Szekvent kalkulus

November 9, 2020

Tartalomjegyzék

1 Feladatok

1.1 Szekvent jelentése

Vizsgáljuk a következő szekventek helyességét, a szekvent jelentése alapján!

1. $\neg(A \vee C), A \longrightarrow B$
2. $\longrightarrow A \supset A$
3. $(\neg A \vee \neg B), \neg A \supset B \longrightarrow A \wedge B, \neg A \wedge \neg B$

1.2 Bizonyítás levezetéssel

1. $\longrightarrow A \supset A$
2. $A \longrightarrow \neg\neg A$
3. $A \wedge C, A \supset B \longrightarrow A \supset B$
4. $\neg(A \vee C), A \longrightarrow B$
5. $\neg(A \supset B) \longrightarrow \neg A \vee \neg B$
6. $(A \vee B) \supset C \longrightarrow (A \supset C) \wedge (B \supset C)$
7. $\neg\forall x P(x) \vee R(y) \longrightarrow \exists x(P(x) \supset R(y))$
8. $\exists x(P(x) \wedge R(x)) \longrightarrow \exists x P(x) \wedge \exists x R(x)$
9. Fifis feladat: {Fifi puli; Minden puli kutya; Minden kutya, ami ugat, az nem harap; Fifi ugat} $\longrightarrow$ Van olyan kutya, amelyik nem harap.

2 Megoldások

2.1 Szekvent jelentése

Vizsgáljuk a következő szekventek helyességét, a szekvent jelentése alapján!

1. $\neg(A \vee C), A \longrightarrow B$

   A de Morgan-azonosság alapján ez a következővel ekvivalens: $\neg A \wedge \neg C \wedge A \supset B$
   Az implikáció bal oldala mindig hamis, így az implikáció mindig igaz, tehát a szekvent teljesül.

2. $\longrightarrow A \supset A$

   Az implikáció bal- és jobb oldalán is azonosan igaz állítás szerepel, így az implikáció mindig igaz, tehát a szekvent teljesül.

3. $(\neg A \vee \neg B), \neg A \supset B \longrightarrow A \wedge B, \neg A \wedge \neg B$

A	B	$(\neg A \vee \neg B) \wedge (\neg A \supset B) \supset A \wedge B \vee \neg A \wedge \neg B$
i	i	i
i	h	i
h	i	h
h	h	i

2.2 Bizonyítás levezetéssel

1. $\longrightarrow A \supset A$

$$(\rightarrow\supset)\ \dfrac{\dfrac{\checkmark}{A \longrightarrow A}}{\longrightarrow A \supset A}$$

2. $A \longrightarrow \neg\neg A$

$$(\rightarrow \neg)\quad (\neg \rightarrow)\ \dfrac{\dfrac{\dfrac{\checkmark}{A \longrightarrow A}}{A, \neg A \longrightarrow}}{A \longrightarrow \neg\neg A}$$

3. $A \wedge C, A \supset B \longrightarrow A \supset B$

   - G-kalkulusban:

$$(b\rightarrow)\,\dfrac{\dfrac{\checkmark}{A\supset B\longrightarrow A\supset B}}{A\wedge C, A\supset B\longrightarrow A\supset B}$$

- C-kalkulusban:

$$\dfrac{\checkmark}{A\wedge C, A\supset B\longrightarrow A\supset B}$$

4. $\neg(A\vee C), A\longrightarrow B$
   - G-kalkulusban:

$$(\neg\rightarrow)\,\dfrac{(\rightarrow\vee)\,\dfrac{\dfrac{?}{A\longrightarrow C, B}}{A\longrightarrow A\vee C, B}}{\neg(A\vee C), A\longrightarrow B}$$

A $(\rightarrow\vee)$ szabály alkalmazásakor rosszul választottunk, emiatt nem sikerült előállítani a levezetést. Jó választás esetén nem akadunk el:

$$(\neg\rightarrow)\,\dfrac{(\rightarrow\vee)\,\dfrac{(\rightarrow b)\,\dfrac{\dfrac{\checkmark}{A\longrightarrow A}}{A\longrightarrow A, B}}{A\longrightarrow A\vee C, B}}{\neg(A\vee C), A\longrightarrow B}$$

5. $\neg(A\supset B)\longrightarrow\neg A\vee\neg B$
   - G-kalkulusban:

$$(\neg\rightarrow)\,\dfrac{(\rightarrow\supset)\,\dfrac{(\rightarrow\vee)\,\dfrac{(\rightarrow\neg)\,\dfrac{(b\rightarrow)\,\dfrac{\dfrac{\checkmark}{B\longrightarrow B}}{A, B\longrightarrow B}}{A\longrightarrow\neg B, B}}{A\longrightarrow\neg A\vee\neg B, B}}{\longrightarrow\neg A\vee\neg B, A\supset B}}{\neg(A\supset B)\longrightarrow\neg A\vee\neg B}$$

   - C-kalkulusban:

$$
\begin{prooftree}
\AxiomC{$\checkmark$}
\UnaryInfC{$A, B \longrightarrow \neg A, B$}
\LeftLabel{$(\rightarrow \neg)$}
\UnaryInfC{$A \longrightarrow \neg A, \neg B, B$}
\LeftLabel{$(\rightarrow\supset)$}
\UnaryInfC{$\longrightarrow \neg A, \neg B, A \supset B$}
\LeftLabel{$(\neg \rightarrow)$}
\UnaryInfC{$\neg(A \supset B) \longrightarrow \neg A, \neg B$}
\LeftLabel{$(\rightarrow \vee)$}
\UnaryInfC{$\neg(A \supset B) \longrightarrow \neg A \vee \neg B$}
\end{prooftree}
$$

6. $(A \vee B) \supset C \longrightarrow (A \supset C) \wedge (B \supset C)$

- G-kalkulusban:

$$
\begin{prooftree}
\AxiomC{$\checkmark$}
\UnaryInfC{$A \longrightarrow A$}
\LeftLabel{$(\rightarrow b)$}
\UnaryInfC{$A \longrightarrow C, A$}
\LeftLabel{$(\rightarrow\supset)$}
\UnaryInfC{$\longrightarrow A \supset C, A$}
\LeftLabel{$(\rightarrow \vee)$}
\UnaryInfC{$\longrightarrow A \supset C, A \vee B$}
\AxiomC{$\checkmark$}
\UnaryInfC{$B \longrightarrow B$}
\LeftLabel{$(\rightarrow b)$}
\UnaryInfC{$B \longrightarrow C, B$}
\LeftLabel{$(\rightarrow\supset)$}
\UnaryInfC{$\longrightarrow B \supset C, B$}
\LeftLabel{$(\rightarrow \vee)$}
\UnaryInfC{$\longrightarrow B \supset C, A \vee B$}
\LeftLabel{$(\rightarrow \wedge)$}
\BinaryInfC{$\longrightarrow (A \supset C) \wedge (B \supset C), A \vee B$}
\AxiomC{$\checkmark$}
\UnaryInfC{$C \longrightarrow C$}
\LeftLabel{$(b \rightarrow)$}
\UnaryInfC{$C, A \longrightarrow C$}
\LeftLabel{$(\rightarrow\supset)$}
\UnaryInfC{$C \longrightarrow A \supset C$}
\AxiomC{$\checkmark$}
\UnaryInfC{$C \longrightarrow C$}
\LeftLabel{$(b \rightarrow)$}
\UnaryInfC{$C, B \longrightarrow C$}
\LeftLabel{$(\rightarrow\supset)$}
\UnaryInfC{$C \longrightarrow B \supset C$}
\LeftLabel{$(\rightarrow \wedge)$}
\BinaryInfC{$C \longrightarrow (A \supset C) \wedge (B \supset C)$}
\LeftLabel{$(\supset\rightarrow)$}
\BinaryInfC{$(A \vee B) \supset C \longrightarrow (A \supset C) \wedge (B \supset C)$}
\end{prooftree}
$$

- C-kalkulusban:

$$
\begin{prooftree}
\AxiomC{$\checkmark$}
\UnaryInfC{$A \longrightarrow A, B, C$}
\LeftLabel{$(\rightarrow\supset)$}
\UnaryInfC{$\longrightarrow A, B, A \supset C$}
\LeftLabel{$(\rightarrow \vee)$}
\UnaryInfC{$\longrightarrow A \vee B, A \supset C$}
\AxiomC{$\checkmark$}
\UnaryInfC{$A, C \longrightarrow C$}
\LeftLabel{$(\rightarrow\supset)$}
\UnaryInfC{$C \longrightarrow A \supset C$}
\LeftLabel{$(\supset\rightarrow)$}
\BinaryInfC{$(A \vee B) \supset C \longrightarrow A \supset C$}
\AxiomC{$\checkmark$}
\UnaryInfC{$B \longrightarrow C, A, B$}
\LeftLabel{$(\rightarrow \vee)$}
\UnaryInfC{$B \longrightarrow C, A \vee B$}
\AxiomC{$\checkmark$}
\UnaryInfC{$C, B \longrightarrow C$}
\LeftLabel{$(\supset\rightarrow)$}
\BinaryInfC{$B, (A \vee B) \supset C \longrightarrow C$}
\LeftLabel{$(\rightarrow\supset)$}
\UnaryInfC{$(A \vee B) \supset C \longrightarrow B \supset C$}
\LeftLabel{$(\rightarrow \wedge)$}
\BinaryInfC{$(A \vee B) \supset C \longrightarrow (A \supset C) \wedge (B \supset C)$}
\end{prooftree}
$$

7. $\neg\forall x P(x) \vee R(y) \longrightarrow \exists x(P(x) \supset R(y))$

- G-kalkulusban:

$$
\begin{prooftree}
\AxiomC{$\checkmark$}
\UnaryInfC{$P(x) \longrightarrow P(x)$}
\LeftLabel{$(\rightarrow b)$}
\UnaryInfC{$P(x) \longrightarrow R(y), P(x)$}
\LeftLabel{$(\rightarrow\supset)$}
\UnaryInfC{$\longrightarrow P(x) \supset R(y), P(x)$}
\LeftLabel{$(\rightarrow \exists)$}
\UnaryInfC{$\longrightarrow \exists x(P(x) \supset R(y)), P(x)$}
\LeftLabel{$(\rightarrow \forall)$}
\UnaryInfC{$\longrightarrow \exists x(P(x) \supset R(y)), \forall x P(x)$}
\LeftLabel{$(\neg \rightarrow)$}
\UnaryInfC{$\neg\forall x P(x) \longrightarrow \exists x(P(x) \supset R(y))$}
\AxiomC{$\checkmark$}
\UnaryInfC{$R(y) \longrightarrow R(y)$}
\LeftLabel{$(b \rightarrow)$}
\UnaryInfC{$R(y), P(x) \longrightarrow R(y)$}
\LeftLabel{$(\rightarrow\supset)$}
\UnaryInfC{$R(y) \longrightarrow P(x) \supset R(y)$}
\LeftLabel{$(\rightarrow \exists)$}
\UnaryInfC{$R(y) \longrightarrow \exists x(P(x) \supset R(y))$}
\LeftLabel{$(\vee \rightarrow)$}
\BinaryInfC{$\neg\forall x P(x) \vee R(y) \longrightarrow \exists x(P(x) \supset R(y))$}
\end{prooftree}
$$

- C-kalkulusban:

$$
{\scriptstyle(\lor\to)}\dfrac{
{\scriptstyle(\neg\to)}\dfrac{
{\scriptstyle(\to\forall)}\dfrac{
{\scriptstyle(\to\exists)}\dfrac{
{\scriptstyle(\to\supset)}\dfrac{
\dfrac{\checkmark}{P(x) \longrightarrow \exists x(P(x) \supset R(y)), R(y), P(x)}
}{\longrightarrow \exists x(P(x) \supset R(y)), P(x) \supset R(y), P(x)}
}{\longrightarrow \exists x(P(x) \supset R(y)), P(x)}
}{\longrightarrow \exists x(P(x) \supset R(y)), \forall x P(x)}
}{\neg\forall x P(x) \longrightarrow \exists x(P(x) \supset R(y))}
\qquad
{\scriptstyle(\to\exists)}\dfrac{
{\scriptstyle(\to\supset)}\dfrac{
\dfrac{\checkmark}{R(y), P(x) \longrightarrow R(y), \exists x(P(x) \supset R(y))}
}{R(y) \longrightarrow P(x) \supset R(y), \exists x(P(x) \supset R(y))}
}{R(y) \longrightarrow \exists x(P(x) \supset R(y))}
}{\neg\forall x P(x) \lor R(y) \longrightarrow \exists x(P(x) \supset R(y))}
$$

8. $\exists x(P(x) \land R(x)) \longrightarrow \exists x P(x) \land \exists x R(x)$

- G-kalkulusban:

$$
{\scriptstyle(\lor\to)}\dfrac{
{\scriptstyle(\neg\to)}\dfrac{
{\scriptstyle(\to\forall)}\dfrac{
{\scriptstyle(\to\exists)}\dfrac{
{\scriptstyle(\to\supset)}\dfrac{
{\scriptstyle(\to b)}\dfrac{
\dfrac{\checkmark}{P(x) \longrightarrow P(x)}
}{P(x) \longrightarrow R(y), P(x)}
}{\longrightarrow P(x) \supset R(y), P(x)}
}{\longrightarrow \exists x(P(x) \supset R(y)), P(x)}
}{\longrightarrow \exists x(P(x) \supset R(y)), \forall x P(x)}
}{\neg\forall x P(x) \longrightarrow \exists x(P(x) \supset R(y))}
\qquad
{\scriptstyle(\to\exists)}\dfrac{
{\scriptstyle(\to\supset)}\dfrac{
{\scriptstyle(b\to)}\dfrac{
\dfrac{\checkmark}{R(y) \longrightarrow R(y)}
}{R(y), P(x) \longrightarrow R(y)}
}{R(y) \longrightarrow P(x) \supset R(y)}
}{R(y) \longrightarrow \exists x(P(x) \supset R(y))}
}{\neg\forall x P(x) \lor R(y) \longrightarrow \exists x(P(x) \supset R(y))}
$$

- C-kalkulusban:

$$
{\scriptstyle(\lor\to)}\dfrac{
{\scriptstyle(\neg\to)}\dfrac{
{\scriptstyle(\to\forall)}\dfrac{
{\scriptstyle(\to\exists)}\dfrac{
{\scriptstyle(\to\supset)}\dfrac{
\dfrac{\checkmark}{P(x) \longrightarrow \exists x(P(x) \supset R(y)), R(y), P(x)}
}{\longrightarrow \exists x(P(x) \supset R(y)), P(x) \supset R(y), P(x)}
}{\longrightarrow \exists x(P(x) \supset R(y)), P(x)}
}{\longrightarrow \exists x(P(x) \supset R(y)), \forall x P(x)}
}{\neg\forall x P(x) \longrightarrow \exists x(P(x) \supset R(y))}
\qquad
{\scriptstyle(\to\exists)}\dfrac{
{\scriptstyle(\to\supset)}\dfrac{
\dfrac{\checkmark}{R(y), P(x) \longrightarrow R(y), \exists x(P(x) \supset R(y))}
}{R(y) \longrightarrow P(x) \supset R(y), \exists x(P(x) \supset R(y))}
}{R(y) \longrightarrow \exists x(P(x) \supset R(y))}
}{\neg\forall x P(x) \lor R(y) \longrightarrow \exists x(P(x) \supset R(y))}
$$

- Fifi-s feladat:

1. ábra

$$
[\supset\to]\,\dfrac{[b\to]\,\dfrac{[b\to]\,\dfrac{[\to b]\,\dfrac{\text{axiómaséma}\,\dfrac{}{P(a)\to P(a)}}{P(a)\to P(a),K(a)}}{P(a),U(a)\to P(a),K(a)}}{P(a),U(a),K(a)\wedge U(a)\supset\neg H(a)\to P(a),K(a)} \qquad [b\to]\,\dfrac{[b\to]\,\dfrac{[b\to]\,\dfrac{\text{axiómaséma}\,\dfrac{}{K(a)\to K(a)}}{K(a)\wedge U(a)\supset\neg H(a),K(a)\to K(a)}}{U(a),K(a)\wedge U(a)\supset\neg H(a),K(a)\to K(a)}}{P(a),U(a),K(a)\wedge U(a)\supset\neg H(a),K(a)\to K(a)}}{...,P(a)\supset K(a)\to K(a)}
$$

$$
[\to\exists]\,\dfrac{[\forall\to]\,\dfrac{[\forall\to]\,\dfrac{[\to\wedge]\,\dfrac{...,P(a)\supset K(a)\to K(a) \qquad \text{2. ábra}}{P(a),U(a),K(a)\wedge U(a)\supset\neg H(a),P(a)\supset K(a)\longrightarrow K(a)\wedge\neg H(a)}}{P(a),U(a),K(a)\wedge U(a)\supset\neg H(a),\forall x(P(x)\supset K(x))\longrightarrow K(a)\wedge\neg H(a)}}{P(a),U(a),\forall x(K(x)\wedge U(x)\supset\neg H(x)),\forall x(P(x)\supset K(x))\longrightarrow K(a)\wedge\neg H(a)}}{P(a),U(a),\forall x(K(x)\wedge U(x)\supset\neg H(x)),\forall x(P(x)\supset K(x))\longrightarrow\exists x(K(x)\wedge\neg H(x))}
$$

2. ábra

$$
\dfrac{\dfrac{\dfrac{\text{Ugyanaz, mint 1. ábra lentről 5. sortól}}{...,P(a)\supset K(a)\longrightarrow\neg H(a),K(a)} \qquad [b\to]\,\dfrac{[b\to]\,\dfrac{[\to b]\,\dfrac{\text{axiómaséma}\,\dfrac{}{U(a)\longrightarrow U(a)}}{U(a)\longrightarrow\neg H(a),U(a)}}{U(a),...\longrightarrow\neg H(a),U(a)}}{...,U(a),...\longrightarrow\neg H(a),U(a)}}{P(a),U(a),P(a)\supset K(a)\longrightarrow\neg H(a),K(a)\wedge U(a)}\,[\to\wedge] \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\neg H(a)\longrightarrow\neg H(a)}\,\text{axiómaséma}}{P(a),\neg H(a)\longrightarrow\neg H(a)}\,[b\to]}{P(a),U(a),\neg H(a)\longrightarrow\neg H(a)}\,[b\to]}{P(a),U(a),\neg H(a),P(a)\supset K(a)\longrightarrow\neg H(a)}\,[b\to]}{P(a),U(a),K(a)\wedge U(a)\supset\neg H(a),P(a)\supset K(a)\longrightarrow\neg H(a)}\,[\supset\to]
$$