

# Hinting for fully applicative STLC

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# Terms

Terms:

$e ::=$

$x \mid (e : H) \mid$   
 $\lambda x. e \mid e_1 \ e_2 \mid$   
 $\text{pair} \mid \text{outl} \mid \text{outr} \mid$   
 $\text{inl} \mid \text{inr} \mid \text{case} \mid$   
 $\text{unit} \mid \mathbf{0}\text{-elim}$

Judgements:

$\Gamma \vdash e \Leftarrow H \Rightarrow A$  – in context  $\Gamma$ , term  $e$  checks with hint  $H$  and infers type  $A$

# Declarative typing – basics

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \text{VAR}$$

$$\frac{\Gamma \vdash e : A \quad H \sqsubseteq A}{\Gamma \vdash (e : H) : A} \text{ANNOT}$$

# Declarative typing – type-directed rules

$$\frac{\Gamma, x : A \vdash e : B}{\Gamma \vdash \lambda x. e : A \rightarrow B} \quad \frac{\Gamma \vdash a : A \quad \Gamma \vdash f : A \rightarrow B}{\Gamma \vdash f \ a : B}$$

$$\overline{\Gamma \vdash \text{pair} : A \rightarrow B \rightarrow A \times B}$$

$$\overline{\Gamma \vdash \text{outl} : A \times B \rightarrow A} \quad \overline{\Gamma \vdash \text{outr} : A \times B \rightarrow B}$$

$$\overline{\Gamma \vdash \text{inl} : A \rightarrow A + B} \quad \overline{\Gamma \vdash \text{inr} : B \rightarrow A + B}$$

$$\overline{\Gamma \vdash \text{case} : (A \rightarrow C) \rightarrow (B \rightarrow C) \rightarrow A + B \rightarrow C}$$

$$\overline{\Gamma \vdash \text{unit} : \mathbf{1}} \quad \overline{\Gamma \vdash \mathbf{0}\text{-elim} : \mathbf{0} \rightarrow A}$$

# Hinting – basic rules

$$\frac{(x : A) \in \Gamma \quad H \sqsubseteq A}{\Gamma \vdash x \Leftarrow H \Rightarrow A} \text{VAR}$$

$$\frac{\Gamma \vdash e \Leftarrow H_1 \sqcup H_2 \Rightarrow A}{\Gamma \vdash (e : H_1) \Leftarrow H_2 \Rightarrow A} \text{ANNOT}$$

# Hinting – type-directed rules

$$\frac{\Gamma, x : A \vdash e \leftarrow H \Rightarrow B}{\Gamma \vdash \lambda x. e \leftarrow A \rightarrow H \Rightarrow A \rightarrow B}$$

$$\frac{\Gamma \vdash a \leftarrow ? \Rightarrow A \quad \Gamma \vdash f \leftarrow A \rightarrow H \Rightarrow A \rightarrow B}{\Gamma \vdash f \ a \leftarrow H \Rightarrow B}$$

$$\frac{H \sqcup ? \rightarrow ? \rightarrow ? \times ? = H_1 \rightarrow H_2 \rightarrow H_3 \times H_4 \quad H_1 \sqcup H_3 = A \quad H_2 \sqcup H_4 = B}{\Gamma \vdash \text{pair} \leftarrow H \Rightarrow A \rightarrow B \rightarrow A \times B}$$

$$\frac{H_1 \sqcup H_2 = A}{\Gamma \vdash \text{outl} \leftarrow H_1 \times B \rightarrow H_2 \Rightarrow A \times B \rightarrow A}$$

$$\frac{H_1 \sqcup H_2 = B}{\Gamma \vdash \text{outr} \leftarrow A \times H_1 \rightarrow H_2 \Rightarrow A \times B \rightarrow B}$$

# Hinting – type-directed rules

$$\frac{H_1 \sqcup H_2 = A}{\Gamma \vdash \text{inl} \leftarrow H_1 \rightarrow H_2 + B \Rightarrow A \rightarrow A + B}$$

$$\frac{H_1 \sqcup H_2 = B}{\Gamma \vdash \text{inr} \leftarrow H_1 \rightarrow A + H_2 \Rightarrow B \rightarrow A + B}$$

$$\frac{\begin{array}{l} H \sqcup (? \rightarrow ?) \rightarrow (? \rightarrow ?) \rightarrow ? + ? \rightarrow ? = \\ (H_1 \rightarrow H_2) \rightarrow (H_3 \rightarrow H_4) \rightarrow H_5 + H_6 \rightarrow H_7 \\ H_1 \sqcup H_5 = A \quad H_3 \sqcup H_6 = B \quad H_2 \sqcup (H_4 \sqcup H_7) = C \end{array}}{\Gamma \vdash \text{case} \leftarrow H \Rightarrow (A \rightarrow C) \rightarrow (B \rightarrow C) \rightarrow A + B \rightarrow C}$$

$$\frac{H \sqsubseteq 1}{\Gamma \vdash \text{unit} \leftarrow H \Rightarrow 1} \quad \frac{H \sqsubseteq 0}{\Gamma \vdash \mathbf{0}\text{-elim} \leftarrow H \rightarrow A \Rightarrow \mathbf{0} \rightarrow A}$$