Homework 4

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Question 1

Instructions

The required packages are numpy and cv2. To run navigate to the functions folder and run the .ipynb file. If that does not work, you can also run main.py. That is what I ran it on, and it will run through each of the input images and then output the masked, and two isolated versions of each image within the data folder.

Implementation

My implementation uses the given read_data function to collect the LAB color values for each pixel. It is then converted to a numpy array then it n by d dimensions. n being the number of pixels in the image and d being 3 for each of the LAB values. I then initialize my weights, mean, and covariances with the kmeanInitialization() function. This function uses one iteration of kmeans to cluster each of the points. From there the weights are initialized by $\frac{n_k}{n}$ for each value of k where k is 2 for the number of objects. In this case background and subject. The mean is initialized by taking the average of the individual L, A, and B values for each cluster. The covariance is initialized the same way with the variance formula.

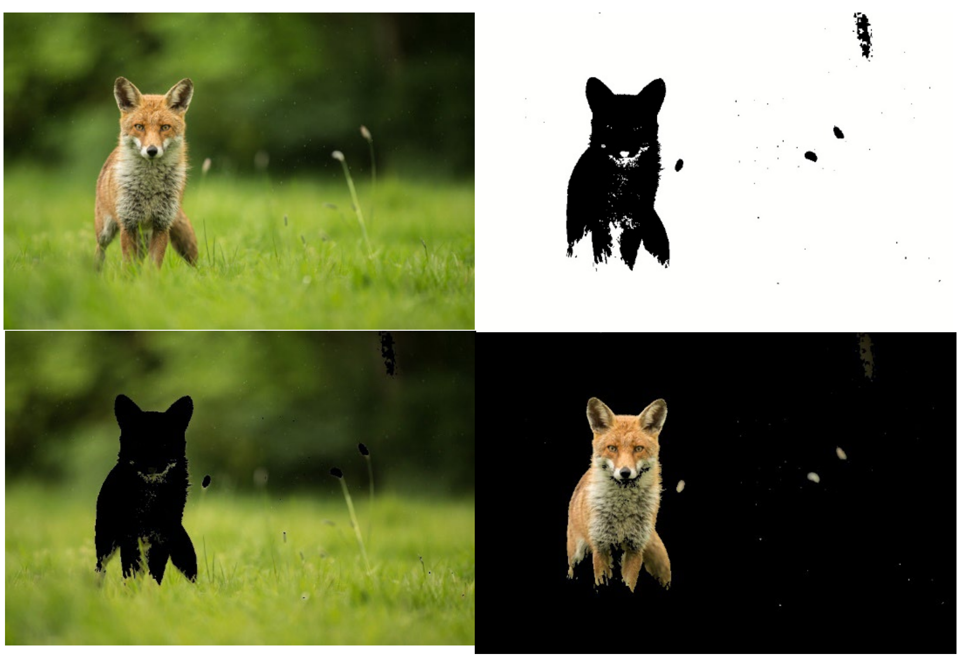
Once the initialization step is done it goes into a loop that checks for convergence. Then the *E-step* occurs. In the *E-step* the responsibilities are calculated using the weights multiplied by the probability distribution function. This is done column by column where each column is an attribute k. Then I divide each row but the sum of the rows values to get a soft classification where the classified one will approach 1 and the unclassified one with approach 0.

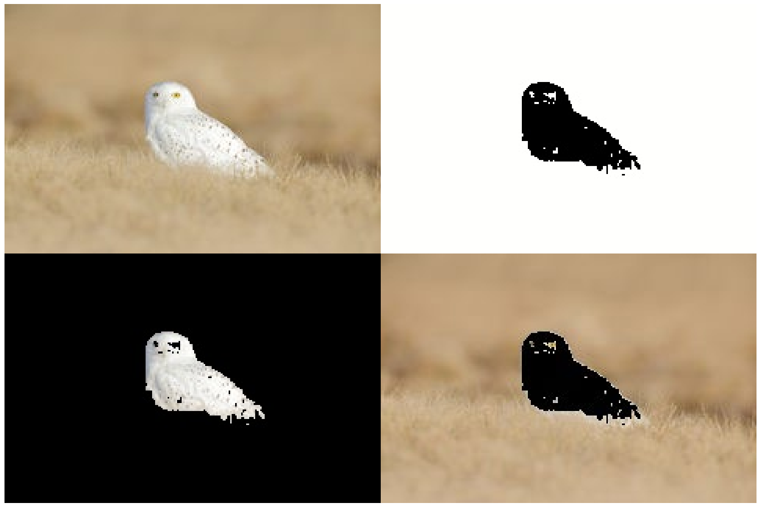
Moving into the *M-step*, I then need to update the respective means, covariances, weights and get the log likelihood to try and maximize that. This is done with the formulas in the slides and similar to that way they were initialized in the first place. Once the values are updated, I check for convergence by comparing the old likelihood with the new likelihood.

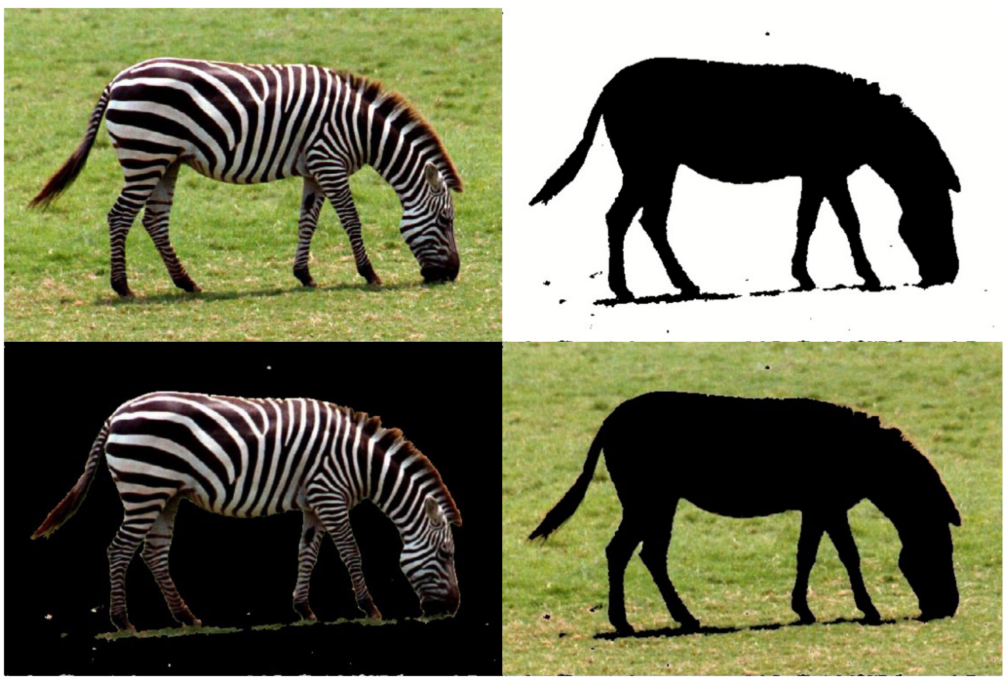
Once the loop is complete, I have a function the gets all of the labels and updates the image data with the according lab values for the write_data function. Those values are either their originals if is the unmasked part or black if its masked and then the masks as well. In the main function, data is collected and stored in jpg with the read_data function.

Overall, this is a more probabilistic way to approach k means. By using a probability distribution function and soft assignments it is less harsh that kmeans would be with its classification and allows for better generalization. Some updates and changes I had to make along the way were the kmean initialization was originally initialized with random centroids in the data. However, made it so the algorithm would get stuck in a local minimum that wasn't a good classification. I decided to hard code some initialization values that would lead to the right minimum and the results improved dramatically. The output images are below.

Output







$\boxed{2}$ → $p(x_1|x_5) = p(x_1)$ by Bayes Rule of Conditional Independence

$x_1 \perp x_5$ Table given by

$p(x_1) = .2$

X_1	X_5	$P(x_1 \mid x_5)$
T	T	.2
T	F	.2
F	T	.8
F	F	.8

→ $P(x_3|x_2) = P(x_3)$ by conditional Independence

$x_3 \perp x_2$

$P(x_3) = .2$

Table

X_3	X_2	$P(X_3 \mid X_2)$
T	T	.2
T	F	.2
F	T	.8
F	F	.8

→ $P(x_5) = P(x_5|x_2 x_3)\,p(x_2)\,p(x_3) + P(x_5|x_2' x_3)\,P(x_2')\,p(x_3) \ldots$
by law of total probability

$= (.44 \cdot .35 \cdot .2) + (.56 \cdot .01 \cdot .2) + (.44 \cdot .6 \cdot .8) + (.95 \cdot .56 \cdot .8)$

$= \boxed{.66872}$

$P(x_5') = 1 - p(x_5) = \boxed{.33128}$

X_5	$P(x_5)$
T	.6687
F	.3313

→ $P(x_4|x_3) = p(x_4)$ by conditional independence

$x_4 \perp x_3$

$p(x_4) = p(x_4|x_1 x_2)\,p(x_1)\,p(x_2) + P(x_4|x_1' x_2)\,p(x_1')\,p(x_2) \ldots$
by law of total probability

$= \underset{TT}{(.35 \cdot .2 \cdot .44)} + \underset{TF}{(.6 \cdot .2 \cdot .56)} + \underset{FT}{(.01 \cdot .8 \cdot .44)} + \underset{FF}{(.95 \cdot .8 \cdot .56)}$

$p(X_4|X_3)$ cont.

$$p(X_4|X_3) = \boxed{\begin{array}{l} p(x_4) = .52712 \\ p(x_4') = 1 - p(x_4) = .47288 \end{array}}$$

X_4	X_3	$P(X_4 \mid X_3)$
T	F	.527
T	F	.527
F	T	.473
F	F	.473

$$P(X_2|X_4) = \frac{p(X_4|X_2)\,p(X_2)}{P(X_4)} \quad \text{by Bayes Rule}$$

$p(x_4|x_2) = p(x_4|x_1 x_2) \cdot P(x_1) + p(x_4|x_1' x_2) \cdot p(x_1')$ by law of total probability

$= .35 \cdot .2 + .01 \cdot .8 = .078$

$p(x_4) = .52712$ from previous problem

$P(X_4') = .47288$

$P(x_4'|x_2') = .6 \cdot .2 + .95 \cdot .8 = .88$

$P(x_4'|x_2) = 1 - .078 = .922$

$P(x_4'|x_2') = 1 - .88 = .12$

X_2	X_4	$P(x_2|x_4)$
T	T	$\frac{.078 \cdot .44}{.52712} = .0651$
T	F	$\frac{.922 \cdot .44}{.47288} = .8579$
F	T	$\frac{.88 \cdot .56}{.52712} = .935$
F	F	$\frac{.12 \cdot .56}{.47288} = .142$