T-Axi (declarative)

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Type conversion

$$\frac{\Gamma\ \mathtt{ctx}_{\mathtt{nc}}}{\Gamma \vdash \mathtt{Unit} \equiv \mathtt{Unit}\ \mathtt{type}} \qquad \frac{\Gamma\ \mathtt{ctx}_{\mathtt{nc}}}{\Gamma \vdash \mathtt{Empty} \equiv \mathtt{Empty}\ \mathtt{type}}$$

$$\frac{\Gamma \vdash A_1 \equiv A_2\ \mathtt{type} \quad \Gamma \vdash B_1 \equiv B_2\ \mathtt{type}}{\Gamma \vdash r\, A_1 \to B_1 \equiv r\, A_2 \to B_2\ \mathtt{type}}$$

$$\frac{\Gamma \vdash A_1 \equiv A_2\ \mathtt{type}}{\Gamma \vdash \mathtt{!}_r\, A_1 \equiv \mathtt{!}_r\, A_2\ \mathtt{type}}$$

$$\frac{\Gamma \vdash A_1 \equiv A_2\ \mathtt{type} \quad \Gamma \vdash B_1 \equiv B_2\ \mathtt{type}}{\Gamma \vdash A_1 \otimes B_1 \equiv A_2 \otimes B_2\ \mathtt{type}}$$

$$\frac{\Gamma \vdash A_1 \equiv A_2\ \mathtt{type} \quad \Gamma \vdash B_1 \equiv B_2\ \mathtt{type}}{\Gamma \vdash A_1 \oplus B_1 \equiv A_2 \oplus B_2\ \mathtt{type}}$$

Type conversion – properties

The rules above are the complete definition of type conversion. For now the context Γ is not used, but soon it will. We don't need to take any closures – type conversion already is an equivalence relation.

- $\Gamma \vdash A \equiv A\ \mathtt{type}$
- If $\Gamma \vdash A \equiv B\ \mathtt{type}$ then $\Gamma \vdash B \equiv A\ \mathtt{type}$
- If $\Gamma \vdash A \equiv B\ \mathtt{type}$ and $\Gamma \vdash B \equiv C\ \mathtt{type}$ then $\Gamma \vdash A \equiv C\ \mathtt{type}$

Proposition conversion

$$\frac{\Gamma\ \mathtt{ctx_{nc}}}{\Gamma \vdash \top \equiv \top\ \mathtt{prop}} \qquad \frac{\Gamma\ \mathtt{ctx_{nc}}}{\Gamma \vdash \bot \equiv \bot\ \mathtt{prop}}$$

$$\frac{\Gamma \vdash P_1 \equiv P_2\ \mathtt{prop} \quad \Gamma \vdash Q_1 \equiv Q_2\ \mathtt{prop}}{\Gamma \vdash P_1 \Rightarrow Q_1 \equiv P_2 \Rightarrow Q_2\ \mathtt{prop}}$$

$$\frac{\Gamma \vdash P_1 \equiv P_2\ \mathtt{prop} \quad \Gamma \vdash Q_1 \equiv Q_2\ \mathtt{prop}}{\Gamma \vdash P_1 \wedge Q_1 \equiv P_2 \wedge Q_2\ \mathtt{prop}}$$

$$\frac{\Gamma \vdash P_1 \equiv P_2\ \mathtt{prop} \quad \Gamma \vdash Q_1 \equiv Q_2\ \mathtt{prop}}{\Gamma \vdash P_1 \vee Q_1 \equiv P_2 \vee Q_2\ \mathtt{prop}}$$

Proposition conversion

$$\frac{\Gamma \vdash A_1 \equiv A_2 \ \mathtt{type} \quad \Gamma, x : A_1 \vdash P_1 \equiv P_2 \ \mathtt{prop}}{\Gamma \vdash \forall x : A_1.\, P_1 \equiv \forall x : A_2.\, P_2 \ \mathtt{prop}}$$

$$\frac{\Gamma \vdash A_1 \equiv A_2 \ \mathtt{type} \quad \Gamma, x : A_1 \vdash P_1 \equiv P_2 \ \mathtt{prop}}{\Gamma \vdash \exists x : A_1.\, P_1 \equiv \exists x : A_2.\, P_2 \ \mathtt{prop}}$$

$$\frac{\Gamma \vdash A_1 \equiv A_2 \ \mathtt{type} \quad \Gamma \vdash e_1 \equiv e_2 : A_1 \quad \Gamma \vdash e_1' \equiv e_2' : A_1}{\Gamma \vdash e_1 =_{A_1} e_1' \equiv e_2 =_{A_2} e_2' \ \mathtt{prop}}$$

Note that we don't worry about α-conversion and assume the variable is the same on both sides.

Term conversion – sanity checks

We'll set up the term conversion judgement so that if $\Gamma \vdash a_1 \equiv a_2 : A$ then $\Gamma \vdash_{\mathtt{nc}} a_1 : A$ and $\Gamma \vdash_{\mathtt{nc}} a_2 : A$.

Term conversion – Empty

$$\frac{\Gamma \vdash_{\mathtt{nc}} e_1 : \mathtt{Empty} \quad \Gamma \vdash_{\mathtt{nc}} e_2 : \mathtt{Empty}}{\Gamma \vdash e_1 \equiv e_2 : \mathtt{Empty}}$$

$$\frac{|\Gamma| \vdash A \ \mathtt{type} \quad \Gamma \vdash e_1 \equiv e_2 : \mathtt{Empty}}{\Gamma \vdash \mathtt{Empty\text{-}elim}\ e_1 \equiv \mathtt{Empty\text{-}elim}\ e_2 : A}$$

Term conversion – Unit

$$\frac{\Gamma \vdash_{\mathtt{nc}} a : A}{\Gamma \vdash \mathtt{let\ unit} = \mathtt{unit\ in}\ a \equiv a : A}$$

$$\frac{\Gamma\ \mathtt{ctx}_{\mathtt{nc}} \qquad \Gamma \sqsubseteq 0\,\Gamma}{\Gamma \vdash \mathtt{unit} \equiv \mathtt{unit} : \mathtt{Unit}}$$

$$\frac{\Gamma \vdash_{\mathtt{nc}} u_1 : \mathtt{Unit} \qquad \Gamma \vdash_{\mathtt{nc}} u_2 : \mathtt{Unit}}{\Gamma \vdash u_1 \equiv u_2 : \mathtt{Unit}}$$

$$\frac{\Gamma \sqsubseteq \Gamma_1 + \Gamma_2 \qquad \Gamma_1 \vdash u_1 \equiv u_2 : \mathtt{Unit} \qquad \Gamma_2 \vdash a_1 \equiv a_2 : A}{\Gamma \vdash \mathtt{let\ unit} = u_1\ \mathtt{in}\ a_1 \equiv \mathtt{let\ unit} = u_2\ \mathtt{in}\ a_2 : A}$$

Term conversion – functions

$$\frac{\Gamma \sqsubseteq \Gamma_1 + r\,\Gamma_2 \quad \Gamma_1, r_A^i\, x : A \vdash_{\mathtt{nc}} b : B \quad \Gamma_2 \vdash_{\mathtt{nc}} a : A}{\Gamma \vdash (\lambda_r\, x : A.\, b)\ a \equiv b\,[x := a] : B}$$

$$\frac{\Gamma, r_A^i\, x : A \vdash f\ x \equiv g\ x : B}{\Gamma \vdash f \equiv g : r\,A \to B}$$

$$\frac{\Gamma \sqsubseteq \Gamma_1 + r\,\Gamma_2 \quad \Gamma_1 \vdash f_1 \equiv f_2 : r\,A \to B \quad \Gamma_2 \vdash a_1 \equiv a_2 : A}{\Gamma \vdash f_1\ a_1 \equiv f_2\ a_2 : B}$$

Term conversion – box

$$\frac{\Gamma \sqsubseteq r\,\Gamma_1 + \Gamma_2 \quad \Gamma_1 \vdash_{\mathtt{nc}} a : A \quad \Gamma_2, r^i_A\, x : A \vdash_{\mathtt{nc}} b : B}{\Gamma \vdash \mathtt{let}\ \ \mathtt{box}\ x = \mathtt{box}_r\ a\ \mathtt{in}\ b \equiv b\,[x := a] : B}$$

$$\frac{\Gamma \sqsubseteq r\,\Gamma' \quad \Gamma' \vdash a_1 \equiv a_2 : A}{\Gamma \vdash \mathtt{box}\ a_1 \equiv \mathtt{box}\ a_2 : !_r\, A}$$

$$\frac{\Gamma \sqsubseteq \Gamma_1 + \Gamma_2 \quad \Gamma_1 \vdash a_1 \equiv a_2 : !_r\, A \quad \Gamma_2, r^i_{A_1}\, x : A_1 \vdash b_1 \equiv b_2 : B}{\Gamma \vdash \mathtt{let}\ \ \mathtt{box}\ x = a_1\ \mathtt{in}\ b_1 \equiv \mathtt{let}\ \ \mathtt{box}\ x = a_2\ \mathtt{in}\ b_2 : B}$$

Term conversion – product

$$\frac{\Gamma \sqsubseteq \Gamma_1 + \Gamma_2 \quad \Gamma_1 \vdash a_1 \equiv a_2 : A \quad \Gamma_2 \vdash b_1 \equiv b_2 : B}{\Gamma \vdash (a_1, b_1) \equiv (a_2, b_2) : A \otimes B}$$

$$\frac{\Gamma \sqsubseteq \Gamma_1 + \Gamma_2 \quad \Gamma_1 \vdash e_1 \equiv e_2 : A \otimes B \quad \Gamma_2, 1^i_A\, x : A, 1^i_B\, y : B \vdash c_1 \equiv c_2 : C}{\Gamma \vdash \mathtt{let}\ (x, y) = e_1\ \mathtt{in}\ c_1 \equiv \mathtt{let}\ (x, y) = e_2\ \mathtt{in}\ c_2 : C}$$

Term conversion – sum

$$\frac{\Gamma \vdash a_1 \equiv a_2 : A \quad |\Gamma| \vdash B \ \mathtt{type}}{\Gamma \vdash \mathtt{inl}\ a_1 \equiv \mathtt{inl}\ a_2 : A \oplus B} \qquad \frac{|\Gamma| \vdash A \ \mathtt{type} \quad \Gamma \vdash b_1 \equiv b_2 : B}{\Gamma \vdash \mathtt{inr}\ b_1 \equiv \mathtt{inr}\ b_2 : A \oplus B}$$

$$\frac{\Gamma \sqsubseteq \Gamma_1 + \Gamma_2 \quad \Gamma_1 \vdash e_1 \equiv e_2 : A \oplus B \quad \begin{array}{c} \Gamma_2 \vdash f_1 \equiv f_2 : 1\, A \to C \\ \Gamma_2 \vdash g_1 \equiv g_2 : 1\, B \to C \end{array}}{\Gamma \vdash \mathtt{case}\ e_1\ \mathtt{of}\ (f_1, g_1) \equiv \mathtt{case}\ e_2\ \mathtt{of}\ (f_2, g_2) : C}$$