Judgements

$\Gamma \vdash_i e \Leftarrow A \Rightarrow e' \dashv \Gamma'$ – in context Γ check that term e has type A and elaborate it to term e', returning the bill Γ'.

$\Gamma \vdash_i e \Rightarrow e' : A \dashv \Gamma'$ – in context Γ infer that A is the type of term e and elaborate it to term e', returning the bill Γ'.

Functions

$$\frac{\Gamma, 0\, x : A \vdash_i e \Leftarrow B \Rightarrow e' \dashv \Gamma', 1^i_A\, x : A}{\Gamma \vdash_i \lambda x.\, e \Leftarrow A \to B \Rightarrow \lambda x : A.\, e' \dashv \Gamma'}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow e_1' : A \to B \dashv \Gamma_1 \quad \Gamma \vdash_i e_2 \Leftarrow A \Rightarrow e_2' \dashv \Gamma_2}{\Gamma \vdash_i e_1\ e_2 \Rightarrow e_1'\ e_2' : B \dashv \Gamma_1 + \Gamma_2}$$

Box

$$\frac{\Gamma \vdash_i e \Leftarrow A \Rightarrow e' \dashv \Gamma'}{\Gamma \vdash_i \texttt{box}\ e \Leftarrow\ !_r\, A \Rightarrow \texttt{box}_r\ e' \dashv r\,\Gamma'}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow e_1' : !_r\, A \dashv \Gamma_1 \quad \Gamma, 0\,x : A \vdash_i e_2 \Rightarrow e_2' : B \dashv \Gamma_2, r_A^i\, x : A}{\Gamma \vdash_i \texttt{let}\ \texttt{box}\ x = e_1\ \texttt{in}\ e_2 \Rightarrow \texttt{let}_B\ \texttt{box}\ x = e_1'\ \texttt{in}\ e_2' : B \dashv \Gamma_1 + \Gamma_2}$$

Empty

$$\frac{\Gamma \vdash_i e \Leftarrow \texttt{Empty} \Rightarrow e' \dashv \Gamma'}{\Gamma \vdash_i \texttt{Empty-elim}\ e \Leftarrow A \Rightarrow \texttt{Empty-elim}_A\ e' \dashv \Gamma'}$$

Unit

$$\frac{}{\Gamma \vdash_i \texttt{unit} \Leftarrow \texttt{Unit} \Rightarrow \texttt{unit} \dashv \Gamma}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow e_1' : \texttt{Unit} \dashv \Gamma_1 \quad \Gamma \vdash_i e_2 \Rightarrow e_2' : A \dashv \Gamma_2}{\Gamma \vdash_i \texttt{let unit} = e_1 \texttt{ in } e_2 \Rightarrow \texttt{let}_A \texttt{ unit} = e_1' \texttt{ in } e_2' : A \dashv \Gamma_1 + \Gamma_2}$$

Product

$$\frac{\Gamma \vdash_i e_1 \Leftarrow A \Rightarrow e_1' \dashv \Gamma_1 \quad \Gamma \vdash_i e_2 \Leftarrow B \Rightarrow e_2' \dashv \Gamma_2}{\Gamma \vdash_i (e_1, e_2) \Leftarrow A \otimes B \Rightarrow (e_1', e_2') \dashv \Gamma_1 + \Gamma_2}$$

$$\frac{\begin{array}{l}\Gamma \vdash_i e_1 \Rightarrow e_1' : A \otimes B \dashv \Gamma_1 \\ \Gamma, 0\, x : A, 0\, y : B \vdash_i e_2 \Rightarrow e_2' : C \dashv \Gamma_2, 1_A^i\, x : A, 1_B^i\, y : B\end{array}}{\Gamma \vdash_i \texttt{let}\ (x, y) = e_1\ \texttt{in}\ e_2 \Rightarrow \texttt{let}_C\ (x, y) = e_1'\ \texttt{in}\ e_2' : C \dashv \Gamma_1 + \Gamma_2}$$

Sum

$$\frac{\Gamma \vdash_i e \Leftarrow A \Rightarrow e' \dashv \Gamma'}{\Gamma \vdash_i \texttt{inl}\ e \Leftarrow A \oplus B \Rightarrow \texttt{inl}_B\ e' \dashv \Gamma'}$$

$$\frac{\Gamma \vdash_i e \Leftarrow B \Rightarrow e' \dashv \Gamma'}{\Gamma \vdash_i \texttt{inr}\ e \Leftarrow A \oplus B \Rightarrow \texttt{inr}_A\ e' \dashv \Gamma'}$$

$$\frac{\Gamma \vdash_i e \Rightarrow e' : A \oplus B \dashv \Gamma_1 \qquad \begin{array}{l} \Gamma, 0\,x : A \vdash_i e_1 \Leftarrow e_1' \Rightarrow C \dashv \Gamma_2, r_1\,x : \\ \Gamma, 0\,y : B \vdash_i e_2 \Leftarrow e_2' \Rightarrow C \dashv \Gamma_3, r_2\,y : \\ r = (r_1/1_A^i) \sqcap (r_2/1_B^i) \end{array}}{\Gamma \vdash_i \texttt{case}\ e\ \texttt{of}\ \{x.e_1;\ y.e_2\} \Leftarrow C \Rightarrow \texttt{case}_C\ e'\ \texttt{of}\ \{x.e_1';\ y.e_2'\} \dashv r\,\Gamma_1 +}$$