Poor Man's Axi: Algorithmic Version

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Syntax

Values (cbv)

$$\frac{}{\lambda x.\, e \ \mathtt{value}}$$

$$\frac{v_1 \ \mathtt{value} \quad v_2 \ \mathtt{value}}{(v_1, v_2) \ \mathtt{value}}$$

$$\frac{v \ \mathtt{value}}{\mathtt{inl}\ v \ \mathtt{value}} \qquad \frac{v \ \mathtt{value}}{\mathtt{inr}\ v \ \mathtt{value}}$$

$$\frac{}{\mathtt{unit}\ \mathtt{value}}$$

Values are the final results of computation. Note that a function is a value whether or not its body is. Other values are pairs of values, values injected into a sum on the left or right, and unit.

Big-step semantics (cbv)

$$\frac{}{\lambda x.\, e \Downarrow \lambda x.\, e} \qquad \frac{e_1 \Downarrow \lambda x.\, e \quad e_2 \Downarrow v \quad e\,[x := v] \Downarrow v'}{e_1\ e_2 \Downarrow v'}$$

$$\frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{(e_1, e_2) \Downarrow (v_1, v_2)} \qquad \frac{e \Downarrow (v_1, v_2)}{\mathtt{outl}\ e \Downarrow v_1} \qquad \frac{e \Downarrow (v_1, v_2)}{\mathtt{outr}\ e \Downarrow v_2}$$

$$\frac{e \Downarrow v}{\mathtt{inl}\ e \Downarrow \mathtt{inl}\ v} \qquad \frac{e \Downarrow v}{\mathtt{inr}\ e \Downarrow \mathtt{inr}\ v}$$

$$\frac{e \Downarrow \mathtt{inl}\ v \quad f\ v \Downarrow v'}{\mathtt{case}\ e\ \mathtt{of}\ (f, g) \Downarrow v'} \qquad \frac{e \Downarrow \mathtt{inr}\ v \quad g\ v \Downarrow v'}{\mathtt{case}\ e\ \mathtt{of}\ (f, g) \Downarrow v'}$$

$$\frac{}{\mathtt{unit} \Downarrow \mathtt{unit}}$$

Small-step semantics (cbv) – basic rules

$$\frac{v \;\texttt{value}}{(\lambda x.\, e)\ v \longrightarrow e\,[x := v]}$$

$$\frac{v_1 \;\texttt{value} \quad v_2 \;\texttt{value}}{\texttt{outl}\ (v_1, v_2) \longrightarrow v_1} \qquad \frac{v_1 \;\texttt{value} \quad v_2 \;\texttt{value}}{\texttt{outr}\ (v_1, v_2) \longrightarrow v_2}$$

$$\frac{v \;\texttt{value}}{\texttt{case}\ (\texttt{inl}\ v)\ \texttt{of}\ (f, g) \longrightarrow f\ v}$$

$$\frac{v \;\texttt{value}}{\texttt{case}\ (\texttt{inr}\ v)\ \texttt{of}\ (f, g) \longrightarrow g\ v}$$

Small-step semantics (cbv) – boring rules

$$\frac{e_1 \longrightarrow e_1'}{e_1\ e_2 \longrightarrow e_1'\ e_2} \qquad \frac{v_1\ \texttt{value} \quad e_2 \longrightarrow e_2'}{v_1\ e_2 \longrightarrow v_1\ e_2'}$$

$$\frac{e_1 \longrightarrow e_1'}{(e_1, e_2) \longrightarrow (e_1', e_2)} \qquad \frac{v_1\ \texttt{value} \quad e_2 \longrightarrow e_2'}{(v_1, e_2) \longrightarrow (v_1, e_2')}$$

$$\frac{e \longrightarrow e'}{\texttt{outl}\ e \longrightarrow \texttt{outl}\ e'} \qquad \frac{e \longrightarrow e'}{\texttt{outr}\ e \longrightarrow \texttt{outr}\ e'}$$

$$\frac{e \longrightarrow e'}{\texttt{inl}\ e \longrightarrow \texttt{inl}\ e'} \qquad \frac{e \longrightarrow e'}{\texttt{inr}\ e \longrightarrow \texttt{inr}\ e'}$$

$$\frac{e \longrightarrow e'}{\texttt{case}\ e\ \texttt{of}\ (f, g) \longrightarrow \texttt{case}\ e'\ \texttt{of}\ (f, g)}$$

Algorithmic typing

Weak head normal forms

$$\frac{}{n \ \mathtt{whnf}}$$

$$\frac{}{\lambda x.\, e \ \mathtt{whnf}}$$

$$\frac{}{(e_1, e_2) \ \mathtt{whnf}}$$

$$\frac{}{\mathtt{inl}\ e \ \mathtt{whnf}} \qquad \frac{}{\mathtt{inr}\ e \ \mathtt{whnf}}$$

$$\frac{}{\mathtt{unit} \ \mathtt{whnf}} \qquad \frac{}{\mathbf{0}\text{-}\mathtt{elim}\ e \ \mathtt{whnf}}$$

Weak head normal forms grammar

Weak head normal forms:

$$
\begin{aligned}
e ::=\ & \\
& n \mid \lambda x.\, e \mid \\
& (e_1, e_2) \mid \\
& \texttt{inl}\ e \mid \texttt{inr}\ e \mid \\
& \texttt{unit} \mid \mathbf{0}\texttt{-elim}\ e
\end{aligned}
$$

Neutral forms:

$$
\begin{aligned}
n ::=\ & \\
& x \mid n\ e \mid \\
& \texttt{outl}\ n \mid \texttt{outr}\ n \mid \\
& \texttt{case}\ n\ \texttt{of}\ (e_1, e_2) \mid
\end{aligned}
$$

Whnf reduction – basic rules

$$\frac{}{(\lambda x.\, e_1)\ e_2 \longrightarrow_{\mathtt{whnf}} e_1\,[x := e_2]}$$

$$\frac{}{\mathtt{outl}\ (e_1, e_2) \longrightarrow_{\mathtt{whnf}} e_1} \qquad \frac{}{\mathtt{outr}\ (e_1, e_2) \longrightarrow_{\mathtt{whnf}} e_2}$$

$$\frac{}{\mathtt{case}\ (\mathtt{inl}\ e)\ \mathtt{of}\ (f, g) \longrightarrow_{\mathtt{whnf}} f\ e}$$

$$\frac{}{\mathtt{case}\ (\mathtt{inr}\ e)\ \mathtt{of}\ (f, g) \longrightarrow_{\mathtt{whnf}} g\ e}$$

Whnf reduction – boring rules

$$\frac{e_1 \longrightarrow_{\mathtt{whnf}} e_1'}{e_1\ e_2 \longrightarrow_{\mathtt{whnf}} e_1'\ e_2} \qquad \frac{e_1\ \mathtt{whnf} \quad e_2 \longrightarrow_{\mathtt{whnf}} e_2'}{e_1\ e_2 \longrightarrow_{\mathtt{whnf}} e_1\ e_2'}$$

$$\frac{e \longrightarrow_{\mathtt{whnf}} e'}{\mathtt{outl}\ e \longrightarrow_{\mathtt{whnf}} \mathtt{outl}\ e'} \qquad \frac{e \longrightarrow_{\mathtt{whnf}} e'}{\mathtt{outr}\ e \longrightarrow_{\mathtt{whnf}} \mathtt{outr}\ e'}$$

$$\frac{e \longrightarrow_{\mathtt{whnf}} e'}{\mathtt{case}\ e\ \mathtt{of}\ (f, g) \longrightarrow_{\mathtt{whnf}} \mathtt{case}\ e'\ \mathtt{of}\ (f, g)}$$

Algorithmic computational equality 0

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x \equiv x \Rightarrow A}\textsc{Var}$$

$$\frac{\Gamma \vdash e \equiv e' \Rightarrow B \quad A = B}{\Gamma \vdash e \equiv e' \Leftarrow A}\textsc{Sub}$$

$$\frac{e_1 \longrightarrow_{\texttt{whnf}} e_1' \quad e_2 \longrightarrow_{\texttt{whnf}} e_2' \quad \Gamma \vdash e_1' \equiv e_2' \Leftarrow A}{\Gamma \vdash e_1 \equiv e_2 \Leftarrow_{\texttt{whnf}} A}$$

$$\frac{e_1 \longrightarrow_{\texttt{whnf}} e_1' \quad e_2 \longrightarrow_{\texttt{whnf}} e_2' \quad \Gamma \vdash e_1' \equiv e_2' \Rightarrow A}{\Gamma \vdash e_1 \equiv e_2 \Rightarrow_{\texttt{whnf}} A}$$

Algorithmic computational equality 1

$$\frac{\Gamma, x : A \vdash e_1 \equiv e_2 \Leftarrow_{\texttt{whnf}} B}{\Gamma \vdash \lambda x.\, e_1 \equiv \lambda x.\, e_2 \Leftarrow A \to B}$$

$$\frac{\Gamma \vdash n_1 \equiv n_2 \Rightarrow A \to B \quad \Gamma \vdash e_1 \equiv e_2 \Leftarrow_{\texttt{whnf}} A}{\Gamma \vdash n_1\ e_1 \equiv n_2\ e_2 \Rightarrow B}$$

$$\frac{\Gamma \vdash a_1 \equiv a_2 \Leftarrow_{\texttt{whnf}} A \quad \Gamma \vdash b_1 \equiv b_2 \Leftarrow_{\texttt{whnf}} B}{\Gamma \vdash (a_1, b_1) \equiv (a_2, b_2) \Leftarrow A \times B}$$

$$\frac{\Gamma \vdash n_1 \equiv n_2 \Rightarrow A \times B}{\Gamma \vdash \texttt{outl}\ n_1 \equiv \texttt{outl}\ n_2 \Rightarrow A} \qquad \frac{\Gamma \vdash n_1 \equiv n_2 \Rightarrow A \times B}{\Gamma \vdash \texttt{outr}\ n_1 \equiv \texttt{outr}\ n_2 \Rightarrow B}$$

Algorithmic computational equality 2

$$\frac{\Gamma \vdash e_1 \equiv e_2 \Leftarrow_{\mathtt{whnf}} A}{\Gamma \vdash \mathtt{inl}\ e_1 \equiv \mathtt{inl}\ e_2 \Leftarrow A + B} \qquad \frac{\Gamma \vdash e_1 \equiv e_2 \Leftarrow_{\mathtt{whnf}} B}{\Gamma \vdash \mathtt{inr}\ e_1 \equiv \mathtt{inr}\ e_2 \Leftarrow A + B}$$

$$\frac{\Gamma \vdash n_1 \equiv n_2 \Rightarrow A + B \qquad \begin{array}{c}\Gamma \vdash f_1 \equiv f_2 \Leftarrow_{\mathtt{whnf}} A \to C \\ \Gamma \vdash g_1 \equiv g_2 \Leftarrow_{\mathtt{whnf}} B \to C\end{array}}{\Gamma \vdash \mathtt{case}\ n_1\ \mathtt{of}\ (f_1, g_1) \equiv \mathtt{case}\ n_2\ \mathtt{of}\ (f_2, g_2) \Rightarrow C}$$

$$\frac{}{\Gamma \vdash \mathtt{unit} \equiv \mathtt{unit} \Leftarrow \mathbf{1}} \qquad \frac{\Gamma \vdash e_1 \equiv e_2 \Leftarrow \mathbf{0}}{\Gamma \vdash \mathbf{0}\text{-}\mathtt{elim}\ e_1 \equiv \mathbf{0}\text{-}\mathtt{elim}\ e_2 \Leftarrow A}$$

Uniqueness rules (asymmetric, contraction-like)

$$\frac{\Gamma \vdash f \Leftarrow A \to B}{\Gamma \vdash f \equiv \lambda x.\, f\ x \Leftarrow A \to B}\textsc{Fun-Uniq}$$

$$\frac{\Gamma \vdash e \Leftarrow A \times B}{\Gamma \vdash e \equiv (\mathtt{outl}\ e, \mathtt{outr}\ e) \Leftarrow A \times B}\textsc{Prod-Uniq}$$

$$\frac{\Gamma \vdash e \Leftarrow \mathbf{1}}{\Gamma \vdash e \equiv \mathtt{unit} \Leftarrow \mathbf{1}}\textsc{Unit-Uniq}$$

Uniqueness rules (symmetric, prop-like)

$$\frac{\Gamma, x : A \vdash f\ x \equiv g\ x \Leftarrow B}{\Gamma \vdash f \equiv g \Leftarrow A \to B}\textsc{Fun-Uniq-Alt}$$

$$\frac{\begin{array}{c}\Gamma \vdash \texttt{outl}\ e_1 \equiv \texttt{outl}\ e_2 \Leftarrow A \\ \Gamma \vdash \texttt{outr}\ e_1 \equiv \texttt{outr}\ e_2 \Leftarrow B\end{array}}{\Gamma \vdash e_1 \equiv e_2 \Leftarrow A \times B}\textsc{Prod-Uniq-Alt}$$

$$\frac{\Gamma \vdash e_1 \Leftarrow \mathbf{1} \quad \Gamma \vdash e_2 \Leftarrow \mathbf{1}}{\Gamma \vdash e_1 \equiv e_2 \Leftarrow \mathbf{1}}\textsc{Unit-Uniq-Alt}$$

$$\frac{\Gamma \vdash e_1 \Leftarrow \mathbf{0} \quad \Gamma \vdash e_2 \Leftarrow \mathbf{0}}{\Gamma \vdash e_1 \equiv e_2 \Leftarrow \mathbf{0}}\textsc{Empty-Uniq-Alt}$$

Subsumption

$$\frac{\Gamma \vdash e \Rightarrow P' \quad P = P'}{\Gamma \vdash e \Leftarrow P}\textsc{Sub}$$

Annotations

$$\frac{\Gamma \vdash P \ \texttt{prop} \quad \Gamma \vdash e \Leftarrow P}{\Gamma \vdash \textbf{have } P \textbf{ from } e \Rightarrow P}\textsc{Proof-Annot}$$

Assumptions

$$\frac{P \in \Gamma}{\Gamma \vdash P \Rightarrow P}\textsc{Ass}$$

True and False

$$\frac{}{\Gamma \vdash \textbf{true} \Rightarrow \top}\textsc{True-Intro}$$

$$\frac{\Gamma \vdash e \Leftarrow \bot}{\Gamma \vdash \textbf{exfalso}\ e \Leftarrow P}\textsc{False-Elim}$$

Implication

$$\frac{\Gamma, P \vdash e \Leftarrow Q}{\Gamma \vdash \textbf{assume } P \textbf{ in } e \Leftarrow P \Rightarrow Q}\textsc{Impl-Intro-Check}$$

$$\frac{\Gamma \vdash P \ \texttt{prop} \quad \Gamma, P \vdash e \Rightarrow Q}{\Gamma \vdash \textbf{assume } P \textbf{ in } e \Rightarrow P \Rightarrow Q}\textsc{Impl-Intro-Infer}$$

$$\frac{\Gamma \vdash e_1 \Rightarrow P \Rightarrow Q \quad \Gamma \vdash e_2 \Leftarrow P}{\Gamma \vdash \textbf{modus-ponens } e_1 \ e_2 \Rightarrow Q}\textsc{Impl-Elim}$$

Negation

$$\frac{\Gamma \vdash P\ \texttt{prop} \quad \Gamma, P \vdash e \Leftarrow \bot}{\Gamma \vdash \textbf{suppose-absurd}\ P\ \textbf{in}\ e \Rightarrow \neg P}\ \textsc{Not-Intro}$$

$$\frac{\Gamma \vdash e_1 \Rightarrow \neg P \quad \Gamma \vdash e_2 \Leftarrow P}{\Gamma \vdash \textbf{absurd}\ e_1\ e_2 \Rightarrow \bot}\ \textsc{Not-Elim}$$

Conjunction

$$\frac{\Gamma \vdash e_1 \Leftarrow P \quad \Gamma \vdash e_2 \Leftarrow Q}{\Gamma \vdash \textbf{both}\ e_1\ e_2 \Leftarrow P \wedge Q}\textsc{And-Intro}$$

$$\frac{\Gamma \vdash e_1 \Rightarrow P \quad \Gamma \vdash e_2 \Rightarrow Q}{\Gamma \vdash \textbf{both}\ e_1\ e_2 \Rightarrow P \wedge Q}\textsc{And-Intro-Infer}$$

$$\frac{\Gamma \vdash e \Rightarrow P \wedge Q}{\Gamma \vdash \textbf{left-and}\ e \Rightarrow P}\textsc{And-Elim-L}$$

$$\frac{\Gamma \vdash e \Rightarrow P \wedge Q}{\Gamma \vdash \textbf{right-and}\ e \Rightarrow Q}\textsc{And-Elim-R}$$

Biconditional

$$\frac{\Gamma \vdash e_1 \Leftarrow P \Rightarrow Q \quad \Gamma \vdash e_2 \Leftarrow Q \Rightarrow P}{\Gamma \vdash \textbf{equivalence}\ e_1\ e_2 \Leftarrow P \Leftrightarrow Q}\textsc{Iff-Intro}$$

$$\frac{\Gamma \vdash e \Rightarrow P \Leftrightarrow Q}{\Gamma \vdash \textbf{left-iff}\ e \Rightarrow P \Rightarrow Q}\textsc{Iff-Elim-L}$$

$$\frac{\Gamma \vdash e \Rightarrow P \Leftrightarrow Q}{\Gamma \vdash \textbf{right-iff}\ e \Rightarrow Q \Rightarrow P}\textsc{Iff-Elim-R}$$

Biconditional – inference

$$\frac{\Gamma \vdash e_1 \color{red}{\Rightarrow} P \Rightarrow Q \quad \Gamma \vdash e_2 \color{blue}{\Leftarrow} Q \Rightarrow P}{\Gamma \vdash \textbf{equivalence}\ e_1\ e_2 \color{red}{\Rightarrow} P \Leftrightarrow Q}\ \textsc{Iff-Intro-Infer-L}$$

$$\frac{\Gamma \vdash e_2 \color{red}{\Rightarrow} Q \Rightarrow P \quad \Gamma \vdash e_1 \color{blue}{\Leftarrow} P \Rightarrow Q}{\Gamma \vdash \textbf{equivalence}\ e_1\ e_2 \color{red}{\Rightarrow} P \Leftrightarrow Q}\ \textsc{Iff-Intro-Infer-R}$$

Disjunction – checking

$$\frac{\Gamma \vdash e \Leftarrow P}{\Gamma \vdash \textbf{left-either}\ Q\ e \Leftarrow P \vee Q}\ \textsc{Or-Intro-L}$$

$$\frac{\Gamma \vdash e \Leftarrow Q}{\Gamma \vdash \textbf{right-either}\ P\ e \Leftarrow P \vee Q}\ \textsc{Or-Intro-R}$$

$$\frac{\Gamma \vdash e_1 \Rightarrow P \vee Q \quad \Gamma \vdash e_2 \Leftarrow P \Rightarrow R \quad \Gamma \vdash e_3 \Leftarrow Q \Rightarrow R}{\Gamma \vdash \textbf{constructive-dilemma}\ e_1\ e_2\ e_3 \Leftarrow R}\ \textsc{Or-Elim}$$

Disjunction – inference

$$\frac{\Gamma \vdash Q \ \texttt{prop} \quad \Gamma \vdash e \Rightarrow P}{\Gamma \vdash \textbf{left-either} \ Q \ e \Rightarrow P \vee Q} \textsc{Or-Intro-L-Infer}$$

$$\frac{\Gamma \vdash P \ \texttt{prop} \quad \Gamma \vdash e \Rightarrow Q}{\Gamma \vdash \textbf{right-either} \ P \ e \Rightarrow P \vee Q} \textsc{Or-Intro-R-Infer}$$

$$\frac{\Gamma \vdash e_1 \Rightarrow P \vee Q \quad \Gamma \vdash e_2 \Rightarrow P \Rightarrow R \quad \Gamma \vdash e_3 \Rightarrow Q \Rightarrow R}{\Gamma \vdash \textbf{constructive-dilemma} \ e_1 \ e_2 \ e_3 \Rightarrow R} \textsc{Or-Elim}$$

Double Negation Elimination

$$\frac{\Gamma \vdash e \Leftarrow \neg\neg P}{\Gamma \vdash \textbf{double-negation}\ e \Leftarrow P}\ \textsc{Classic-Check}$$

$$\frac{\Gamma \vdash e \Rightarrow \neg\neg P}{\Gamma \vdash \textbf{double-negation}\ e \Rightarrow P}\ \textsc{Classic-Infer}$$

Cut rule

$$\frac{\Gamma \vdash e_1 \Rightarrow P \quad \Gamma, P \vdash e_2 \Leftarrow Q}{\Gamma \vdash e_1; e_2 \Leftarrow Q}\textsc{Cut}$$

$$\frac{\Gamma \vdash e_1 \Rightarrow P \quad \Gamma, P \vdash e_2 \Rightarrow Q}{\Gamma \vdash e_1; e_2 \Rightarrow Q}\textsc{Cut-Infer}$$

Universal quantifier

$$\frac{\Gamma, y : A \vdash e \Leftarrow P\,[x := y]}{\Gamma \vdash \textbf{pick-any } y \textbf{ in } e \Leftarrow \forall x : A.\, P}\textsc{Forall-Intro}$$

$$\frac{\Gamma, x : A \vdash e \Rightarrow P}{\Gamma \vdash \textbf{pick-any } x : A \textbf{ in } e \Rightarrow \forall x : A.\, P}\textsc{Forall-Intro-Infer}$$

$$\frac{\Gamma \vdash e \Rightarrow \forall x : A.\, P \quad \Gamma \vdash t \Leftarrow A}{\Gamma \vdash \textbf{specialize } e \textbf{ with } t \Rightarrow P\,[x := t]}\textsc{Forall-Elim}$$

Existential quantifier

$$\dfrac{\Gamma \vdash t \Leftarrow A \quad \Gamma \vdash e \Leftarrow P\,[x := t]}{\Gamma \vdash \textbf{exists } t \textbf{ such that } e \Leftarrow \exists x : A.\, P}\,\textsc{Exists-Intro}$$

$$\dfrac{\Gamma \vdash e_1 \Rightarrow \exists x : A.\, P \quad \Gamma, y : A, P\,[x := y] \vdash e_2 \Rightarrow R}{\Gamma \vdash \textbf{pick-witness } y \textbf{ for } e_1 \textbf{ in } e_2 \Rightarrow R}\,\textsc{Exists-Elim}$$

$$\dfrac{\Gamma \vdash e_1 \Rightarrow \exists x : A.\, P \quad \Gamma, y : A, P\,[x := y] \vdash e_2 \Leftarrow R}{\Gamma \vdash \textbf{pick-witness } y \textbf{ for } e_1 \textbf{ in } e_2 \Leftarrow R}\,\textsc{Exists-Elim-Check}$$

Conversion rule – TODO

$$\frac{\Gamma, x : A \vdash P\ \mathtt{prop} \quad \Gamma \vdash t_1 \equiv t_2 : A \quad \Gamma \vdash e : P\,[x := t_1]}{\Gamma \vdash e : P\,[x := t_2]}\,\textsc{Conv}$$

Equality introduction and elimination

$$\frac{\Gamma \vdash t \Leftarrow A}{\Gamma \vdash \textbf{refl}\ t \Leftarrow t =_A t}\ \textsc{Eq-Intro}$$

$$\frac{\Gamma \vdash t \Rightarrow A}{\Gamma \vdash \textbf{refl}\ t \Rightarrow t =_A t}\ \textsc{Eq-Intro-Infer}$$

$$\frac{\Gamma, x : A \vdash P\ \mathtt{prop} \quad \Gamma \vdash e \Rightarrow t_1 =_A t_2 \quad \Gamma \vdash e' \Leftarrow P\,[x := t_1]}{\Gamma \vdash \textbf{rewrite}\ e\ \textbf{in}\ e' \Rightarrow P\,[x := t_2]}\ \textsc{Eq-Elim}$$

Experimenting with **refl**

$$\frac{\Gamma \vdash t_1 \equiv t_2 \Leftarrow A}{\Gamma \vdash \mathbf{refl} \Leftarrow t_1 =_A t_2}\textsc{Eq-Intro-Alt}$$

Function extensionality

$$\frac{\Gamma \vdash e \Leftarrow \forall x : A.\, f\ x =_B g\ x}{\Gamma \vdash \textbf{funext}\ e \Leftarrow f =_{A \to B} g}\,\textsc{Funext}$$

Reasoning by cases (for sums)

$$\frac{\Gamma, x : A + B \vdash P \ \texttt{prop} \quad \Gamma \vdash t \Rightarrow A + B \quad \begin{array}{c}\Gamma, a : A \vdash e_1 \Leftarrow P\,[x := \texttt{inl}\ a \\ \Gamma, b : B \vdash e_2 \Leftarrow P\,[x := \texttt{inr}\ b\end{array}}{\Gamma \vdash \textbf{case}\ t\ \textbf{of}\ (\texttt{inl}\ a \to e_1, \texttt{inr}\ b \to e_2) \Rightarrow P\,[x := t]}$$