Hinting for fully applicative STLC

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Terms

Terms:

$e ::=$

$x \mid (e : H) \mid$

$\lambda x.\, e \mid e_1\ e_2 \mid$

$\texttt{pair} \mid \texttt{outl} \mid \texttt{outr} \mid$

$\texttt{inl} \mid \texttt{inr} \mid \texttt{case} \mid$

$\texttt{unit} \mid \mathbf{0}\texttt{-elim}$

Judgements:

$\Gamma \vdash e \Leftarrow H \Rightarrow A$ – in context Γ, term e checks with hint H and infers type A

Declarative typing – basics

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A}\textsc{Var}$$

$$\frac{\Gamma \vdash e : A \quad H \sqsubseteq A}{\Gamma \vdash (e : H) : A}\textsc{Annot}$$

Declarative typing – type-directed rules

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x.\, e : A \to B} \qquad \frac{\Gamma \vdash a : A \quad \Gamma \vdash f : A \to B}{\Gamma \vdash f\ a : B}$$

$$\frac{}{\Gamma \vdash \mathtt{pair} : A \to B \to A \times B}$$

$$\frac{}{\Gamma \vdash \mathtt{outl} : A \times B \to A} \qquad \frac{}{\Gamma \vdash \mathtt{outr} : A \times B \to B}$$

$$\frac{}{\Gamma \vdash \mathtt{inl} : A \to A + B} \qquad \frac{}{\Gamma \vdash \mathtt{inr} : B \to A + B}$$

$$\frac{}{\Gamma \vdash \mathtt{case} : (A \to C) \to (B \to C) \to A + B \to C}$$

$$\frac{}{\Gamma \vdash \mathtt{unit} : \mathbf{1}} \qquad \frac{}{\Gamma \vdash \mathbf{0}\text{-}\mathtt{elim} : \mathbf{0} \to A}$$

Hinting – basic rules

$$\frac{(x : A) \in \Gamma \quad H \sqsubseteq A}{\Gamma \vdash x \Leftarrow H \Rightarrow A}\textsc{Var}$$

$$\frac{\Gamma \vdash e \Leftarrow H_1 \sqcup H_2 \Rightarrow A}{\Gamma \vdash (e : H_1) \Leftarrow H_2 \Rightarrow A}\textsc{Annot}$$

Hinting – type-directed rules

$$\frac{\Gamma, x : A \vdash e \Leftarrow H \Rightarrow B}{\Gamma \vdash \lambda x.\, e \Leftarrow A \to H \Rightarrow A \to B}$$

$$\frac{\Gamma \vdash a \Leftarrow \mathbf{?} \Rightarrow A \quad \Gamma \vdash f \Leftarrow A \to H \Rightarrow A \to B}{\Gamma \vdash f\ a \Leftarrow H \Rightarrow B}$$

$$\frac{\begin{array}{c} H \sqcup \mathbf{?} \to \mathbf{?} \to \mathbf{?} \times \mathbf{?} = H_1 \to H_2 \to H_3 \times H_4 \\ H_1 \sqcup H_3 = A \quad H_2 \sqcup H_4 = B \end{array}}{\Gamma \vdash \texttt{pair} \Leftarrow H \Rightarrow A \to B \to A \times B}$$

$$\frac{H_1 \sqcup H_2 = A}{\Gamma \vdash \texttt{outl} \Leftarrow H_1 \times B \to H_2 \Rightarrow A \times B \to A}$$

$$\frac{H_1 \sqcup H_2 = B}{\Gamma \vdash \texttt{outr} \Leftarrow A \times H_1 \to H_2 \Rightarrow A \times B \to B}$$

Hinting – type-directed rules

$$\frac{H_1 \sqcup H_2 = A}{\Gamma \vdash \texttt{inl} \Leftarrow H_1 \to H_2 + B \Rightarrow A \to A + B}$$

$$\frac{H_1 \sqcup H_2 = B}{\Gamma \vdash \texttt{inr} \Leftarrow H_1 \to A + H_2 \Rightarrow B \to A + B}$$

$$\frac{\begin{array}{c} H \sqcup (\textbf{?} \to \textbf{?}) \to (\textbf{?} \to \textbf{?}) \to \textbf{?} + \textbf{?} \to \textbf{?} = \\ (H_1 \to H_2) \to (H_3 \to H_4) \to H_5 + H_6 \to H_7 \\ H_1 \sqcup H_5 = A \quad H_3 \sqcup H_6 = B \quad H_2 \sqcup (H_4 \sqcup H_7) = C \end{array}}{\Gamma \vdash \texttt{case} \Leftarrow H \Rightarrow (A \to C) \to (B \to C) \to A + B \to C}$$

$$\frac{H \sqsubseteq \mathbf{1}}{\Gamma \vdash \texttt{unit} \Leftarrow H \Rightarrow \mathbf{1}} \qquad \frac{H \sqsubseteq \mathbf{0}}{\Gamma \vdash \mathbf{0}\texttt{-elim} \Leftarrow H \to A \Rightarrow \mathbf{0} \to A}$$