T-Axi

Wojciech Kołowski
Mateusz Pyzik

Quantities

Quantities:
$r ::= 0 \mid 1 \mid \omega$

The default quantity is ω, so it's not written most of the time.

Quantities are ordered:

$$\omega \leq r$$
$$0 \leq 0$$
$$1 \leq 1$$

Operations on quantities

Addition of quantities:

$$
\begin{aligned}
0 + r &= r \\
\omega + r &= \omega \\
1 + 0 &= 1 \\
1 + 1 &= \omega \\
1 + \omega &= \omega
\end{aligned}
$$

Multiplication:

$$
\begin{aligned}
0 \cdot r &= 0 \\
1 \cdot r &= r \\
\omega \cdot 0 &= 0 \\
\omega \cdot 1 &= \omega \\
\omega \cdot \omega &= \omega
\end{aligned}
$$

Types

Types:

$$A, B ::= r\ A \to B \mid !_r A \mid A \otimes B \mid A \oplus B \mid \mathbf{1} \mid \mathbf{0}$$

Terms

Terms:

$e ::=$

$x \mid \lambda_r\, x : A.\, e \mid e_1\ e_2 \mid$

$\mathtt{box}_r\ e \mid \mathtt{letbox}\ x = e_1\ \mathtt{in}\ e_2$

$\mathtt{let}_r\ x = e_1\ \mathtt{in}\ e_2 \mid$

Contexts

Contexts:

$\Gamma ::= \cdot \mid \Gamma, r\, x : A \mid \Gamma, r\, x : A := e \mid \Gamma, p : P$

Operations on contexts:

$\Gamma_1 + \Gamma_2$

$r\,\Gamma$

$\Gamma_1 \leq \Gamma_2$

Contexts addition

$$\cdot + \cdot = \cdot$$
$$(\Gamma_1, r_1\, x : A) + (\Gamma_2, r_2\, x : A) = (\Gamma_1 + \Gamma_2), (r_1 + r_2)\, x : A$$
$$(\Gamma_1, r_1\, x : A := e) + (\Gamma_2, r_2\, x : A := e) = (\Gamma_1 + \Gamma_2), (r_1 + r_2)\, x : A := e$$
$$(\Gamma_1, p : P) + (\Gamma_2, p : P) = (\Gamma_1 + \Gamma_2), p : P$$

Context scaling

$$
\begin{aligned}
s\,\cdot &= \cdot \\
s\,(\Gamma, r\,x : A) &= s\,\Gamma, (s \cdot r)\,x : A \\
s\,(\Gamma, r\,x : A := e) &= s\,\Gamma, (s \cdot r)\,x : A := e \\
s\,(\Gamma, p : P) &= s\,\Gamma, p : P
\end{aligned}
$$

Context ordering

$$\frac{}{\cdot \leq \cdot}$$

$$\frac{\Gamma_1 \leq \Gamma_2 \quad r_1 \leq r_2}{\Gamma_1, r_1\, x : A \leq \Gamma_2, r_2\, x : A}$$

$$\frac{\Gamma_1 \leq \Gamma_2 \quad r_1 \leq r_2}{\Gamma_1, r_1\, x : A := e \leq \Gamma_2, r_2\, x : A := e}$$

$$\frac{\Gamma_1 \leq \Gamma_2}{\Gamma_1, p : P \leq \Gamma_2, p : P}$$

Judgements

Functions

$$\frac{\Gamma, r\,x : A \vdash e : B}{\Gamma \vdash \lambda_r\,x : A.\,e : r\ A \to B}$$

$$\frac{\Gamma \leq \Gamma_1 + r\,\Gamma_2 \quad \Gamma_1 \vdash e_1 : r\ A \to B \quad \Gamma_2 \vdash e_2 : B}{\Gamma \vdash e_1\ e_2 : B}$$

Box

$$\frac{\Gamma \leq r\,\Gamma' \quad \Gamma' \vdash e : A}{\Gamma \vdash \mathtt{box}_r\ e : \,!_r\, A}$$

$$\frac{\Gamma \leq \Gamma_1 + \Gamma_2 \quad \Gamma_1 \vdash e_1 : \,!_r\, A \quad \Gamma_2, r\,x : A \vdash e_2 : B}{\Gamma \vdash \mathtt{letbox}\ x = e_1\ \mathtt{in}\ e_2 : B}$$

Let

$$\frac{\Gamma \leq \Gamma_1 + r\,\Gamma_2 \quad \Gamma_1 \vdash e_1 : A \quad \Gamma_2, r\,x : A \vdash e_2 : B}{\Gamma \vdash \texttt{let}_r\ x = e_1\ \texttt{in}\ e_2 : B}$$