

# Axi Design: Language Features

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# What is a function, classically?

In classical mathematics, a relation  $R$  between  $A$  and  $B$  is called a function when it is:

- Right-unique (also called deterministic): for all  $a : A$  and  $b_1, b_2 : B$ , if  $R(a, b_1)$  and  $R(a, b_2)$ , then  $b_1 = b_2$ .
- Right-total (also called total): for all  $a : A$ , there exists  $b : B$  such that  $R(a, b)$ .

# What is a function, constructively?

In constructive mathematics, TODO

# What is totality checking?

Totality checking is the following task: we are given the definition of a well-typed function  $f : A \rightarrow B$  and we need to tell whether it is total, i.e. whether  $\forall a : A, \exists b : B, f(a) = b$ . Spelled out in words: for all  $a$  of type  $A$ , there exists  $b$  of type  $B$  such that  $f(a)$  equals  $b$ .

# Why should we care?

Why should we care about totality checking? TODO

# Why should we care?

We already know that all functions must terminate for proof checking to be decidable. However, this isn't termination checking's *raison d'être*.

Neither has it anything to do with gas, or even what happens at runtime at all. After all, the universe is going to last a finite amount of time, so all functions will eventually terminate, right? Wrong.

Termination is first and foremost a logical notion. It's evil twin, non-termination, manifests itself most strikingly not at runtime, where it cannot be observed at all (because it would take forever), but in the logic – TODO

# Termination checking is undecidable

In general, termination checking is undecidable – it means literally solving the Halting Problem! That's hard!