Tablókalkulus

ELTE

2020/2021 1.félév

Bevezető

A tablók módszere

A tablókalkulus módszeréhez kapcsolható szemantikus eldöntésprobléma a $\{A_1, A_2, ..., A_n\}$ formulahalmaz és egy B formula esetén a

(a) $\neg(A_1 \supset A_2 \supset ... \supset A_{n-1} \supset A_n \supset B)$ formula,

(b) $\{A_1, A_2, ..., A_n, \neg B\}$ formulahalmaz

kielégíthetetlenségének vizsgálata.

Ítéletlogika - Jelölt tabló

Az ítéletlogikában tanult igazságértékelés függvényhez hasonlóan fogjuk a jelölt tablót is felépíteni ítéletlogikában.

Jelölt tabló

Egy C formula jelölt tablója egy olyan bináris fa,amelynek csúcsai jelölt formulák. A gyökérbe elhelyezzük a C jelölt formulát. Előállítjuk a C közvetlen tablóját, majd folytatjuk a "feldolgozást" a kapott tablók közvetlen kiterjesztésével,amíg van nem "feldolgozott" jelölt formula.

Ítéletlogika

Közvetlen tablók jelölt formulákra

$T\neg A$
|
FA

$F\neg A$
|
TA

$TA \wedge B$
|
TA
|
TB

$FA \wedge B$
/\
FA FB

$TA \vee B$
/\
TA TB

$FA \vee B$
|
FA
|
FB

$TA \supset B$
/\
FA TB

$FA \supset B$
|
TA
|
FB

Ítéletlogika - Feladatok

1. Helyes-e a következő szemantikus következmény?

$$\{(X \supset Y) \wedge (X \supset Z), X\} \models_0 (Y \wedge Z)$$

Ítéletlogika - Feladatok

1. Helyes-e a következő szemantikus következmény?

$$\{(X \supset Y) \land (X \supset Z), X\} \models_0 (Y \land Z)$$

Induljunk ki a dedukciós tétel használatával:

Ítéletlogika - Feladatok

1. Helyes-e a következő szemantikus következmény?

$\{(X \supset Y) \land (X \supset Z), X\} \models_0 (Y \land Z)$

Induljunk ki a dedukciós tétel használatával:

Tautológia-e a következő formula?

Ítéletlogika - Feladatok

1. Helyes-e a következő szemantikus következmény?

$$\{(X \supset Y) \wedge (X \supset Z), X\} \models_0 (Y \wedge Z)$$

Induljunk ki a dedukciós tétel használatával:

Tautológia-e a következő formula?

$$((X \supset Y) \wedge (X \supset Z)) \supset (X \supset (Y \wedge Z))$$

Vagyis létezik-e zárt tabló a formulához, van-e a formulának tablócáfolata?

Ítéletlogika - Feladatok

$F((X \supset Y) \land (X \supset Z)) \supset (X \supset (Y \land Z))$ (1)

Ítéletlogika - Feladatok

$$F((X \supset Y) \land (X \supset Z)) \supset (X \supset (Y \land Z)) \quad (1)$$
$$|$$
$$T((X \supset Y) \land (X \supset Z))$$
$$|$$
$$F(X \supset (Y \land Z))$$

Ítéletlogika - Feladatok

$$F((X \supset Y) \wedge (X \supset Z)) \supset (X \supset (Y \wedge Z)) \quad (1)$$
$$|$$
$$T((X \supset Y) \wedge (X \supset Z)) \quad (2)$$
$$|$$
$$F(X \supset (Y \wedge Z))$$
$$|$$
$$T(X \supset Y)$$
$$|$$
$$T(X \supset Z)$$

Ítéletlogika - Feladatok

$$F((X \supset Y) \land (X \supset Z)) \supset (X \supset (Y \land Z)) \quad (1)$$
$$|$$
$$T((X \supset Y) \land (X \supset Z)) \quad (2)$$
$$|$$
$$F(X \supset (Y \land Z)) \quad (3)$$
$$|$$
$$T(X \supset Y)$$
$$|$$
$$T(X \supset Z)$$
$$|$$
$$\underline{TX}$$
$$|$$
$$F(Y \land Z)$$

Ítéletlogika - Feladatok

$$F((X \supset Y) \land (X \supset Z)) \supset (X \supset (Y \land Z)) \quad (1)$$

$$T((X \supset Y) \land (X \supset Z)) \quad (2)$$

$$F(X \supset (Y \land Z)) \quad (3)$$

$$T(X \supset Y) \quad (4)$$

$$T(X \supset Z)$$

$$\underline{TX}$$

$$F(Y \land Z)$$

$$\underline{FX} \qquad \underline{TY}$$

Ítéletlogika - Feladatok

$$F((X \supset Y) \wedge (X \supset Z)) \supset (X \supset (Y \wedge Z)) \quad (1)$$

$$T((X \supset Y) \wedge (X \supset Z)) \quad (2)$$

$$F(X \supset (Y \wedge Z)) \quad (3)$$

$$T(X \supset Y) \quad (4)$$

$$T(X \supset Z)$$

$$\underline{TX}$$

$$F(Y \wedge Z)$$

$$\underline{FX} \not\quad \underline{TY}$$

Ítéletlogika - Feladatok

$$F((X \supset Y) \wedge (X \supset Z)) \supset (X \supset (Y \wedge Z)) \quad (1)$$

$$T((X \supset Y) \wedge (X \supset Z)) \quad (2)$$

$$F(X \supset (Y \wedge Z)) \quad (3)$$

$$T(X \supset Y) \quad (4)$$

$$T(X \supset Z)$$

$$\underline{TX}$$

$$F(Y \wedge Z) \quad (5)$$

$$\underline{FX}\, \cancel{\;} \qquad \underline{TY}$$

$$\underline{FY} \qquad \underline{FZ}$$

Ítéletlogika - Feladatok

$$F((X \supset Y) \land (X \supset Z)) \supset (X \supset (Y \land Z)) \quad (1)$$

$$T((X \supset Y) \land (X \supset Z)) \quad (2)$$

$$F(X \supset (Y \land Z)) \quad (3)$$

$$T(X \supset Y) \quad (4)$$

$$T(X \supset Z)$$

$$\underline{TX}$$

$$F(Y \land Z) \quad (5)$$

$$\underline{FX} \, \natural \qquad \underline{TY}$$

$$\underline{FY} \, \natural \qquad \underline{FZ}$$

Ítéletlogika - Feladatok

$F((X \supset Y) \land (X \supset Z)) \supset (X \supset (Y \land Z))$ (1)

$T((X \supset Y) \land (X \supset Z))$ (2)

$F(X \supset (Y \land Z))$ (3)

$T(X \supset Y)$ (4)

$T(X \supset Z)$ (6)

$\underline{TX}$

$F(Y \land Z)$ (5)

$\underline{FX \lightning} \qquad \underline{TY}$

$\underline{FY \lightning} \qquad \underline{FZ}$

$\underline{FX} \qquad \underline{TZ}$

Ítéletlogika - Feladatok

$$F((X \supset Y) \land (X \supset Z)) \supset (X \supset (Y \land Z)) \quad (1)$$

$$T((X \supset Y) \land (X \supset Z)) \quad (2)$$

$$F(X \supset (Y \land Z)) \quad (3)$$

$$T(X \supset Y) \quad (4)$$

$$T(X \supset Z) \quad (6)$$

$$\underline{TX}$$

$$F(Y \land Z) \quad (5)$$

$$\underline{FX} \not\downarrow \qquad \underline{TY}$$

$$\underline{FY} \not\downarrow \qquad \underline{FZ}$$

$$\underline{FX} \not\downarrow \qquad \underline{TZ} \not\downarrow$$

Ítéletlogika - Feladatok

$$F((X \supset Y) \land (X \supset Z)) \supset (X \supset (Y \land Z)) \quad (1)$$

$$T((X \supset Y) \land (X \supset Z)) \quad (2)$$

$$F(X \supset (Y \land Z)) \quad (3)$$

$$T(X \supset Y) \quad (4)$$

$$T(X \supset Z) \quad (6)$$

$$\underline{TX}$$

$$F(Y \land Z) \quad (5)$$

$$\underline{FX \text{\textonehalf}} \qquad \underline{TY}$$

$$\underline{FY \text{\textonehalf}} \qquad \underline{FZ}$$

$$\underline{FX \text{\textonehalf}} \qquad \underline{TZ \text{\textonehalf}}$$

Mivel a tabló zárt, így a negált formula kielégíthetetlen, az eredeti implikációs lánc tautológia és a szemantikus következmény teljesül.

Ítéletlogika - Feladatok

2. Tautológia-e a következő formula?

$$(\neg X \vee \neg Y) \supset \neg(X \wedge Y)$$

Vagyis létezik-e zárt tabló a formulához, van-e a formulának tablócáfolata?

$$F((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \ (1)$$
$$|$$
$$T(\neg X \vee \neg Y)$$
$$|$$
$$F(\neg(X \wedge Y))$$

Ítéletlogika - Feladatok

$$F((\neg X \lor \neg Y) \supset \neg(X \land Y)) \ (1)$$
$$\big|$$
$$T(\neg X \lor \neg Y)$$
$$\big|$$
$$F(\neg(X \land Y)) \ (2)$$
$$\big|$$
$$T(X \land Y)$$

Ítéletlogika - Feladatok

$$F((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \ (1)$$
$$|$$
$$T(\neg X \vee \neg Y) \ (4)$$
$$|$$
$$F(\neg(X \wedge Y)) \ (2)$$
$$|$$
$$T(X \wedge Y) \ (3)$$
$$|$$
$$\underline{TX}$$
$$|$$
$$\underline{TY}$$

$$T\neg X \quad T\neg Y$$

Ítéletlogika - Feladatok

$$F((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \ (1)$$

$$\mid$$

$$T(\neg X \vee \neg Y) \ (4)$$

$$\mid$$

$$F(\neg(X \wedge Y)) \ (2)$$

$$\mid$$

$$T(X \wedge Y) \ (3)$$

$$\mid$$

$$\underline{TX}$$

$$\mid$$

$$\underline{TY}$$

$$T\neg X \ (5) \qquad T\neg Y \ (6)$$

$$\mid \qquad\qquad \mid$$

$$\underline{FX} \lightning \qquad\qquad \underline{FY} \lightning$$

Ítéletlogika

Közvetlen tablók jelöletlen formulákra

$A \wedge B$
|
A
|
B

$\neg(A \vee B)$
|
$\neg A$
|
$\neg B$

$\neg(A \supset B)$
|
A
|
$\neg B$

$\neg\neg A$
|
A

$\neg(A \wedge B)$

$\neg A$ $\neg B$

$A \vee B$

A B

$A \supset B$

$\neg A$ B

Ítéletlogika - Feladatok

3. Tautológia-e a következő formula?

$$(\neg X \vee \neg Y) \supset \neg(X \wedge Y)$$

Ítéletlogika - Feladatok

$$\neg((\neg X \vee \neg Y) \supset \neg(X \wedge Y))$$

Ítéletlogika - Feladatok

$$\neg((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \quad (1)$$

Ítéletlogika - Feladatok

$$\neg((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \quad (1)$$
$$|$$

Ítéletlogika - Feladatok

$$\neg((\neg X \lor \neg Y) \supset \neg(X \land Y)) \quad (1)$$
$$|$$
$$\neg X \lor \neg Y$$
$$|$$
$$\neg\neg(X \land Y)$$

Ítéletlogika - Feladatok

$$\neg((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \quad (1)$$
$$|$$
$$\neg X \vee \neg Y$$
$$|$$
$$\neg\neg(X \wedge Y) \quad (2)$$

Ítéletlogika - Feladatok

$$\neg((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \quad (1)$$
$$|$$
$$\neg X \vee \neg Y$$
$$|$$
$$\neg\neg(X \wedge Y) \quad (2)$$
$$|$$

Ítéletlogika - Feladatok

$$\neg((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \quad (1)$$
$$|$$
$$\neg X \vee \neg Y$$
$$|$$
$$\neg\neg(X \wedge Y) \quad (2)$$
$$|$$
$$X \wedge Y$$

Ítéletlogika - Feladatok

$$\neg((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \quad (1)$$
$$|$$
$$\neg X \vee \neg Y$$
$$|$$
$$\neg\neg(X \wedge Y) \quad (2)$$
$$|$$
$$X \wedge Y \quad (3)$$

$$\neg((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \quad (1)$$
$$|$$
$$\neg X \vee \neg Y$$
$$|$$
$$\neg\neg(X \wedge Y) \quad (2)$$
$$|$$
$$X \wedge Y \quad (3)$$
$$|$$

Ítéletlogika - Feladatok

$$\neg((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \quad (1)$$
$$|$$
$$\neg X \vee \neg Y$$
$$|$$
$$\neg\neg(X \wedge Y) \quad (2)$$
$$|$$
$$X \wedge Y \quad (3)$$
$$|$$
$$\underline{X}$$
$$|$$
$$\underline{Y}$$

Ítéletlogika - Feladatok

$$\neg((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \quad (1)$$
$$|$$
$$\neg X \vee \neg Y \quad (4)$$
$$|$$
$$\neg\neg(X \wedge Y) \quad (2)$$
$$|$$
$$X \wedge Y \quad (3)$$
$$|$$
$$\underline{X}$$
$$|$$
$$\underline{Y}$$

Ítéletlogika - Feladatok

$$\neg((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \quad (1)$$
$$|$$
$$\neg X \vee \neg Y \quad (4)$$
$$|$$
$$\neg\neg(X \wedge Y) \quad (2)$$
$$|$$
$$X \wedge Y \quad (3)$$
$$|$$
$$\underline{X}$$
$$|$$
$$\underline{Y}$$

Ítéletlogika - Feladatok

$$\neg((\neg X \vee \neg Y) \supset \neg(X \wedge Y)) \quad (1)$$

$$\mid$$

$$\neg X \vee \neg Y \quad (4)$$

$$\mid$$

$$\neg\neg(X \wedge Y) \quad (2)$$

$$\mid$$

$$X \wedge Y \quad (3)$$

$$\mid$$

$$\underline{X}$$

$$\mid$$

$$\underline{Y}$$

$$\underline{\neg X} \lightning \qquad \underline{\neg Y} \lightning$$

Ítéletlogika - Feladatok

4. Helyes-e a következő szemantikus következmény?

$$\{X \lor (Y \land Z)\} \models_0 (X \lor Y) \land (X \lor Z)$$

Induljunk el a visszakövetkeztetéssel:

Kielégíthetetlen-e a következő formulahalmaz?

$$\{X \lor (Y \land Z), \neg((X \lor Y) \land (X \lor Z))\}$$

Vagyis létezik-e zárt tabló a formulához, van-e a formulának tablócáfolata?

$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z))$

$$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z)) \quad (1)$$

$$(X \vee (Y \wedge Z)) \wedge \neg((X \vee Y) \wedge (X \vee Z)) \quad (1)$$

$$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z)) \quad (1)$$

$$X \lor (Y \land Z)$$

$$\neg((X \lor Y) \land (X \lor Z))$$

$$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z)) \quad (1)$$

$$\vert$$

$$X \lor (Y \land Z) \quad (2)$$

$$\vert$$

$$\neg((X \lor Y) \land (X \lor Z))$$

$$(X \vee (Y \wedge Z)) \wedge \neg((X \vee Y) \wedge (X \vee Z)) \quad (1)$$

$$X \vee (Y \wedge Z) \quad (2)$$

$$\neg((X \vee Y) \wedge (X \vee Z))$$

$$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z)) \quad (1)$$

$$\mid$$

$$X \lor (Y \land Z) \quad (2)$$

$$\mid$$

$$\neg((X \lor Y) \land (X \lor Z))$$

$$\underline{X} \qquad\qquad\qquad Y \land Z$$

$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z))$ (1)

$X \lor (Y \land Z)$ (2)

$\neg((X \lor Y) \land (X \lor Z))$

$\underline{X}$ $Y \land Z$ (3)

$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z))$ (1)

$X \lor (Y \land Z)$ (2)

$\neg((X \lor Y) \land (X \lor Z))$

$\underline{X}$ $Y \land Z$ (3)

$$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z)) \quad (1)$$

$$|$$

$$X \lor (Y \land Z) \quad (2)$$

$$|$$

$$\neg((X \lor Y) \land (X \lor Z))$$

$$\underline{X} \qquad\qquad Y \land Z \quad (3)$$

$$|$$

$$\underline{Y}$$

$$|$$

$$\underline{Z}$$

$(X \vee (Y \wedge Z)) \wedge \neg((X \vee Y) \wedge (X \vee Z))$ (1)
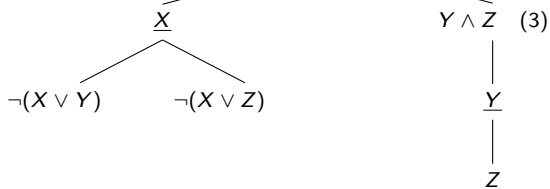
$X \vee (Y \wedge Z)$ (2)

$\neg((X \vee Y) \wedge (X \vee Z))$ (4)

$\underline{X}$ $Y \wedge Z$ (3)

$\underline{Y}$

$\underline{Z}$

$$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z)) \quad (1)$$

$$|$$

$$X \lor (Y \land Z) \quad (2)$$

$$|$$

$$\neg((X \lor Y) \land (X \lor Z)) \quad (4)$$

$$\underline{X} \qquad\qquad Y \land Z \quad (3)$$

$$\neg(X \lor Y) \qquad \neg(X \lor Z) \qquad\qquad \underline{Y}$$

$$|$$

$$\underline{Z}$$

$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z))$ (1)

$X \lor (Y \land Z)$ (2)

$\neg((X \lor Y) \land (X \lor Z))$ (4)

$\underline{X}$ $Y \land Z$ (3)

$\neg(X \lor Y)$ $\neg(X \lor Z)$

$\underline{Y}$

$\underline{Z}$

$\neg(X \lor Y)$ $\neg(X \lor Z)$

$(X \vee (Y \wedge Z)) \wedge \neg((X \vee Y) \wedge (X \vee Z))$ (1)

$X \vee (Y \wedge Z)$ (2)

$\neg((X \vee Y) \wedge (X \vee Z))$ (4)

$\underline{X}$

$Y \wedge Z$ (3)

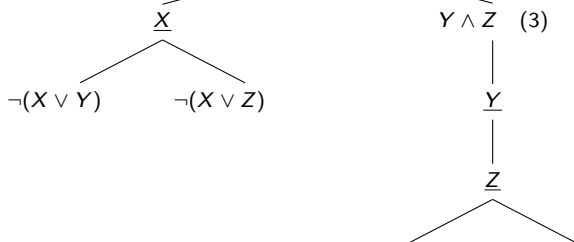$\neg(X \vee Y)$ (5) $\neg(X \vee Z)$

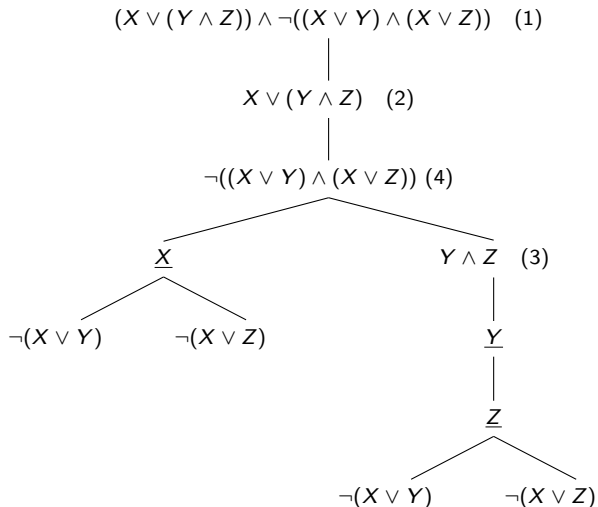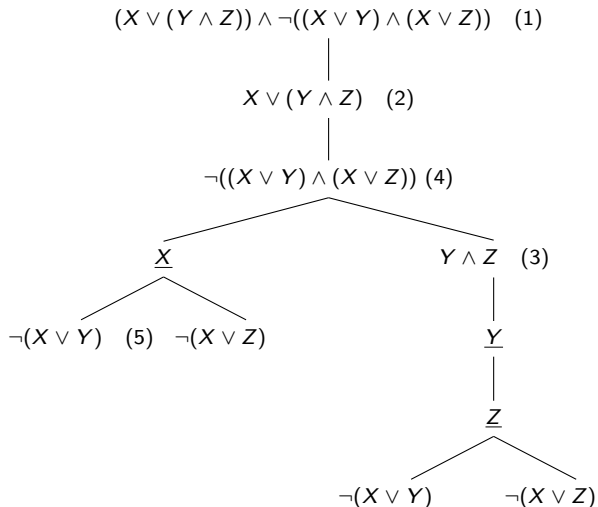$\underline{Y}$

$\underline{Z}$

$\neg(X \vee Y)$ $\neg(X \vee Z)$

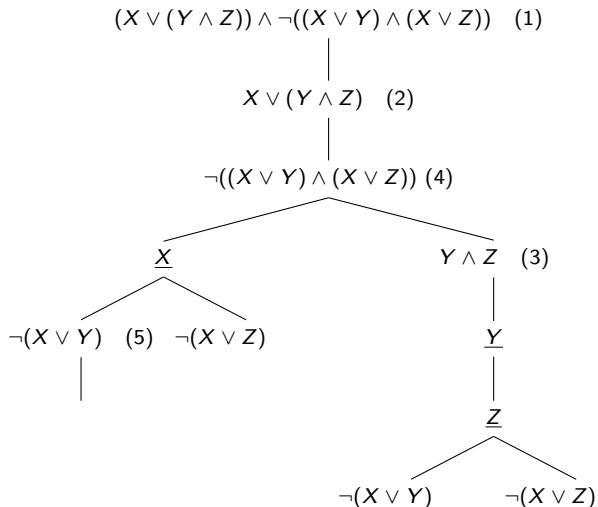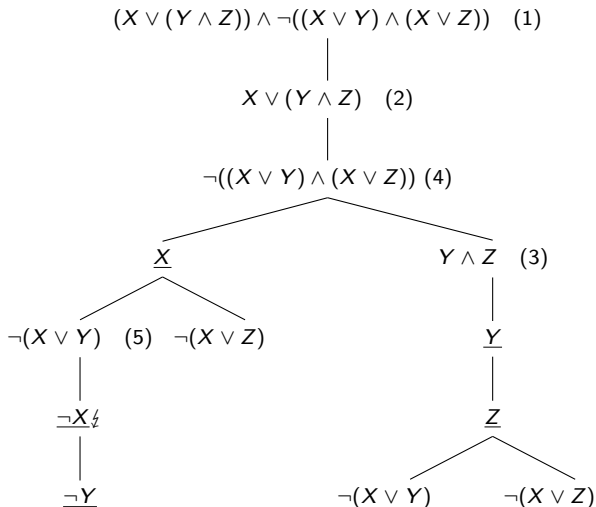$(X \vee (Y \wedge Z)) \wedge \neg((X \vee Y) \wedge (X \vee Z))$ (1)

$X \vee (Y \wedge Z)$ (2)

$\neg((X \vee Y) \wedge (X \vee Z))$ (4)

$\underline{X}$ $Y \wedge Z$ (3)
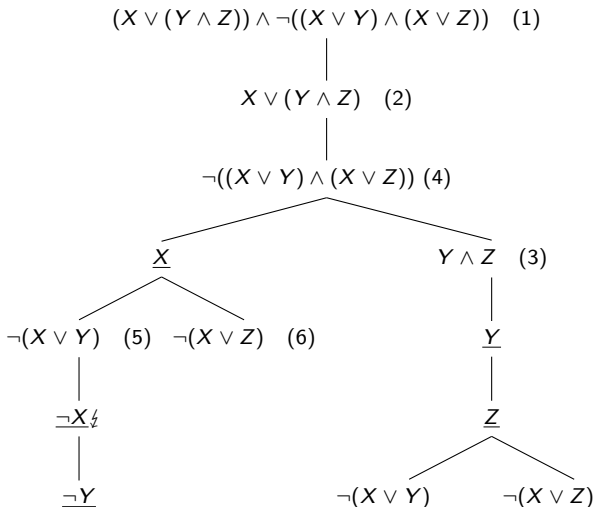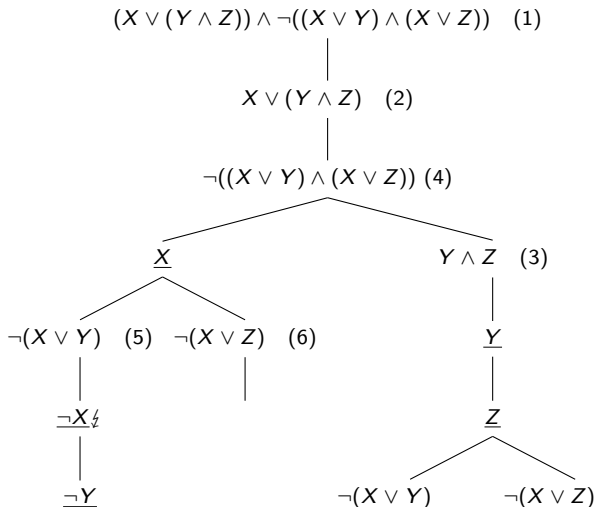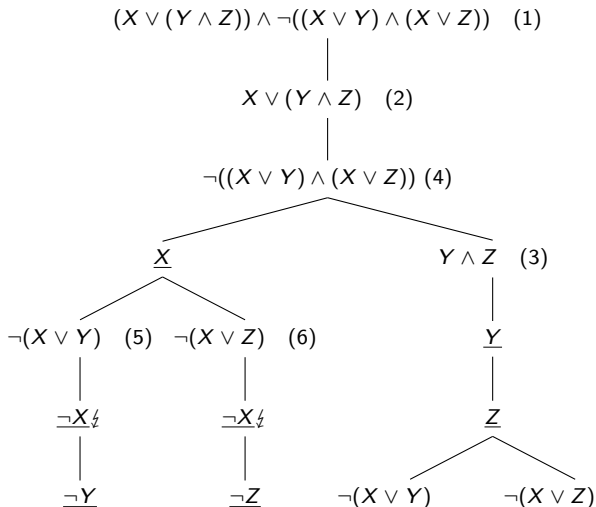
$\neg(X \vee Y)$ (5) $\neg(X \vee Z)$ (6) $\underline{Y}$

$\underline{\neg X}$ $\underline{Z}$

$\underline{\neg Y}$ $\neg(X \vee Y)$ $\neg(X \vee Z)$

$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z))$ (1)

$X \lor (Y \land Z)$ (2)

$\neg((X \lor Y) \land (X \lor Z))$ (4)

$\underline{X}$ $Y \land Z$ (3)

$\neg(X \lor Y)$ (5) $\neg(X \lor Z)$ (6) $\underline{Y}$

$\underline{\neg X} \lightning$ $\underline{\neg X} \lightning$ $\underline{Z}$

$\underline{\neg Y}$ $\underline{\neg Z}$ $\neg(X \lor Y)$ $\neg(X \lor Z)$

$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z))$ (1)

$X \lor (Y \land Z)$ (2)

$\neg((X \lor Y) \land (X \lor Z))$ (4)

$\underline{X}$

$Y \land Z$ (3)

$\neg(X \lor Y)$ (5) $\neg(X \lor Z)$ (6)

$\underline{Y}$

$\underline{\neg X} \not z$

$\underline{\neg X} \not z$

$\underline{Z}$

$\underline{\neg Y}$

$\underline{\neg Z}$

$\neg(X \lor Y)$ (7) $\neg(X \lor Z)$ (8)

$\underline{\neg X}$

$\underline{\neg Y} \not z$

$$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z)) \quad (1)$$

$$X \lor (Y \land Z) \quad (2)$$

$$\neg((X \lor Y) \land (X \lor Z)) \quad (4)$$

$$\underline{X} \qquad Y \land Z \quad (3)$$

$$\neg(X \lor Y) \quad (5) \qquad \neg(X \lor Z) \quad (6) \qquad \underline{Y}$$

$$\underline{\neg X} \lightning \qquad \underline{\neg X} \lightning \qquad \underline{Z}$$

$$\underline{\neg Y} \qquad \underline{\neg Z} \qquad \neg(X \lor Y) \quad (7) \qquad \neg(X \lor Z) \quad (8)$$

$$\underline{\neg X}$$

$$\underline{\neg Y} \lightning$$

$$(X \lor (Y \land Z)) \land \neg((X \lor Y) \land (X \lor Z)) \quad (1)$$

$$X \lor (Y \land Z) \quad (2)$$

$$\neg((X \lor Y) \land (X \lor Z)) \quad (4)$$

$$\underline{X} \qquad\qquad Y \land Z \quad (3)$$

$$\neg(X \lor Y) \quad (5) \qquad \neg(X \lor Z) \quad (6) \qquad\qquad \underline{Y}$$

$$\underline{\neg X}\, \text{\textonequarter} \qquad\qquad \underline{\neg X}\, \text{\textonequarter} \qquad\qquad \underline{Z}$$

$$\underline{\neg Y} \qquad\qquad \underline{\neg Z} \qquad\qquad \neg(X \lor Y) \quad (7) \qquad \neg(X \lor Z) \quad (8)$$

$$\underline{\neg X} \qquad\qquad \underline{\neg X}$$

$$\underline{\neg Y}\, \text{\textonequarter} \qquad\qquad \underline{\neg Z}\, \text{\textonequarter}$$
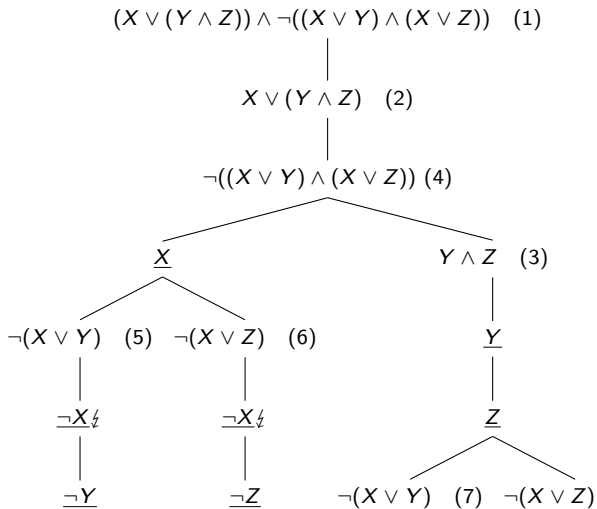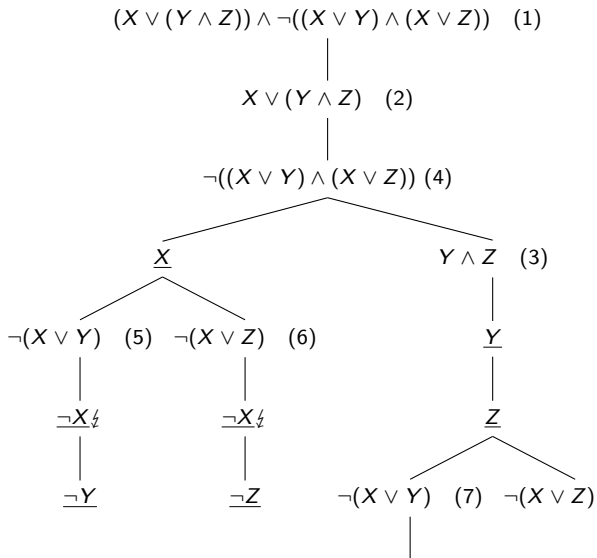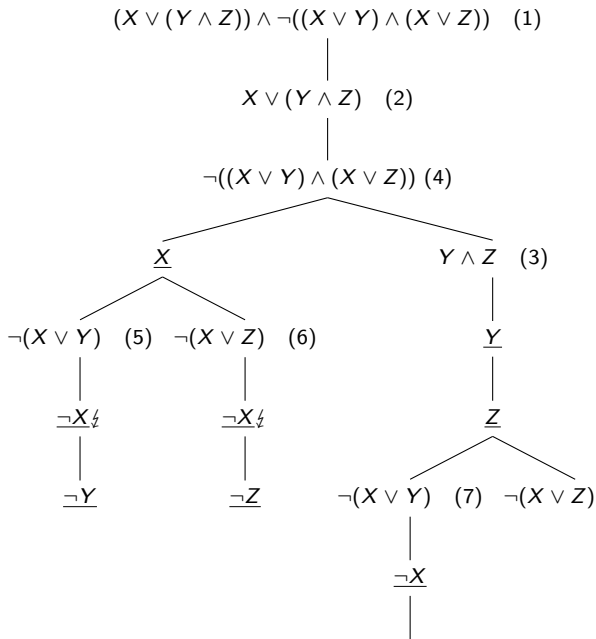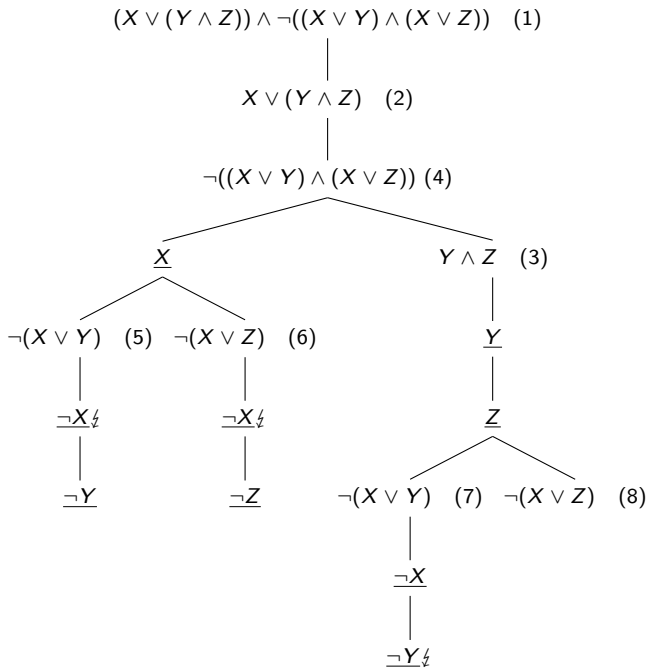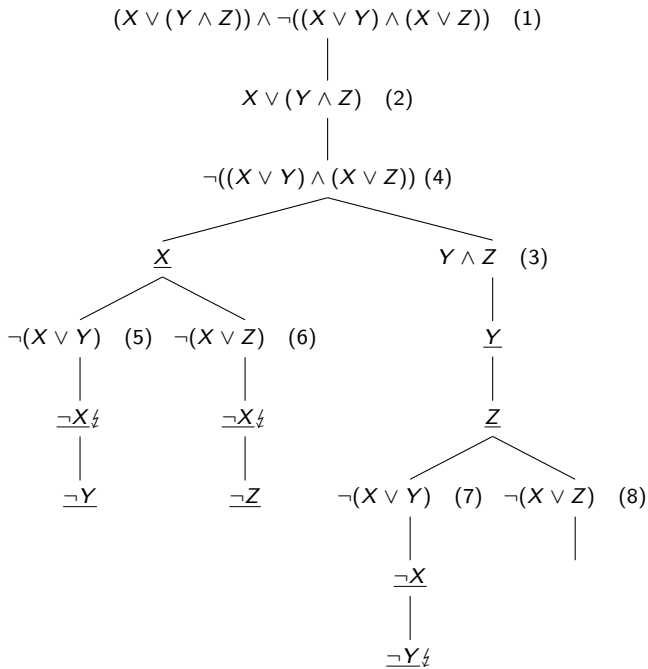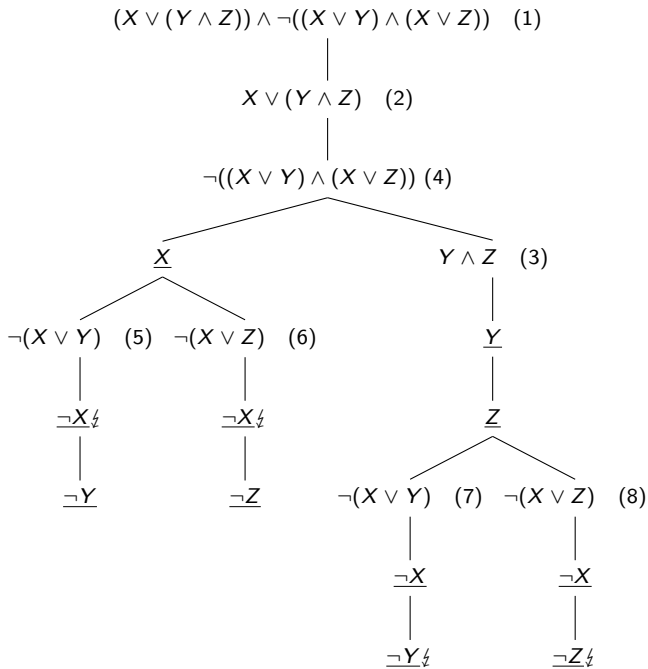
Elsőrendű logika - Jelölt tabló

Hasonlóan az ítéletlogikához elsőrendű logikában is kielégíthetetlenséget vizsgálunk. Az elsőrendű tablókalkulus helyessége alapján: ha az elsőrendű A formula tablója zárt, akkor A kielégíthetetlen.

Közvetlen tablók α- ,β- ,γ- és δ formulákra

α
|
α_1
|
α_2

β
/\
β_1 β_2

γ
|
$\gamma(a)$

δ
|
$\delta(a)$

(Megkötéssel)

Közvetlen tablók jelölt formulákra

(γ)

$T\forall xA$
|
$TA(x||a)$

$F\exists xA$
|
$FA(x||a)$

(δ)

$T\exists xA$
|
$TA(x||a)$

$F\forall xA$
|
$FA(x||a)$

γ - Valamilyen helyettesítést végzünk el.

δ - Mindenképpen egy új szimbólumot vezetünk be az adott ágon!

Érdemes bontási sorrend: $\alpha, \delta, \beta, \gamma$

Elsőrendű logika - Feladatok

1. Helyes-e a következő szemantikus következmény?

$$\{\forall x P(x) \supset Q(\bar{a})\} \models \exists y (P(y) \supset Q(\bar{a}))$$

Használjuk a dedukciós tételt:

Logikailag igaz-e a következő elsőrendű formula?

$$(\forall x P(x) \supset Q(\bar{a})) \supset \exists y (P(y) \supset Q(\bar{a}))$$

Vagyis létezik-e zárt tabló a formulához,van-e a formulának tablócáfolata?

$F(\forall x P(x) \supset Q(\bar{a})) \supset \exists y (P(y) \supset Q(\bar{a}))$

$F(\forall x P(x) \supset Q(\bar{a})) \supset \exists y (P(y) \supset Q(\bar{a})) \quad (\alpha) \quad (1)$

Elsőrendű logika - Feladatok

$$F(\forall x P(x) \supset Q(\overline{a})) \supset \exists y (P(y) \supset Q(\overline{a})) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x P(x) \supset Q(\overline{a}))$$
$$|$$
$$F(\exists y (P(y) \supset Q(\overline{a})))$$

Elsőrendű logika - Feladatok

$$F(\forall x P(x) \supset Q(\overline{a})) \supset \exists y (P(y) \supset Q(\overline{a})) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x P(x) \supset Q(\overline{a})) \quad (\beta)$$
$$|$$
$$F(\exists y (P(y) \supset Q(\overline{a})))$$

$$F(\forall x P(x) \supset Q(\bar{a})) \supset \exists y (P(y) \supset Q(\bar{a})) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x P(x) \supset Q(\bar{a})) \quad (\beta)$$
$$|$$
$$F(\exists y (P(y) \supset Q(\bar{a}))) \quad (\gamma)$$

$$F(\forall x P(x) \supset Q(\overline{a})) \supset \exists y(P(y) \supset Q(\overline{a})) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x P(x) \supset Q(\overline{a})) \quad (\beta) \quad (2)$$
$$|$$
$$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma)$$

$$F\forall x P(x) \qquad \underline{T(Q(\overline{a}))}$$

$$F(\forall x P(x) \supset Q(\overline{a})) \supset \exists y (P(y) \supset Q(\overline{a})) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x P(x) \supset Q(\overline{a})) \quad (\beta) \quad (2)$$
$$|$$
$$F(\exists y (P(y) \supset Q(\overline{a}))) \quad (\gamma)$$

$$F\forall x P(x) \quad (\delta) \qquad \underline{T(Q(\overline{a}))}$$

Elsőrendű logika - Feladatok

$$F(\forall x P(x) \supset Q(\overline{a})) \supset \exists y(P(y) \supset Q(\overline{a})) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x P(x) \supset Q(\overline{a})) \quad (\beta) \quad (2)$$
$$|$$
$$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma)$$

$$F\forall x P(x) \quad (\delta) \quad (3) \qquad \underline{T(Q(\overline{a}))}$$
$$|$$
$$\underline{F(P(\overline{b}))}$$

Elsőrendű logika - Feladatok

$$F(\forall x P(x) \supset Q(\overline{a})) \supset \exists y(P(y) \supset Q(\overline{a})) \quad (\alpha) \quad (1)$$

$$T(\forall x P(x) \supset Q(\overline{a})) \quad (\beta) \quad (2)$$

$$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma(y\|a)) \quad (4)$$

$$F\forall x P(x) \quad (\delta) \quad (3) \qquad \underline{T(Q(\overline{a}))}$$

$$\underline{F(P(\overline{b}))} \qquad F(P(\overline{a}) \supset Q(\overline{a}))$$

$$F(P(\overline{a}) \supset Q(\overline{a}))$$

$$F(\forall x P(x) \supset Q(\bar{a})) \supset \exists y (P(y) \supset Q(\bar{a})) \quad (\alpha) \quad (1)$$

$$T(\forall x P(x) \supset Q(\bar{a})) \quad (\beta) \quad (2)$$

$$F(\exists y (P(y) \supset Q(\bar{a}))) \quad (\gamma(y\|a)) \quad (4)$$

$$F\forall x P(x) \quad (\delta) \quad (3) \qquad \underline{T(Q(\bar{a}))}$$

$$\underline{F(P(\bar{b}))} \qquad F(P(\bar{a}) \supset Q(\bar{a}))$$

$$F(P(\bar{a}) \supset Q(\bar{a})) \quad (\alpha)$$

$F(\forall x P(x) \supset Q(\bar{a})) \supset \exists y (P(y) \supset Q(\bar{a}))$ (α) (1)

$T(\forall x P(x) \supset Q(\bar{a}))$ (β) (2)

$F(\exists y (P(y) \supset Q(\bar{a})))$ $(\gamma(y \| a))$ (4)

$F \forall x P(x)$ (δ) (3) $\underline{T(Q(\bar{a}))}$

$\underline{F(P(\bar{b}))}$ $F(P(\bar{a}) \supset Q(\bar{a}))$ (α)

$F(P(\bar{a}) \supset Q(\bar{a}))$ (α)

$F(\forall x P(x) \supset Q(\overline{a})) \supset \exists y (P(y) \supset Q(\overline{a}))$ (α) (1)
|
$T(\forall x P(x) \supset Q(\overline{a}))$ (β) (2)
|
$F(\exists y (P(y) \supset Q(\overline{a})))$ $(\gamma(y \| a))$ (4)

$F(\forall x P(x)$ (δ) (3) $\underline{T(Q(\overline{a}))}$
| |
$\underline{F(P(\overline{b}))}$ $F(P(\overline{a}) \supset Q(\overline{a}))$ (α)
| |
$F(P(\overline{a}) \supset Q(\overline{a}))$ (α) $F(\exists y (P(y) \supset Q(\overline{a})))$ (γ)
|
$F(\exists y (P(y) \supset Q(\overline{a})))$ (γ)

$$F(\forall x P(x) \supset Q(\overline{a})) \supset \exists y(P(y) \supset Q(\overline{a})) \quad (\alpha) \quad (1)$$

$$T(\forall x P(x) \supset Q(\overline{a})) \quad (\beta) \quad (2)$$

$$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma(y\|a)) \quad (4)$$

$$F(\forall x P(x) \quad (\delta) \quad (3)) \qquad \underline{T(Q(\overline{a}))}$$

$$\underline{F(P(\overline{b}))} \qquad F(P(\overline{a}) \supset Q(\overline{a})) \quad (\alpha) \quad (5)$$

$$F(P(\overline{a}) \supset Q(\overline{a})) \quad (\alpha) \qquad F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma)$$

$$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma) \qquad \underline{T(P(\overline{a}))}$$

$$\underline{F(Q(\overline{a}))}$$

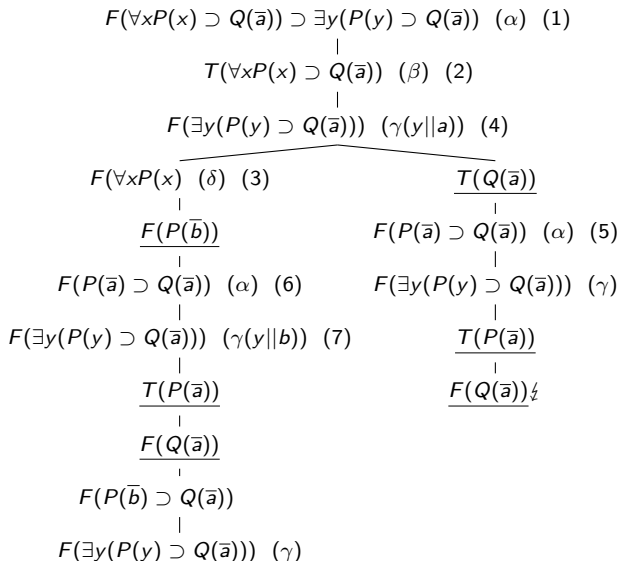# Elsőrendű logika - Feladatok

$$F(\forall x P(x) \supset Q(\bar{a})) \supset \exists y (P(y) \supset Q(\bar{a})) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x P(x) \supset Q(\bar{a})) \quad (\beta) \quad (2)$$
$$|$$
$$F(\exists y (P(y) \supset Q(\bar{a}))) \quad (\gamma(y\|a)) \quad (4)$$

$$F(\forall x P(x) \quad (\delta) \quad (3) \qquad\qquad \underline{T(Q(\bar{a}))}$$
$$| \qquad\qquad\qquad\qquad\qquad\qquad |$$
$$\underline{F(P(\bar{b}))} \qquad\qquad\qquad F(P(\bar{a}) \supset Q(\bar{a})) \quad (\alpha) \quad (5)$$
$$| \qquad\qquad\qquad\qquad\qquad\qquad\qquad |$$
$$F(P(\bar{a}) \supset Q(\bar{a})) \quad (\alpha) \qquad F(\exists y (P(y) \supset Q(\bar{a}))) \quad (\gamma)$$
$$| \qquad\qquad\qquad\qquad\qquad\qquad\qquad |$$
$$F(\exists y (P(y) \supset Q(\bar{a}))) \quad (\gamma) \qquad\qquad \underline{T(P(\bar{a}))}$$
$$| $$
$$\underline{F(Q(\bar{a}))} \,\not{}$$

$$F(\forall x P(x) \supset Q(\bar{a})) \supset \exists y(P(y) \supset Q(\bar{a})) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x P(x) \supset Q(\bar{a})) \quad (\beta) \quad (2)$$
$$|$$
$$F(\exists y(P(y) \supset Q(\bar{a}))) \quad (\gamma(y\|a)) \quad (4)$$

$$\overbrace{F(\forall x P(x) \quad (\delta) \quad (3) \qquad\qquad \underline{T(Q(\bar{a}))}}$$
$$| \qquad\qquad\qquad\qquad\qquad |$$
$$\underline{F(P(\bar{b}))} \qquad\qquad F(P(\bar{a}) \supset Q(\bar{a})) \quad (\alpha) \quad (5)$$
$$| \qquad\qquad\qquad\qquad\qquad |$$
$$F(P(\bar{a}) \supset Q(\bar{a})) \quad (\alpha) \quad (6) \quad F(\exists y(P(y) \supset Q(\bar{a}))) \quad (\gamma)$$
$$| \qquad\qquad\qquad\qquad\qquad |$$
$$F(\exists y(P(y) \supset Q(\bar{a}))) \quad (\gamma) \qquad\qquad \underline{T(P(\bar{a}))}$$
$$| \qquad\qquad\qquad\qquad\qquad |$$
$$\underline{T(P(\bar{a}))} \qquad\qquad\qquad \underline{F(Q(\bar{a}))}\nmid$$
$$|$$
$$\underline{F(Q(\bar{a}))}$$

Elsőrendű logika - Feladatok

$$F(\forall x P(x) \supset Q(\overline{a})) \supset \exists y(P(y) \supset Q(\overline{a})) \quad (\alpha) \quad (1)$$

$$T(\forall x P(x) \supset Q(\overline{a})) \quad (\beta) \quad (2)$$

$$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma(y||a)) \quad (4)$$

$$F(\forall x P(x)) \quad (\delta) \quad (3) \qquad\qquad \underline{T(Q(\overline{a}))}$$

$$\underline{F(P(\overline{b}))} \qquad\qquad F(P(\overline{a}) \supset Q(\overline{a})) \quad (\alpha) \quad (5)$$

$$F(P(\overline{a}) \supset Q(\overline{a})) \quad (\alpha) \quad (6) \qquad F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma)$$

$$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma(y||b)) \quad (7) \qquad\qquad \underline{T(P(\overline{a}))}$$

$$\underline{T(P(\overline{a}))} \qquad\qquad\qquad \underline{F(Q(\overline{a}))} \notni$$

$$\underline{F(Q(\overline{a}))}$$

$$F(P(\overline{b}) \supset Q(\overline{a}))$$

$$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma)$$

Elsőrendű logika - Feladatok

$$F(\forall x P(x) \supset Q(\overline{a})) \supset \exists y(P(y) \supset Q(\overline{a})) \quad (\alpha) \quad (1)$$

$$T(\forall x P(x) \supset Q(\overline{a})) \quad (\beta) \quad (2)$$

$$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma(y\|a)) \quad (4)$$

$F(\forall x P(x)) \quad (\delta) \quad (3)$ $\underline{T(Q(\overline{a}))}$

$\underline{F(P(\overline{b}))}$ $F(P(\overline{a}) \supset Q(\overline{a})) \quad (\alpha) \quad (5)$

$F(P(\overline{a}) \supset Q(\overline{a})) \quad (\alpha) \quad (6)$ $F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma)$

$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma(y\|b)) \quad (7)$ $\underline{T(P(\overline{a}))}$

$\underline{T(P(\overline{a}))}$ $\underline{F(Q(\overline{a}))}\,\nmid$

$\underline{F(Q(\overline{a}))}$

$F(P(\overline{b}) \supset Q(\overline{a})) \quad (\alpha)$

$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma)$

$$F(\forall x P(x) \supset Q(\overline{a})) \supset \exists y(P(y) \supset Q(\overline{a})) \quad (\alpha) \quad (1)$$

$$\mid$$

$$T(\forall x P(x) \supset Q(\overline{a})) \quad (\beta) \quad (2)$$

$$\mid$$

$$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma(y\|a)) \quad (4)$$

$$F(\forall x P(x)) \quad (\delta) \quad (3) \qquad\qquad \underline{T(Q(\overline{a}))}$$

$$\underline{F(P(\overline{b}))} \qquad\qquad F(P(\overline{a}) \supset Q(\overline{a})) \quad (\alpha) \quad (5)$$

$$F(P(\overline{a}) \supset Q(\overline{a})) \quad (\alpha) \quad (6) \qquad F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma)$$

$$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma(y\|b)) \quad (7) \qquad \underline{T(P(\overline{a}))}$$

$$\underline{T(P(\overline{a}))} \qquad\qquad \underline{F(Q(\overline{a}))} \natural$$

$$\underline{F(Q(\overline{a}))}$$

$$F(P(\overline{b}) \supset Q(\overline{a})) \quad (\alpha) \quad (8)$$

$$\mid$$

$$F(\exists y(P(y) \supset Q(\overline{a}))) \quad (\gamma)$$

$$\mid$$

$$\underline{T(P(\overline{b}))}$$

$$\mid$$

$$\underline{F(Q(\overline{a}))}$$

$$F(\forall x P(x) \supset Q(\overline{a})) \supset \exists y (P(y) \supset Q(\overline{a})) \quad (\alpha) \quad (1)$$

$$\mid$$

$$T(\forall x P(x) \supset Q(\overline{a})) \quad (\beta) \quad (2)$$

$$\mid$$

$$F(\exists y (P(y) \supset Q(\overline{a}))) \quad (\gamma(y\|a)) \quad (4)$$

$$F(\forall x P(x)) \quad (\delta) \quad (3) \qquad\qquad \underline{T(Q(\overline{a}))}$$

$$\underline{F(P(\overline{b}))} \qquad\qquad F(P(\overline{a}) \supset Q(\overline{a})) \quad (\alpha) \quad (5)$$

$$F(P(\overline{a}) \supset Q(\overline{a})) \quad (\alpha) \quad (6) \qquad F(\exists y (P(y) \supset Q(\overline{a}))) \quad (\gamma)$$

$$F(\exists y (P(y) \supset Q(\overline{a}))) \quad (\gamma(y\|b)) \quad (7) \qquad \underline{T(P(\overline{a}))}$$

$$\underline{T(P(\overline{a}))} \qquad\qquad \underline{F(Q(\overline{a}))}\not

$$

$$\underline{F(Q(\overline{a}))}$$

$$F(P(\overline{b}) \supset Q(\overline{a})) \quad (\alpha) \quad (8)$$

$$F(\exists y (P(y) \supset Q(\overline{a}))) \quad (\gamma)$$

$$\underline{T(P(\overline{b}))}\notni$$

$$\underline{F(Q(\overline{a}))}$$

Elsőrendű logika - Feladatok

2. Helyes-e a következő szemantikus következmény?

$$\{(\forall x \exists y Q(x, y) \supset \forall x P(x)), \neg \forall x P(x)\} \models \neg \forall x \exists y Q(x, y)$$

Használjuk a visszakövetkeztetést, vagyis kielégíthetetlen-e a formulahalmaz:

$$\{(\forall x \exists y Q(x, y) \supset \forall x P(x)), \neg \forall x P(x), \neg \neg \forall x \exists y Q(x, y)\} =$$
$$\{(\forall x \exists y Q(x, y) \supset \forall x P(x)), \neg \forall x P(x), \forall x \exists y Q(x, y)\}$$

Elsőrendű logika - Feladatok

$T(\forall x \exists y Q(x, y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x, y)$

$T(\forall x \exists y Q(x, y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x, y) \quad (\alpha) \quad (1)$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \wedge \neg\forall x P(x) \wedge \forall x \exists y Q(x,y) \quad (\alpha) \quad (1)$$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x))$$

$$T\neg\forall x P(x)$$

$$T\forall x \exists y Q(x,y)$$

Elsőrendű logika - Feladatok

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x,y) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \quad (\beta)$$
$$|$$
$$T \neg \forall x P(x)$$
$$|$$
$$T \forall x \exists y Q(x,y)$$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \land \neg \forall x P(x) \land \forall x \exists y Q(x,y) \quad (\alpha) \quad (1)$$

$$|$$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \quad (\beta)$$

$$|$$

$$T \neg \forall x P(x) \quad (\alpha)$$

$$|$$

$$T \forall x \exists y Q(x,y)$$

Elsőrendű logika - Feladatok

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x,y) \quad (\alpha) \quad (1)$$

$$|$$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \quad (\beta)$$

$$|$$

$$T \neg \forall x P(x) \quad (\alpha)$$

$$|$$

$$T \forall x \exists y Q(x,y) \quad (\gamma)$$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \wedge \neg\forall x P(x) \wedge \forall x \exists y Q(x,y) \quad (\alpha) \quad (1)$$

$$|$$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \quad (\beta)$$

$$|$$

$$T\neg\forall x P(x) \quad (\alpha) \quad (2)$$

$$|$$

$$T\forall x \exists y Q(x,y) \quad (\gamma)$$

$$|$$

$$F\forall x P(x)$$

Elsőrendű logika - Feladatok

$$T(\forall x \exists y Q(x, y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x, y) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x \exists y Q(x, y) \supset \forall x P(x)) \quad (\beta)$$
$$|$$
$$T \neg \forall x P(x) \quad (\alpha) \quad (2)$$
$$|$$
$$T \forall x \exists y Q(x, y) \quad (\gamma)$$
$$|$$
$$F \forall x P(x) \quad (\delta)$$

$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x,y)$ (α) (1)

$T(\forall x \exists y Q(x,y) \supset \forall x P(x))$ (β)

$T \neg \forall x P(x)$ (α) (2)

$T \forall x \exists y Q(x,y)$ (γ)

$F \forall x P(x)$ $(\delta(x \| \overline{a}))$ (3)

$\underline{F P(\overline{a})}$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x,y) \quad (\alpha) \quad (1)$$

$$|$$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \quad (\beta) \quad (4)$$

$$|$$

$$T \neg \forall x P(x) \quad (\alpha) \quad (2)$$

$$|$$

$$T \forall x \exists y Q(x,y) \quad (\gamma)$$

$$|$$

$$F \forall x P(x) \quad (\delta(x||\bar{a})) \quad (3)$$

$$|$$

$$\underline{F P(\bar{a})}$$

$$F(\forall x \exists y Q(x,y) \qquad T(\forall x P(x))$$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x,y) \quad (\alpha) \quad (1)$$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \quad (\beta) \quad (4)$$

$$T\neg\forall x P(x) \quad (\alpha) \quad (2)$$

$$T\forall x \exists y Q(x,y) \quad (\gamma)$$

$$F\forall x P(x) \quad (\delta(x||\bar{a})) \quad (3)$$

$$\underline{FP(\bar{a})}$$

$$F(\forall x \exists y Q(x,y) \quad (\delta) \qquad T(\forall x P(x))$$

Elsőrendű logika - Feladatok

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \land \neg \forall x P(x) \land \forall x \exists y Q(x,y) \quad (\alpha) \quad (1)$$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \quad (\beta) \quad (4)$$

$$T \neg \forall x P(x) \quad (\alpha) \quad (2)$$

$$T \forall x \exists y Q(x,y) \quad (\gamma)$$

$$F \forall x P(x) \quad (\delta(x \| \overline{a})) \quad (3)$$

$$\underline{F P(\overline{a})}$$

$$F(\forall x \exists y Q(x,y) \quad (\delta) \qquad T(\forall x P(x)) \quad (\gamma)$$

Elsőrendű logika - Feladatok

$$T(\forall x \exists y Q(x, y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x, y) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x \exists y Q(x, y) \supset \forall x P(x)) \quad (\beta) \quad (4)$$
$$|$$
$$T \neg \forall x P(x) \quad (\alpha) \quad (2)$$
$$|$$
$$T \forall x \exists y Q(x, y) \quad (\gamma)$$
$$|$$
$$F \forall x P(x) \quad (\delta(x||\overline{a})) \quad (3)$$
$$|$$
$$F P(\overline{a})$$

$$F(\forall x \exists y Q(x, y) \quad \delta(x||\overline{b}) \quad (5) \qquad T(\forall x P(x)) \quad (\gamma)$$
$$|$$
$$F(\exists y Q(\overline{b}, y))$$

Elsőrendű logika - Feladatok

$$T(\forall x\exists y Q(x,y) \supset \forall x P(x)) \land \neg\forall x P(x) \land \forall x\exists y Q(x,y) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x\exists y Q(x,y) \supset \forall x P(x)) \quad (\beta) \quad (4)$$
$$|$$
$$T\neg\forall x P(x) \quad (\alpha) \quad (2)$$
$$|$$
$$T\forall x\exists y Q(x,y) \quad (\gamma)$$
$$|$$
$$F\forall x P(x) \quad (\delta(x||\overline{a})) \quad (3)$$
$$|$$
$$FP(\overline{a})$$

$$F(\forall x\exists y Q(x,y) \quad \delta(x||\overline{b})) \quad (5) \qquad T(\forall x P(x)) \quad (\gamma)$$
$$|$$
$$F(\exists y Q(\overline{b},y)) \quad (\gamma)$$

Elsőrendű logika - Feladatok

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x,y) \quad (\alpha) \quad (1)$$

$$\mid$$

$$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \quad (\beta) \quad (4)$$

$$\mid$$

$$T\neg\forall x P(x) \quad (\alpha) \quad (2)$$

$$\mid$$

$$T\forall x \exists y Q(x,y) \quad (\gamma)$$

$$\mid$$

$$F\forall x P(x) \quad (\delta(x||\overline{a})) \quad (3)$$

$$\mid$$

$$\underline{FP(\overline{a})}$$

$$F(\forall x \exists y Q(x,y) \quad \delta(x||\overline{b})) \quad (5) \qquad T(\forall x P(x)) \quad (\gamma(x||\overline{a})) \quad (6)$$

$$\mid \qquad\qquad\qquad\qquad\qquad\qquad \mid$$

$$F(\exists y Q(\overline{b},y)) \quad (\gamma) \qquad\qquad\qquad\qquad \underline{TP(\overline{a})}$$

$$T(\forall x \exists y Q(x, y) \supset \forall x P(x)) \land \neg \forall x P(x) \land \forall x \exists y Q(x, y) \quad (\alpha) \quad (1)$$

$$\mid$$

$$T(\forall x \exists y Q(x, y) \supset \forall x P(x)) \quad (\beta) \quad (4)$$

$$\mid$$

$$T\neg \forall x P(x) \quad (\alpha) \quad (2)$$

$$\mid$$

$$T\forall x \exists y Q(x, y) \quad (\gamma)$$

$$\mid$$

$$F\forall x P(x) \quad (\delta(x\|\overline{a})) \quad (3)$$

$$\mid$$

$$\underline{FP(\overline{a})}$$

$$F(\forall x \exists y Q(x, y) \quad \delta(x\|\overline{b}) \quad (5) \qquad T(\forall x P(x)) \quad (\gamma(x\|\overline{a})) \quad (6)$$

$$\mid \qquad\qquad\qquad\qquad\qquad\qquad \mid$$

$$F(\exists y Q(\overline{b}, y)) \quad (\gamma) \qquad\qquad\qquad \underline{TP(\overline{a})}\lightning$$

$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x,y)$ (α)   (1)

$T(\forall x \exists y Q(x,y) \supset \forall x P(x))$ (β)   (4)

$T \neg \forall x P(x)$ (α)   (2)

$T \forall x \exists y Q(x,y)$ ($\gamma(x||\overline{b})$)   (7)

$F \forall x P(x)$ ($\delta(x||\overline{a})$)   (3)

$\underline{FP(\overline{a})}$

$F(\forall x \exists y Q(x,y)$ $\delta(x||\overline{b})$ (5) $T(\forall x P(x))$ ($\gamma(x||\overline{a})$)   (6)

$F(\exists y Q(\overline{b}, y))$ (γ) $\underline{TP(\overline{a})} \notin$

$T(\exists y Q(\overline{b}, y))$

$T \forall x \exists y Q(x,y)$ (γ)

$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \land \neg \forall x P(x) \land \forall x \exists y Q(x,y) \quad (\alpha) \quad (1)$

$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \quad (\beta) \quad (4)$

$T \neg \forall x P(x) \quad (\alpha) \quad (2)$

$T \forall x \exists y Q(x,y) \quad (\gamma(x||\overline{b})) \quad (7)$

$F \forall x P(x) \quad (\delta(x||\overline{a})) \quad (3)$

$\underline{FP(\overline{a})}$

$F(\forall x \exists y Q(x,y) \quad \delta(x||\overline{b}) \quad (5) \qquad T(\forall x P(x)) \quad (\gamma(x||\overline{a})) \quad (6)$

$F(\exists y Q(\overline{b}, y)) \quad (\gamma) \qquad\qquad\qquad \underline{TP(\overline{a})} \notin$

$T(\exists y Q(\overline{b}, y)) \quad (\delta)$

$T \forall x \exists y Q(x,y) \quad (\gamma)$

$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \land \neg \forall x P(x) \land \forall x \exists y Q(x,y)$ (α) (1)

$|$

$T(\forall x \exists y Q(x,y) \supset \forall x P(x))$ (β) (4)

$|$

$T\neg \forall x P(x)$ (α) (2)

$|$

$T\forall x \exists y Q(x,y)$ $(\gamma(x||\overline{b}))$ (7)

$|$

$F\forall x P(x)$ $(\delta(x||\overline{a}))$ (3)

$|$

$\underline{FP(\overline{a})}$

$F(\forall x \exists y Q(x,y))$ $\delta(x||\overline{b})$ (5) $T(\forall x P(x))$ $(\gamma(x||\overline{a}))$ (6)

$|$ $|$

$F(\exists y Q(\overline{b},y))$ (γ) $\underline{TP(\overline{a})}\not\downarrow$

$|$

$T(\exists y Q(\overline{b},y))$ $(\delta(y||\overline{c}))$ (8)

$|$

$T\forall x \exists y Q(x,y)$ (γ)

$|$

$\underline{TQ(\overline{b},\overline{c})}$

Elsőrendű logika - Feladatok

$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x,y)$ (α) (1)

$T(\forall x \exists y Q(x,y) \supset \forall x P(x))$ (β) (4)

$T\neg \forall x P(x)$ (α) (2)

$T\forall x \exists y Q(x,y)$ $(\gamma(x||\overline{b}))$ (7)

$F\forall x P(x)$ $(\delta(x||\overline{a}))$ (3)

$\underline{FP(\overline{a})}$

$F(\forall x \exists y Q(x,y)$ $\delta(x||\overline{b})$ (5) $T(\forall x P(x))$ $(\gamma(x||\overline{a}))$ (6)

$F(\exists y Q(\overline{b},y))$ $(\gamma(y||\overline{c}))$ (9) $\underline{TP(\overline{a})}\lightning$

$T(\exists y Q(\overline{b},y))$ $(\delta(y||\overline{c}))$ (8)

$T\forall x \exists y Q(x,y)$ (γ)

$\underline{TQ(\overline{b},\overline{c})}$

$\underline{FQ(\overline{b},\overline{c})}$

$T(\forall x \exists y Q(x, y) \supset \forall x P(x)) \land \neg \forall x P(x) \land \forall x \exists y Q(x, y)$ (α) (1)

$T(\forall x \exists y Q(x, y) \supset \forall x P(x))$ (β) (4)

$T \neg \forall x P(x)$ (α) (2)

$T \forall x \exists y Q(x, y)$ $(\gamma(x || \overline{b}))$ (7)

$F \forall x P(x)$ $(\delta(x || \overline{a}))$ (3)

$\underline{FP(\overline{a})}$

$F(\forall x \exists y Q(x, y))$ $\delta(x || \overline{b})$ (5) $\quad$ $T(\forall x P(x))$ $(\gamma(x || \overline{a}))$ (6)

$F(\exists y Q(\overline{b}, y))$ $(\gamma(y || \overline{c}))$ (9) $\quad\quad\quad$ $\underline{TP(\overline{a})} \downarrow$

$T(\exists y Q(\overline{b}, y))$ $(\delta(y || \overline{c}))$ (8)

$T \forall x \exists y Q(x, y)$ (γ)

$\underline{TQ(\overline{b}, \overline{c})}$

$\underline{FQ(\overline{b}, \overline{c})}$

$F(\exists y Q(\overline{b}, y))$ (γ)

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$T(\forall x \exists y Q(x,y) \supset \forall x P(x)) \wedge \neg \forall x P(x) \wedge \forall x \exists y Q(x,y)$ (α) (1)

$T(\forall x \exists y Q(x,y) \supset \forall x P(x))$ (β) (4)

$T\neg \forall x P(x)$ (α) (2)

$T\forall x \exists y Q(x,y)$ $(\gamma(x||\overline{b}))$ (7)

$F\forall x P(x)$ $(\delta(x||\overline{a}))$ (3)

$\underline{FP(\overline{a})}$

$F(\forall x \exists y Q(x,y)$ $\delta(x||\overline{b})$ (5) $T(\forall x P(x))$ $(\gamma(x||\overline{a}))$ (6)

$F(\exists y Q(\overline{b},y))$ $(\gamma(y||\overline{c}))$ (9) $\underline{TP(\overline{a})}\notin$

$T(\exists y Q(\overline{b},y))$ $(\delta(y||\overline{c}))$ (8)

$T\forall x \exists y Q(x,y)$ (γ)

$\underline{TQ(\overline{b},\overline{c})}$

$\underline{FQ(\overline{b},\overline{c})}\notin$

$F(\exists y Q(\overline{b},y))$ (γ)

Elsőrendű logika - Feladatok

3. Helyes-e a következő szemantikus következmény?

$$\{\forall x(P(x) \lor Q(x))\} \models (\forall x P(x) \lor \forall x Q(x))$$

Használjunk dedukciós tételt:

$$(\forall x(P(x) \lor Q(x)) \supset \neg(\forall x P(x) \lor \forall x Q(x)))$$

Vagyis létezik-e zárt tabló a formulához,van-e a formulának tablócáfolata?

Elsőrendű logika - Feladatok

$F(\forall x(P(x) \lor Q(x)) \supset (\forall x P(x) \lor \forall x Q(x)))$

$F(\forall x(P(x) \lor Q(x)) \supset (\forall x P(x) \lor \forall x Q(x)))$ (α) (1)

Elsőrendű logika - Feladatok

$$F(\forall x(P(x) \lor Q(x)) \supset (\forall x P(x) \lor \forall x Q(x))) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x(P(x) \lor Q(x))$$
$$|$$
$$F(\forall x P(x) \lor \forall x Q(x))$$

Elsőrendű logika - Feladatok

$$F(\forall x(P(x) \lor Q(x)) \supset (\forall xP(x) \lor \forall xQ(x))) \ \ (\alpha) \ \ (1)$$
$$|$$
$$T(\forall x(P(x) \lor Q(x)) \ \ (\gamma)$$
$$|$$
$$F(\forall xP(x) \lor \forall xQ(x))$$

Elsőrendű logika - Feladatok

$F(\forall x(P(x) \vee Q(x)) \supset (\forall x P(x) \vee \forall x Q(x)))$ (α) (1)

$T(\forall x(P(x) \vee Q(x))$ (γ)

$F(\forall x P(x) \vee \forall x Q(x))$ (α)

Elsőrendű logika - Feladatok

$$F(\forall x(P(x) \vee Q(x)) \supset (\forall x P(x) \vee \forall x Q(x))) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x(P(x) \vee Q(x)) \quad (\gamma)$$
$$|$$
$$F(\forall x P(x) \vee \forall x Q(x)) \quad (\alpha) \quad (2)$$
$$|$$
$$F\forall x P(x)$$
$$|$$
$$F\forall x Q(x)$$

$F(\forall x(P(x) \vee Q(x)) \supset (\forall xP(x) \vee \forall xQ(x)))$ (α) (1)

|

$T(\forall x(P(x) \vee Q(x))$ (γ)

|

$F(\forall xP(x) \vee \forall xQ(x))$ (α) (2)

|

$F\forall xP(x)$ (δ)

|

$F\forall xQ(x)$

Elsőrendű logika - Feladatok

$$F(\forall x(P(x) \vee Q(x)) \supset (\forall x P(x) \vee \forall x Q(x)))\ \ (\alpha)\ \ (1)$$
$$|$$
$$T(\forall x(P(x) \vee Q(x))\ \ (\gamma)$$
$$|$$
$$F(\forall x P(x) \vee \forall x Q(x))\ \ (\alpha)\ \ (2)$$
$$|$$
$$F\forall x P(x)\ \ (\delta)$$
$$|$$
$$F\forall x Q(x)\ \ (\delta)$$

Elsőrendű logika - Feladatok

$$F(\forall x(P(x) \vee Q(x)) \supset (\forall xP(x) \vee \forall xQ(x))) \ (\alpha) \ (1)$$
$$|$$
$$T(\forall x(P(x) \vee Q(x))) \ (\gamma)$$
$$|$$
$$F(\forall xP(x) \vee \forall xQ(x)) \ (\alpha) \ (2)$$
$$|$$
$$F\forall xP(x) \ (\delta) \ (3)$$
$$|$$
$$F\forall xQ(x) \ (\delta)$$
$$|$$
$$\underline{FP(\overline{a})}$$

Elsőrendű logika - Feladatok

$$F(\forall x(P(x) \lor Q(x)) \supset (\forall xP(x) \lor \forall xQ(x))) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x(P(x) \lor Q(x)) \quad (\gamma)$$
$$|$$
$$F(\forall xP(x) \lor \forall xQ(x)) \quad (\alpha) \quad (2)$$
$$|$$
$$F\forall xP(x) \quad (\delta) \quad (3)$$
$$|$$
$$F\forall xQ(x) \quad (\delta) \quad (4)$$
$$|$$
$$\underline{FP(\overline{a})}$$
$$|$$
$$\underline{FQ(\overline{b})}$$

Elsőrendű logika - Feladatok

$$F(\forall x(P(x) \vee Q(x)) \supset (\forall x P(x) \vee \forall x Q(x))) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x(P(x) \vee Q(x)) \quad (\gamma(x||a)) \quad (5)$$
$$|$$
$$F(\forall x P(x) \vee \forall x Q(x)) \quad (\alpha) \quad (2)$$
$$|$$
$$F \forall x P(x) \quad (\delta) \quad (3)$$
$$|$$
$$F \forall x Q(x) \quad (\delta) \quad (4)$$
$$|$$
$$\underline{FP(\overline{a})}$$
$$|$$
$$\underline{FQ(\overline{b})}$$
$$|$$
$$T(P(\overline{a}) \vee Q(\overline{a}))$$

$F(\forall x(P(x) \lor Q(x)) \supset (\forall x P(x) \lor \forall x Q(x)))$ (α) (1)

$T(\forall x(P(x) \lor Q(x))$ $(\gamma(x||a))$ (5)

$F(\forall x P(x) \lor \forall x Q(x))$ (α) (2)

$F \forall x P(x)$ (δ) (3)

$F \forall x Q(x)$ (δ) (4)

$\underline{FP(\overline{a})}$

$\underline{FQ(\overline{b})}$

$T(P(\overline{a}) \lor Q(\overline{a}))$ (β)

Elsőrendű logika - Feladatok

$$F(\forall x(P(x) \lor Q(x)) \supset (\forall xP(x) \lor \forall xQ(x)))\ \ (\alpha)\ \ (1)$$

$$T(\forall x(P(x) \lor Q(x))\ \ (\gamma(x\|a))\ \ (5)$$

$$F(\forall xP(x) \lor \forall xQ(x))\ \ (\alpha)\ \ (2)$$

$$F\forall xP(x)\ \ (\delta)\ \ (3)$$

$$F\forall xQ(x)\ \ (\delta)\ \ (4)$$

$$\underline{FP(\overline{a})}$$

$$\underline{FQ(\overline{b})}$$

$$T(P(\overline{a}) \lor Q(\overline{a}))\ \ (\beta)$$

$$T\forall x(P(x) \lor Q(x))$$

Elsőrendű logika - Feladatok

$$F(\forall x(P(x) \lor Q(x)) \supset (\forall xP(x) \lor \forall xQ(x))) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x(P(x) \lor Q(x)) \quad (\gamma(x\|a)) \quad (5)$$
$$|$$
$$F(\forall xP(x) \lor \forall xQ(x)) \quad (\alpha) \quad (2)$$
$$|$$
$$F\forall xP(x) \quad (\delta) \quad (3)$$
$$|$$
$$F\forall xQ(x) \quad (\delta) \quad (4)$$
$$|$$
$$\underline{FP(\overline{a})}$$
$$|$$
$$\underline{FQ(\overline{b})}$$
$$|$$
$$T(P(\overline{a}) \lor Q(\overline{a})) \quad (\beta)$$
$$|$$
$$T\forall x(P(x) \lor Q(x)) \quad (\gamma)$$

Elsőrendű logika - Feladatok

$$F(\forall x(P(x) \lor Q(x)) \supset (\forall x P(x) \lor \forall x Q(x))) \quad (\alpha) \quad (1)$$
$$|$$
$$T(\forall x(P(x) \lor Q(x)) \quad (\gamma(x\|a)) \quad (5)$$
$$|$$
$$F(\forall x P(x) \lor \forall x Q(x)) \quad (\alpha) \quad (2)$$
$$|$$
$$F\forall x P(x) \quad (\delta) \quad (3)$$
$$|$$
$$F\forall x Q(x) \quad (\delta) \quad (4)$$
$$|$$
$$\underline{FP(\bar{a})}$$
$$|$$
$$\underline{FQ(\bar{b})}$$
$$|$$
$$T(P(\bar{a}) \lor Q(\bar{a})) \quad (\beta) \quad (6)$$
$$|$$
$$T\forall x(P(x) \lor Q(x)) \quad (\gamma)$$

$$\underline{TP(\bar{a})} \qquad \underline{TQ(\bar{a})}$$

Elsőrendű logika - Feladatok

$F(\forall x(P(x) \lor Q(x)) \supset (\forall x P(x) \lor \forall x Q(x)))$ (α) (1)
|
$T(\forall x(P(x) \lor Q(x)))$ $(\gamma(x\|a))$ (5)
|
$F(\forall x P(x) \lor \forall x Q(x))$ (α) (2)
|
$F \forall x P(x)$ (δ) (3)
|
$F \forall x Q(x)$ (δ) (4)
|
$\underline{FP(\overline{a})}$
|
$\underline{FQ(\overline{b})}$
|
$T(P(\overline{a}) \lor Q(\overline{a}))$ (β) (6)
|
$T \forall x(P(x) \lor Q(x))$ (γ)

$\underline{TP(\overline{a})} \notmid \quad \underline{TQ(\overline{a})}$

Elsőrendű logika - Feladatok

$F(\forall x(P(x) \lor Q(x)) \supset (\forall xP(x) \lor \forall xQ(x)))$ (α) (1)
|
$T(\forall x(P(x) \lor Q(x))$ $(\gamma(x\|a))$ (5)
|
$F(\forall xP(x) \lor \forall xQ(x))$ (α) (2)
|
$F\forall xP(x)$ (δ) (3)
|
$F\forall xQ(x)$ (δ) (4)
|
$\underline{FP(\overline{a})}$
|
$\underline{FQ(\overline{b})}$
|
$T(P(\overline{a}) \lor Q(\overline{a}))$ (β) (6)
|
$T\forall x(P(x) \lor Q(x))$ $(\gamma(x\|b))$ (7)

$\overbrace{\qquad\qquad\qquad\qquad}$

$\underline{TP(\overline{a})}\xi$ $\qquad$ $\underline{TQ(\overline{a})}$
|
$T(P(\overline{b}) \lor Q(\overline{b}))$
|
$T\forall x(P(x) \lor Q(x))$ (γ)

Elsőrendű logika - Feladatok

$F(\forall x(P(x) \vee Q(x)) \supset (\forall xP(x) \vee \forall xQ(x)))$ (α) (1)

$T(\forall x(P(x) \vee Q(x))$ $(\gamma(x\|a))$ (5)

$F(\forall xP(x) \vee \forall xQ(x))$ (α) (2)

$F\forall xP(x)$ (δ) (3)

$F\forall xQ(x)$ (δ) (4)

$\underline{FP(\overline{a})}$

$\underline{FQ(\overline{b})}$

$T(P(\overline{a}) \vee Q(\overline{a}))$ (β) (6)

$T\forall x(P(x) \vee Q(x))$ $(\gamma(x\|b))$ (7)

$\underbrace{\underline{TP(\overline{a})}_{\not z} \qquad \underline{TQ(\overline{a})}}$

$T(P(\overline{b}) \vee Q(\overline{b}))$ (β)

$T\forall x(P(x) \vee Q(x))$ (γ)

$F(\forall x(P(x) \lor Q(x)) \supset (\forall x P(x) \lor \forall x Q(x)))$ (α) (1)

$T(\forall x(P(x) \lor Q(x)))$ $(\gamma(x\|a))$ (5)

$F(\forall x P(x) \lor \forall x Q(x))$ (α) (2)

$F \forall x P(x)$ (δ) (3)

$F \forall x Q(x)$ (δ) (4)

$\underline{FP(\overline{a})}$

$\underline{FQ(\overline{b})}$

$T(P(\overline{a}) \lor Q(\overline{a}))$ (β) (6)

$T \forall x(P(x) \lor Q(x))$ $(\gamma(x\|b))$ (7)

$\underline{TP(\overline{a})} \notin$ $\qquad$ $\underline{TQ(\overline{a})}$

$T(P(\overline{b}) \lor Q(\overline{b}))$ (β) (8)

$T \forall x(P(x) \lor Q(x))$ (γ)

$\underline{TP(\overline{b})} \qquad \underline{TQ(\overline{b})} \notin$

Elsőrendű logika - Feladatok

A 2. ágon még lehetne folytatni a γ típusú csúcs kibontásával, azonban ez végtelen számításhoz vezethet.