T-Axi (declarative)

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Universes

$$U ::= \mathtt{Type}_1 \mid \mathtt{Type}_? \mid \mathtt{Type}_+ \mid \mathtt{Type}_*$$

The default quantity is $*$, so Type without any index means $\mathtt{Type}_*$.

$$\frac{\Gamma \ \mathtt{ctx}_c \quad r \neq 0}{\Gamma, a : \mathtt{Type}_r \ \mathtt{ctx}_c}$$

Kinding

$$\frac{\Gamma\ \mathtt{ctx}_{\mathtt{nc}}}{\Gamma \vdash \mathtt{Unit} : \mathtt{Type}} \qquad \frac{\Gamma\ \mathtt{ctx}_{\mathtt{nc}}}{\Gamma \vdash \mathtt{Empty} : \mathtt{Type}}$$

$$\frac{\Gamma \vdash A : \mathtt{Type}_{r_1} \quad \Gamma \vdash B : \mathtt{Type}_{r_2}}{\Gamma \vdash s\, A \to B : \mathtt{Type}_1}$$

$$\frac{\Gamma \vdash A : \mathtt{Type}_r}{\Gamma \vdash !_0\, A : \mathtt{Type}} \qquad \frac{\Gamma \vdash A : \mathtt{Type}_r \quad r \neq 0}{\Gamma \vdash !_s\, A : \mathtt{Type}_{r \cdot s}}$$

$$\frac{\Gamma \vdash A : \mathtt{Type}_{r_1} \quad \Gamma \vdash B : \mathtt{Type}_{r_2}}{\Gamma \vdash A \otimes B : \mathtt{Type}_{r_1 \sqcup r_2}} \qquad \frac{\Gamma \vdash A : \mathtt{Type}_{r_1} \quad \Gamma \vdash B : \mathtt{Type}_{r_2}}{\Gamma \vdash A \oplus B : \mathtt{Type}_{r_1 \sqcup r_2}}$$

Kinding

$$\frac{\Gamma\ \mathtt{ctx}_{\mathtt{nc}} \quad (a : \mathtt{Type}_r) \in \Gamma}{\Gamma \vdash a : \mathtt{Type}_r}$$

$$\frac{\Gamma, A : \mathtt{Type}_r \vdash B : \mathtt{Type}_s}{\Gamma \vdash \forall A : \mathtt{Type}_r.\, B : \mathtt{Type}_s}$$

$$\frac{\Gamma, A : \mathtt{Type}_r \vdash B : \mathtt{Type}_s}{\Gamma \vdash \forall \{A : \mathtt{Type}_r\}.\, B : \mathtt{Type}_s}$$

Polymorphism (decl)

$$\frac{\Gamma, a : \mathtt{Type}_r \vdash_i e : B}{\Gamma \vdash_i \Lambda\, a : \mathtt{Type}_r.\, e : \forall a : \mathtt{Type}_r.\, B}$$

$$\frac{\Gamma, a : \mathtt{Type}_r \vdash_i e : B}{\Gamma \vdash_i \Lambda\, \{a : \mathtt{Type}_r\}.\, e : \forall \{a : \mathtt{Type}_r\}.\, B}$$

$$\frac{\Gamma \vdash_i e : \forall a : \mathtt{Type}_r.\, B \quad |\Gamma| \vdash A : \mathtt{Type}_r}{\Gamma \vdash_i e\ A : B\,[a := A]}$$

$$\frac{\Gamma \vdash_i e : \forall \{a : \mathtt{Type}_r\}.\, B \quad |\Gamma| \vdash A : \mathtt{Type}_r}{\Gamma \vdash_i e\ @A : B\,[a := A]}$$