T-Axi (declarative)

Wojciech Kołowski
Mateusz Pyzik

Type conversion – sanity checks

We'll set up the type conversion judgement so that if $\Gamma \vdash A \equiv B\ \texttt{type}$ then $\Gamma \vdash A\ \texttt{type}$ and $\Gamma \vdash B\ \texttt{type}$.

Type conversion

$$\frac{}{\Gamma \vdash \texttt{Unit} \equiv \texttt{Unit}\ \texttt{type}} \qquad \frac{}{\Gamma \vdash \texttt{Empty} \equiv \texttt{Empty}\ \texttt{type}}$$

$$\frac{\Gamma \vdash A_1 \equiv A_2\ \texttt{type} \quad \Gamma \vdash B_1 \equiv B_2\ \texttt{type}}{\Gamma \vdash r\, A_1 \to B_1 \equiv r\, A_2 \to B_2\ \texttt{type}}$$

$$\frac{\Gamma \vdash A_1 \equiv A_2\ \texttt{type}}{\Gamma \vdash \,!_r\, A_1 \equiv \,!_r\, A_2\ \texttt{type}}$$

$$\frac{\Gamma \vdash A_1 \equiv A_2\ \texttt{type} \quad \Gamma \vdash B_1 \equiv B_2\ \texttt{type}}{\Gamma \vdash A_1 \otimes B_1 \equiv A_2 \otimes B_2\ \texttt{type}}$$

$$\frac{\Gamma \vdash A_1 \equiv A_2\ \texttt{type} \quad \Gamma \vdash B_1 \equiv B_2\ \texttt{type}}{\Gamma \vdash A_1 \oplus B_1 \equiv A_2 \oplus B_2\ \texttt{type}}$$

Type conversion – properties

The rules above are the complete definition of type conversion. For now the context Γ is not used, but soon it will. We don't need to take any closures – type conversion already is an equivalence relation.

- $\Gamma \vdash A \equiv A \ \mathtt{type}$
- If $\Gamma \vdash A \equiv B \ \mathtt{type}$ then $\Gamma \vdash B \equiv A \ \mathtt{type}$
- If $\Gamma \vdash A \equiv B \ \mathtt{type}$ and $\Gamma \vdash B \equiv C \ \mathtt{type}$ then $\Gamma \vdash A \equiv C \ \mathtt{type}$

Proposition conversion – sanity checks

We'll set up the proposition conversion judgement so that if $\Gamma \vdash P \equiv Q \texttt{ prop}$ then $\Gamma \vdash P \texttt{ prop}$ and $\Gamma \vdash Q \texttt{ prop}$.

Proposition conversion

$$\frac{}{\Gamma \vdash \top \equiv \top \ \texttt{prop}} \qquad \frac{}{\Gamma \vdash \bot \equiv \bot \ \texttt{prop}}$$

$$\frac{\Gamma \vdash P_1 \equiv P_2 \ \texttt{prop} \quad \Gamma \vdash Q_1 \equiv Q_2 \ \texttt{prop}}{\Gamma \vdash P_1 \Rightarrow Q_1 \equiv P_2 \Rightarrow Q_2 \ \texttt{prop}}$$

$$\frac{\Gamma \vdash P_1 \equiv P_2 \ \texttt{prop} \quad \Gamma \vdash Q_1 \equiv Q_2 \ \texttt{prop}}{\Gamma \vdash P_1 \wedge Q_1 \equiv P_2 \wedge Q_2 \ \texttt{prop}}$$

$$\frac{\Gamma \vdash P_1 \equiv P_2 \ \texttt{prop} \quad \Gamma \vdash Q_1 \equiv Q_2 \ \texttt{prop}}{\Gamma \vdash P_1 \vee Q_1 \equiv P_2 \vee Q_2 \ \texttt{prop}}$$

Proposition conversion

$$\frac{\Gamma \vdash A_1 \equiv A_2 \ \texttt{type} \qquad \Gamma, x : A_1 \vdash P_1 \equiv P_2 \ \texttt{prop}}{\Gamma \vdash \forall x : A_1 .\, P_1 \equiv \forall x : A_2 .\, P_2 \ \texttt{prop}}$$

$$\frac{\Gamma \vdash A_1 \equiv A_2 \ \texttt{type} \qquad \Gamma, x : A_1 \vdash P_1 \equiv P_2 \ \texttt{prop}}{\Gamma \vdash \exists x : A_1 .\, P_1 \equiv \exists x : A_2 .\, P_2 \ \texttt{prop}}$$

$$\frac{\Gamma \vdash A_1 \equiv A_2 \ \texttt{type} \qquad \Gamma \vdash e_1 \equiv e_2 : A_1 \qquad \Gamma \vdash e_1' \equiv e_2' : A_1}{\Gamma \vdash e_1 =_{A_1} e_1' \equiv e_2 =_{A_2} e_2' \ \texttt{prop}}$$

Term conversion – sanity checks

We'll set up the term conversion judgement so that if $\Gamma \vdash a_1 \equiv a_2 : A$ then $\Gamma \vdash a_1 : A$ and $\Gamma \vdash a_2 : A$.

Term conversion – Empty

$$\frac{\Gamma \vdash e_1 : \texttt{Empty} \quad \Gamma \vdash e_2 : \texttt{Empty}}{\Gamma \vdash e_1 \equiv e_2 : \texttt{Empty}}$$

$$\frac{|\Gamma| \vdash A \ \texttt{type} \quad \Gamma \vdash e_1 \equiv e_2 : \texttt{Empty}}{\Gamma \vdash \texttt{Empty-elim}\ e_1 \equiv \texttt{Empty-elim}\ e_2 : A}$$

Term conversion – Unit

$$\frac{\Gamma \vdash a : A}{\Gamma \vdash \texttt{let unit} = \texttt{unit in } a \equiv a : A}$$

$$\frac{\Gamma \ \texttt{ctx} \quad \Gamma \leq 0\,\Gamma}{\Gamma \vdash \texttt{unit} \equiv \texttt{unit} : \texttt{Unit}}$$

$$\frac{\Gamma \vdash u_1 : \texttt{Unit} \quad \Gamma \vdash u_2 : \texttt{Unit}}{\Gamma \vdash u_1 \equiv u_2 : \texttt{Unit}}$$

$$\frac{\Gamma \leq \Gamma_1 + \Gamma_2 \quad \Gamma_1 \vdash u_1 \equiv u_2 : \texttt{Unit} \quad \Gamma_2 \vdash a_1 \equiv a_2 : A}{\Gamma \vdash \texttt{let unit} = u_1 \texttt{ in } a_1 \equiv \texttt{let unit} = u_2 \texttt{ in } a_2 : A}$$

Term conversion – functions

$$\frac{\Gamma \leq \Gamma_1 + r\,\Gamma_2 \quad \Gamma_1, r^i_A\, x : A \vdash b : B \quad \Gamma_2 \vdash a : A}{\Gamma \vdash (\lambda_r\, x : A.\, b)\ a \equiv b\,[x := a] : B}$$

$$\frac{\Gamma, r^i_A\, x : A \vdash f\ \ x \equiv g\ \ x : B}{\Gamma \vdash f \equiv g : r\,A \to B}$$

$$\frac{\Gamma \leq \Gamma_1 + r\,\Gamma_2 \quad \Gamma_1 \vdash f_1 \equiv f_2 : r\,A \to B \quad \Gamma_2 \vdash a_1 \equiv a_2 : A}{\Gamma \vdash f_1\ \ a_1 \equiv f_2\ \ a_2 : B}$$

Term conversion – box

$$\frac{\Gamma \leq r\,\Gamma_1 + \Gamma_2 \quad \Gamma_1 \vdash a : A \quad \Gamma_2, r^i_A\, x : A \vdash b : B}{\Gamma \vdash \texttt{let box}\ x = \texttt{box}_r\ a\ \texttt{in}\ b \equiv b\,[x := a] : B}$$

$$\frac{\Gamma \leq r\,\Gamma' \quad \Gamma' \vdash a_1 \equiv a_2 : A}{\Gamma \vdash \texttt{box}\ a_1 \equiv \texttt{box}\ a_2 : !_r\, A}$$

$$\frac{\Gamma \leq \Gamma_1 + \Gamma_2 \quad \Gamma_1 \vdash a_1 \equiv a_2 : !_r\, A \quad \Gamma_2, r^i_{A_1}\, x : A_1 \vdash b_1 \equiv b_2 : B}{\Gamma \vdash \texttt{let box}\ x = a_1\ \texttt{in}\ b_1 \equiv \texttt{let box}\ x = a_2\ \texttt{in}\ b_2 : B}$$

Term conversion – product

$$\frac{\Gamma \leq \Gamma_1 + \Gamma_2 \quad \Gamma_1 \vdash a_1 \equiv a_2 : A \quad \Gamma_2 \vdash b_1 \equiv b_2 : B}{\Gamma \vdash (a_1, b_1) \equiv (a_2, b_2) : A \otimes B}$$

$$\frac{\Gamma \leq \Gamma_1 + \Gamma_2 \quad \Gamma_1 \vdash e_1 \equiv e_2 : A \otimes B \quad \Gamma_2, 1^i_A\, x : A, 1^i_B\, y : B \vdash c_1 \equiv c_2 : C}{\Gamma \vdash \mathtt{let}\ (x, y) = e_1\ \mathtt{in}\ c_1 \equiv \mathtt{let}\ (x, y) = e_2\ \mathtt{in}\ c_2 : C}$$

Term conversion – sum

$$\frac{\Gamma \vdash a_1 \equiv a_2 : A \quad |\Gamma| \vdash B\ \texttt{type}}{\Gamma \vdash \texttt{inl}\ a_1 \equiv \texttt{inl}\ a_2 : A \oplus B} \qquad \frac{|\Gamma| \vdash A\ \texttt{type} \quad \Gamma \vdash b_1 \equiv b_2 : B}{\Gamma \vdash \texttt{inr}\ b_1 \equiv \texttt{inr}\ b_2 : A \oplus B}$$

$$\frac{\Gamma \leq \Gamma_1 + \Gamma_2 \quad \Gamma_1 \vdash e_1 \equiv e_2 : A \oplus B \quad \begin{array}{l} \Gamma_2 \vdash f_1 \equiv f_2 : 1\,A \to C \\ \Gamma_2 \vdash g_1 \equiv g_2 : 1\,B \to C \end{array}}{\Gamma \vdash \texttt{case}\ e_1\ \texttt{of}\ (f_1, g_1) \equiv \texttt{case}\ e_2\ \texttt{of}\ (f_2, g_2) : C}$$

Environments

Global environments:

$$\Sigma ::= \emptyset \mid \Sigma, h : P := p \mid \Sigma, x : A := e \mid \Sigma, \texttt{partial}\ x : A := e \mid \Sigma, \texttt{totality}\ x\ p$$

Well-formed environments

$$\frac{}{\emptyset\ \texttt{env}}$$

$$\frac{\Sigma\ \texttt{env} \quad h \notin \Sigma \quad \Sigma \mid \cdot \vdash p : P}{\Sigma, h : P := p\ \texttt{env}}$$

$$\frac{\Sigma\ \texttt{env} \quad x \notin \Sigma \quad \Sigma \mid \cdot \vdash_{c} e : A}{\Sigma, x : A := e\ \texttt{env}}$$

$$\frac{\Sigma\ \texttt{env} \quad x \notin \Sigma \quad \Sigma \mid \cdot \vdash_{c} e : A \quad e\ \textbf{fails syntactic check}}{\Sigma, \texttt{partial}\ x : A := e\ \texttt{env}}$$

$$\frac{\Sigma\ \texttt{env} \quad \Sigma = \Sigma_1, \texttt{partial}\ x : A := e, \Sigma_2 \quad \Sigma \mid \cdot \vdash p : \exists r : A.\, e =?= r}{\Sigma, \texttt{totality}\ x\ p\ \texttt{env}}$$