

T-Axi (algorithmic)

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Decrement variable in context

· $-x = \mathbf{undefined}$

$$(\Gamma, rx : A) - x = \Gamma, (r-1)x : A$$

$$(\Gamma, ry : A) - x = \Gamma - x, ry : A$$

$$(\Gamma, rx : A := e) - x = \Gamma, (r-1)x : A := e$$

$$(\Gamma, ry : A := e) - x = \Gamma - x, ry : A := e$$

$$(\Gamma, h : P) - x = \Gamma - x, h : P$$

Using variables

$$\frac{\Gamma(x) = A}{\Gamma \vdash x \Rightarrow A \dashv \vdash \Gamma - x}$$

Functions

$$\frac{\Gamma, rx : A \vdash e \Leftarrow B \dashv \Gamma', r'x : A \quad r' \leq 0}{\Gamma \vdash \lambda x. e \Leftarrow rA \rightarrow B \dashv \Gamma'}$$

$$\frac{\Gamma \vdash f \Rightarrow rA \rightarrow B \dashv \Gamma_1 \quad \Gamma_1/r = (\Gamma_2, \Gamma_3) \quad \Gamma_2 \vdash a \Leftarrow A \dashv \Gamma_4}{\Gamma \vdash f a \Rightarrow B \dashv \Gamma_3 + r\Gamma_4}$$

Useful shorthand (when not enough space):

$$\frac{\Gamma, rx : A \vdash e \Leftarrow B \dashv \Gamma', 0x : A}{\Gamma \vdash \lambda x. e \Leftarrow rA \rightarrow B \dashv \Gamma'}$$

Box

$$\frac{\Gamma/r = (\Gamma_1, \Gamma_2) \quad \Gamma_1 \vdash e \Leftarrow A \dashv \Gamma_3}{\Gamma \vdash \text{box } e \Leftarrow !_r A \dashv \Gamma_2 + r \Gamma_3}$$

$$\frac{\Gamma \vdash e_1 \Rightarrow !_r A \dashv \Gamma_1 \quad \Gamma_1, r x : A \vdash e_2 \Rightarrow B \dashv \Gamma_2, r' x : A \quad r' \leq 0}{\Gamma \vdash \text{let box } x = e_1 \text{ in } e_2 \Rightarrow B \dashv \Gamma_2}$$

Empty

$$\frac{\Gamma \vdash e \Leftarrow \text{Empty} \dashv \Gamma'}{\Gamma \vdash \text{Empty-elim } e \Leftarrow A \dashv \Gamma'}$$

Unit

$$\overline{\Gamma \vdash \text{unit} \Leftarrow \text{Unit} \dashv \Gamma}$$

$$\frac{\Gamma \vdash e_1 \Rightarrow \text{Unit} \dashv \Gamma_1 \quad \Gamma_1 \vdash e_2 \Rightarrow A \dashv \Gamma_2}{\Gamma \vdash \text{let unit} = e_1 \text{ in } e_2 \Rightarrow A \dashv \Gamma_2}$$

Products

$$\frac{\Gamma \vdash a \Leftarrow A \dashv \Gamma_1 \quad \Gamma_1 \vdash b \Leftarrow B \dashv \Gamma_2}{\Gamma \vdash (a, b) \Leftarrow A \otimes B \dashv \Gamma_2}$$

$$\frac{\Gamma \vdash e_1 \Rightarrow A \otimes B \dashv \Gamma_1 \quad \Gamma_1, 1x : A, 1y : B \vdash e_2 \Rightarrow C \dashv \Gamma_2, 0x : A, 0y : B}{\Gamma \vdash \text{let } (x, y) = e_1 \text{ in } e_2 \Rightarrow C \dashv \Gamma_2}$$

Sums

$$\frac{\Gamma \vdash e \Leftarrow A \dashv \Gamma'}{\Gamma \vdash \text{inl } e \Leftarrow A \oplus B \dashv \Gamma'} \quad \frac{\Gamma \vdash e \Leftarrow B \dashv \Gamma'}{\Gamma \vdash \text{inr } e \Leftarrow A \oplus B \dashv \Gamma'}$$

$$\frac{\Gamma \vdash e \Rightarrow A \oplus B \dashv \Gamma_1 \quad \Gamma_1 \vdash f \Leftarrow 1A \rightarrow C \dashv \Gamma_2 \quad \Gamma_1 \vdash g \Leftarrow 1B \rightarrow C \dashv \Gamma_3}{\Gamma \vdash \text{case } e \text{ of } (f, g) \Leftarrow C \dashv \Gamma_2 \sqcup \Gamma_3}$$

$$\frac{\Gamma \vdash e \Rightarrow A \oplus B \dashv \Gamma_1 \quad \Gamma_1 \vdash f \Rightarrow 1A \rightarrow C \dashv \Gamma_2 \quad \Gamma_1 \vdash g \Rightarrow 1B \rightarrow C \dashv \Gamma_3}{\Gamma \vdash \text{case } e \text{ of } (f, g) \Rightarrow C \dashv \Gamma_2 \sqcup \Gamma_3}$$

Let

$$\frac{\Gamma/r = (\Gamma_1, \Gamma_2) \quad \Gamma_1 \vdash e_1 \Rightarrow A \dashv \Gamma_3 \quad \Gamma_2 + r \Gamma_3, r x : A \vdash e_2 \Rightarrow B \dashv \Gamma_4, 0 x}{\Gamma \vdash \text{let}_r x = e_1 \text{ in } e_2 \Rightarrow B \dashv \Gamma_4}$$

Conversion of propositions

$$\overline{\Gamma \vdash \top \equiv \top \dashv \Gamma} \quad \overline{\Gamma \vdash \perp \equiv \perp \dashv \Gamma}$$

$$\frac{\Gamma \vdash P_1 \triangleq P_2 \dashv \Gamma_1 \quad \Gamma_1 \vdash Q_1 \triangleq Q_2 \dashv \Gamma_2}{\Gamma \vdash P_1 \Rightarrow Q_1 \equiv P_2 \Rightarrow Q_2 \dashv \Gamma_2}$$

$$\frac{\Gamma \vdash P_1 \triangleq P_2 \dashv \Gamma_1 \quad \Gamma_1 \vdash Q_1 \triangleq Q_2 \dashv \Gamma_2}{\Gamma \vdash P_1 \wedge Q_1 \equiv P_2 \wedge Q_2 \dashv \Gamma_2}$$

$$\frac{\Gamma \vdash P_1 \triangleq P_2 \dashv \Gamma_1 \quad \Gamma_1 \vdash Q_1 \triangleq Q_2 \dashv \Gamma_2}{\Gamma \vdash P_1 \vee Q_1 \equiv P_2 \vee Q_2 \dashv \Gamma_2}$$

Conversion of propositions

$$\frac{\Gamma \vdash A_1 \triangleq A_2 \dashv \Gamma_1 \quad \Gamma_1, x_1 : A_1 \vdash P_1 \triangleq P_2 [x_2 := x_1] \dashv \Gamma_2, x_1 : A_1}{\Gamma \vdash \forall x_1 : A_1. P_1 \equiv \forall x_2 : A_2. P_2 \dashv \Gamma_2}$$

$$\frac{\Gamma \vdash A_1 \triangleq A_2 \dashv \Gamma_1 \quad \Gamma_1, x_1 : A_1 \vdash P_1 \triangleq P_2 [x_2 := x_1] \dashv \Gamma_2, x_1 : A_1}{\Gamma \vdash \exists x_1 : A_1. P_1 \equiv \exists x_2 : A_2. P_2 \dashv \Gamma_2}$$

$$\frac{\Gamma \vdash A_1 \triangleq A_2 \dashv \Gamma_1 \quad \Gamma_1 \vdash e_1 \triangleq e_2 \Leftarrow A_1 \dashv \Gamma_2 \quad \Gamma_2 \vdash e'_1 \triangleq e'_2 \Leftarrow A_1 \dashv \Gamma_3}{\Gamma \vdash e_1 =_{A_1} e'_1 \equiv e_2 =_{A_2} e'_2 \dashv \Gamma_3}$$

Conversion of propositions

$$\frac{\Gamma \vdash N_1 \sim N_2 \dashv \Gamma'}{\Gamma \vdash N_1 \equiv N_2 \dashv \Gamma'}$$

Conversion of types

$$\overline{\Gamma \vdash \text{Unit} \equiv \text{Unit} \dashv \Gamma} \quad \overline{\Gamma \vdash \text{Empty} \equiv \text{Empty} \dashv \Gamma}$$

$$\frac{\Gamma \vdash A_1 \hat{=} A_2 \dashv \Gamma_1 \quad \Gamma_1 \vdash B_1 \hat{=} B_2 \dashv \Gamma_2}{\Gamma \vdash A_1 \otimes B_1 \equiv A_2 \otimes B_2 \dashv \Gamma_2}$$

$$\frac{\Gamma \vdash A_1 \hat{=} A_2 \dashv \Gamma_1 \quad \Gamma_1 \vdash B_1 \hat{=} B_2 \dashv \Gamma_2}{\Gamma \vdash A_1 \oplus B_1 \equiv A_2 \oplus B_2 \dashv \Gamma_2}$$

$$\frac{\Gamma \vdash A_1 \hat{=} A_2 \dashv \Gamma_1 \quad \Gamma_1 \vdash B_1 \hat{=} B_2 \dashv \Gamma_2}{\Gamma \vdash r A_1 \rightarrow B_1 \equiv r A_2 \rightarrow B_2 \dashv \Gamma_2}$$

$$\frac{\Gamma \vdash A_1 \hat{=} A_2 \dashv \Gamma'}{\Gamma \vdash !_r A_1 \equiv !_r A_2 \dashv \Gamma'}$$

Conversion of terms

$$\frac{\Gamma(x) = A}{\Gamma \vdash x \hat{\sim} x \Rightarrow A \vdash \Gamma}$$

$$\frac{\Gamma, x : A \vdash f \ x \hat{=} g \ x \Leftarrow B \vdash \Gamma', x : A}{\Gamma \vdash f \equiv g \Leftarrow r A \rightarrow B \vdash \Gamma'}$$

$$\frac{\Gamma \vdash n_1 \sim n_2 \Rightarrow r A \rightarrow B \vdash \Gamma_1 \quad \Gamma_1 \vdash u_1 \hat{=} u_2 \Leftarrow A \vdash \Gamma_2}{\Gamma \vdash n_1 \ u_1 \hat{\sim} n_2 \ u_2 \Rightarrow B \vdash \Gamma_2}$$