Judgements

- $\Gamma \vdash_i e \Rightarrow A \dashv \Gamma'$ – in context Γ, term e infers type A, with output context Γ'
- $\Gamma \vdash A \rightsquigarrow B \dashv \Gamma'$ – in context Γ, type A matches (output) type B, with output context Γ'
- $\Gamma \vdash r \Rightarrow A <: B \Rightarrow s \dashv \Gamma'$ – in context Γ, s copies of subtype A are needed to produce r copies of supertype B, with output context Γ'.
- $\Gamma \vdash r \Rightarrow \hat{a} <: A \Rightarrow s \dashv \Gamma'$ – in context Γ, instantiate the existential variable $\hat{a}$ to a subtype of $!_r A$, lacking s, returning context Γ'
- $\Gamma \vdash r \Rightarrow A <: \hat{a} \Rightarrow s \dashv \Gamma'$ – in context Γ, instantiate the existential variable $\hat{a}$, so that $A <: !_r \hat{a}$, lacking s, returning context Γ

Variables and annotations

$$\frac{\Gamma(x) = A \quad 0\,\Gamma + x \vdash \langle 0\,\Gamma + x\rangle A \rightsquigarrow B \dashv \Gamma'}{\Gamma \vdash_{\mathtt{c}} x \Rightarrow B \dashv \Gamma' + \Gamma}$$

$$\frac{\Gamma(x) = A \quad \Gamma \vdash \langle \Gamma\rangle A \rightsquigarrow B \dashv \Gamma'}{\Gamma \vdash_{\mathtt{nc}} x \Rightarrow B \dashv \Gamma'}$$

$$\frac{\Gamma \vdash A \Rightarrow \mathtt{Type}_r \dashv \Gamma_1 \quad \begin{array}{l} 0\,\Gamma_1, \Box : A \vdash_i e \Rightarrow A' \dashv \Gamma_2 \\ \Gamma_2 \vdash \langle \Gamma_2\rangle A' \rightsquigarrow B \dashv \Gamma_3 \end{array}}{\Gamma \vdash_i (e : A) \Rightarrow B \dashv \Gamma_3 + \Gamma_1}$$

Stationary rules

$$\frac{\Gamma, \Delta, a, \square : A \vdash_i e \Rightarrow A' \dashv \Gamma', a, \Theta}{\Gamma, \square : \forall a.\, A, \Delta \vdash_i e \Rightarrow \forall a.\, A \dashv \Gamma'}$$

$$\frac{0\,\Gamma, 0\,\Delta, \square : A \vdash_i e \Rightarrow A' \dashv \Gamma'}{\Gamma, \square : \,!_r\, A, \Delta \vdash_i e \Rightarrow\, !_r\, A \dashv r\,\Gamma' + \Gamma, \Delta}$$

Application

$$\frac{\Gamma, \square\ e_2 \vdash_i e_1 \Rightarrow A \to B \dashv \Gamma'}{\Gamma \vdash_i e_1\ e_2 \Rightarrow B \dashv \Gamma'}$$

Functions

$$\frac{\Gamma, \Delta, 0\,x : A, \Box : B \vdash_i e \Rightarrow B' \dashv \Gamma_1, r\,x : A, \Theta \quad \Gamma_1 \vdash A \Leftarrow \mathtt{Type}_r \dashv \Gamma_2}{\Gamma, \Box : A \to B, \Delta \vdash_i \lambda\, x.\, e \Rightarrow A \to B \dashv \Gamma_2}$$

$$\frac{\begin{array}{l}\Gamma \vdash_i e' \Rightarrow A \dashv \Gamma_1 \quad \Gamma_1, \Delta, 0\,x : A \vdash_i e \Rightarrow B \dashv \Gamma_2, r\,x : A, \Theta \\ \Gamma_2 \vdash A \Leftarrow \mathtt{Type}_r \dashv \Gamma_3\end{array}}{\Gamma, \Box\ e', \Delta \vdash_i \lambda\, x.\, e \Rightarrow A \to B \dashv \Gamma_3}$$

$$\frac{\begin{array}{l}\Gamma\left[\hat{a}_1, \hat{a}_2, \hat{a} = \hat{a}_1 \to \hat{a}_2\right], \Delta, 0\,x : \hat{a}_1, \Box : \hat{a}_2 \vdash_i e \Rightarrow A \dashv \Gamma_1, r\,x : \hat{a}_1, \Theta \\ \Gamma_1 \vdash \hat{a}_1 \Leftarrow \mathtt{Type}_r \dashv \Gamma_2\end{array}}{\Gamma\left[\hat{a}\right], \Box : \hat{a}, \Delta \vdash_i \lambda\, x.\, e \Rightarrow \hat{a} \dashv \Gamma_2}$$

$$\frac{\begin{array}{l}\Gamma, \blacktriangleright_{\hat{a}}, \hat{a}, 0\,x : \hat{a}, \Box \vdash_i e \Rightarrow A \dashv \Gamma_1, r\,x : \hat{a}, \Gamma_4 \\ \Gamma_1 \vdash \hat{a} \Leftarrow \mathtt{Type}_r \dashv \Gamma_2, \blacktriangleright_{\hat{a}}, \Gamma_3 \\ B = \forall \mathtt{unsolved}(\Gamma_3).\, \langle \Gamma_3 \rangle (\hat{a} \to \forall \mathtt{unsolved}(\Gamma_4).\, \langle \Gamma_4 \rangle A)\end{array}}{\Gamma, \Box, \Delta \vdash_i \lambda\, x.\, e \Rightarrow B \dashv \Gamma_2}$$

Matching

$$\frac{}{\Gamma, \square, \Delta \vdash A \rightsquigarrow A \dashv \Gamma, \Delta}$$

$$\frac{\Gamma, \Delta \vdash 1 \Rightarrow A <: B \Rightarrow r \dashv \Gamma'}{\Gamma, \square : B, \Delta \vdash A \rightsquigarrow B \dashv r\, \Gamma'}$$

$$\frac{\Gamma, \square : A \vdash_i e \Rightarrow A' \dashv \Gamma_1 \quad \Gamma_1, \Delta \vdash B \rightsquigarrow B' \dashv \Gamma_2}{\Gamma, \square\ e, \Delta \vdash A \rightarrow B \rightsquigarrow A \rightarrow B' \dashv \Gamma_2}$$

Matching

$$\frac{\Gamma, \Box\ e, \Delta \vdash A \rightsquigarrow B \dashv \Gamma'}{\Gamma, \Box\ e, \Delta \vdash\ !_r\, A \rightsquigarrow B \dashv \Gamma'}$$

$$\frac{\Gamma, \Delta, \Box\ e, \hat{a} \vdash A\,[a := \hat{a}] \rightsquigarrow B \dashv \Gamma'}{\Gamma, \Box\ e, \Delta \vdash \forall a.\, A \rightsquigarrow B \dashv \Gamma'}$$

$$\frac{\Gamma \vdash B \Rightarrow \mathtt{Type}_r \dashv \Gamma_1 \quad \Gamma_1, \Delta \vdash A\,[a := B] \rightsquigarrow C \dashv \Gamma'}{\Gamma, \Box\ B, \Delta \vdash \forall @a.\, A \rightsquigarrow C \dashv \Gamma'}$$

Subtyping

$$\frac{}{\Gamma \vdash r \Rightarrow \texttt{Unit} <: \texttt{Unit} \Rightarrow 1 \dashv \Gamma}$$

$$\frac{}{\Gamma \vdash r \Rightarrow \texttt{Empty} <: \texttt{Empty} \Rightarrow 1 \dashv \Gamma}$$

$$\frac{\Gamma \vdash r \Rightarrow A_1 <: A_2 \Rightarrow s_A \dashv \Gamma_1 \quad \Gamma_1 \vdash r \Rightarrow B_1 <: B_2 \Rightarrow s_B \dashv \Gamma_2}{\Gamma \vdash r \Rightarrow A_1 \otimes B_1 <: A_2 \otimes B_2 \Rightarrow s_A \sqcap s_B \dashv \Gamma_2}$$

$$\frac{\Gamma \vdash r \Rightarrow A_1 <: A_2 \Rightarrow s_A \dashv \Gamma_1 \quad \Gamma_1 \vdash r \Rightarrow B_1 <: B_2 \Rightarrow s_B \dashv \Gamma_2}{\Gamma \vdash r \Rightarrow A_1 \oplus B_1 <: A_2 \oplus B_2 \Rightarrow s_A \sqcap s_B \dashv \Gamma_2}$$

$$\frac{\Gamma \vdash 1 \Rightarrow A_2 <: A_1 \Rightarrow 1 \dashv \Gamma_1 \quad \Gamma_1 \vdash 1 \Rightarrow B_1 <: B_2 \Rightarrow 1 \dashv \Gamma_2}{\Gamma \vdash r \Rightarrow A_1 \to B_1 <: A_2 \to B_2 \Rightarrow r \dashv \Gamma_2}$$

Subtyping

$$\frac{}{\Gamma[\hat{a}] \vdash r \Rightarrow \hat{a} <: \hat{a} \Rightarrow r \dashv \Gamma[\hat{a}]}$$

$$\frac{}{\Gamma[a : \mathtt{Type}_s] \vdash r \Rightarrow a <: a \Rightarrow r/s \dashv \Gamma[a : \mathtt{Type}_s]}$$

$$\frac{\Gamma, a : \mathtt{Type}_s \vdash r \Rightarrow A <: B \Rightarrow t \dashv \Gamma', a : \mathtt{Type}_s, \Theta}{\Gamma \vdash r \Rightarrow A <: \forall a : \mathtt{Type}_s.\, B \Rightarrow t \dashv \Gamma'}$$

$$\frac{\Gamma, \blacktriangleright_{\hat{a}}, \hat{a} \vdash r \Rightarrow A[a := \,!_s\, \hat{a}] <: B \Rightarrow t \dashv \Gamma', \blacktriangleright_{\hat{a}}, \Theta}{\Gamma \vdash r \Rightarrow \forall a : \mathtt{Type}_s.\, A <: B \Rightarrow t \dashv \Gamma'}$$

$$\frac{\Gamma, a : \mathtt{Type}_s \vdash r \Rightarrow A <: B \Rightarrow t \dashv \Gamma', a : \mathtt{Type}_s, \Theta}{\Gamma \vdash r \Rightarrow \forall @a : \mathtt{Type}_s.\, A <: \forall @a : \mathtt{Type}_s.\, B \Rightarrow t \dashv \Gamma'}$$

Subtyping

$$\frac{\Gamma \vdash r \cdot s \Rightarrow A <: B \Rightarrow t \dashv \Gamma'}{\Gamma \vdash r \Rightarrow A <: \,!_s\, B \Rightarrow t \dashv \Gamma'}$$

$$\frac{B \neq \,!_\_\, B' \quad \Gamma \vdash r/s \Rightarrow A <: B \Rightarrow t \dashv \Gamma'}{\Gamma \vdash r \Rightarrow \,!_s\, A <: B \Rightarrow t \dashv \Gamma'}$$