Assignment#2

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Question 2.1

The process can be written as:

$$(1 - 0.8B)X_t = (1 + 0.8B - 0.5B^2)\epsilon_t$$

##1

Stationary:

$$\phi(z^{-1}) = 1 - 0.8z^{-1} = 0$$

The roots of this equation :

$$z - 0.8 = 0$$
$$z = 0.8$$

The z value is lower than a unit circle therifere the process is *staionary*.

Invertibility

$$\theta(z^{-1}) = 1 + 0.8z^{-1} - 0.5z^{-2} = 0$$

The roots of this equation :

$$z^2 + 0.8z - 0.5 = 0$$
$$z_1 = 0.41$$
$$z_2 = -1.21$$

The z_2 does no lie in the unit circle, so the process is not invertible.

##2

The mean of ARMA model (p,q) can be written as:

$$E[X_t] = 0.8E[X_{t-1}] + E[\epsilon_t] + 0.8E[\epsilon_{t-1}] - 0.5E[\epsilon_{t-2}]$$

Since, staionary, the$E[X_t] = E[X_{t-1}]$ and the white noise, $\mu = 0$

$$E[X_t] = 0.8E[X_t] + 0$$
$$E[X_t] = 0$$

The autocovariance for the ARMA(p,q) process, can be written as:

$$\gamma_{\epsilon X}(0) = \sigma^2 = 0.16$$

$$\gamma_{\epsilon X}(1) = (\theta_1 - \phi_1)\sigma^2 = (0.8 + 0.8) * 0.16 = 0.256$$
$$\gamma_{\epsilon X}(2) = -\phi_1\gamma_{\epsilon X}(1) + \theta_2\sigma^2 = +0.8 * 0.256 - 0.5 * 0.16 = -0.12$$

Now based on Eq (5-101):

$$\gamma(0) + \phi_1\gamma(1) = \gamma_{\epsilon X}(0) + \theta_1\gamma_{\epsilon X}(1) + \theta_2\gamma_{\epsilon X}(2)$$

$$\gamma(1) + \phi_1\gamma(0) = \theta_1\gamma_{\epsilon X}(0) + \theta_2\gamma_{\epsilon X}(1)$$

Threfore,

$$\gamma(1) = \frac{\gamma(0)(\theta_1 - \phi_1)}{1 - \theta_2}$$

$$\gamma(0) = \frac{\gamma_{\epsilon X}(0) + \theta_1\gamma_{\epsilon X}(1) + \theta_2\gamma_{\epsilon X}(2)}{1 + \frac{\theta_1 - \phi_1}{1 - \theta_2}\phi_1}$$

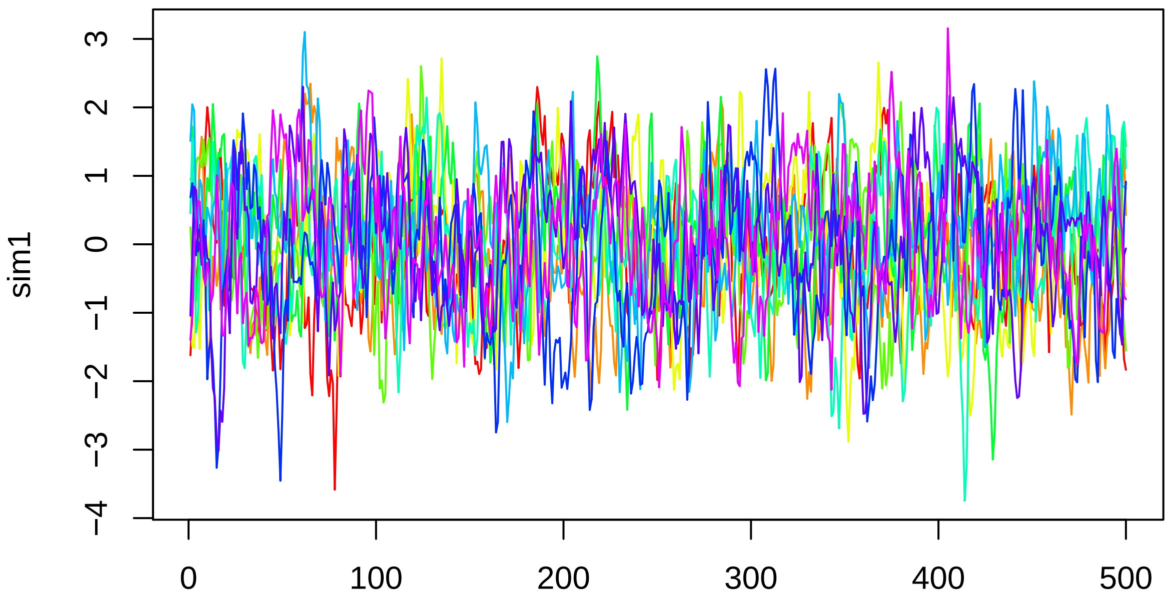
$$\gamma(0) = \frac{0.16 + 0.8 * 0.256 + 0.5 * 0.28}{1 + \frac{0.8+0.8}{1+0.5} * -0.8} = \frac{0.50}{0.14} = 3.4$$
$$\gamma(1) = \frac{\gamma(0) * (\theta 1 - \phi 1)}{1 - \theta 2} = \frac{3.4 * (0.8 + 0.8)}{1 + 0.5} = 3.63$$

The Varinace is:

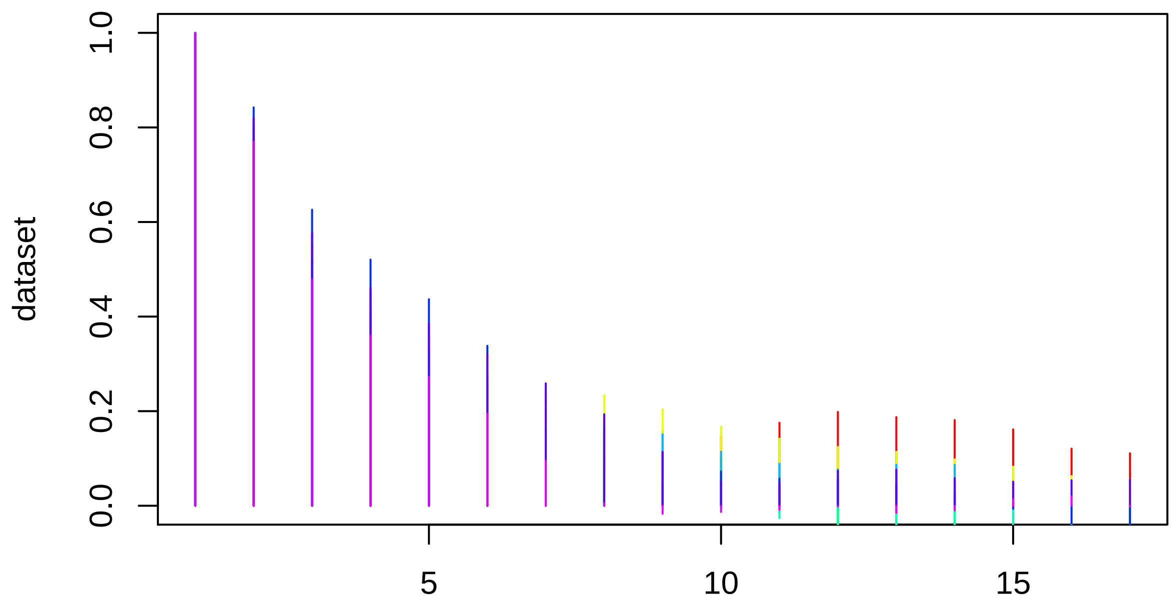
$$\sigma^2 = \gamma(0) = 3.4$$

3

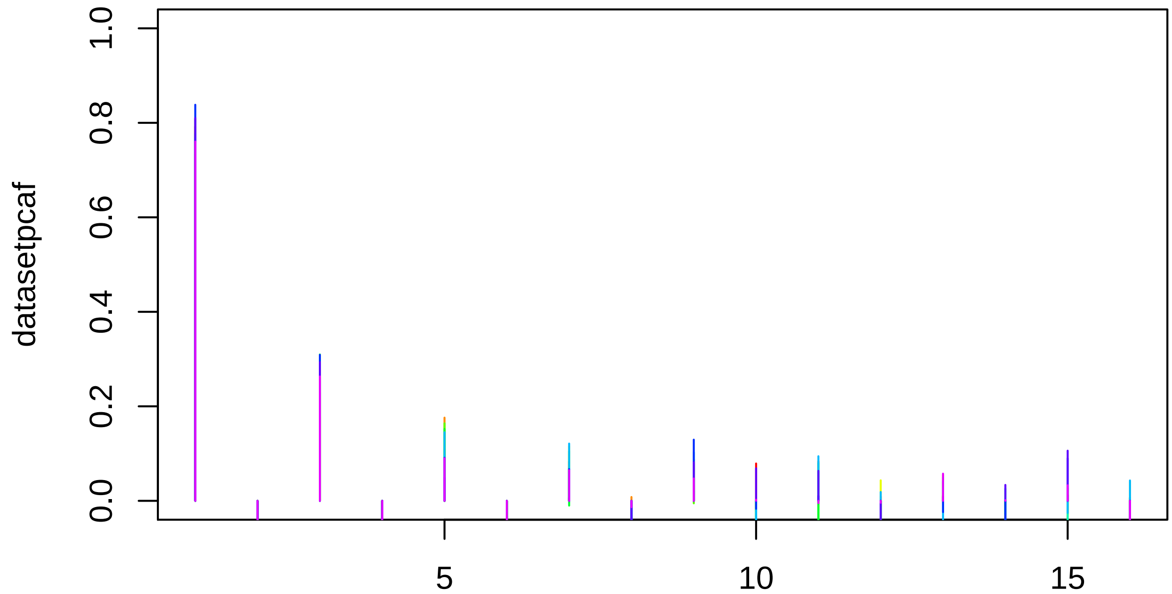
```
sim1 <- replicate(10,arima.sim(model = list(ar=c(0.8),ma=c(0.8,-0.5), order=c(1,0,2)), n = 500 ,sd=0.4))
matplot(sim1, lty=1, type="l", col=rainbow(11))
```



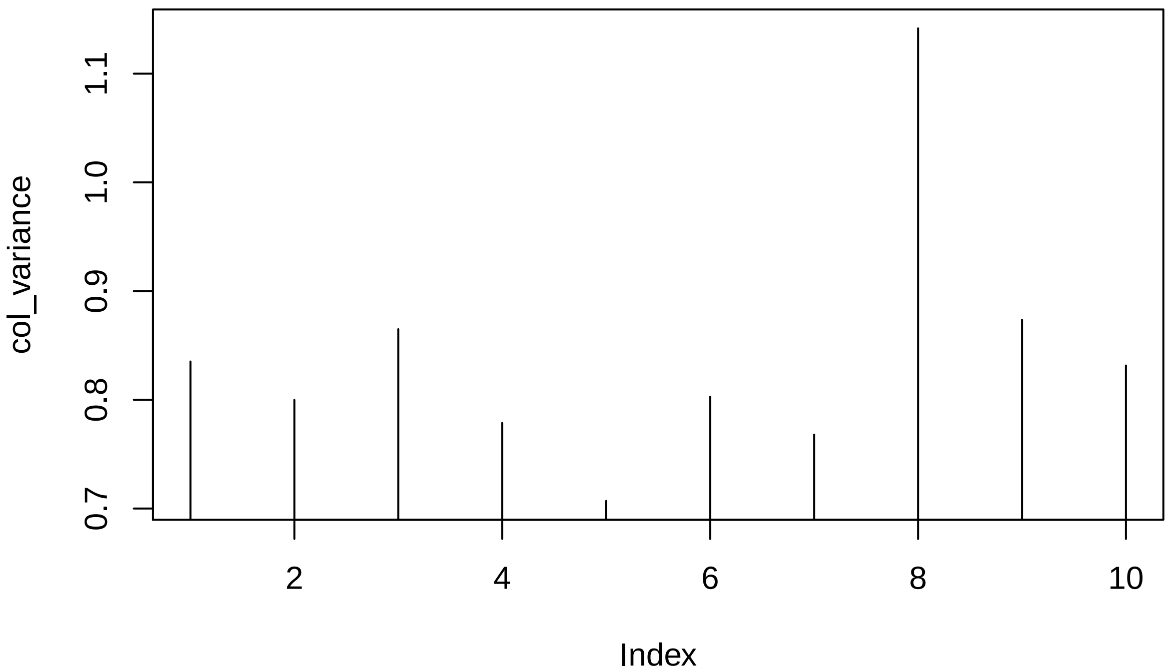

```
sim1acf <- acf(sim1, plot = F)
simacf <- sim1acf$acf
dataset <- cbind(simacf[,1,1],simacf[,2,2],simacf[,3,3],simacf[,4,4],simacf[,5,5],simacf[,6,6],simacf[,7
matplot(dataset, lty=1, type="h", col=rainbow(11), ylim = c(0,1))
```




```
sim1pacf <- pacf(sim1, plot = F)
simpacf <- sim1pacf$acf
datasetpcaf <- cbind(simpacf[,1,1],simpacf[,2,2],simpacf[,3,3],simpacf[,4,4],simpacf[,5,5],simpacf[,6,6
matplot(datasetpcaf, lty=1, type="h", col=rainbow(11), ylim = c(0,1))
```




```
col_variance <- apply(sim1,2,var)
plot(col_variance, type = 'h')
```



Question 2.2

Considering the $Y - \mu = Z_t$, we have:

$$(1 - 1.04B + 0.2B^2)(1 - 0.86B^4)(Z_t) = (1 - 0.42B^4)\epsilon$$

Multyplying the polonomyal terms:

$$(1 - 0.86B^4 - 1.04B + 0.89B^5 + 0.2B^2 - 0.17B^6)Z_t = (1 - 0.42B^4)\epsilon_t$$

Reordering the terms:

$$\varphi(B) = 1 - 1.04B + 0.2B^2 - 0.86B^4 + 0.89B^5 - 0.17B^6$$

Using the conditional term, we have:

$$Y_{t+1|t} = 1.04Z_t - 0.2Z_{t-1} + 0.86Z_{t-3} - 0.89Z_{t-4} + 0.17Z_{t-5}$$

```
data <- read.delim('A2_sales.txt', sep = "")
data['Z_t'] <- data['Sales']-2070
data
```

```
##    Quarter Sales  Z_t
## 1   2014K1  1784 -286
## 2   2014K2  2066   -4
## 3   2014K3  2221  151
## 4   2014K4  2306  236
## 5   2015K1  2748  678
## 6   2015K2  2910  840
## 7   2015K3  2674  604
## 8   2015K4  2253  183
## 9   2016K1  2426  356
## 10  2016K2  2747  677
## 11  2016K3  2540  470
## 12  2016K4  2275  205
## 13  2017K1  2652  582
## 14  2017K2  2623  553
## 15  2017K3  2532  462
## 16  2017K4  2377  307
## 17  2018K1  2351  281
## 18  2018K2  2385  315
## 19  2018K3  2259  189
## 20  2018K4  2088   18
```

$$Z_{2019Q1|2018K4} = 1.04 * 18 - 0.2 * 189$$
$$+0.86 * 281 - 0.89 * 307 + 0.17 * 462$$

$$= 28$$

Therefore, the

$$Y_t = 2070 + 28 = 2098$$

95 % prediction interval is:

$$2098(+)(-)1.96 * 200 = [1706, 2490]$$

$$Y_{t+1|t} = 1.04Z_{t+1|t} - 0.2Z_t + 0.86Z_{t-2} - 0.89Z_{t-3} + 0.17Z_{t-4}$$

$$Z_{2019Q2|2019Q1} = 1.04 * 28 - 0.2 * 18$$
$$+0.86 * 315 - 0.89 * 281 + 0.17 * 307$$

$$= 99$$

Therefore, the

$$Y_t = 2070 + 99 = 2169$$

95 % prediction interval is:

$$2169(+)(-)\sqrt(1 + 1.04^2) * 1.96 * 200 =$$

$$[2169 - 565, 2169 + 565]$$

$$[1604, 2734]$$

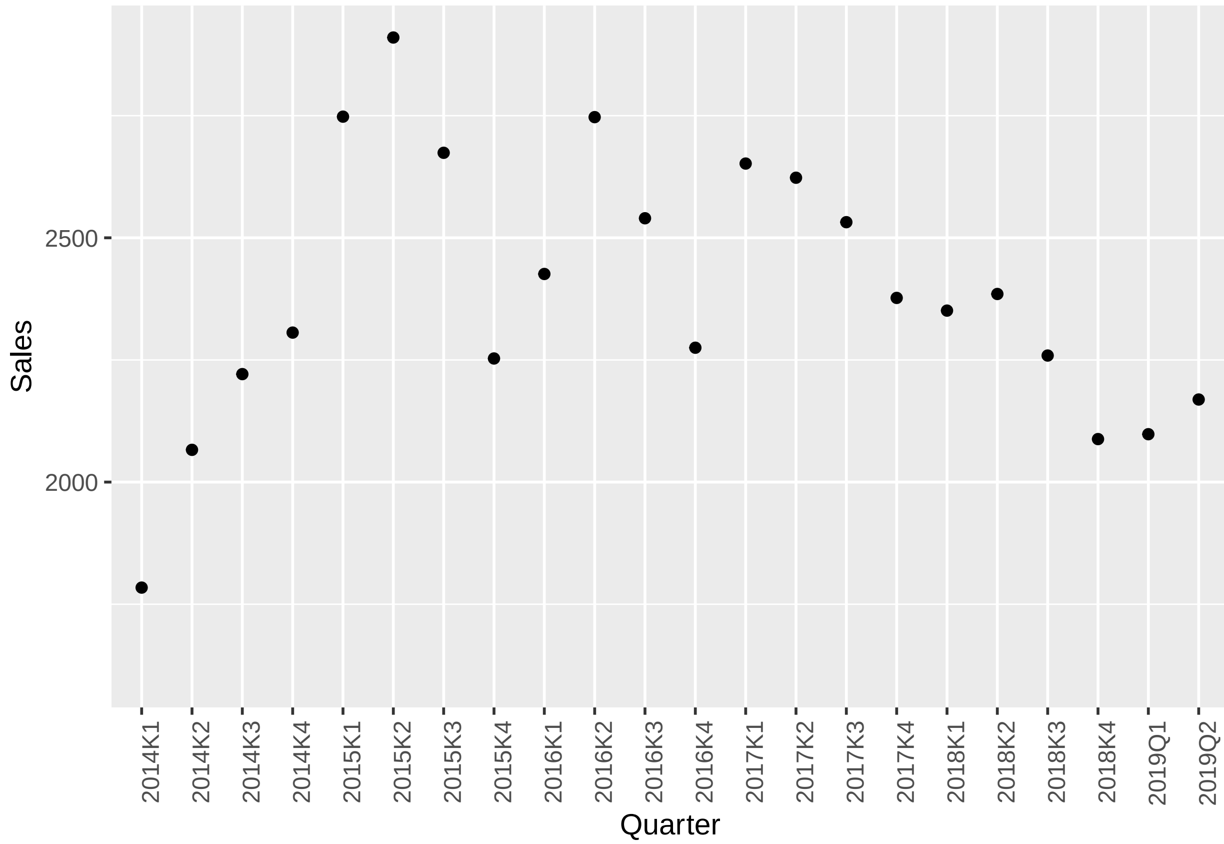
```r
prediction <- data.frame(Quarter=c("2019Q1","2019Q2"), Sales = c(2098,2169),Z_t=c(28,99))
past_prediction <- rbind(data, prediction)
past_prediction[,'Quartern'] <- as.character(past_prediction$Quarter)
newxaxis <- as.character(past_prediction$Quarter)
```

```r
vardatf <- data.frame(Quarter = c("2019Q1","2019Q2"), Sales=c(1706,1604))
past_prediction[21,'lower'] <- 2000
past_prediction[21,'Upper'] <- 2100
past_prediction[22,'lower'] <- 2100
past_prediction[22,'Upper'] <- 2300
library(tidyverse)
```

```
## -- Attaching packages --------------------------------------------------------------- tidyverse 1.2.1

## v ggplot2 3.2.1     v purrr   0.3.2
## v tibble  2.1.3     v dplyr   0.8.3
## v tidyr   0.8.3     v stringr 1.4.0
## v readr   1.3.1     v forcats 0.4.0

## -- Conflicts ------------------------------------------------------------------ tidyverse_conflicts()
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```r
gg <- ggplot(past_prediction, aes(Quarter,Sales))
gg <- gg+geom_point()
gg <- gg+geom_line(data=vardatf,aes(x=Quarter,y=Sales))
gg <- gg + theme(axis.text.x = element_text(angle = 90, hjust = 1))
gg
```

```
## geom_path: Each group consists of only one observation. Do you need to
## adjust the group aesthetic?
```



Question 2.3

Roots:

Calculation of the roots of the $\phi(Z^{-1})$:

For the first value of the $\phi 2$:

```
polyroot(rev(c(1,-1.5,0.52)))
```

```
## [1] 0.5438447-0i 0.9561553+0i
```

For the second value of the $\phi 2$:

```
polyroot(rev(c(1,-1.5,0.98)))
```

```
## [1] 0.75+0.6461424i 0.75-0.6461424i
```

Histogram

```r
estphi <- function(phi2,sd) {
  phi2 <- phi2
  sd <- sd
  estimatephi2 <- rep(0,100)
  estimatephi1 <- rep(0,100)

  for (i in 1:100){
    sim1 <- arima.sim(model = list(ar=c(1.5,phi2), order=c(2,0,0)), n = 300, sd=sd)
    arima200 <- arima(sim1, order = c(2,0,0)) # May give a warning due to unit root
    estimatephi2[i] <- arima200$coef[2]
    estimatephi1[i] <- arima200$coef[1]
  }
  estph12 <- data.frame(estimatephi1,estimatephi2)
  return(estph12)
}

par(mfrow=c(2,2))
Hist_Proces1 <- estphi(-0.52,0.1)
hist(Hist_Proces1$estimatephi2, breaks = 10, main = 'ph2 =0.52, sd=0.1')
abline(v=quantile(Hist_Proces1$estimatephi2,0.95))
var(Hist_Proces1$estimatephi2)
```

```
## [1] 0.001841179
```

```r
mtext("0.002",4)


Hist_Proces2 <- estphi(-0.52,5)
hist(Hist_Proces2$estimatephi2, breaks = 10,main='ph2 =0.52, sd=5')
abline(v=quantile(Hist_Proces2$estimatephi2,0.95))
var(Hist_Proces2$estimatephi2)
```

```
## [1] 0.002438434
```

```r
mtext("0.002",4)



Hist_Proces3 <- estphi(-0.98,0.1)
hist(Hist_Proces3$estimatephi2, breaks = 10,main='ph2 =0.98, sd=0.1')
abline(v=quantile(Hist_Proces3$estimatephi2,0.95))
var(Hist_Proces3$estimatephi2)
```

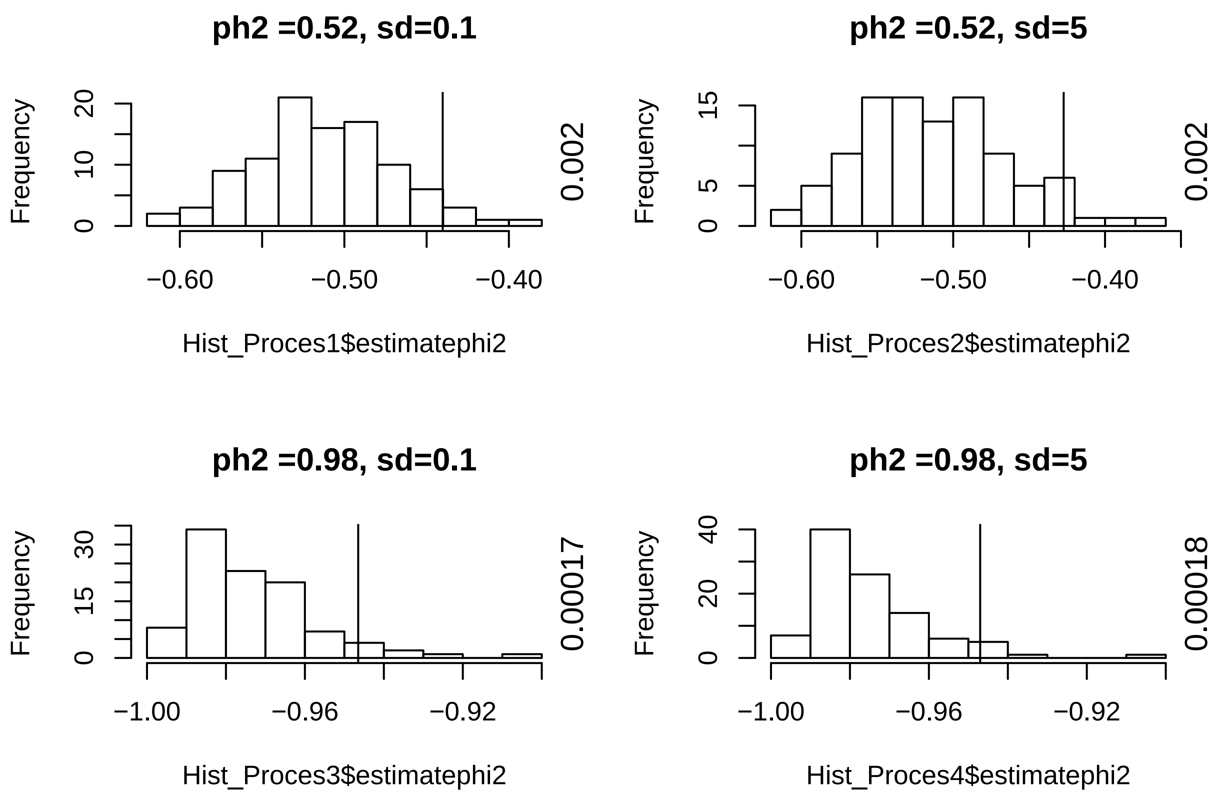
```
## [1] 0.0002389394
```

```r
mtext("0.00017",4)



Hist_Proces4 <- estphi(-0.98,5)
hist(Hist_Proces4$estimatephi2, breaks = 10,main='ph2 =0.98, sd=5')
```

```r
abline(v=quantile(Hist_Proces4$estimatephi2,0.95))
var(Hist_Proces4$estimatephi2)
```

```
## [1] 0.0002034888
```

```r
mtext("0.00018",4)
```



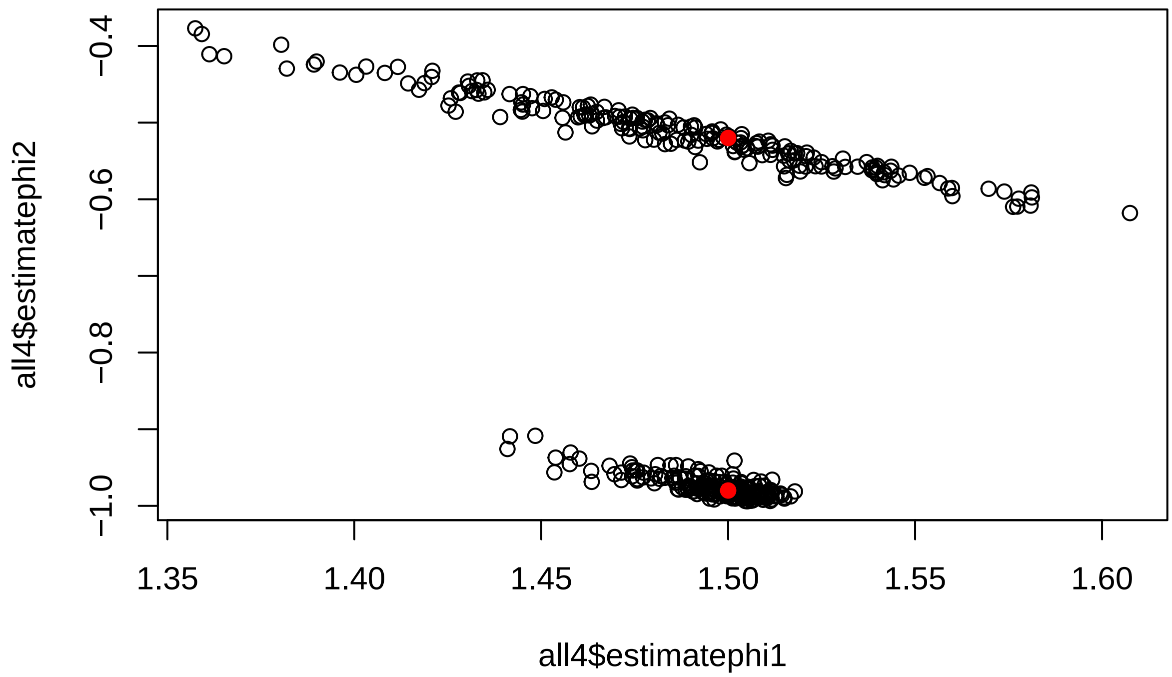
As we can see from the above plot, change in $\phi 2$:

dramatically change the variance of the paramter estimation, as we can see on the two figure in the first column, the chnage of the $\phi 2$ decreased the variance in theorder of ten times.

On the other hand:

We can see that even change in the σ^2 has a negligible effect on the estimate of variance of estimation of the parameter.

```r
all4 <- rbind(Hist_Proces1,Hist_Proces2,Hist_Proces3,Hist_Proces4)
plot(all4$estimatephi1,all4$estimatephi2)
points(1.5,-0.52, col='red', cex=1,pch=19)
points(1.5,-0.98, col='red', cex=1,pch=19)
```



We see the declining trend. So, as the estimation of ϕ_1 increases, the estimate of the $\phi 2$ decrrases. Threfore, the trend of these two parameters works in the opposite way.