**02417: Time Series Analysis**
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# Assignment 2: ARMA Processes and Seasonal Processes

NOTE that the model parametrizations in the assignment and in your favorite software package may not be identical. You will study how the choice of coefficients (of the operator polynomials in ARMA processes) affects the structure of the process, through simulated data and empirical autocorrelation functions.

**Question 2.1: Stability** Let the process $X_t$ be given by

$$X_t - 0.8X_{t-1} = \epsilon_t + 0.8\epsilon_{t-1} - 0.5\epsilon_{t-2}$$

where $\epsilon_t$ is a white noise process with $\sigma = 0.4$.

1. Is the process stationary and invertible?
2. Investigate analytically the second order moment representation of the process.
3. Simulate 10 realisations with 200 observations from the process and plot them (Preferably in the same plot).
4. Estimate the ACF for each realisation and plot those (Again, preferably in the same plot). Comment on the results.
5. Repeat for the PACF of the same realisations.
6. Calculate the variance of each of the realisations.
7. Discuss and compare the analytical and numerical results.

**Question 2.2: Predicting the the number of sales of apartments** The quarterly number of sales of apartments in the capital region of Denmark has been modelled.

Based on historical data the following model has been identified:

$$(1 - 1.04B + 0.2B^2)(1 - 0.86B^4)(Y_t - \mu) = (1 - 0.42B^4)\epsilon_t$$

where $\epsilon_t$ is a white-noise process with variance $\sigma_\epsilon^2$. Based on six days of data, it is found that $\sigma_\epsilon^2 = 36963$. Furthermore, $\mu$ was estimated to 2070. The file `A2_sales.txt` contains the sales for the last five years of $Y_t$.

Predict the values of $Y_t$ corresponding to t = 2019Q1 and 2019Q2 (Two steps ahead), together with 95% prediction intervals for the predictions.

Plot the observations along with the two predictions and their 95% prediction intervals.

**Question 2.3:** Consider the following ARMA(2,0) process

$$\phi(B)X_t = X_t - 1.5X_{t-1} + \phi_2 X_{t-2} = \epsilon_t$$

Consider the four variations of the process with $\phi_2 \in \{0.52, 0.98\}$ and $\sigma^2 \in \{0.1^2, 5^2\}$. Simulate 300 observations of each of the four processes 100 times. Subsequently, estimate the model parameters based on the simulated sequences.

1. For both values of $\phi_2$ you should calculate the roots of $\phi(z^{-1}) = 0$
2. For each process, make a histogram plot of the estimates of parameter $\phi_2$ and indicate the 95% quantiles.
3. How do different values of $\phi_2$ affect the variance/distribution of the estimated $\phi_2$?
4. How do different values of $\sigma$ affect the variance/distribution of the estimated $\phi_2$?
5. Plot all the estimated pairs of parameters $(\phi_1, \phi_2)$ for the four variations. Comment on what you see and compare with the true values.