

## algoritmy založené na porovnaníach

- ekvivalentné okolia
- $c$ -symetrický reťazec: pre každé  $\sqrt{n} \leq l \leq n$  a pre každý segment  $S$  je  $\lfloor \frac{cn}{l} \rfloor$  ekvivalentných
- bit-reversal je  $1/2$ -symetrický
- $c$ -symetrický reťazec, alg. nemôže skončiť po  $k$  kolách pre  $\lfloor \frac{cn}{2k+1} \rfloor \geq 2$
- počet správ:  $k = \lfloor \frac{cn-2}{4} \rfloor$ , je aspoň  $k + 1$  kôl
- aspoň  $\lfloor \frac{cn}{2r-1} \rfloor$  aktívnych v  $r$ -tom kole pošle správu

## algoritmy využívajúce integer ID

- rôzne rýchlosti
- v  $i$ -tom kroku test

V toruse rozmerov  $n \times n$  (t.j. zacyklenej mriežke) sú na začiatku zobudené dva vrcholy. Napíšte algoritmus, pomocou ktorého sa každý z nich dozvie identifikátor druhého s použitím  $O(n)$  správ.

Nájdite asymptoticky optimálny (čo do počtu správ) algoritmus na voľbu šéfa v úplnom bipartitnom grafe  $K_{n,n}$ . Dokážte jeho zložitosť a optimalitu.

★

broadcasting a voľba šéfa na (ne?)orientovanej hyperkocke s lineárnym počtom správ

cieľ: doručovať srávy medzi ľubovoľnou dvojicou

## modely

- destination based
- splittable
- connections (wormhole)
- buffering
- selfish

## ciele

- statické váhy (najkratšie cesty)
- dynamické váhy (hot potato)
- deadlock

## najkratšie cesty

```
begin (* Initialize  $S$  to  $\emptyset$  and  $D$  to  $\emptyset$ -distance *)  
   $S := \emptyset$  ;  
  forall  $u, v$  do  
    if  $u = v$  then  $D[u, v] := 0$   
    else if  $uv \in E$  then  $D[u, v] := \omega_{uv}$   
    else  $D[u, v] := \infty$  ;  
  (* Expand  $S$  by pivoting *)  
  while  $S \neq V$  do  
    (* Loop invariant:  $\forall u, v : D[u, v] = d^S(u, v)$  *)  
    begin pick  $w$  from  $V \setminus S$  ;  
      (* Execute a global  $w$ -pivot *)  
      forall  $u \in V$  do  
        (* Execute a local  $w$ -pivot at  $u$  *)  
        forall  $v \in V$  do  
           $D[u, v] := \min ( D[u, v], D[u, w] + D[w, v] )$  ;  
         $S := S \cup \{w\}$   
      end (*  $\forall u, v : D[u, v] = d(u, v)$  *)  
  end
```

## najkratšie cesty

```
var  $S_u$  : set of nodes ;  
     $D_u$  : array of weights ;  
     $Nb_u$  : array of nodes ;  
  
begin  $S_u := \emptyset$  ;  
    forall  $v \in V$  do  
        if  $v = u$   
            then begin  $D_u[v] := 0$  ;  $Nb_u[v] := u$  end  
        else if  $v \in Neigh_u$   
            then begin  $D_u[v] := \omega_{uv}$  ;  $Nb_u[v] := v$  end  
        else begin  $D_u[v] := \infty$  ;  $Nb_u[v] := u$  end ;  
    while  $S_u \neq V$  do  
        begin pick  $w$  from  $V \setminus S_u$  ;  
            (* All nodes must pick the same node  $w$  here *)  
            if  $u = w$   
                then “broadcast the table  $D_w$ ”  
            else “receive the table  $D_w$ ”  
            forall  $v \in V$  do  
                if  $D_u[w] + D_w[v] < D_u[v]$  then  
                    begin  $D_u[v] := D_u[w] + D_w[v]$  ;  
                         $Nb_u[v] := Nb_u[w]$   
                    end ;  
                 $S_u := S_u \cup \{w\}$   
            end  
        end  
    end  
end
```

## najkratšie cesty

```
var  $S_u$  : set of nodes ;  
     $D_u$  : array of weights ;  
     $Nb_u$  : array of nodes ;  
  
begin  $S_u := \emptyset$  ;  
    forall  $v \in V$  do  
        if  $v = u$   
            then begin  $D_u[v] := 0$  ;  $Nb_u[v] := undef$  end  
        else if  $v \in Neigh_u$   
            then begin  $D_u[v] := \omega_{uv}$  ;  $Nb_u[v] := v$  end  
        else begin  $D_u[v] := \infty$  ;  $Nb_u[v] := undef$  end ;  
    while  $S_u \neq V$  do  
        begin pick  $w$  from  $V \setminus S_u$  ;  
            (* Construct the tree  $T_w$  *)  
            forall  $x \in Neigh_u$  do  
                if  $Nb_u[w] = x$  then send  $\langle \mathbf{ys}, w \rangle$  to  $x$   
                else send  $\langle \mathbf{nys}, w \rangle$  to  $x$  ;  
            num_rec_u := 0 ; (*  $u$  must receive  $|Neigh_u|$  messages *)  
            while num_rec_u <  $|Neigh_u|$  do  
                begin receive  $\langle \mathbf{ys}, w \rangle$  or  $\langle \mathbf{nys}, w \rangle$  message ;  
                    num_rec_u := num_rec_u + 1  
                end ;  
            if  $D_u[w] < \infty$  then (* participate in pivot round *)  
                begin if  $u \neq w$   
                    then receive  $\langle \mathbf{dtab}, w, D \rangle$  from this  $Nb_u[w]$  ;  
                    forall  $x \in Neigh_u$  do  
                        if  $\langle \mathbf{ys}, w \rangle$  was received from  $x$   
                            then send  $\langle \mathbf{dtab}, w, D \rangle$  to  $x$  ;  
                    forall  $v \in V$  do (* local  $w$ -pivot *)  
                        if  $D_u[w] + D[v] < D_u[v]$  then  
                            begin  $D_u[v] := D_u[w] + D[v]$  ;  
                                 $Nb_u[v] := Nb_u[w]$   
                            end  
                        end  
                    end ;  
                end ;  
            end ;
```

# Netchange

```
var  $Neigh_u$       : set of nodes ;      (* The neighbors of  $u$  *)  
     $D_u$            : array of 0..  $N$  ;    (*  $D_u[v]$  estimates  $d(u, v)$  *)  
     $Nb_u$          : array of nodes ;    (*  $Nb_u[v]$  is preferred neighbor for  $v$  *)  
     $ndis_u$        : array of 0..  $N$  ;    (*  $ndis_u[w, v]$  estimates  $d(w, v)$  *)
```

Initialization:

```
begin forall  $w \in Neigh_u, v \in V$  do  $ndis_u[w, v] := N$  ;  
    forall  $v \in V$  do  
        begin  $D_u[v] := N$  ;  $Nb_u[v] := undef$  end ;  
         $D_u[u] := 0$  ;  $Nb_u[u] := local$  ;  
        forall  $w \in Neigh_u$  do send  $\langle mydist, u, 0 \rangle$  to  $w$   
    end
```

Procedure *Recompute* ( $v$ ):

```
begin if  $v = u$   
    then begin  $D_u[v] := 0$  ;  $Nb_u[v] := local$  end  
    else begin (* Estimate distance to  $v$  *)  
         $d := 1 + \min\{ndis_u[w, v] : w \in Neigh_u\}$  ;  
        if  $d < N$  then  
            begin  $D_u[v] := d$  ;  
                 $Nb_u[v] := w$  with  $1 + ndis_u[w, v] = d$   
            end  
        else begin  $D_u[v] := N$  ;  $Nb_u[v] := undef$  end  
    end ;  
    if  $D_u[v]$  has changed then  
        forall  $x \in Neigh_u$  do send  $\langle mydist, v, D_u[v] \rangle$  to  $x$ 
```

end

Processing a  $\langle mydist, v, d \rangle$  message from neighbor  $w$ :

```
{ A  $\langle mydist, v, d \rangle$  is at the head of  $Q_{wv}$  }  
begin receive  $\langle mydist, v, d \rangle$  from  $w$  ;  
     $ndis_u[w, v] := d$  ; Recompute ( $v$ )  
end
```

Upon failure of channel  $uw$ :

```
begin receive  $\langle fail, w \rangle$  ;  $Neigh_u := Neigh_u \setminus \{w\}$  ;  
    forall  $v \in V$  do Recompute ( $v$ )  
end
```

Upon repair of channel  $uw$ :

```
begin receive  $\langle repair, w \rangle$  ;  $Neigh_u := Neigh_u \cup \{w\}$  ;  
    forall  $v \in V$  do  
        begin  $ndis_u[w, v] := N$  ;  
            send  $\langle mydist, v, D_u[v] \rangle$  to  $w$   
        end  
    end
```

end

## korektnost'

lexikograficky klesá hodnota  $[t_0, t_1, \dots, t_N]$

kde  $t_i$  je počet správ  $\langle mydist, i \rangle$  + počet dvojíc  $u, v$  kde  $D_u[v] = i$