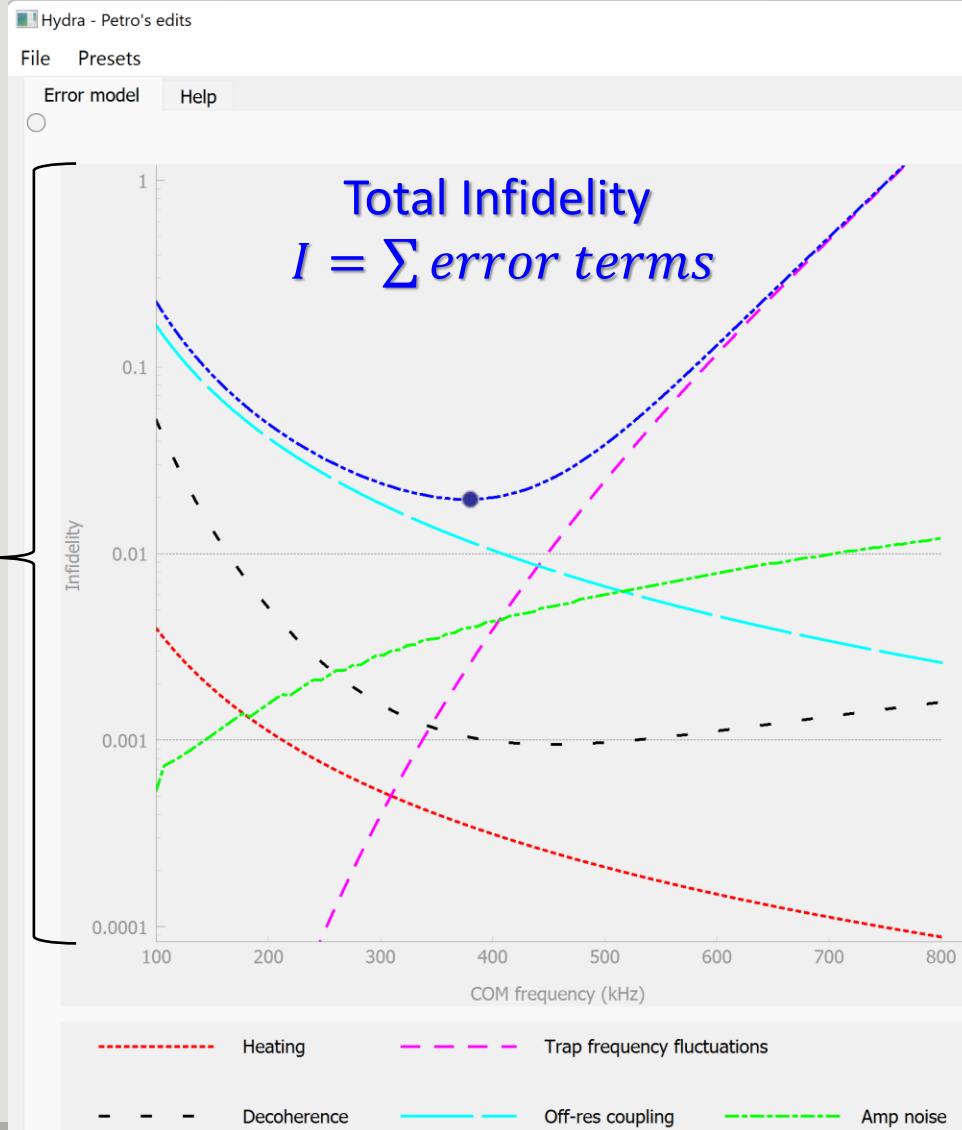
# HYDRA

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*“For every head chopped off, the Hydra would regrow two” →  
For every noise source eliminated, two more pop up.*

- A software which visualises the error models for two-qubit microwave gates using trapped ions in a static magnetic field gradient.
- Assumes perfect calibration of experimental parameters (no mis-set errors).
- Allows the option to optimise the variable parameter for minimising errors (maximising fidelity).
- Calculates maximum achievable fidelity due to inherent error/noise mechanisms.
- Also calculates expected heating rate, gate time and coherence time.

# User Interface



Parameters

Variable parameter: COM frequency

Update :

Architecture : Chip

Voltage noise : Correlated

Vibrational mode : Axial STR

Trace : One

Update :

Hide :

Optimize Fidelity :

Fidelity	98.048 %
Optimal Frequency	380.0 kHz
Gate Time	1.985 ms
Coherence Time	1.262 s
STR Heating Rate	0.348 quanta/s

dB/dz : 50 T/m

$\Omega$  : 50 kHz

$\omega_{SE}$  : 1.00E-05

SBa : 1.00E-22

SV : 2.51E-17

$\omega_{XY}$  : 1.5 MHz

nbar : 0.10

K : 1

$\omega_{COM}$  : 296 kHz

Amplitude noise :

SNR : 1.26E-02

CCW Current noise :

SA : 1.00E-12

Trap frequency fluctuations (from voltage noise) :

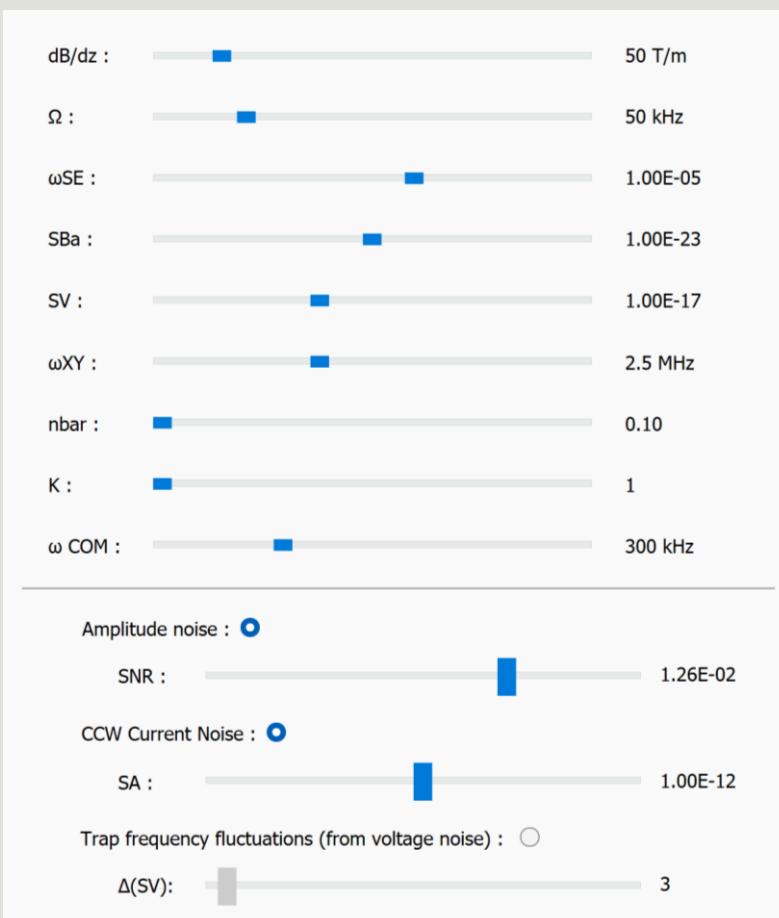
$\Delta(SV)$  : 3

Off-resonant coupling :  Pulse Shaping  
 Show  
 Include Error

Independent Variable

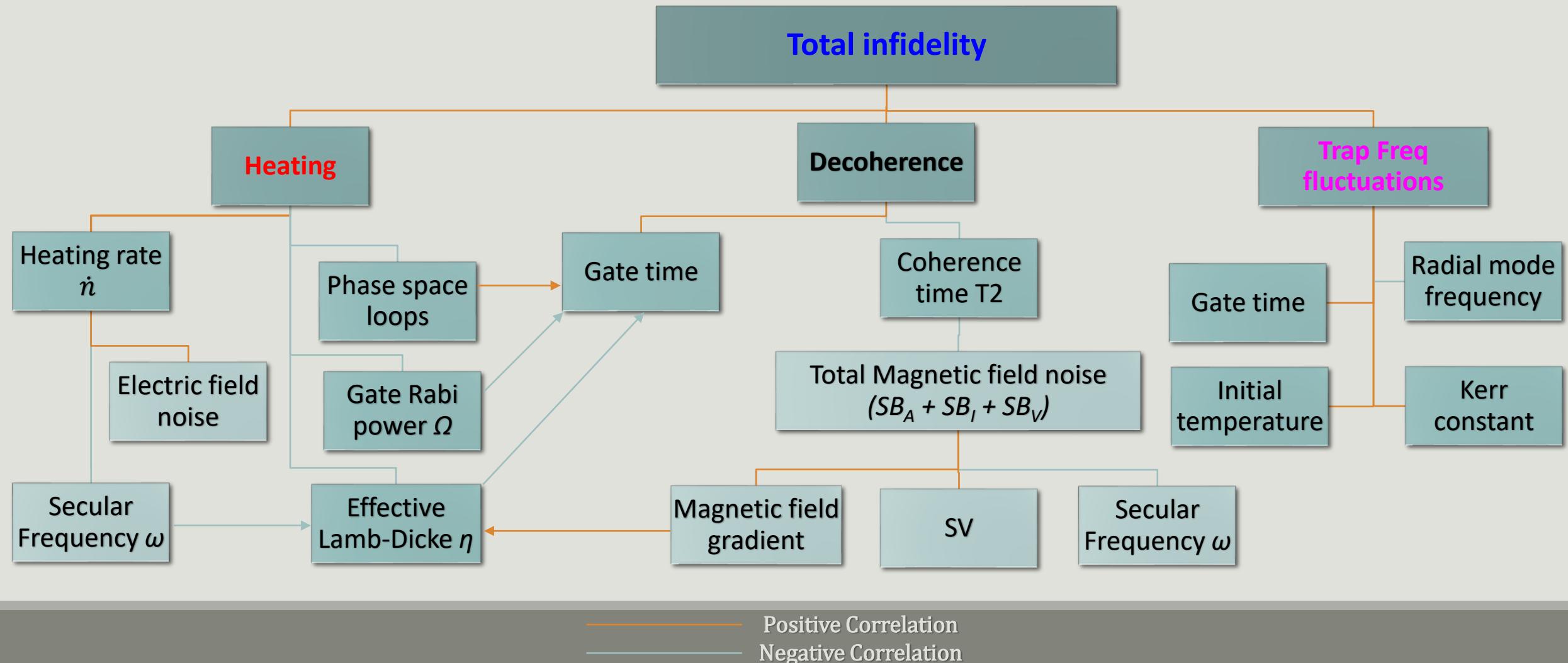
Optimised Output

# Parameters



- Magnetic field gradient
- Gate Rabi frequency/power
- Scaled Electric field noise PSD (V/m)<sup>2</sup>
- Ambient Magnetic field noise PSD (T<sup>2</sup>/Hz)
- Voltage noise PSD (V<sup>2</sup>/Hz)
- Radial mode frequency
- Initial temperature  $\bar{n}$
- Loops in phase space
- Secular COM frequency
- Signal-to-Noise Ratio
- Current noise PSD (A<sup>2</sup>/Hz)
- Variance from voltage noise

# Dependencies



# Heating errors

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- Motional heating changes the populations of the Fock states  $|n\rangle$
- Caused by stray electric fields matching the frequency of the ion's vibrational mode.
- The infidelity due to heating is derived analytically in the original MS paper:

$$\textcolor{red}{err}_{\text{heating}} : 1 - \frac{1}{8} ( 3 + 4e^{-\frac{\dot{n}\pi}{2\varphi\eta\Omega}} + e^{-\frac{2\dot{n}\pi}{\varphi\eta\Omega}} )$$

Where  $\dot{n}$  is the heating rate,  $\varphi$  is the heating factor ( $\sqrt{k}$  for a k-loop MS gate; tone-dependent for Multi-Tone MS gate),  $\eta$  is the effective Lamb-Dicke parameter and  $\Omega$  is the gate Rabi frequency.

# Heating errors

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- For 2 ions, there are two vibrational modes: the centre-of-mass (COM) mode and the stretch (STR) mode.
- Heating the STR mode is more difficult since the two ions vibrate out of phase.
- Also, electric field noise PSD usually scales as  $1/\omega$ , and since  $\omega_{STR} = \sqrt{3} \omega_{COM}$ , the STR mode experiences less noise.

$$\dot{n}_{COM} = \frac{e^2}{4 m \hbar \omega_z^2} * \omega SE \quad \text{or} \quad \dot{n}_{STR} = \frac{e^2}{4 m \hbar \omega_z^2} * \omega SE * \left(\frac{\Delta z}{d}\right)^2$$

Where  $\omega_z$  is the secular frequency,  $d$  is the ion-electrode distance,

$$\text{and the ion-ion spacing } \Delta z = \frac{2}{2^{2/3}} \left( \frac{e^2}{4\pi\epsilon_0 m \omega_{com}^2} \right)^{1/3}$$

# Decoherence errors

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- Depolarisation and dephasing arise from magnetic field noise which fluctuates the energy levels of the qubit.
- Exact expression is dependent on the chosen gate scheme.
- For the dressed state scheme, the decoherence of the  $|D\rangle$  state is analytically derived and numerically confirmed by simulating the full Hamiltonians.

$$err_{decoherence} : \frac{1}{3} \left(1 - e^{-\frac{t_{gate}}{T_2}}\right)$$

$$\eta_{eff} = \frac{\mu_B}{\sqrt{2} \hbar \omega_z} * \frac{\partial B}{\partial z} * \sqrt{\frac{\hbar}{2 m \omega_z}}$$

$$\text{Coherence time } T_2 = \frac{\hbar^2}{\mu_B^2 S B}$$

$$t_{gate} = \frac{\pi}{\eta * \Omega} * Cost \longrightarrow$$

*Trade-off for using MTMS or PM gates ( $\sqrt{k}$  for a k-loop MS gate; tone-dependent for Multi-Tone MS gate)*

# Decoherence errors

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Total PSD of B-field noise  $SB = SB_A + SB_I + SB_V$

$SB_A \rightarrow$  Input parameter

$$SB_I = \left( \frac{\partial B}{\partial I} \right)^2 * SA$$

$$\frac{\partial B}{\partial I} = \frac{(1.6827 * 10^{-5}) * dx}{10 z_0^{1.78}}$$

$$dx = 4 \times 10^{-9}$$

(Ion's position resolution  
allowable with DAC)

$$SB_V = \left( \frac{\partial B}{\partial V} \right)^2 * SV$$

$$\frac{\partial B}{\partial V} = \frac{e g}{m \omega_z^2} * \frac{\partial B}{\partial z}$$

( $g$  = Geometric factor for voltage noise  
found by numerically simulating  
chip geometries)

# Geometric factor

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$$g = \frac{\partial E}{\partial V}$$

1<sup>st</sup> nn: 25 m<sup>-1</sup>, 2<sup>nd</sup> nn: 15 m<sup>-1</sup>, 3<sup>rd</sup> nn: 8 m<sup>-1</sup>, 4<sup>th</sup> nn: 5 m<sup>-1</sup>, etc...

$$g \approx \sqrt{25^2 \times 4 + 15^2 \times 4 + 8^2 \times 4 + 5^2 \times 4} \approx 62 \text{ m}^{-1}$$



Incoherent sum of independent sources

# Trap frequency fluctuation errors

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- Non-linear coupling between the axial stretch mode ( $\omega_{STR}$ ) and the radial rocking modes ( $\omega_r$ ) causes a Kerr-type effect.
- The Kerr constant  $\chi$  describes the coupling strength and is derived analytically.

$$\textcolor{magenta}{err}_{\text{trap\_freq\_fluct}} : \left( 2 * K_{var} + (2\pi \Delta V)^2 \right) * t_{gate}^2 * \left( \frac{1+2(1+2 \bar{n})}{16} \right)$$

$$K_{var} = \chi^2 (\bar{n}_r)(2 \bar{n}_r + 1)$$

where the Kerr coefficient  $\chi = -\omega_{STR} \left( \frac{1}{2} + \frac{\omega_{STR}^2}{2(4\omega_r^2 - \omega_{STR}^2)} \right) \left( \frac{\omega_{COM}}{\omega_r} \right) \left( \frac{2 \hbar \omega_{COM}}{\alpha^2 m c^2} \right)^{1/3}$

$$\omega_r = \sqrt{\omega_{xy}^2 - \omega_{COM}^2} \quad \text{and the Radial mode temperature } \bar{n}_r = \frac{19.6}{2 \omega_{xy}} \longrightarrow S \rightarrow P \text{ linewidth in MHz}$$

$\longrightarrow$  Radial mode frequency

# Amplitude noise errors

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- Noise in the dynamical decoupling drive amplitude will eventually cause decoherence in the same way that magnetic field fluctuations lead to dephasing.
- The associated infidelity is derived by evaluating the errors due to amplitude noise during a Rabi oscillation, using the PSD of Rabi frequency noise.

$$\text{err}_{\text{amp\_noise}} : 1 - e^{-\frac{t_{\text{gate}} * \text{amp\_noise} * \omega_{dd}^2}{2}}$$

$$\text{amp\_noise} = \text{Amp\_PSD}(f) = \frac{A}{f^2 + f_0^2} = \frac{2 \text{SNR}^2 f_0}{\pi (f^2 + f_0^2)}$$

with sampling frequency  $f_s = \frac{n_{\text{rot}}}{t_{\text{gate}}}$  and  $n_{\text{rot}} = \text{int} \left( \frac{t_{\text{gate}}}{T_{\text{rot}}} \right) = \text{int} \left( \frac{t_{\text{gate}}}{2\pi/\omega_{dd}} \right)$

$f_0 = 1000 \text{ Hz}$ , Power of continuous DD drives  $\omega_{dd} = 2\pi 30 \times 10^3$

# Off-resonant coupling errors

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- The MS sideband fields can couple to the carrier transitions and induce errors.
- Not a priority since not dominant (at low powers) & can be eliminated by pulse shaping.
- Not accurately modelled in HYDRA, but can be included by numerically simulating the Hamiltonians.

$$\textcolor{cyan}{err_{off-res}} : 2 \left( \frac{\Omega}{\delta} \right)^2$$

*where the detuning  $\delta = \omega_{STR}$*

# Preliminary Results

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# Setting the parameters

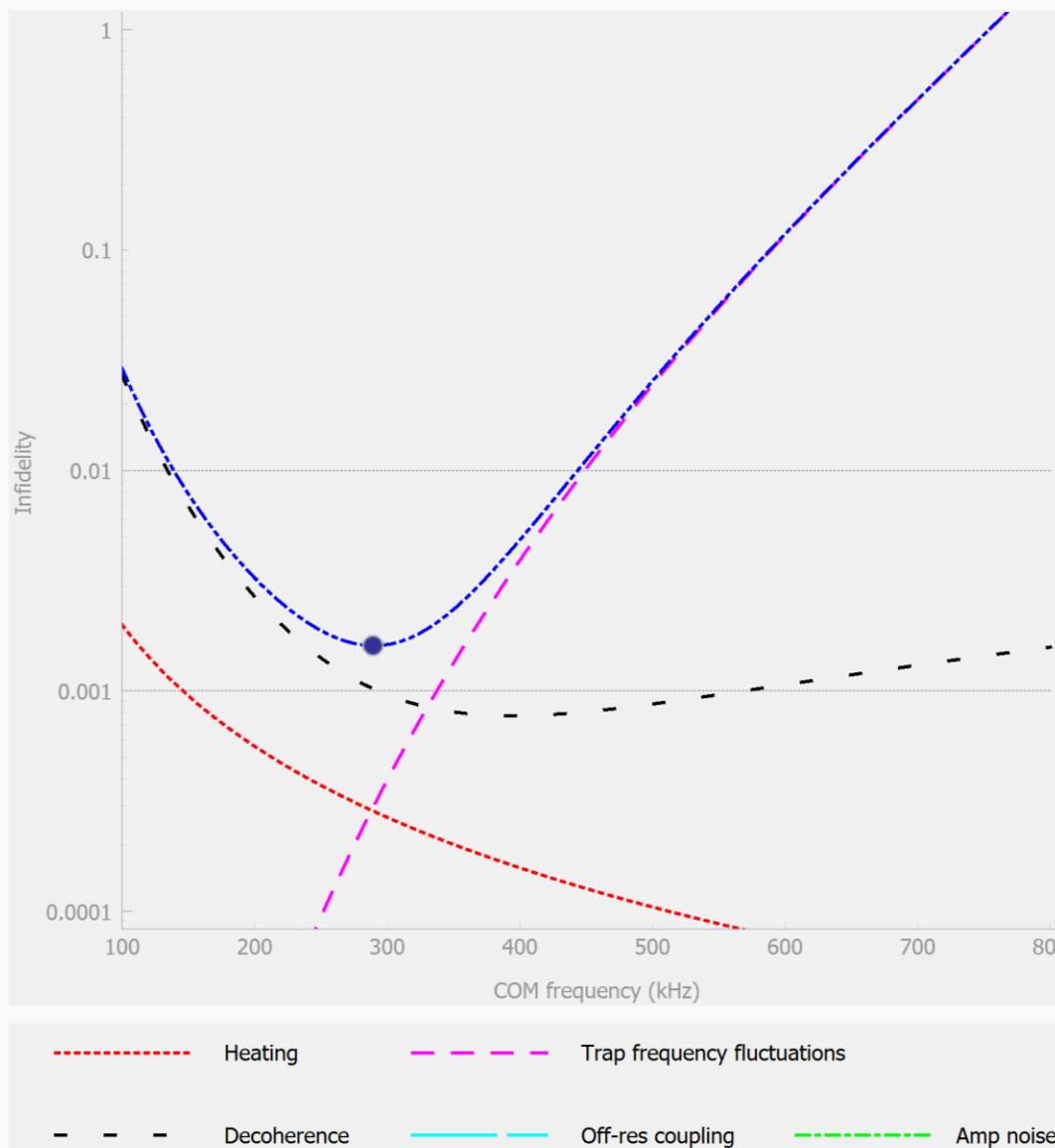
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- B-field gradient : **50 T/m**
- Gate Rabi frequency : **50 kHz**
- Scaled E-noise :  **$5 \times 10^{-6} \text{ V}^2/\text{m}^2$**  (*estimated from literature* using  $d=125 \mu\text{m}$  &  $T=70 \text{ K}$ )
- Ambient B-noise :  **$1 \times 10^{-22} \text{ T}^2/\text{Hz}$**  (*corresponds to ambient B-noise of  $10 \text{ pT}/\text{Hz}^{1/2}$* )
- Voltage noise :  **$1 \times 10^{-17} \text{ V}^2/\text{Hz}$**  (*corresponds to electrode noise  $\sim 3 \text{ nV}/\text{Hz}^{1/2}$* )
- Radial frequency : **1.5 MHz**
- Initial temperature : **0.1**
- Phase space loops : **1** (*primitive MS gate*)
- Current noise :  **$1 \times 10^{-12} \text{ A}^2/\text{Hz}$**  (*corresponds to CCW noise of  $1 \mu\text{A}/\text{Hz}^{1/2}$* )
  
- **Using STR mode and assuming correlated noise in the electrodes**

File Presets

Error model

Help



Variable parameter: COM frequency

Update :

---

Architecture : Chip

Voltage noise : Correlated

Vibrational mode : Axial STR

---

Trace : One

Update :

Hide :

---

Optimize Fidelity :

Fidelity	99.840 %
Optimal Frequency	289.0 kHz
Gate Time	1.316 ms
Coherence Time	1.277 s
STR Heating Rate	0.434 quanta/s

**Expected Optimum**

dB/dz :  50 T/m

$\Omega$  :  50 kHz

$\omega_{SE}$  :  5.01E-06

SBa :  1.00E-22

SV :  1.00E-17

$\omega_{XY}$  :  1.5 MHz

nbar :  0.10

K :  1

$\omega_{COM}$  :  300 kHz

Amplitude noise :

SNR :  1.58E-02

CCW Current noise :

SA :

Trap frequency fluctuations (from voltage) :

$\Delta(SV)$  :  3

Off-resonant coupling :  Pulse Shaping  
 Show  
 Include Error

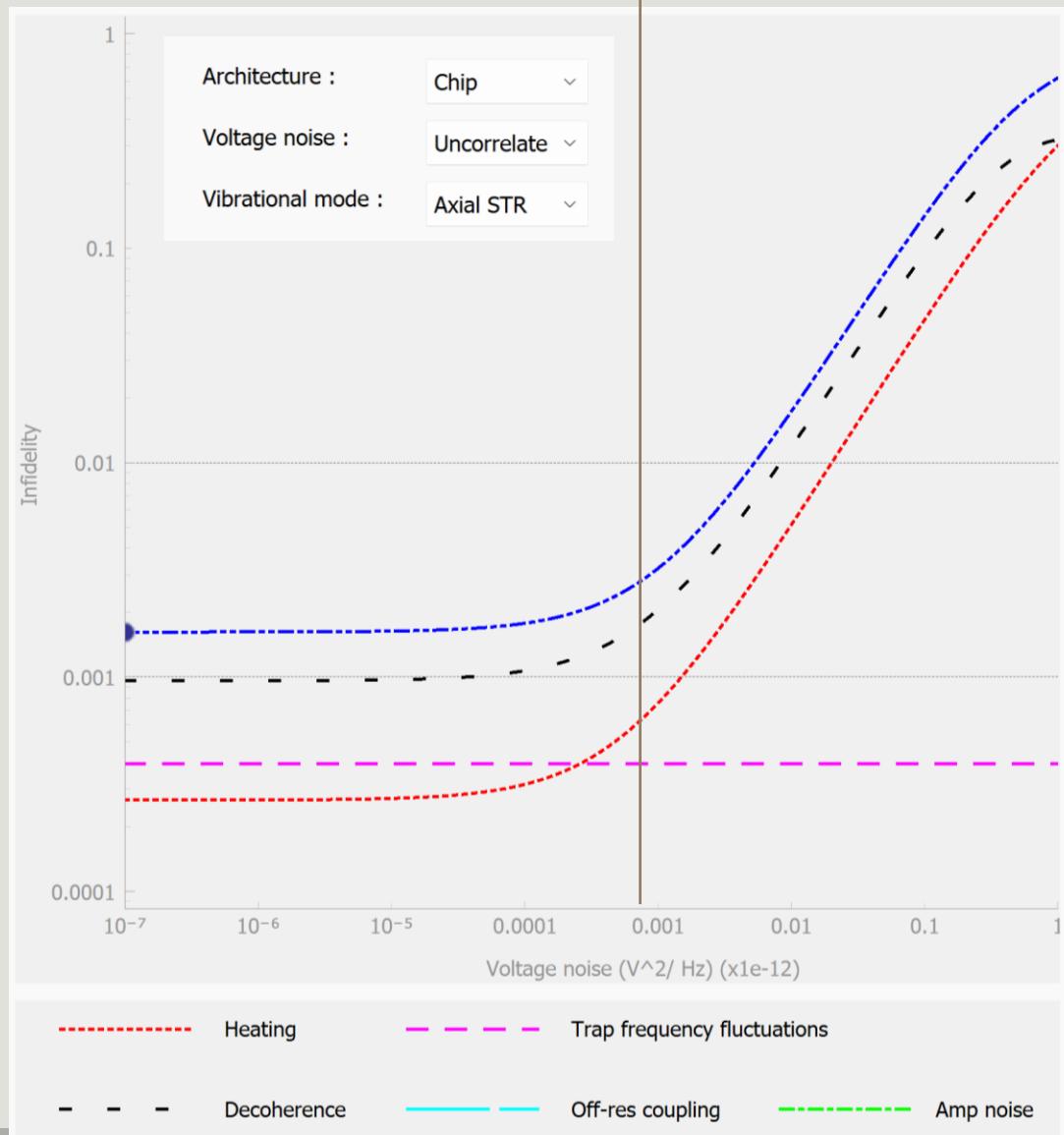
&gt; 99% theoretically achievable!!!

# Electrode pairing

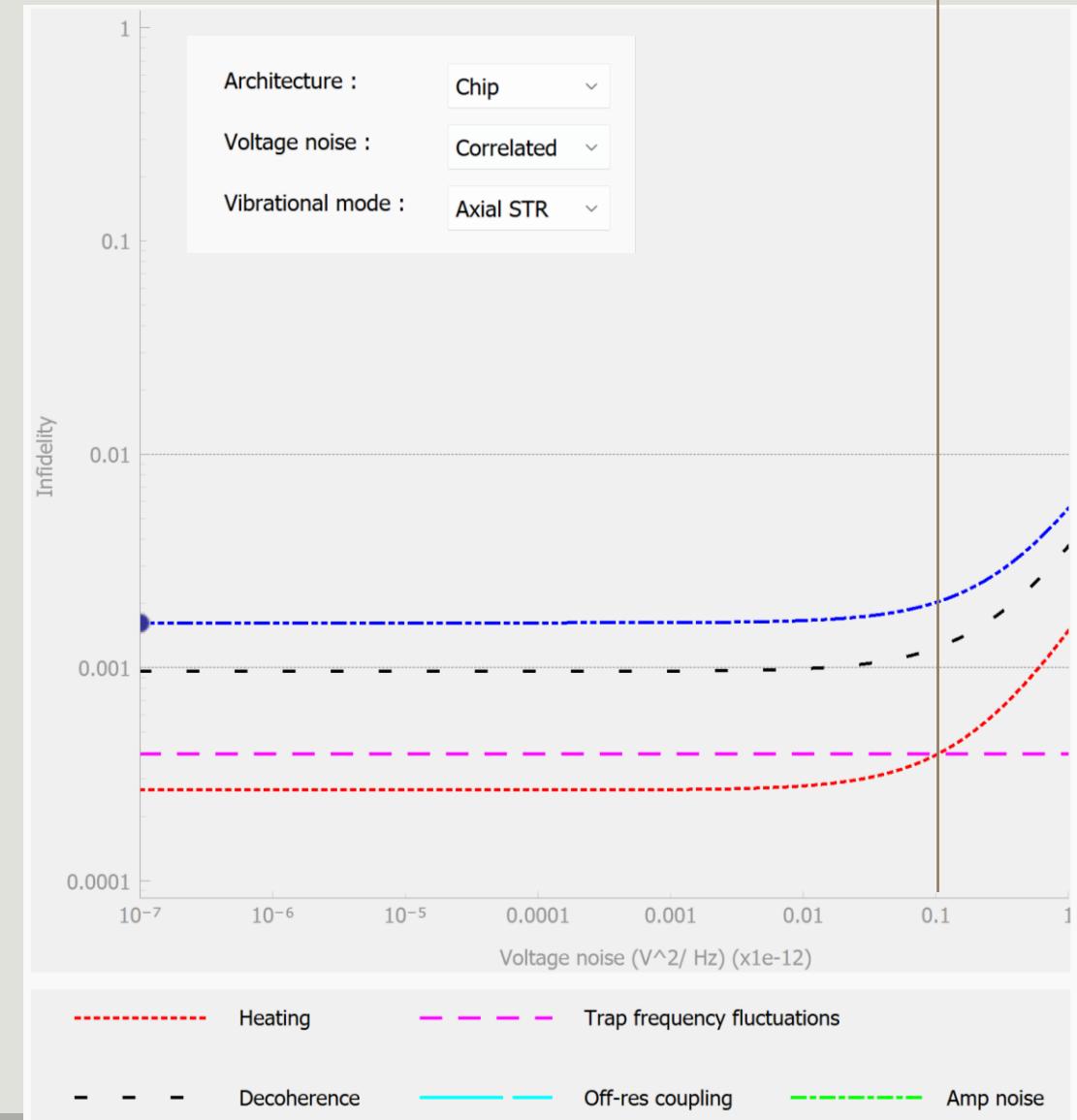
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- In the case of uncorrelated noise between all electrodes, the E-field PSD is an incoherent sum of independent noise sources.
- In the case of correlated noise, the E-field PSD is derived from a coherent sum of the geometric factors for each electrode.
- Therefore, in the latter case, one can engineer the electrode geometry such that the geometric factor summation reduces to  $\sim 0$ .
- Numerical simulations show that this reduction is by a factor of  $\sim 20$ . (*Alex Owens*)
- The resulting effect is increased “resilience” to voltage noise.

➤ Voltage noise above  $30 \text{ nV}/\text{Hz}^{1/2}$  starts causing decoherence.



➤ Voltage noise above  $300 \text{ nV}/\text{Hz}^{1/2}$  starts causing decoherence.



# Filter redesign

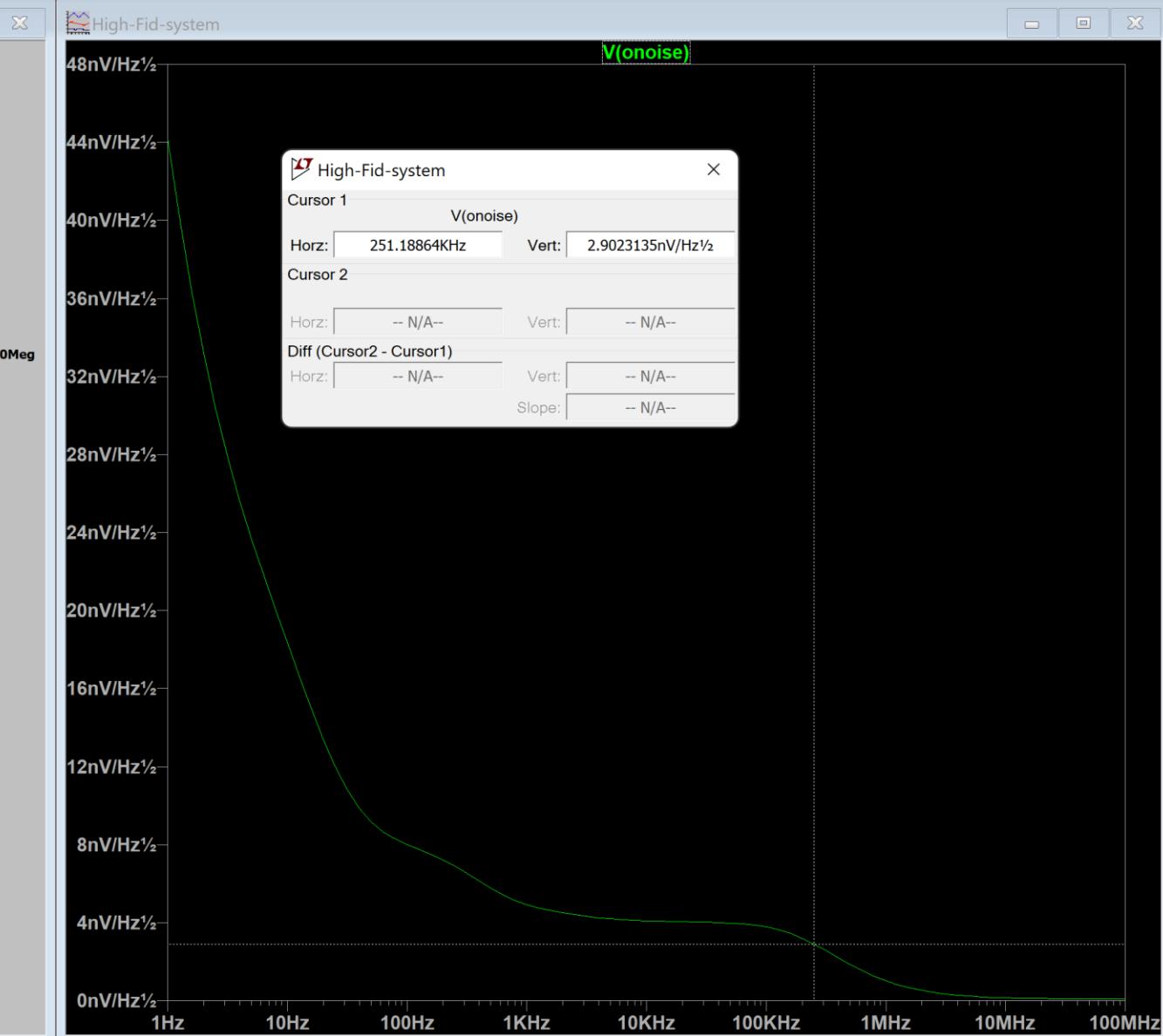
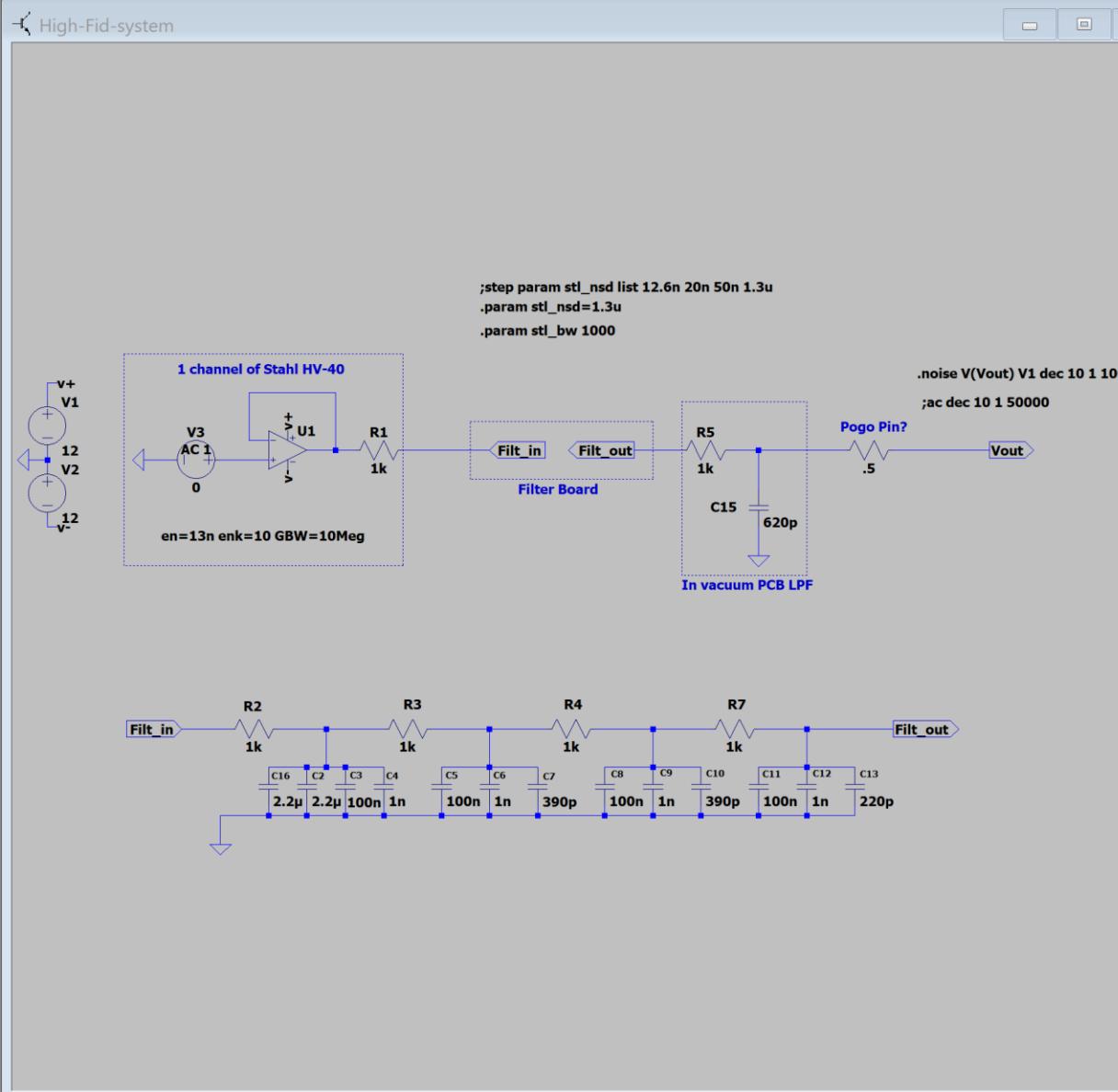
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- Voltage noise output from a channel of the *Stahl HV-40* is  $\sim 13 \text{ nV/Hz}^{1/2}$ , which corresponds to setting the SV slider at  $1.7 \times 10^{-16} \text{ V}^2/\text{Hz}$ . (relatively high noise)
- After passing through the filter box and the in-vacuum filters ( $R = 1k\Omega$  &  $C = 620pF$ ), the voltage noise at the chip electrodes is  $\sim 2.9 \text{ nV/Hz}^{1/2}$  (SV slider at  $8.4 \times 10^{-18} \text{ V}^2/\text{Hz}$ ).
- Assuming we redesign and change the in-vacuum capacitors with same size but  $C = 1000pF$ , the electrode noise drops to  $\sim 2.2 \text{ nV/Hz}^{1/2}$  (SV slider at  $4.8 \times 10^{-18} \text{ V}^2/\text{Hz}$ ).  
*(LTspice simulation screenshots in the next slides)*
- This leads to a negligible fidelity improvement due to the effect of electrode pairing.
- Thus, we have to focus on our dominant noise source : surface/anomalous heating.

File View Plot Settings Simulation Tools Window Help



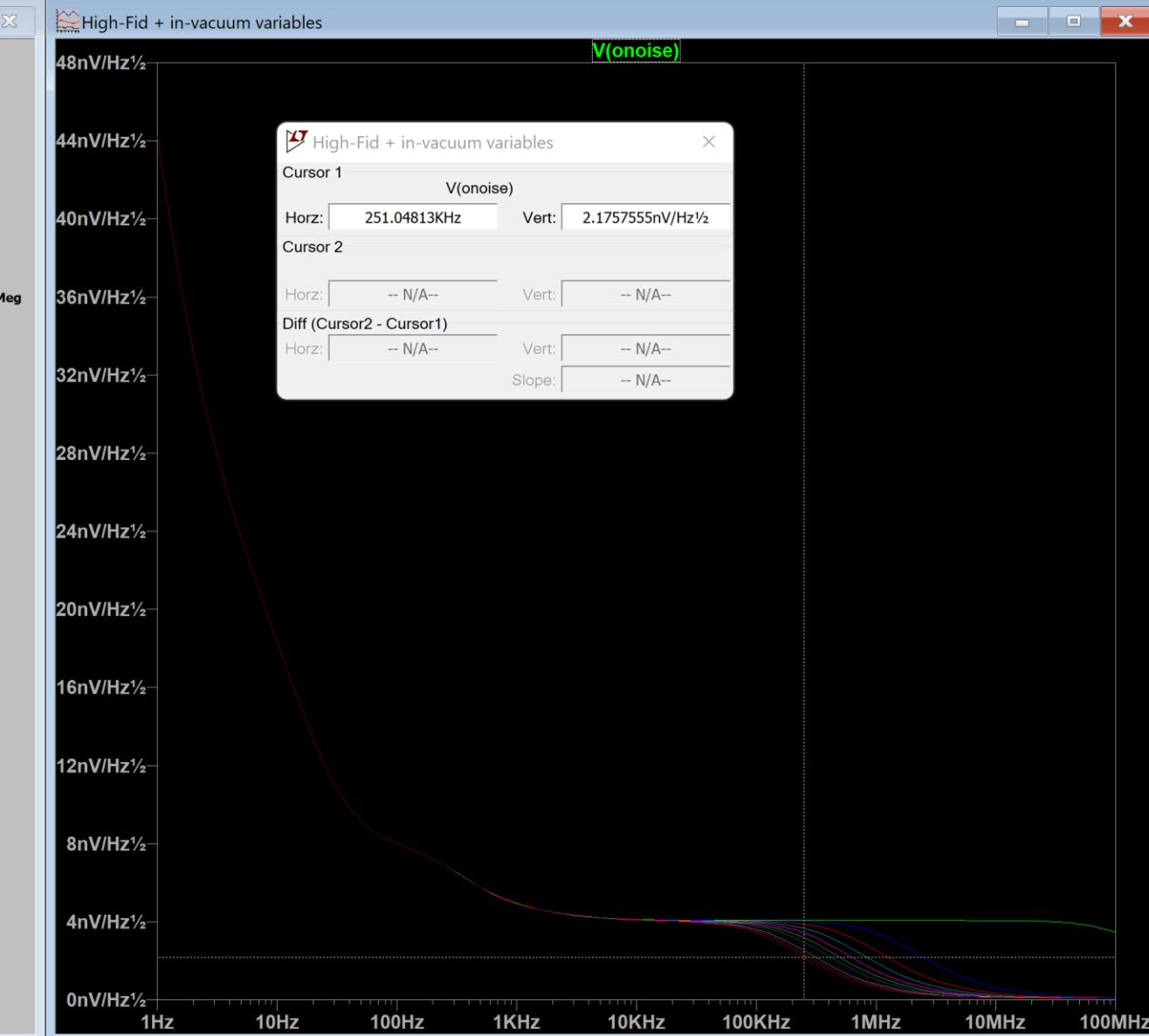
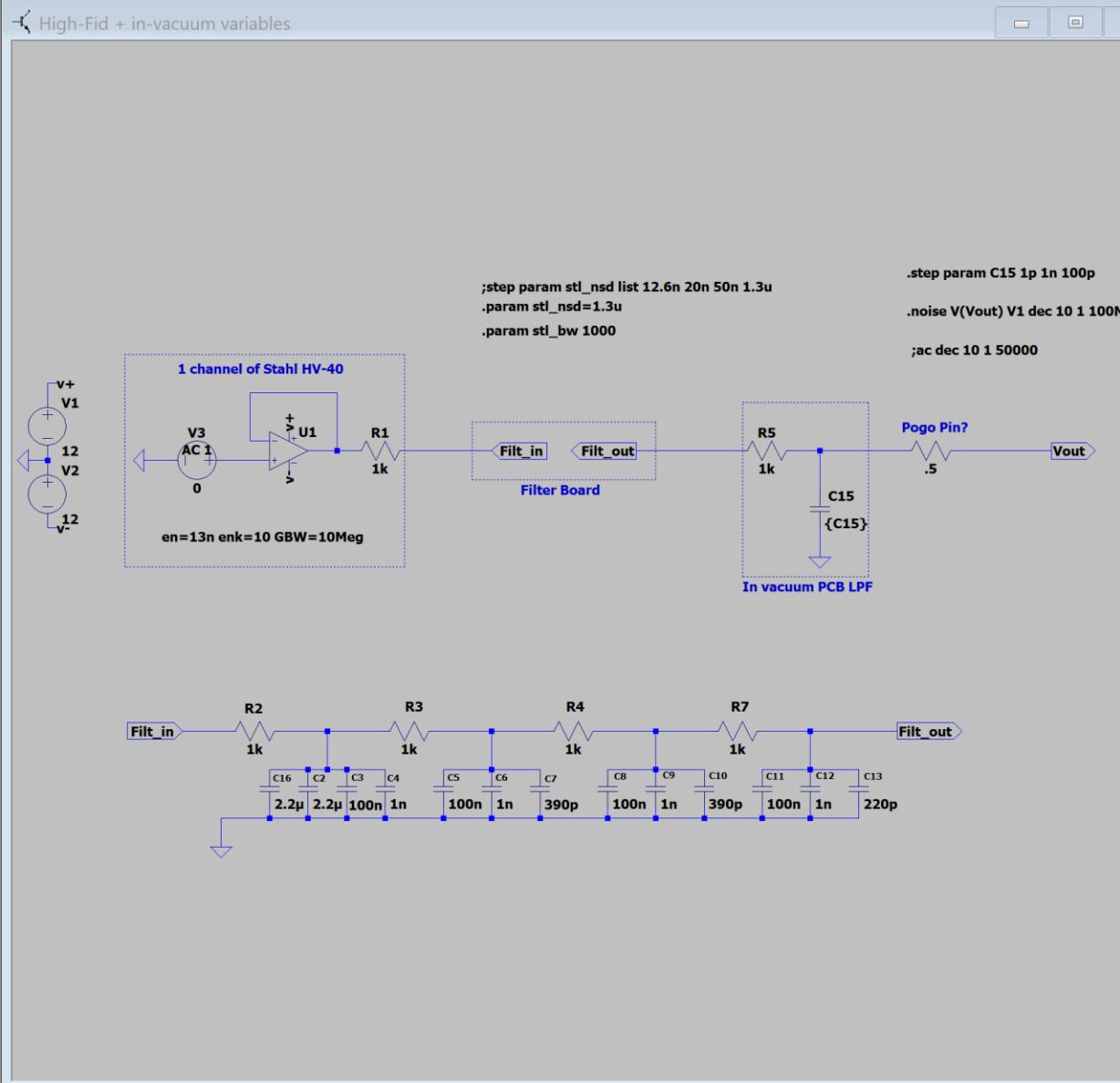
&lt; High-Fid-system High-Fid-system



File View Plot Settings Simulation Tools Window Help



< High-Fid + in-vacuum variables ■ High-Fid + in-vacuum variables

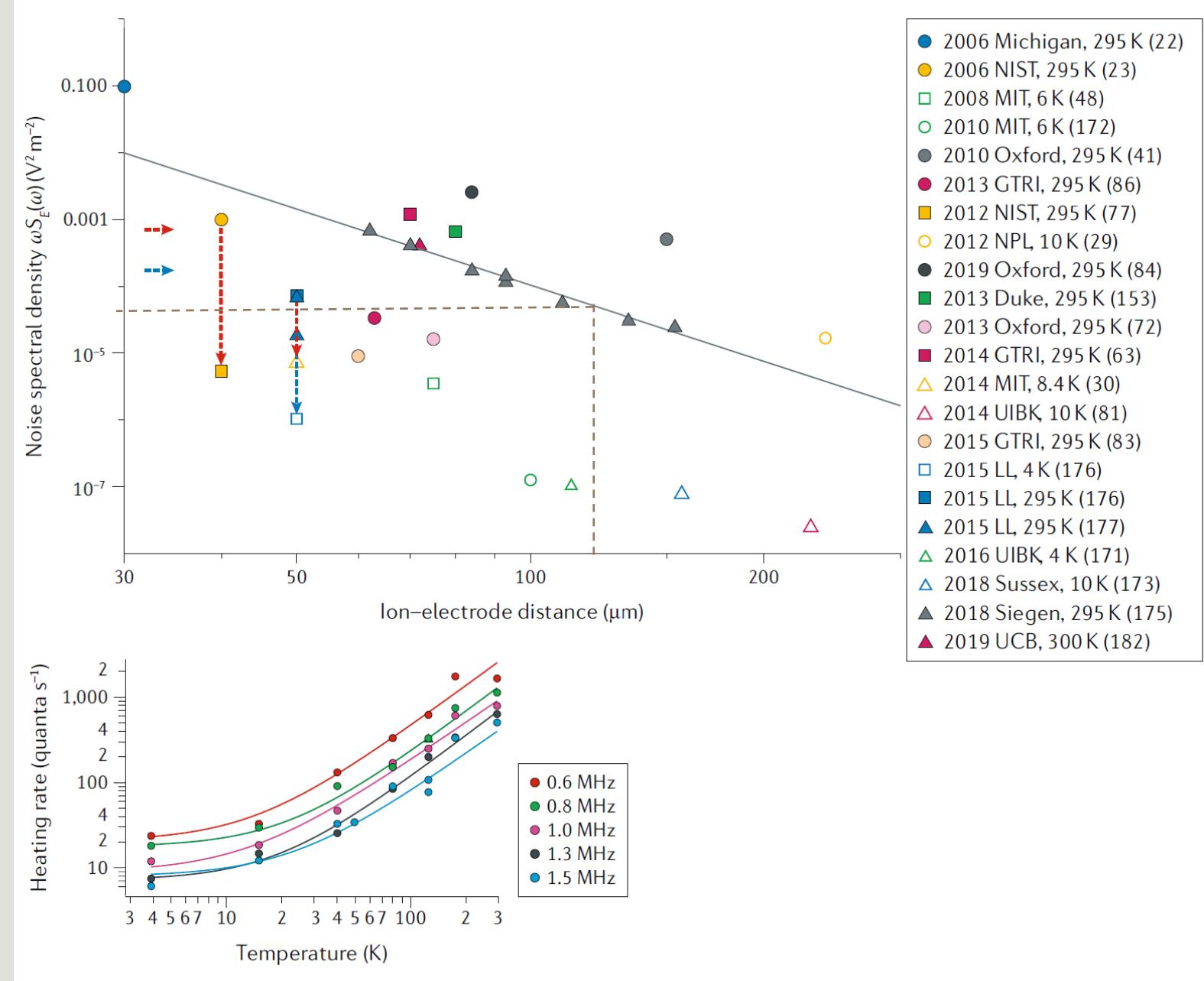


# Anomalous Heating

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- Thought to be induced by electric field noise from the surface of the chip.
- Modelled as random distribution of adsorbed atoms (fluctuating charges) producing  $SE \propto \omega^{-1}$ .
- A good measure of anomalous heating is the ions' heating rates.
- Methods to reduce the heating rate:
  - Increase the distance  $d$  between the ion and the chip surface ( since  $\omega SE \propto d^{-4}$  ).  
The disadvantage of this is that it leads to a shallower trap depth due to hardware limitations.
  - Cooling the ion-trap to cryogenic temperatures.
  - In-situ cleaning of the chip's surface to remove adsorbed contaminants.

- At 125  $\mu\text{m}$  and 295 K, we should expect  $\omega_{\text{SE}} \sim 5 \times 10^{-5}$ .  
(brown dashed line)
- Following the plot below, by cooling the trap to 70 K, we should expect the heating rate (and thus  $\omega_{\text{SE}}$ ) to decrease by an order of magnitude.  
(also blue dashed line in top plot)  
(so  $\omega_{\text{SE}} \sim 5 \times 10^{-6}$ )
- We can also expect to drop by another order of magnitude with proper surface cleaning  
(red dashed line in top plot)  
(so best-case scenario  $\omega_{\text{SE}} \sim 5 \times 10^{-7}$ )

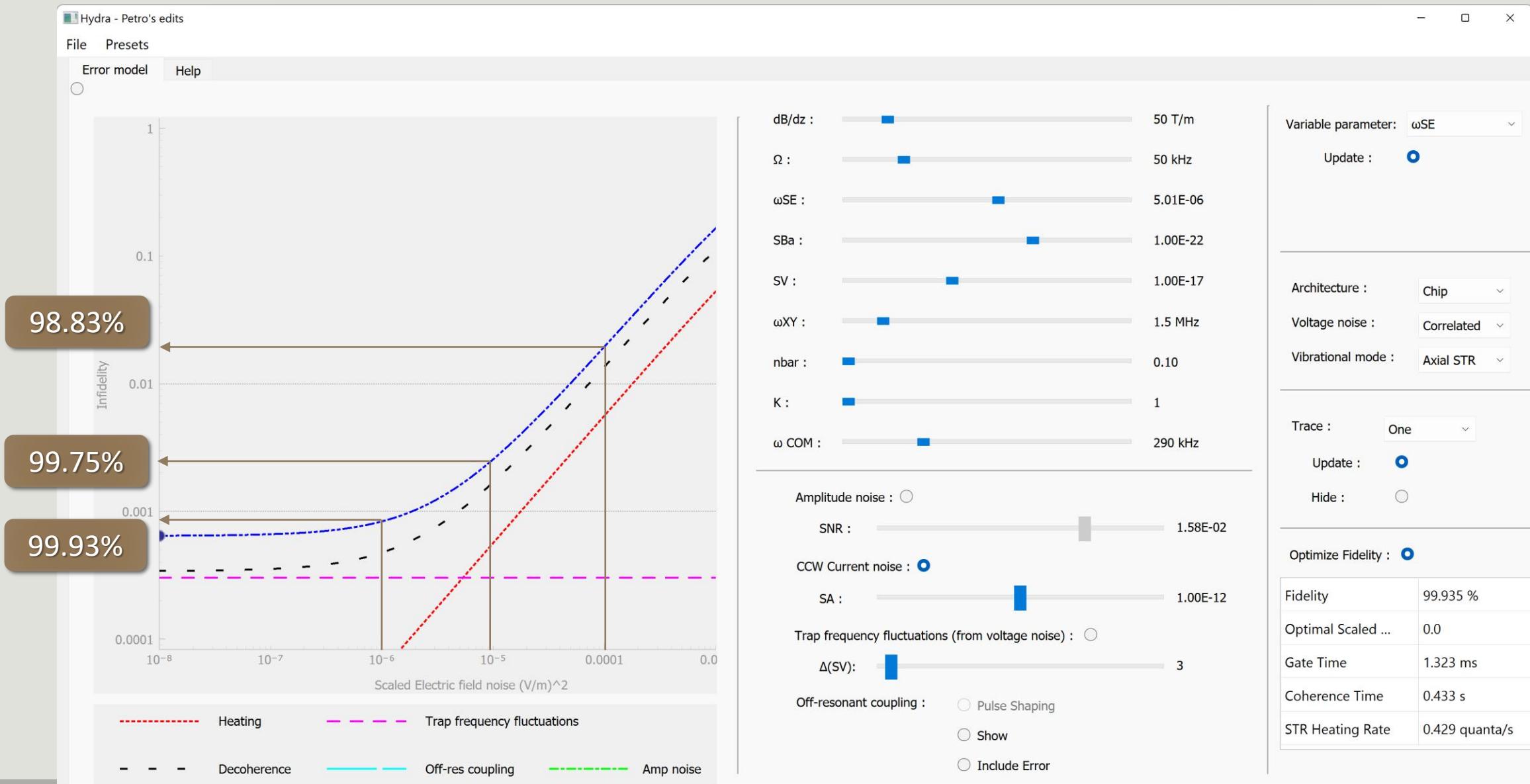




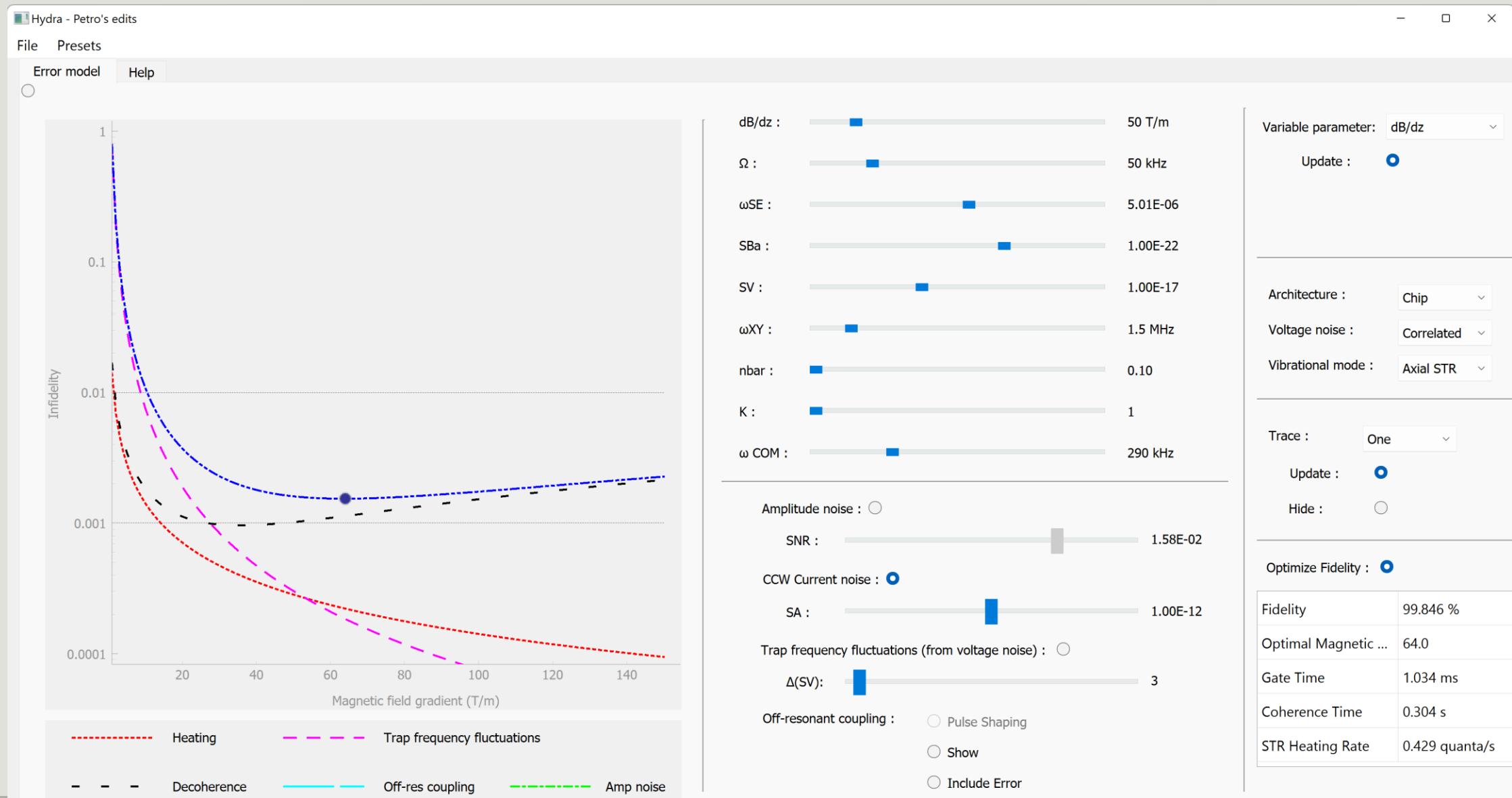
# Parameter Sensitivity

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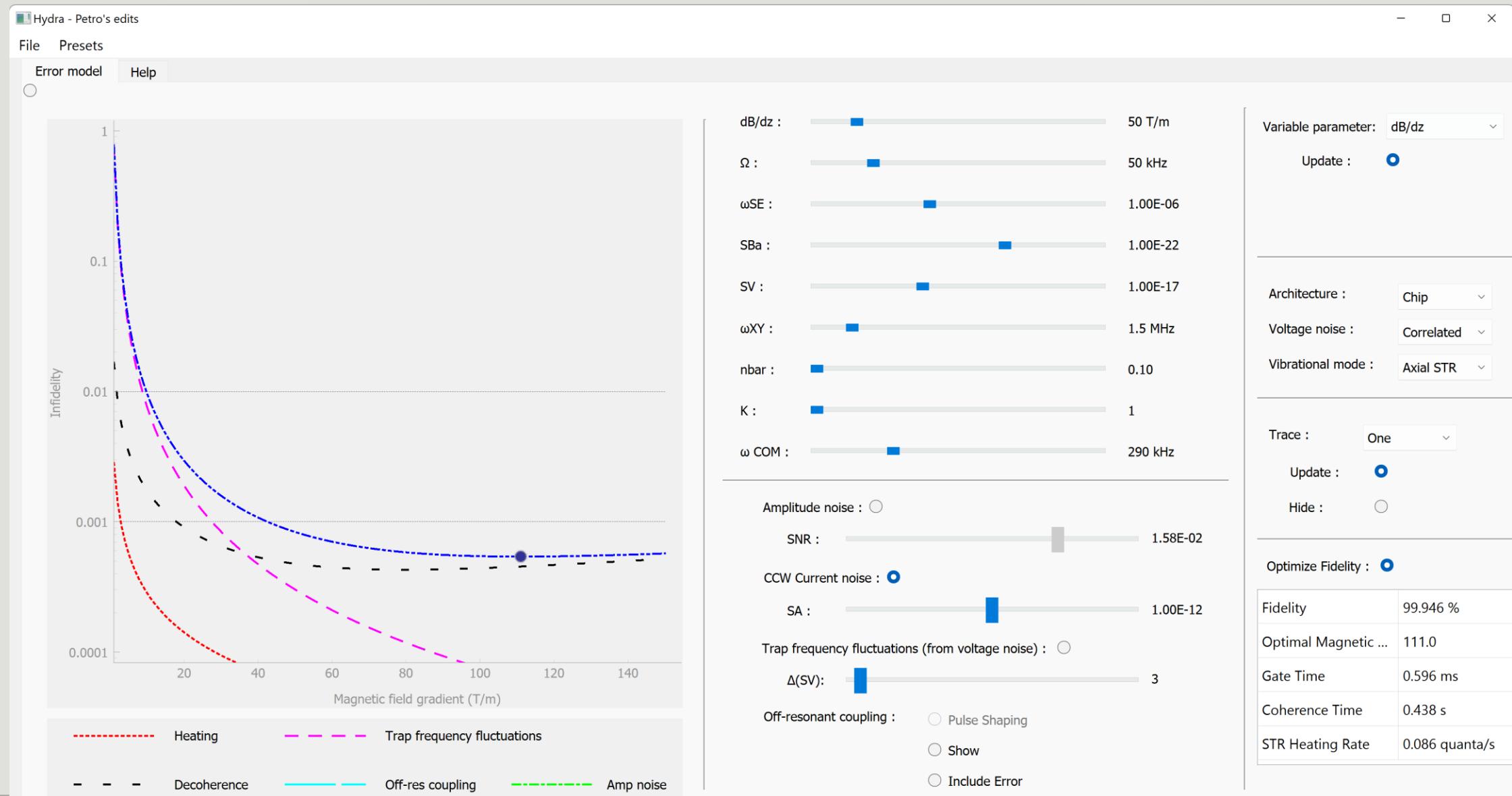
# Scaled Electric field noise



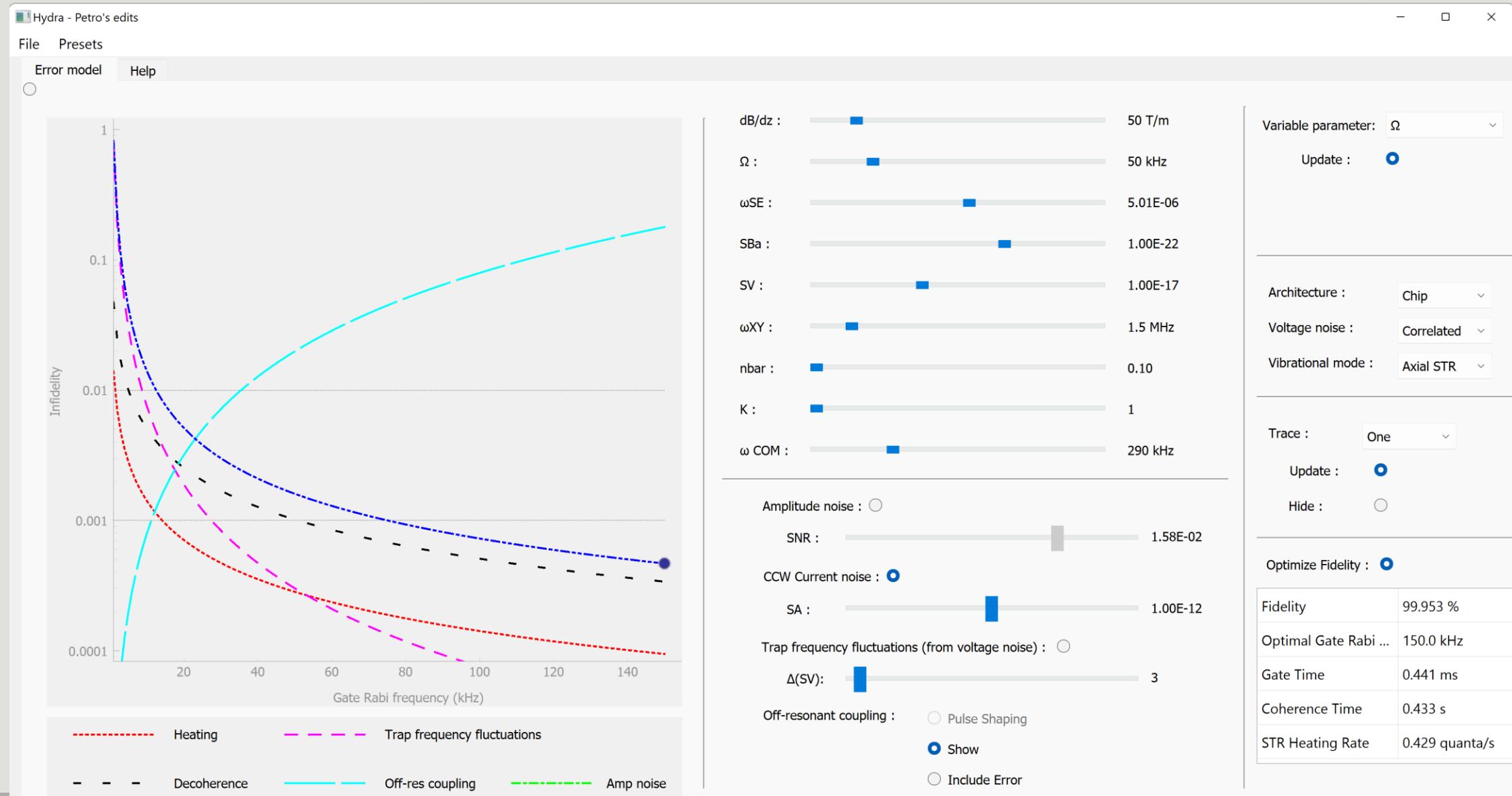
# Magnetic field gradient



# Higher gradients?

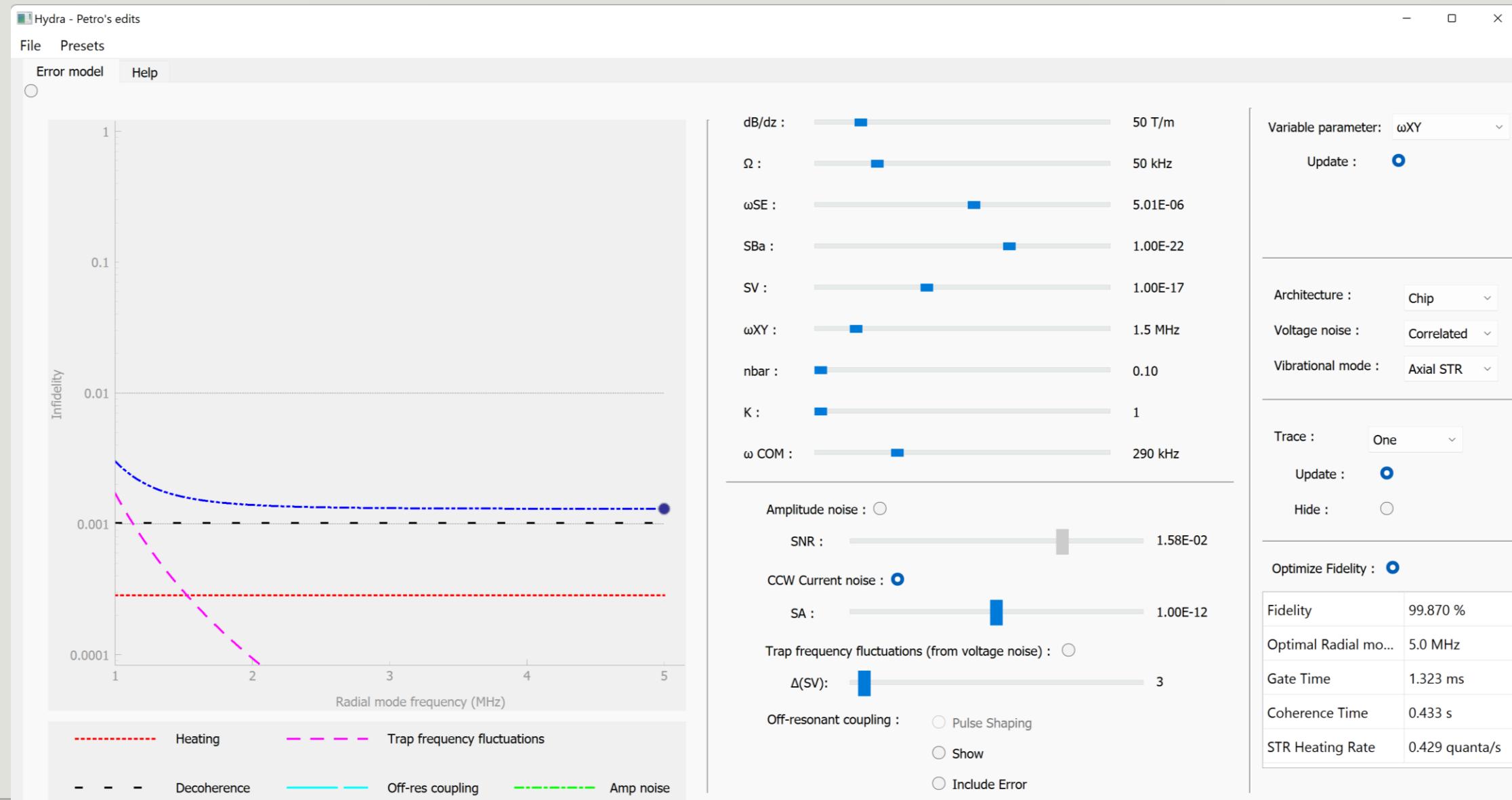


# Gate Rabi frequency (power)

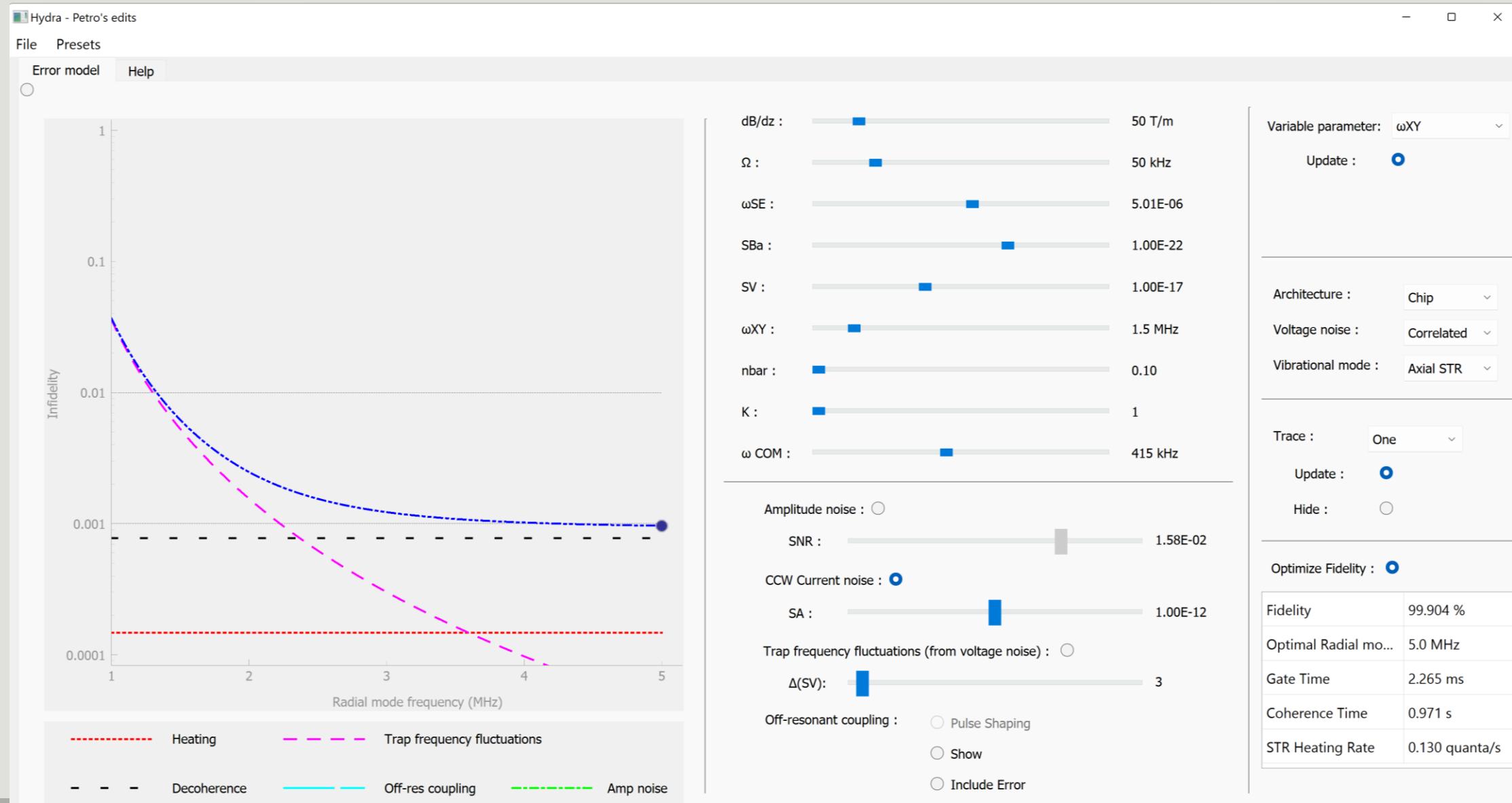


Investigate Power Broadening?

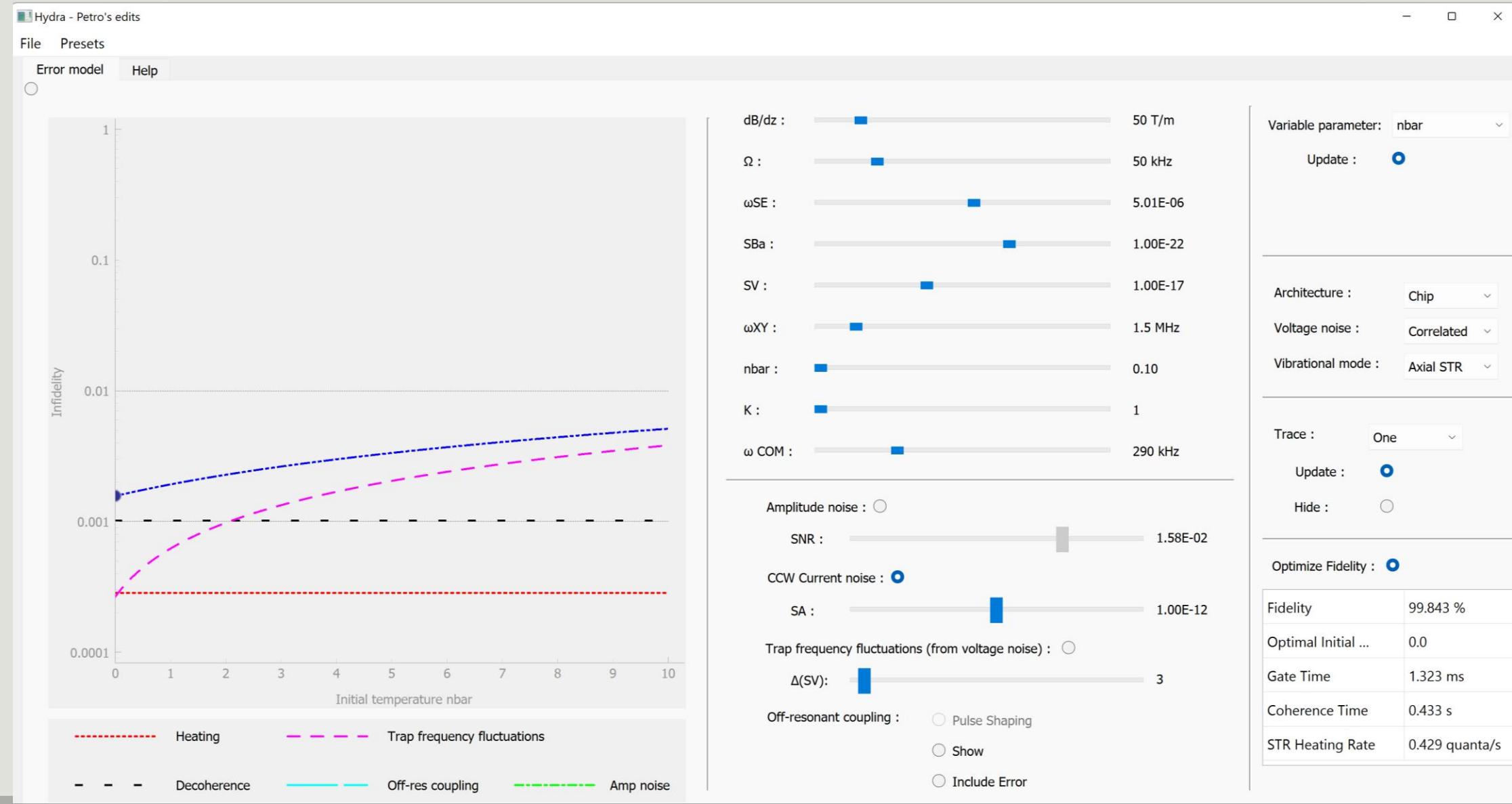
# Radial frequency



# Higher radial + secular frequencies



# Initial temperature



# Phase space loops



# Future

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- ❖ Investigate whether other terms from literature become dominant when changing parameters (e.g. increasing  $\Omega$ )
- ❖ Make sure the input parameters (e.g. B-ambient, heating rates!) closely resemble our lab/experiment conditions. If not, measure them and modify accordingly .
- ❖ Extend to other MS gate schemes (currently Dressed States, but could include PDD & CDD along with phase, frequency and amplitude modulations: PM, FM & AM)
- ❖ Possibility to add LO noise, but found to be dominated by voltage noise and magnetic field noise.
- ❖ Include effects of mis-set parameters (e.g. incorrect symmetric/asymmetric detuning). This will enhance HYDRA to serve as a sanity checking tool when actually running experiments, and not just as a prediction tool.