

# Compatibility

$$\frac{}{\Gamma \vdash \square \approx A \dashv \Gamma}$$

$$\frac{\Gamma \vdash A \equiv B \Rightarrow \text{Type}_r \dashv \Gamma'}{\Gamma \vdash A \approx B \dashv \Gamma'}$$

$$\frac{\Gamma \mid A \vdash; e \Rightarrow A' \dashv \Gamma_1 \quad \Gamma_1 \vdash \Sigma \approx B \dashv \Gamma_2}{\Gamma \vdash \Sigma, e \approx A \rightarrow B \dashv \Gamma_2}$$

# Variables and annotations

$$\frac{(x : A) \in \Gamma \quad \Gamma - x \vdash \Sigma \approx A \dashv \Gamma'}{\Gamma \mid \Sigma \vdash_c x \Rightarrow A \dashv \Gamma'}$$

$$\frac{(x : A) \in \Gamma \quad \Gamma \vdash \Sigma \approx A \dashv \Gamma'}{\Gamma \mid \Sigma \vdash_{nc} x \Rightarrow A \dashv \Gamma'}$$

$$\frac{\Gamma \vdash A \Rightarrow \text{Type}_r \dashv \Gamma_1 \quad \Gamma_1 \mid A \vdash_i e \Rightarrow B \dashv \Gamma_2 \quad \Gamma_2 \vdash \Sigma \approx B \dashv \Gamma_3}{\Gamma \mid \Sigma \vdash_i (e : A) \Rightarrow A \dashv \Gamma_3}$$

## Functions

$$\frac{\Gamma \mid \Sigma, e_2 \vdash_i e_1 \Rightarrow A \rightarrow B \dashv \Gamma'}{\Gamma \mid \Sigma \vdash_i e_1 \ e_2 \Rightarrow B \dashv \Gamma'}$$

$$\frac{\Gamma, 1_A^i x : A \mid B \vdash_i e \Rightarrow B' \dashv \Gamma', 0x : A}{\Gamma \mid A \rightarrow B \vdash_i \lambda x. e \Rightarrow A \rightarrow B' \dashv \Gamma'}$$

$$\frac{\begin{array}{c} \Gamma \mid \square \vdash_i e' \Rightarrow A \dashv \Gamma_1 \\ \Gamma_1, 1_A^i x : A \mid \Sigma \vdash_i e \Rightarrow B \dashv \Gamma_2, 0x : A \end{array}}{\Gamma \mid \Sigma, e' \vdash_i \lambda x. e \Rightarrow A \rightarrow B \dashv \Gamma_2}$$

## Box

$$\frac{\Gamma \mid \square \vdash_i e_1 \Rightarrow !_r A \dashv \Gamma_1 \quad \Gamma_1, r_A^i x : A \mid \Sigma \vdash_i e_2 \Rightarrow B \dashv \Gamma_2, 0 x : A}{\Gamma \mid \Sigma \vdash_i \text{let box } x = e_1 \text{ in } e_2 \Rightarrow B \dashv \Gamma_2}$$

$$\frac{\Gamma / r = (\Gamma_1, \Gamma_2) \quad \Gamma_1 \mid A \vdash_i e \Rightarrow A' \dashv \Gamma_3}{\Gamma \mid !_r A \vdash_i \text{box } e \Rightarrow !_r A' \dashv \Gamma_2 + r \Gamma_3}$$

$$\frac{\Gamma / r = (\Gamma_1, \Gamma_2) \quad \Gamma_1 \mid \square \vdash_i e \Rightarrow A \dashv \Gamma_3}{\Gamma \mid \square \vdash_i \text{box}_r e \Rightarrow !_r A \dashv \Gamma_2 + r \Gamma_3}$$

## Unit

$$\frac{\Gamma \mid \square \vdash; e_1 \Rightarrow \text{Unit} \dashv \Gamma_1 \quad \Gamma \mid \Sigma \vdash; e_2 \Rightarrow A \dashv \Gamma_2}{\Gamma \mid \Sigma \vdash; \text{let unit} = e_1 \text{ in } e_2 \Rightarrow A \dashv \Gamma_2}$$

$$\frac{\Gamma \vdash \Sigma \approx \text{Unit} \dashv \Gamma'}{\Gamma \mid \Sigma \vdash; \text{unit} \Rightarrow \text{Unit} \dashv \Gamma'}$$

## Empty

$$\frac{\Gamma \mid \text{Empty} \vdash; e \Rightarrow \text{Empty} \dashv \Gamma'}{\Gamma \mid A \vdash; \text{Empty-elim } e \Rightarrow A \dashv \Gamma'}$$

$$\frac{\Gamma \mid \Sigma \vdash; \text{Empty-elim } e \Rightarrow B \dashv \Gamma_1 \quad \Gamma_1 \mid \square \vdash; e' \Rightarrow A' \dashv \Gamma_2}{\Gamma \mid \Sigma, e' \vdash; \text{Empty-elim } e \Rightarrow A \rightarrow B \dashv \Gamma_2}$$

## Product

$$\frac{\Gamma \mid \square \vdash_i e_1 \Rightarrow A \otimes B \dashv \Gamma_1 \quad \Gamma_1, 1_A^i x : A, 1_B^i y : B \mid \Sigma \vdash_i e_2 \Rightarrow C \dashv \Gamma_2, 0x : A, 0y : B}{\Gamma \mid \Sigma \vdash_i \text{let } (x, y) = e_1 \text{ in } e_2 \Rightarrow C \dashv \Gamma_2}$$

$$\frac{\Gamma \mid A \vdash_i e_1 \Rightarrow A' \dashv \Gamma_1 \quad \Gamma_1 \mid B \vdash_i e_2 \Rightarrow B' \dashv \Gamma_2}{\Gamma \mid A \otimes B \vdash_i (e_1, e_2) \Rightarrow A' \otimes B' \dashv \Gamma_2}$$

$$\frac{\Gamma \mid \square \vdash_i e_1 \Rightarrow A \dashv \Gamma_1 \quad \Gamma_1 \mid \square \vdash_i e_2 \Rightarrow B \dashv \Gamma_2}{\Gamma \mid \square \vdash_i (e_1, e_2) \Rightarrow A \otimes B \dashv \Gamma_2}$$

## Sum

$$\frac{\Gamma \mid \square \vdash_i e \Rightarrow A \oplus B \dashv \Gamma_1 \quad \begin{array}{l} \Gamma_1, 1_A^i x : A \mid \Sigma \vdash; e_1 \Rightarrow C \dashv \Gamma_2, 0x : A \\ \Gamma_1, 1_B^i y : B \mid \Sigma \vdash; e_2 \Rightarrow C \dashv \Gamma_3, 0y : B \end{array}}{\Gamma \mid \Sigma \vdash; \text{case } e \text{ of } \{x.e_1; y.e_2\} \Rightarrow C \dashv \Gamma_2 \sqcup \Gamma_3}$$

$$\frac{\Gamma \mid A \vdash; e \Rightarrow A' \dashv \Gamma'}{\Gamma \mid A \oplus B \vdash; \text{inl } e \Rightarrow A' \oplus B \dashv \Gamma'}$$

$$\frac{\Gamma \mid B \vdash; e \Rightarrow B' \dashv \Gamma'}{\Gamma \mid A \oplus B \vdash; \text{inr } e \Rightarrow A \oplus B' \dashv \Gamma'}$$