Compatibility

$$\frac{}{\Gamma \vdash \square \approx A \dashv \Gamma}$$

$$\frac{\Gamma \vdash A \equiv B \Rightarrow \mathtt{Type}_r \dashv \Gamma'}{\Gamma \vdash A \approx B \dashv \Gamma'}$$

$$\frac{\Gamma / r = (\Gamma_1, \Gamma_2) \quad \Gamma_1 \mid A \vdash_i e \Rightarrow A' \dashv \Gamma_3 \quad \Gamma_2 + r\,\Gamma_3 \vdash \Sigma \approx B \dashv \Gamma_4}{\Gamma \vdash \Sigma, e \approx r\,A \to B \dashv \Gamma_4}$$

Typing

$$\frac{(x : A) \in \Gamma \quad \Gamma - x \vdash \Sigma \approx A \dashv \Gamma'}{\Gamma \mid \Sigma \vdash_{\mathtt{c}} x \Rightarrow A \dashv \Gamma'}$$

$$\frac{(x : A) \in \Gamma \quad \Gamma \vdash \Sigma \approx A \dashv \Gamma'}{\Gamma \mid \Sigma \vdash_{\mathtt{nc}} x \Rightarrow A \dashv \Gamma'}$$

$$\frac{\Gamma \vdash A \Rightarrow \mathtt{Type}_r \dashv \Gamma_1 \quad \Gamma_1 \mid A \vdash_i e \Rightarrow B \dashv \Gamma_2 \quad \Gamma_2 \vdash \Sigma \approx B \dashv \Gamma_3}{\Gamma \mid \Sigma \vdash_i (e : A) \Rightarrow A \dashv \Gamma_3}$$

Functions

$$\frac{\Gamma \mid \Sigma, e_2 \vdash_i e_1 \Rightarrow r\, A \to B \dashv \Gamma'}{\Gamma \mid \Sigma \vdash_i e_1\ e_2 \Rightarrow B \dashv \Gamma'}$$

$$\frac{\Gamma, r^i_A\, x : A \mid B \vdash_i e \Rightarrow B' \dashv \Gamma', 0\, x : A}{\Gamma \mid r\, A \to B \vdash_i \lambda\, x.\, e \Rightarrow r\, A \to B' \dashv \Gamma'}$$

$$\frac{\begin{array}{c}\Gamma / r = (\Gamma_1, \Gamma_2) \quad \Gamma_1 \mid \Box \vdash_i e' \Rightarrow A \dashv \Gamma_3 \\ (\Gamma_2 + r\,\Gamma_3), r^i_A\, x : A \mid \Sigma \vdash_i e \Rightarrow B \dashv \Gamma_4, 0\, x : A\end{array}}{\Gamma \mid \Sigma, e' \vdash_i \lambda_r\, x.\, e \Rightarrow r\, A \to B \dashv \Gamma_4}$$

Inferring quantities

$$\frac{\Gamma \mid \square \vdash_i e' \Rightarrow A \dashv \Gamma' \quad \Gamma, -0\, x : A \mid \Sigma \vdash_i e \Rightarrow B \dashv \Gamma_1, -r'\, x : A \\ r^i_A - r' \sqsubseteq 0 \quad \Gamma_1 / r = (\Gamma_2, \Gamma_3) \quad \Gamma_2 \mid \square \vdash_i e' \Rightarrow A \dashv \Gamma_4}{\Gamma \mid \Sigma, e' \vdash_i \lambda x.\, e \Rightarrow r\, A \rightarrow B \dashv \Gamma_3 + r\, \Gamma_4}$$

$$\frac{\Gamma \mid \square \vdash_i e' \Rightarrow A \dashv \Gamma_1 \quad \Gamma, -0\, x : A \mid \Sigma \vdash_i e \Rightarrow B \dashv \Gamma_2, -r'\, x : A \\ r^i_A - r' \sqsubseteq 0}{\Gamma \mid \Sigma, e' \vdash_i \lambda x.\, e \Rightarrow r\, A \rightarrow B \dashv \Gamma_2 - r\, (\Gamma - \Gamma_1)}$$

Box

$$\frac{\Gamma \mid \Box \vdash_i e_1 \Rightarrow\ !_r\, A \dashv \Gamma_1 \quad \Gamma_1, r^i_A\, x : A \mid \Sigma \vdash_i e_2 \Rightarrow B \dashv \Gamma_2, 0\, x : A}{\Gamma \mid \Sigma \vdash_i \texttt{let box}\ x = e_1\ \texttt{in}\ e_2 \Rightarrow B \dashv \Gamma_2}$$

$$\frac{\Gamma / r = (\Gamma_1, \Gamma_2) \quad \Gamma_1 \mid A \vdash_i e \Rightarrow A' \dashv \Gamma_3}{\Gamma \mid\ !_r\, A \vdash_i \texttt{box}\ e \Rightarrow\ !_r\, A' \dashv \Gamma_2 + r\,\Gamma_3}$$

$$\frac{\Gamma / r = (\Gamma_1, \Gamma_2) \quad \Gamma_1 \mid \Box \vdash_i e \Rightarrow A \dashv \Gamma_3}{\Gamma \mid \Box \vdash_i \texttt{box}_r\ e \Rightarrow\ !_r\, A \dashv \Gamma_2 + r\,\Gamma_3}$$

Unit

$$\frac{\Gamma \mid \square \vdash_i e_1 \Rightarrow \texttt{Unit} \dashv \Gamma_1 \quad \Gamma \mid \Sigma \vdash_i e_2 \Rightarrow A \dashv \Gamma_2}{\Gamma \mid \Sigma \vdash_i \texttt{let unit} = e_1 \texttt{ in } e_2 \Rightarrow A \dashv \Gamma_2}$$

$$\frac{\Gamma \vdash \Sigma \approx \texttt{Unit} \dashv \Gamma'}{\Gamma \mid \Sigma \vdash_i \texttt{unit} \Rightarrow \texttt{Unit} \dashv \Gamma'}$$

Empty

$$\frac{\Gamma \mid \texttt{Empty} \vdash_i e \Rightarrow \texttt{Empty} \dashv \Gamma'}{\Gamma \mid A \vdash_i \texttt{Empty-elim}\ e \Rightarrow A \dashv \Gamma'}$$

$$\frac{\begin{array}{c}\Gamma \mid \Sigma \vdash_i \texttt{Empty-elim}\ e \Rightarrow r\,A \to B \dashv \Gamma_1 \\ \Gamma_1/ = (\Gamma_2, \Gamma_3) \quad \Gamma_2 \mid A \vdash_i e' \Rightarrow A' \dashv \Gamma_4\end{array}}{\Gamma \mid \Sigma, e' \vdash_i \texttt{Empty-elim}\ e \Rightarrow r\,A \to B \dashv \Gamma_3 + r\,\Gamma_4}$$

Product

$$\frac{\Gamma \mid \Box \vdash_i e_1 \Rightarrow A \otimes B \dashv \Gamma_1 \qquad \Gamma_1, 1^i_A\, x : A, 1^i_B\, y : B \mid \Sigma \vdash_i e_2 \Rightarrow C \dashv \Gamma_2, 0\, x : A, 0\, y : B}{\Gamma \mid \Sigma \vdash_i \mathtt{let}\ (x, y) = e_1\ \mathtt{in}\ e_2 \Rightarrow C \dashv \Gamma_2}$$

$$\frac{\Gamma \mid A \vdash_i e_1 \Rightarrow A' \dashv \Gamma_1 \qquad \Gamma_1 \mid B \vdash_i e_2 \Rightarrow B' \dashv \Gamma_2}{\Gamma \mid A \otimes B \vdash_i (e_1, e_2) \Rightarrow A' \otimes B' \dashv \Gamma_2}$$

$$\frac{\Gamma \mid \Box \vdash_i e_1 \Rightarrow A \dashv \Gamma_1 \qquad \Gamma_1 \mid \Box \vdash_i e_2 \Rightarrow B \dashv \Gamma_2}{\Gamma \mid \Box \vdash_i (e_1, e_2) \Rightarrow A \otimes B \dashv \Gamma_2}$$

Sum

$$\frac{\Gamma \mid \square \vdash_i e \Rightarrow A \oplus B \dashv \Gamma_1 \qquad \begin{array}{c} \Gamma_1 \mid \Sigma \vdash_i f \Rightarrow 1\, A \to C \dashv \Gamma_2 \\ \Gamma_1 \mid \Sigma \vdash_i g \Rightarrow 1\, B \to C \dashv \Gamma_3 \end{array}}{\Gamma \mid \Sigma \vdash_i \mathsf{case}\ e\ \mathsf{of}\ (f, g) \Rightarrow C \dashv \Gamma_2 \sqcup \Gamma_3}$$

$$\frac{\Gamma \mid A \vdash_i e \Rightarrow A' \dashv \Gamma'}{\Gamma \mid A \oplus B \vdash_i \mathtt{inl}\ e \Rightarrow A' \oplus B \dashv \Gamma'}$$

$$\frac{\Gamma \mid B \vdash_i e \Rightarrow B' \dashv \Gamma'}{\Gamma \mid A \oplus B \vdash_i \mathtt{inr}\ e \Rightarrow A \oplus B' \dashv \Gamma'}$$