

# voľba šéfa: jednosmerné kruhy

## Chang Roberts

**const:**    *ID*        : integer  
             *l<sub>in</sub>*, *l<sub>out</sub>* : link  
**var:**        *leader* : integer

Init:

*leader* := NULL

Code:

**send**  $\langle ID \rangle$   
**wait** until *leader*  $\neq$  NULL

On receipt  $\langle i \rangle$ :

**if**  $i < ID$  **then send**  $\langle i \rangle$   
**if**  $i = ID$  **then**  
    *leader* := *ID*  
    **send**  $\langle \text{leader}, ID \rangle$

On receipt  $\langle \text{leader}, x \rangle$ :

*leader* := *x*  
**send**  $\langle \text{leader}, ID \rangle$

$O(n \log n)$  správ v priemernom prípade,  $\Omega(n^2)$  v najhoršom

## Hirschberg-Sinclair

- level  $l$ : dobýjať územie  $2^l$
- $\log n$  levelov
- $n/2^l$  vrcholov na leveli
- každý vrchol pošle  $2^l$  správ

## Franklin

- level  $l$ : poraziť susedov (na rovnakom leveli; “synchronizácia”)
- $\log n$  levelov
- $n$  správ na level

## Dolev – simulácia na 1-smernom kruhu

- idea: presunúť identitu

## dolný odhad

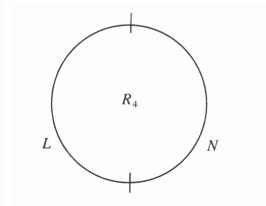
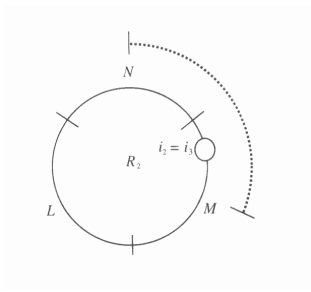
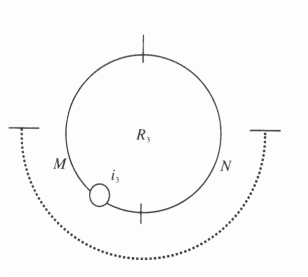
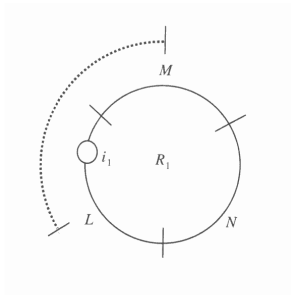
zapojiť do čiary  $L$ , vymení sa  $C(L)$  správ

### lema

Pre každé  $r$  existuje nekonečne veľa čiar dĺžky  $2^r$ , kde  $C(L) \geq r2^{r-2}$

- indukcia
- dve vrecia: vyberám trojice, chcem dve spojiť item pri spojení: dĺžka  $2^{r+1}$ , počet správ  $\geq r2^{r-1}$
- ešte treba  $2^{r-1}$  správ; sporom

# dolný odhad



- ľubovoľná topológia
- buduje sa kostra
- “segmenty”
- spájanie po najlacnejšej odchádzajúcej hrane:
  - A menší sa pripojí k väčšiemu
  - B rovnakí sa spoja
  - C väčší čaká
- veľkosť  $\Rightarrow$  level

---

```

var  $state_p$  : (sleep, find, found) ;
       $stach_p[q]$  : (basic, branch, reject) for each  $q \in Neigh_p$  ;
       $name_p, bestwt_p$  : real ;
       $level_p$  : integer ;
       $testch_p, bestch_p, father_p$  :  $Neigh_p$  ;
       $rec_p$  : integer ;

```

- (1) As the first action of each process, the algorithm must be initialized:

```

begin let  $pq$  be the channel of  $p$  with smallest weight ;
       $stach_p[q] := branch$  ;  $level_p := 0$  ;
       $state_p := found$  ;  $rec_p := 0$  ;
      send  $\langle \mathbf{connect}, 0 \rangle$  to  $q$ 
end

```

- (2) Upon receipt of  $\langle \mathbf{connect}, L \rangle$  from  $q$ :

```

begin if  $L < level_p$  then (* Combine with Rule A *)
      begin  $stach_p[q] := branch$  ;
            send  $\langle \mathbf{initiate}, level_p, name_p, state_p \rangle$  to  $q$ 
      end
      else if  $stach_p[q] = basic$ 
            then (* Rule C *) process the message later
            else (* Rule B *) send  $\langle \mathbf{initiate}, level_p + 1, \omega(pq), find \rangle$  to  $q$ 
end

```

- (3) Upon receipt of  $\langle \mathbf{initiate}, L, F, S \rangle$  from  $q$ :

```

begin  $level_p := L$  ;  $name_p := F$  ;  $state_p := S$  ;  $father_p := q$  ;
       $bestch_p := undef$  ;  $bestwt_p := \infty$  ;
      forall  $r \in Neigh_p$  :  $stach_p[r] = branch \wedge r \neq q$  do
            send  $\langle \mathbf{initiate}, L, F, S \rangle$  to  $r$  ;
      if  $state_p = find$  then begin  $rec_p := 0$  ; test end
end

```

```

(4) procedure test:
  begin if  $\exists q \in \text{Neigh}_p : \text{stach}_p[q] = \text{basic}$  then
    begin testchp := q with  $\text{stach}_p[q] = \text{basic}$  and  $\omega(pq)$  minimal ;
      send  $\langle \text{test}, \text{level}_p, \text{name}_p \rangle$  to testchp
    end
  else begin testchp := undef ; report end
  end

```

```

(5) Upon receipt of  $\langle \text{test}, L, F \rangle$  from q:
  begin if  $L > \text{level}_p$  then (* Answer must wait! *)
    process the message later
  else if  $F = \text{name}_p$  then (* internal edge *)
    begin if  $\text{stach}_p[q] = \text{basic}$  then  $\text{stach}_p[q] := \text{reject}$  ;
      if  $q \neq \text{testch}_p$ 
        then send  $\langle \text{reject} \rangle$  to q
      else test
    end
  else send  $\langle \text{accept} \rangle$  to q
  end

```

```

(6) Upon receipt of  $\langle \text{accept} \rangle$  from q:
  begin testchp := undef ;
    if  $\omega(pq) < \text{bestwt}_p$ 
      then begin  $\text{bestwt}_p := \omega(pq)$  ;  $\text{bestch}_p := q$  end ;
    report
  end

```

```

(7) Upon receipt of  $\langle \text{reject} \rangle$  from q:
  begin if  $\text{stach}_p[q] = \text{basic}$  then  $\text{stach}_p[q] := \text{reject}$  ;
    test
  end

```

- 
- (8) **procedure** *report*:
- ```

begin if  $rec_p = \#\{q : stach_p[q] = branch \wedge q \neq father_p\}$ 
    and  $testch_p = undef$  then
        begin  $state_p := found$  ; send  $\langle report, bestwt_p \rangle$  to  $father_p$  end
    end

```
- (9) Upon receipt of  $\langle report, \omega \rangle$  from  $q$ :
- ```

begin if  $q \neq father_p$ 
    then (* reply for initiate message *)
        begin if  $\omega < bestwt_p$  then
            begin  $bestwt_p := \omega$  ;  $bestch_p := q$  end ;
             $rec_p := rec_p + 1$  ; report
        end
    else (*  $pq$  is the core edge *)
        if  $state_p = find$ 
            then process this message later
        else if  $\omega > bestwt_p$ 
            then changeroot
        else if  $\omega = bestwt_p = \infty$  then stop
    end

```
- (10) **procedure** *changeroot*:
- ```

begin if  $stach_p[bestch_p] = branch$ 
    then send  $\langle changeroot \rangle$  to  $bestch_p$ 
    else begin send  $\langle connect, level_p \rangle$  to  $bestch_p$  ;
         $stach_p[bestch_p] := branch$ 
    end
end

```
- (11) Upon receipt of  $\langle changeroot \rangle$ :
- ```

begin changeroot end

```



## správnosť

ukázať, že sa zvolí práve jeden šéf: nenastane deadlock

## počet správ

- testovacie správy: jeden test po každej hrane
- kostrové správy: fragment s  $n_i$  vrcholmi pri postupe o level  $O(n_i)$  správ
- postupy na level  $l$ : dizjunktné vrcholy

- $f(x)$ -traverzovanie
- tokeny traverzujú/označujú územia
- levely: keď sa stretnú dva, vznikne nový
- naháňanie

```

var  $lev_p$       : integer      init -1 ;
       $cat_p, wait_p$  :  $\mathcal{P}$         init undef ;
       $last_p$       :  $Neigh_p$     init undef ;

begin if  $p$  is initiator then
  begin  $lev_p := lev_p + 1$  ;  $last_p := trav(p, lev_p)$  ;
       $cat_p := p$  ; send  $\langle \text{annex}, p, lev_p \rangle$  to  $last_p$ 
  end ;
  while ... (* Termination condition, see text *) do
    begin receive token  $(q, l)$  ;
      if token is annexing then  $t := A$  else  $t := C$  ;
      if  $l > lev_p$  then (* Case I *)
        begin  $lev_p := l$  ;  $cat_p := q$  ;
             $wait_p := undef$  ;  $last_p := trav(q, l)$  ;
            send  $\langle \text{annex}, q, l \rangle$  to  $last_p$ 
        end
      else if  $l = lev_p$  and  $wait_p \neq undef$  then (* Case II *)
        begin  $wait_p := undef$  ;  $lev_p := lev_p + 1$  ;
             $last_p := trav(p, lev_p)$  ;  $cat_p := p$  ;
            send  $\langle \text{annex}, p, lev_p \rangle$  to  $last_p$ 
        end
      else if  $l = lev_p$  and  $last_p = undef$  then (* Case III *)
         $wait_p := q$ 
      else if  $l = lev_p$  and  $t = A$  and  $q = cat_p$  then (* Case IV *)
        begin  $last_p := trav(q, l)$  ;
            if  $last_p = decide$ 
              then  $p$  announces itself leader
            else send  $\langle \text{annex}, q, l \rangle$  to  $last_p$ 
        end
      else if  $l = lev_p$  and  $((t = A$  and
         $q > cat_p)$  or  $t = C)$  then (* Case V *)
        begin send  $\langle \text{chase}, q, l \rangle$  to  $last_p$  ;  $last_p := undef$  end
      else if  $l = lev_p$  then (* Case VI *)
         $wait_p := q$ 
    end
  end

```

## počet správ

- naháňacie: 1 na vrchol a level, spolu  $n$  za level
- objavovacie:  $\sum_i f(n_i)$
- ak  $f$  je konvexná, t.j.  $f(a) + f(b) \leq f(a + b)$ , tak je  $O(\log n(n + f(n)))$  správ