$u \in V$

Make_Set(u)

$u.\pi = u;\ u.s = 1$

$u \in V$

Find_Set(u):V

$u.\pi \neq u$	
$u.\pi = \text{Find\_Set}(u.\pi)$	X
return $u.\pi$	

Precond: $\begin{cases} x.\pi == x \\ y.\pi == y \end{cases}$

union(x, y : V)

$x.s < y.s$	
$x.\pi = y$ $y.s += x.s$	$y.\pi = x$ $x.s += y.s$

$s(x) := x.s$

$h(x) :=$ height of the tree rooted by x } before union

$\boxed{I}$, $\lg(s(x)) \geq h(x)$ for each $x \in V$

Proof, Initially: ~~assert~~

$\left.\begin{aligned} s(u) &= 1 \\ h(u) &= 0 \end{aligned}\right\} \lg(s(u)) = \lg(1) = 0 = h(u) \Rightarrow \lg(s(u)) \geq h(u)$ $(u \in V)$

It is enough to prove that if

$\Rightarrow$ Before union: $\left.\begin{aligned} \lg(s(x)) &\geq h(x) \\ \lg(s(y)) &\geq h(y) \end{aligned}\right\} \Rightarrow \lg(s(x \cup y)) \geq h(x \cup y)$

(where $(x \cup y)$ is the root of x and y after union.

In order to prove this we can suppose that $s(x) \geq s(y)$

Thus $h(x \cup y) = \max(h(x), h(y)+1)$

a.) $h(x) > h(y) \Rightarrow h(x \cup y) = h(x)$

$s(x \cup y) \geq s(x)$

$\lg(s(x \cup y)) \geq \lg(s(x)) \geq h(x) = h(x \cup y)$

$\lg(s(x \cup y)) \geq h(x \cup y)$: ✓

b.) $h(x) \leq h(y) \Rightarrow h(x \cup y) = h(y) + 1$

$s(x \cup y) = s(x) + s(y) \geq 2 \cdot s(y)$

$\lg(s(x \cup y)) \geq \lg(2 \cdot s(y)) = 1 + \lg(s(y)) \geq 1 + h(y) =$

$= h(x \cup y)$

$\Rightarrow \lg(s(x \cup y)) \geq h(x \cup y)$: ✓

MST_KRUSKAL(G, w)

```
∀u ∈ G.V
  | Make_Set(u)
A = {}; m = |G.V|
Sort G.E nondecreasingly according to weights
∀(u,v) ∈ G.E sorted, while m > 1
  | x = Find_Set(u)
  | y = Find_Set(v)
  | x ≠ y
  |   A = A ∪ {(u,v)}     | X
  |   union(x, y)         |
  |   m--                 |
```

$G = (V, E)$ $\begin{cases} \text{undirected} \\ \text{connected} \\ \text{weighted} \end{cases}$

$w : E \to \mathbb{R}$