

Judgements

$\Gamma \vdash; e \Leftarrow A \Rightarrow e' \dashv \Gamma'$ – in context Γ check that term e has type A and elaborate it to term e' , returning the bill Γ' .

$\Gamma \vdash; e \Rightarrow e' : A \dashv \Gamma'$ – in context Γ infer that A is the type of term e and elaborate it to term e' , returning the bill Γ' .

Functions

$$\frac{\Gamma, 0 \ x : A \vdash_i e \Leftarrow B \Rightarrow e' \dashv \Gamma', 1_A^i x : A}{\Gamma \vdash_i \lambda x. e \Leftarrow A \rightarrow B \Rightarrow \lambda x : A. e' \dashv \Gamma'}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow e'_1 : A \rightarrow B \dashv \Gamma_1 \quad \Gamma \vdash_i e_2 \Leftarrow A \Rightarrow e'_2 \dashv \Gamma_2}{\Gamma \vdash_i e_1 \ e_2 \Rightarrow e'_1 \ e'_2 : B \dashv \Gamma_1 + \Gamma_2}$$

Box

$$\frac{\Gamma \vdash_i e \Leftarrow A \Rightarrow e' \dashv \Gamma'}{\Gamma \vdash_i \text{box } e \Leftarrow !_r A \Rightarrow \text{box}_r e' \dashv r \Gamma'}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow e'_1 : !_r A \dashv \Gamma_1 \quad \Gamma, 0x : A \vdash_i e_2 \Rightarrow e'_2 : B \dashv \Gamma_2, r_A^i x : A}{\Gamma \vdash_i \text{let box } x = e_1 \text{ in } e_2 \Rightarrow \text{let}_B \text{ box } x = e'_1 \text{ in } e'_2 : B \dashv \Gamma_1 + \Gamma_2}$$

Empty

$$\frac{\Gamma \vdash; e \Leftarrow \text{Empty} \Rightarrow e' \dashv \Gamma'}{\Gamma \vdash; \text{Empty-elim } e \Leftarrow A \Rightarrow \text{Empty-elim}_A e' \dashv \Gamma'}$$

Unit

$$\frac{}{\Gamma \vdash_i \text{unit} \Leftarrow \text{Unit} \Rightarrow \text{unit} \dashv \Gamma}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow e'_1 : \text{Unit} \dashv \Gamma_1 \quad \Gamma \vdash_i e_2 \Rightarrow e'_2 : A \dashv \Gamma_2}{\Gamma \vdash_i \text{let unit} = e_1 \text{ in } e_2 \Rightarrow \text{let}_A \text{unit} = e'_1 \text{ in } e'_2 : A \dashv \Gamma_1 + \Gamma_2}$$

Product

$$\frac{\Gamma \vdash_i e_1 \Leftarrow A \Rightarrow e'_1 \dashv \Gamma_1 \quad \Gamma \vdash_i e_2 \Leftarrow B \Rightarrow e'_2 \dashv \Gamma_2}{\Gamma \vdash_i (e_1, e_2) \Leftarrow A \otimes B \Rightarrow (e'_1, e'_2) \dashv \Gamma_1 + \Gamma_2}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow e'_1 : A \otimes B \dashv \Gamma_1 \quad \Gamma, 0x : A, 0y : B \vdash_i e_2 \Rightarrow e'_2 : C \dashv \Gamma_2, 1_A^i x : A, 1_B^i y : B}{\Gamma \vdash_i \text{let } (x, y) = e_1 \text{ in } e_2 \Rightarrow \text{let}_C (x, y) = e'_1 \text{ in } e'_2 : C \dashv \Gamma_1 + \Gamma_2}$$

Sum

$$\frac{\Gamma \vdash; e \Leftarrow A \Rightarrow e' \dashv \Gamma'}{\Gamma \vdash; \text{inl } e \Leftarrow A \oplus B \Rightarrow \text{inl}_B e' \dashv \Gamma'}$$

$$\frac{\Gamma \vdash; e \Leftarrow B \Rightarrow e' \dashv \Gamma'}{\Gamma \vdash; \text{inr } e \Leftarrow A \oplus B \Rightarrow \text{inr}_A e' \dashv \Gamma'}$$

$$\frac{\Gamma \vdash; e \Rightarrow e' : A \oplus B \dashv \Gamma_1 \quad \Gamma, 0x : A \vdash; e_1 \Leftarrow e'_1 \Rightarrow C \dashv \Gamma_2, r_1 x : r = (r_1 / 1_A^i) \sqcap (r_2 / 1_B^i)}{\Gamma, 0y : B \vdash; e_2 \Leftarrow e'_2 \Rightarrow C \dashv \Gamma_3, r_2 y : r = (r_1 / 1_A^i) \sqcap (r_2 / 1_B^i)}$$

$$\Gamma \vdash; \text{case } e \text{ of } \{x.e_1; y.e_2\} \Leftarrow C \Rightarrow \text{case}_C e' \text{ of } \{x.e'_1; y.e'_2\} \dashv r \Gamma_1 + \Gamma_2$$