Judgements

$\Gamma \vdash_i e \Leftarrow A \dashv \Gamma'$ - in context Γ (which has zero resources), e checks to have type A and consumes resources according to quantities from Γ'.

$\Gamma \vdash_i e \Rightarrow A \dashv \Gamma'$ - as above, but with type inference.

Notation

$\Gamma \vdash_i e \Leftarrow A \dashv \Gamma', r_A^i\, x : A$ is a shorthand for $\Gamma \vdash_i e \Leftarrow A \dashv \Gamma', r\, x : A$ with the condition $1_A^i \sqsubseteq r$.

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Subsumption and annotations

$$\frac{\Gamma \vdash_i e \Rightarrow A \dashv \Gamma_1 \quad \Gamma \vdash A \equiv B \Rightarrow \texttt{Type}_r \dashv \Gamma_2}{\Gamma \vdash_i e \Leftarrow B \dashv \Gamma_1 + \Gamma_2}\textsc{Subsumption}$$

$$\frac{\Gamma \vdash A \Rightarrow \texttt{Type}_r \dashv \Gamma_1 \quad \Gamma \vdash_i e \Leftarrow A \dashv \Gamma_2}{\Gamma \vdash_i (e : A) \Rightarrow A \dashv \Gamma_1 + \Gamma_2}\textsc{Annot}$$

Using variables

$$\frac{\Gamma(x) = A}{\Gamma \vdash_{\mathtt{c}} x \Rightarrow A \dashv |\Gamma|_x}$$

$$\frac{\Gamma(x) = A}{\Gamma \vdash_{\mathtt{nc}} x \Rightarrow A \dashv 0\,\Gamma}$$

Functions

$$\frac{\Gamma, 0\,x : A \vdash_i e \Leftarrow B \dashv \Gamma', 1^i_A\, x : A}{\Gamma \vdash_i \lambda x.\, e \Leftarrow A \to B \dashv \Gamma'}$$

$$\frac{\Gamma \vdash_i f \Rightarrow A \to B \dashv \Gamma_1 \quad \Gamma \vdash_i a \Leftarrow A \dashv \Gamma_2}{\Gamma \vdash_i f\ a \Rightarrow B \dashv \Gamma_1 + \Gamma_2}$$

Box

$$\frac{\Gamma \vdash_i e \Leftarrow A \dashv \Gamma'}{\Gamma \vdash_i \texttt{box}\ e \Leftarrow\ !_r A \dashv r\,\Gamma'}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow\ !_r A \dashv \Gamma_1 \quad \Gamma, 0\,x : A \vdash_i e_2 \Rightarrow B \dashv \Gamma_2, r^i_A\,x : A}{\Gamma \vdash_i \texttt{let box}\ x = e_1\ \texttt{in}\ e_2 \Rightarrow B \dashv \Gamma_1 + \Gamma_2}$$

Empty

$$\frac{\Gamma \vdash_i e \Leftarrow \texttt{Empty} \dashv \Gamma'}{\Gamma \vdash_i \texttt{Empty-elim}\ e \Leftarrow A \dashv \Gamma'}$$

Unit

$$\frac{}{\Gamma \vdash_i \texttt{unit} \Leftarrow \texttt{Unit} \dashv 0\,\Gamma}$$

$$\frac{\Gamma \vdash_i e_1 \Rightarrow \texttt{Unit} \dashv \Gamma_1 \quad \Gamma \vdash_i e_2 \Rightarrow A \dashv \Gamma_2}{\Gamma \vdash_i \texttt{let unit} = e_1 \texttt{ in } e_2 \Rightarrow A \dashv \Gamma_1 + \Gamma_2}$$

Products

$$\frac{\Gamma \vdash_i e_1 \Leftarrow A \dashv \Gamma_1 \quad \Gamma \vdash_i e_2 \Leftarrow B \dashv \Gamma_2}{\Gamma \vdash_i (e_1, e_2) \Leftarrow A \otimes B \dashv \Gamma_1 + \Gamma_2}$$

$$\frac{\begin{array}{l}\Gamma \vdash_i e_1 \Rightarrow A \otimes B \dashv \Gamma_1 \\ \Gamma, 0\, x : A, 0\, y : B \vdash_i e_2 \Rightarrow C \dashv \Gamma_2, 1^i_A\, x : A, 1^i_B\, y : B\end{array}}{\Gamma \vdash_i \mathtt{let}\ (x, y) = e_1\ \mathtt{in}\ e_2 \Rightarrow C \dashv \Gamma_1 + \Gamma_2}$$

Division

$$r_1/r_2 = \sup\{s \in \mathcal{Q} \mid s \cdot r_2 \sqsubseteq r_1\}$$

r_1/r_2	0	1	?	+	*
0	0	0	0	0	0
1		1	1	1	1
?		?	1	?	1
+		+	+	1	1
*		*	+	?	1

Sums

$$\frac{\Gamma \vdash_i e \Leftarrow A \dashv \Gamma'}{\Gamma \vdash_i \mathtt{inl}\ e \Leftarrow A \oplus B \dashv \Gamma'} \qquad \frac{\Gamma \vdash_i e \Leftarrow B \dashv \Gamma'}{\Gamma \vdash_i \mathtt{inr}\ e \Leftarrow A \oplus B \dashv \Gamma'}$$

$$\frac{\Gamma \vdash_i e \Rightarrow A \oplus B \dashv \Gamma_1 \qquad \begin{array}{l} \Gamma, 0\, x : A \vdash_i e_1 \Leftarrow C \dashv \Gamma_2, r_1\, x : A \\ \Gamma, 0\, y : B \vdash_i e_2 \Leftarrow C \dashv \Gamma_3, r_2\, y : B \\ r = (r_1 / 1^i_A) \sqcap (r_2 / 1^i_B) \end{array}}{\Gamma \vdash_i \mathsf{case}\ e\ \mathsf{of}\ \{x.e_1;\ y.e_2\} \Leftarrow C \dashv r\, \Gamma_1 + (\Gamma_2 \sqcap \Gamma_3)}$$

$$\frac{\Gamma \vdash_i e \Rightarrow A \oplus B \dashv \Gamma_1 \qquad \begin{array}{l} \Gamma, 0\, x : A \vdash_i e_1 \Rightarrow C \dashv \Gamma_2, r_1\, x : A \\ \Gamma, 0\, y : B \vdash_i e_2 \Rightarrow C \dashv \Gamma_3, r_2\, y : B \\ r = (r_1 / 1^i_A) \sqcap (r_2 / 1^i_B) \end{array}}{\Gamma \vdash_i \mathsf{case}\ e\ \mathsf{of}\ \{x.e_1;\ y.e_2\} \Rightarrow C \dashv r\, \Gamma_1 + (\Gamma_2 \sqcap \Gamma_3)}$$

Let

$$\frac{\Gamma \vdash_i e_1 \Rightarrow A \dashv \Gamma_1 \quad \Gamma, 0\,x : A \vdash_i e_2 \Rightarrow B \dashv \Gamma_2, r\,x : A \quad r' = r/1^i_A}{\Gamma \vdash_i \texttt{let}\ x = e_1\ \texttt{in}\ e_2 \Rightarrow B \dashv r'\,\Gamma_1 + \Gamma_2}$$

Natural numbers

$$\frac{}{\Gamma \vdash_i 0 \Leftarrow \mathbb{N} \dashv 0\,\Gamma}$$

$$\frac{\Gamma \vdash_i n \Leftarrow \mathbb{N} \dashv \Gamma'}{\Gamma \vdash_i \mathtt{succ}\ n \Leftarrow \mathbb{N} \dashv \Gamma'}$$

$$\frac{\Gamma \vdash_i n \Rightarrow \mathbb{N} \dashv \Gamma_1 \quad \Gamma \vdash_i z \Rightarrow A \dashv \Gamma_2 \quad \Gamma \vdash_i s \Rightarrow A \to A \dashv \Gamma_3}{\Gamma \vdash_i \mathtt{elim}_{\mathbb{N}}\ z\ s\ n \Rightarrow A \dashv \Gamma_1 + \Gamma_2 + *\Gamma_3}$$