Notation

$\Gamma \vdash_i e \Leftarrow A \dashv \Gamma', r_A^i x : A$ is a shorthand for $\Gamma \vdash_i e \Leftarrow A \dashv \Gamma', r x : A$ with the condition $1_A^i \sqsubseteq r$.

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Subsumption and annotations

$$\frac{\Gamma \vdash_i e \Rightarrow A \dashv \Gamma_1 \quad \Gamma \vdash A \equiv B \Rightarrow \mathrm{Type}_r \dashv \Gamma_2}{\Gamma \vdash_i e \Leftarrow B \dashv \Gamma_1 + \Gamma_2} \textsc{Subsumption}$$

$$\frac{\Gamma \vdash A \Rightarrow \mathrm{Type}_r \dashv \Gamma_1 \quad \Gamma \vdash_i e \Leftarrow A \dashv \Gamma_2}{\Gamma \vdash_i (e : A) \Rightarrow A \dashv \Gamma_1 + \Gamma_2} \textsc{Annot}$$

Using variables

$$\frac{\Gamma(x) = A}{\Gamma \vdash_{\mathrm{c}} x \Rightarrow A \dashv |\Gamma|_x}$$

$$\frac{\Gamma(x) = A}{\Gamma \vdash_{\mathrm{nc}} x \Rightarrow A \dashv 0\,\Gamma}$$

Functions

$$\frac{\Gamma, 0\,x : A \vdash_i e \Leftarrow B \dashv \Gamma', 1_A^i\,x : A}{\Gamma \vdash_i \lambda\,x.\,e \Leftarrow A \rightarrow B \dashv \Gamma'}$$

$$\frac{\Gamma \vdash_i f \Rightarrow A \rightarrow B \dashv \Gamma_1 \quad \Gamma \vdash_i a \Leftarrow A \dashv \Gamma_2}{\Gamma \vdash_i f\ a \Rightarrow B \dashv \Gamma_1 + \Gamma_2}$$

Box

$$\frac{\Gamma \vdash_i e \Leftarrow A \dashv \Gamma'}{\Gamma \vdash_i \text{box } e \Leftarrow \,!_r A \dashv r \, \Gamma'}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow \,!_r A \dashv \Gamma_1 \quad \Gamma, 0 \, x : A \vdash_i e_2 \Rightarrow B \dashv \Gamma_2, r_A^i \, x : A}{\Gamma \vdash_i \text{let box } x = e_1 \text{ in } e_2 \Rightarrow B \dashv \Gamma_1 + \Gamma_2}$$

Empty

$$\frac{\Gamma \vdash_i e \Leftarrow \texttt{Empty} \dashv \Gamma'}{\Gamma \vdash_i \texttt{Empty-elim } e \Leftarrow A \dashv \Gamma'}$$

Unit

$$\overline{\Gamma \vdash_i \texttt{unit} \Leftarrow \texttt{Unit} \dashv 0\,\Gamma}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow \texttt{Unit} \dashv \Gamma_1 \quad \Gamma \vdash_i e_2 \Rightarrow A \dashv \Gamma_2}{\Gamma \vdash_i \texttt{let unit} = e_1 \texttt{ in } e_2 \Rightarrow A \dashv \Gamma_1 + \Gamma_2}$$

Products

$$\frac{\Gamma \vdash_i e_1 \Leftarrow A \dashv \Gamma_1 \quad \Gamma \vdash_i e_2 \Leftarrow B \dashv \Gamma_2}{\Gamma \vdash_i (e_1, e_2) \Leftarrow A \otimes B \dashv \Gamma_1 + \Gamma_2}$$

$$\frac{\begin{array}{l} \Gamma \vdash_i e_1 \Rightarrow A \otimes B \dashv \Gamma_1 \\ \Gamma, 0\, x : A, 0\, y : B \vdash_i e_2 \Rightarrow C \dashv \Gamma_2, 1_A^i\, x : A, 1_B^i\, y : B \end{array}}{\Gamma \vdash_i \texttt{let } (x, y) = e_1 \texttt{ in } e_2 \Rightarrow C \dashv \Gamma_1 + \Gamma_2}$$

Division

$r_1/r_2 = \sup\{s \in \mathcal{Q} \mid s \cdot r_2 \sqsubseteq r_1\}$

r_1/r_2	0	1	?	$+$	$*$
0	0	0	0	0	0
1		1	1	1	1
?		?	1	?	1
$+$		$+$	$+$	1	1
$*$		$*$	$+$	?	1

Sums

$$\frac{\Gamma \vdash_i e \Leftarrow A \dashv \Gamma'}{\Gamma \vdash_i \mathtt{inl}\ e \Leftarrow A \oplus B \dashv \Gamma'} \qquad \frac{\Gamma \vdash_i e \Leftarrow B \dashv \Gamma'}{\Gamma \vdash_i \mathtt{inr}\ e \Leftarrow A \oplus B \dashv \Gamma'}$$

$$\frac{\Gamma \vdash_i e \Rightarrow A \oplus B \dashv \Gamma_1 \quad \begin{array}{c} \Gamma, 0\,x : A \vdash_i e_1 \Leftarrow C \dashv \Gamma_2, r_1\,x : A \\ \Gamma, 0\,y : B \vdash_i e_2 \Leftarrow C \dashv \Gamma_3, r_2\,y : B \\ r = (r_1/1_A^i) \sqcap (r_2/1_B^i) \end{array}}{\Gamma \vdash_i \mathtt{case}\ e\ \mathtt{of}\ \{\mathtt{inl}\ x \rightarrow e_1;\ \mathtt{inr}\ y \rightarrow e_2\} \Leftarrow C \dashv r\,\Gamma_1 + (\Gamma_2 \sqcap \Gamma_3)}$$

$$\frac{\Gamma \vdash_i e \Rightarrow A \oplus B \dashv \Gamma_1 \quad \begin{array}{c} \Gamma, 0\,x : A \vdash_i e_1 \Rightarrow C \dashv \Gamma_2, r_1\,x : A \\ \Gamma, 0\,y : B \vdash_i e_2 \Rightarrow C \dashv \Gamma_3, r_2\,y : B \\ r = (r_1/1_A^i) \sqcap (r_2/1_B^i) \end{array}}{\Gamma \vdash_i \mathtt{case}\ e\ \mathtt{of}\ \{\mathtt{inl}\ x \rightarrow e_1;\ \mathtt{inr}\ y \rightarrow e_2\} \Rightarrow C \dashv r\,\Gamma_1 + (\Gamma_2 \sqcap \Gamma_3)}$$

Let

$$\frac{\Gamma \vdash_i e_1 \Rightarrow A \dashv \Gamma_1 \quad \Gamma, 0\, x : A \vdash_i e_2 \Rightarrow B \dashv \Gamma_2, r\, x : A \quad r' = r/1_A^i}{\Gamma \vdash_i \mathtt{let}\ x = e_1\ \mathtt{in}\ e_2 \Rightarrow B \dashv r'\,\Gamma_1 + \Gamma_2}$$