

# Judgements

$\Gamma \vdash_i e \Leftarrow A \dashv \Gamma'$  - in context  $\Gamma$  (which has zero resources),  $e$  checks to have type  $A$  and consumes resources according to quantities from  $\Gamma'$ .

$\Gamma \vdash_i e \Rightarrow A \dashv \Gamma'$  - as above, but with type inference.

# Notation

$\Gamma \vdash_i e \Leftarrow A \dashv \Gamma', s_A^i x : A$  is a shorthand for  $\Gamma \vdash_i e \Leftarrow A \dashv \Gamma', r x : A$  with the condition  $s_A^i \sqsubseteq r$ .

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# Subsumption and annotations

$$\frac{\Gamma \vdash_i e \Rightarrow A \dashv \Gamma_1 \quad \Gamma \vdash A \equiv B \Rightarrow \text{Type}_r \dashv \Gamma_2}{\Gamma \vdash_i e \Leftarrow B \dashv \Gamma_1 + \Gamma_2} \text{SUBSUMPTION}$$

$$\frac{\Gamma \vdash A \Rightarrow \text{Type}_r \dashv \Gamma_1 \quad \Gamma \vdash_i e \Leftarrow A \dashv \Gamma_2}{\Gamma \vdash_i (e : A) \Rightarrow A \dashv \Gamma_1 + \Gamma_2} \text{ANNOT}$$

## Using variables

$$\frac{\Gamma(x) = A}{\Gamma \vdash_c x \Rightarrow A \dashv \vdash |\Gamma|_x}$$

$$\frac{\Gamma(x) = A}{\Gamma \vdash_{\text{nc}} x \Rightarrow A \vdash 0 \Gamma}$$

# Functions

$$\frac{\Gamma, 0x : A \vdash_i e \Leftarrow B \dashv \Gamma', 1_A^i x : A}{\Gamma \vdash_i \lambda x. e \Leftarrow A \rightarrow B \dashv \Gamma'}$$

$$\frac{\Gamma \vdash_i f \Rightarrow A \rightarrow B \dashv \Gamma_1 \quad \Gamma \vdash_i a \Leftarrow A \dashv \Gamma_2}{\Gamma \vdash_i f a \Rightarrow B \dashv \Gamma_1 + \Gamma_2}$$

# Box

$$\frac{\Gamma \vdash_i e \Leftarrow A \dashv \Gamma'}{\Gamma \vdash_i \text{box } e \Leftarrow !_r A \dashv r \Gamma'}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow !_r A \dashv \Gamma_1 \quad \Gamma, 0x : A \vdash_i e_2 \Rightarrow B \dashv \Gamma_2, r_A^i x : A}{\Gamma \vdash_i \text{let box } x = e_1 \text{ in } e_2 \Rightarrow B \dashv \Gamma_1 + \Gamma_2}$$

# Empty

$$\frac{\Gamma \vdash_i e \Leftarrow \text{Empty} \dashv \Gamma'}{\Gamma \vdash_i \text{Empty-elim } e \Leftarrow A \dashv \Gamma'}$$

# Unit

$$\overline{\Gamma \vdash_i \text{unit} \Leftarrow \text{Unit} \dashv 0 \Gamma}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow \text{Unit} \dashv \Gamma_1 \quad \Gamma \vdash_i e_2 \Rightarrow A \dashv \Gamma_2}{\Gamma \vdash_i \text{let unit} = e_1 \text{ in } e_2 \Rightarrow A \dashv \Gamma_1 + \Gamma_2}$$



# Products

$$\frac{\Gamma \vdash_i e_1 \Leftarrow A \dashv \Gamma_1 \quad \Gamma \vdash_i e_2 \Leftarrow B \dashv \Gamma_2}{\Gamma \vdash_i (e_1, e_2) \Leftarrow A \otimes B \dashv \Gamma_1 + \Gamma_2}$$

$$\frac{\begin{array}{l} \Gamma \vdash_i e_1 \Rightarrow A \otimes B \dashv \Gamma_1 \\ \Gamma, 0x : A, 0y : B \vdash_i e_2 \Rightarrow C \dashv \Gamma_2, 1_A^i x : A, 1_B^i y : B \end{array}}{\Gamma \vdash_i \text{let } (x, y) = e_1 \text{ in } e_2 \Rightarrow C \dashv \Gamma_1 + \Gamma_2}$$

## Division

$$r_1/r_2 = \sup\{s \in \mathcal{Q} \mid s \cdot r_2 \sqsubseteq r_1\}$$

$r_1/r_2$	0	1	?	+	*
0	0	0	0	0	0
1		1	1	1	1
?		?	1	?	1
+		+	+	1	1
*		*	+	?	1

# Sums

$$\frac{\Gamma \vdash_i e \Leftarrow A \dashv \Gamma'}{\Gamma \vdash_i \text{inl } e \Leftarrow A \oplus B \dashv \Gamma'} \quad \frac{\Gamma \vdash_i e \Leftarrow B \dashv \Gamma'}{\Gamma \vdash_i \text{inr } e \Leftarrow A \oplus B \dashv \Gamma'}$$

$$\frac{\Gamma \vdash_i e \Rightarrow A \oplus B \dashv \Gamma_1 \quad \begin{array}{l} \Gamma, 0x : A \vdash_i e_1 \Leftarrow C \dashv \Gamma_2, r_1 x : A \\ \Gamma, 0y : B \vdash_i e_2 \Leftarrow C \dashv \Gamma_3, r_2 y : B \\ r = (r_1/1_A^i) \sqcap (r_2/1_B^i) \end{array}}{\Gamma \vdash_i \text{case } e \text{ of } \{x.e_1; y.e_2\} \Leftarrow C \dashv r\Gamma_1 + (\Gamma_2 \sqcap \Gamma_3)}$$

$$\frac{\Gamma \vdash_i e \Rightarrow A \oplus B \dashv \Gamma_1 \quad \begin{array}{l} \Gamma, 0x : A \vdash_i e_1 \Rightarrow C \dashv \Gamma_2, r_1 x : A \\ \Gamma, 0y : B \vdash_i e_2 \Rightarrow C \dashv \Gamma_3, r_2 y : B \\ r = (r_1/1_A^i) \sqcap (r_2/1_B^i) \end{array}}{\Gamma \vdash_i \text{case } e \text{ of } \{x.e_1; y.e_2\} \Rightarrow C \dashv r\Gamma_1 + (\Gamma_2 \sqcap \Gamma_3)}$$

Let

$$\frac{\Gamma \vdash_i e_1 \Rightarrow A \dashv \Gamma_1 \quad \Gamma, 0x : A \vdash_i e_2 \Rightarrow B \dashv \Gamma_2, rx : A \quad r' = r/1_A^i}{\Gamma \vdash_i \text{let } x = e_1 \text{ in } e_2 \Rightarrow B \dashv r' \Gamma_1 + \Gamma_2}$$

# Natural numbers

$$\overline{\Gamma \vdash_i 0 \Leftarrow \mathbb{N} \dashv 0 \Gamma}$$

$$\frac{\Gamma \vdash_i n \Leftarrow \mathbb{N} \dashv \Gamma'}{\Gamma \vdash_i \text{succ } n \Leftarrow \mathbb{N} \dashv \Gamma'}$$

$$\frac{\Gamma \vdash_i n \Rightarrow \mathbb{N} \dashv \Gamma_1 \quad \Gamma \vdash_i z \Rightarrow A \dashv \Gamma_2 \quad \Gamma \vdash_i s \Rightarrow A \rightarrow A \dashv \Gamma_3}{\Gamma \vdash_i \text{elim}_{\mathbb{N}} z s n \Rightarrow A \dashv \Gamma_1 + \Gamma_2 + * \Gamma_3}$$