

Beam Mechanics

Wind & Loading

Wind profile (power law):

U(z) = U_ref * (z/z_ref)^epsilon

epsilon approx 0.1 - 0.3 (terrain dep.)
Distributed drag:

q(z) = 1/2 * rho * C_D(z) * D(z) * U(z)^2

Nacelle load:

q_n(z) = { F_L / h_n, z in [z_t, z_t + h_n] / 0, else

Shear & Moment

Differential relations:

dV/dz = -q(z), dM/dz = -V(z)

Integration:

V(z) = integral from z to H of q(xi) dxi, M(z) = integral from z to H of V(xi) dxi

Deflection

Curvature:

kappa(z) = M(z) / (EI(z))

Slope:

theta(z) = integral from 0 to z of kappa(xi) dxi

Deflection (BC: w(0) = 0, theta(0) = 0):

w(z) = integral from 0 to z of theta(xi) dxi

Integration method: trapezoidal (cumtrapz)

Stress

Bending stress (at base):

sigma_b = Mc / I

c = R_outer for tubes
Principal stresses (uniaxial):

sigma_1 = sigma_b, sigma_2 = 0, tau_max = sigma_b / 2

Directional factor:

sigma_case2 = sigma_case1 * cos(delta psi)

Max occurs at: z = 0 (tower base)

Aerodynamics

Basic Parameters

Tip-speed ratio (fundamental): lambda = omega * R / V_infinity (Typical lambda = 8-13)
Angular velocity: omega = 2 * pi * n / 60 [rad/s] (n in RPM)
Local TSR: lambda_r = lambda * r / R

Induction Factors

Axial induction: a = 1/3
Tangential induction:

a' = -1/2 + 1/2 * sqrt(1 + 4 / (lambda_r^2 * a * (1 - a)))

Flow Angles

Inflow angle:

phi = tan^-1((1 - a) / ((1 + a') * lambda_r))

Angle of attack: alpha = phi - (theta + beta) (theta = twist, beta = pitch)
Relative velocity: V_rel = sqrt((V_infinity * (1 - a))^2 + (omega * r * (1 + a'))^2)

Force Coefficients

Normal:

C_n = C_L * cos(phi) + C_D * sin(phi)

Tangential:

C_t = C_L * sin(phi) - C_D * cos(phi)

Elemental Loads

Thrust:

dT = 1/2 * rho * V_rel^2 * c * C_n

Torque:

dQ = 1/2 * rho * V_rel^2 * c * C_t * r

Power:

dP = dQ * omega

Total Loads

Integration (trapezoidal):

T = B * integral from r_h to R of T dr = B * trapz(r, dT)

P = B * integral from r_h to R of P dr = B * trapz(r, dP)

B = number of blades

Performance Coefficients

Power coefficient (key metric):

C_P = P / (1/2 * rho * A * V_infinity^3)

Thrust coefficient:

C_T = T / (1/2 * rho * A * V_infinity^2)

Swept area: A = pi * R^2 Bets limit: C_P,max = 0.5926 (theoretical max)
Power from C_P: P = C_P * (1/2 * rho * A * V_infinity^3)

Material Strength

Process: (1) Calculate max stress sigma_max, (2) Obtain material properties S_y, S_ut, (3) Compute n = strength/stress, (4) Check n >= 2.0

Static Failure

Max Normal Stress:

n_MNST = S_y / sigma_max

Max Shear Stress:

n_MSST = (S_y / 2) / (sigma_max / 2) = S_y / sigma_max

Distortion Energy:

n_DET = S_y / (sqrt(sigma_max^2 + 3 * tau_max^2))

Equivalent stress:

sigma_eq = sqrt(sigma^2 + 3 * tau^2)

For uniaxial: all yield n = S_y / sigma_max

Fatigue Analysis

Mean stress:

sigma_m = 1/2 * (sigma_max + sigma_min)

Alternating stress:

sigma_a = 1/2 * |sigma_max - sigma_min|

Base endurance:

S'_e = C_se * S_ut

Modified endurance:

S_e = C_size * C_surf * C_rel * S'_e

Goodman safety factor:

n_G = 1 / (sigma_a / S_e + sigma_m / S_ut)

Requires: n_G >= 2.0

Other Relations

Bernoulli (Incompressible)

Along a streamline, steady, inviscid:

p / rho_g + V^2 / 2g + z = const

Equivalent energy form:

p + 1/2 * rho * V^2 + rho * g * z = const

Reynolds Number

Definition:

Re = (rho * V_rel * D) / mu

Key Constants

Typical values:

- C_se = 0.5 (endurance factor)
- C_size approx 0.9 (size factor)
- C_surf approx 0.7 (surface factor)
- C_rel = 1.0 (reliability)

Air Properties

Typical values:

- rho = 1.225 kg/m^3 (sea level)
- mu = 1.81 x 10^-5 Pa.s

Power Relations

Power from C_P:

P = C_P * 1/2 * rho * A * V_infinity^3

Mohr's Circle

Radius:

R = sqrt(((sigma_x - sigma_y) / 2)^2 + tau_xy^2)

Center:

C = (sigma_x + sigma_y) / 2

Second Moment of Area

Solid cylinder:

I = (pi * D^4) / 64 = (pi * R^4) / 4

Ring (hollow):

I = (pi / 64) * (D_o^4 - D_i^4) = (pi / 4) * (R_o^4 - R_i^4)

Rectangle:

I = (b * h^3) / 12

Material Properties

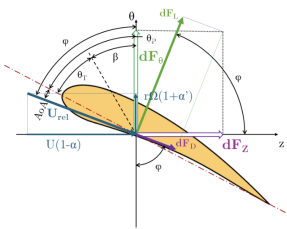
- S_y = yield strength [Pa]
- S_ut = ultimate strength [Pa]
- E = elastic modulus [Pa]
- I = second moment of area [m^4]
- Typical steel: S_y approx 345 MPa, S_ut approx 450 MPa, E approx 200 GPa

Assumptions

- Steady-state, linear-elastic
- Pure bending (uniaxial)
- Rigid nacelle
- No blade interference
- Lumped/Actuator-disk (aero): uniform axisymmetric inflow/outflow
- Inviscid, incompressible, steady far-field; pressure jump at disk
- No swirl in far-wake (for basic BEM); momentum applies
- Betz-optimal: a = 1/3 (upper bound for ideal power extraction)

Visual Notes

Blade element schematic (r, dr, forces)



- AoA - Angle of attack
- dF_D - Drag Force
- dF_L - Lift Force
- dF_a - Axial Force
- dF_z - Angular Force
- r - Radial position
- U_rel - Relative velocity vector
- U - Freestream wind velocity
- z - axial coordinate
- alpha - Axial induction factor
- alpha' - Angular induction factor
- beta - Blade angle pitch
- theta - cylindrical coordinate
- theta_p - Section pitch angle
- theta_t - Section twist angle
- phi - Angle of relative wind
- Omega - Rotational velocity

Tower shear, moment, slope, deflection distributions

