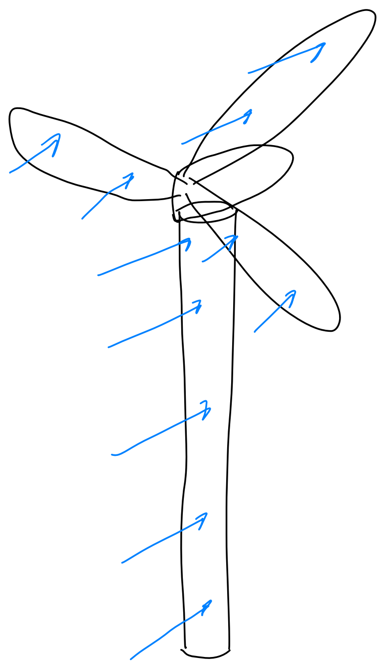
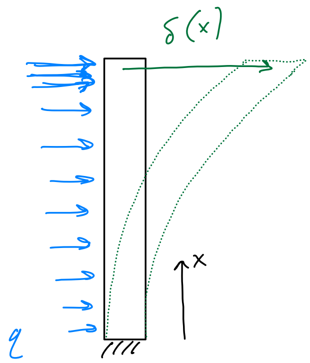
Goals

1) Review beam theory

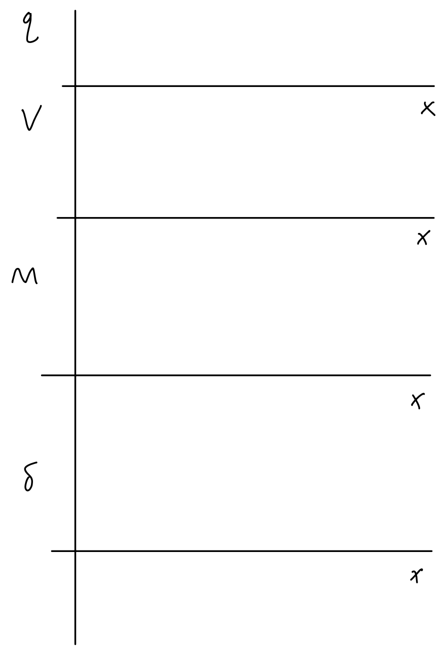
2) Use beam theory to derive shear force bending moment diagram and deflection curve for a wind turbine

Our structure → Model





Beam theory



Today, we are going to talk about making these plots

Assumptions

- Beam is linearly elastic (follows Hooke's law)
- "small" deflection $< 4-5°$ → small angle approximation
- Statically determinate

Overview of theory

Statics - AEM 2021

External loads

method of sections

method of integration

Today

Internal loads → deflection analysis

"deform" AEM 3031

Stresses

Later

Stress analysis

Mohr's circle

max stress

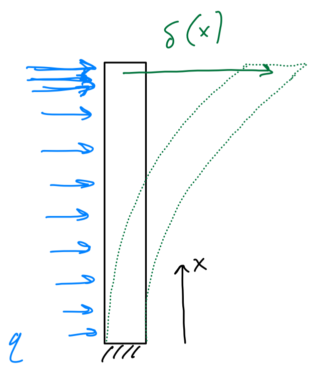
compare

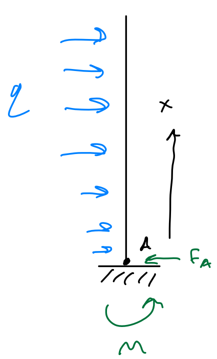
failure theory

ME3221, ME3222

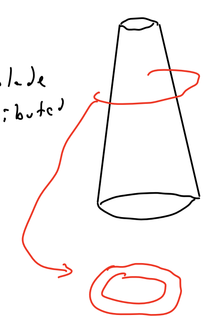
Strength of material

Our problem is a cantilevered beam

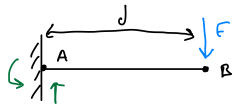




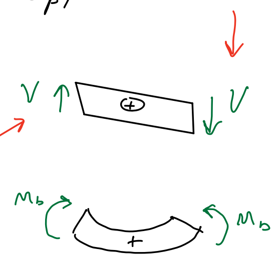
- Cross-section changes with height x
- Model turbine blade load as a distributed force over the Nacel height



Cantilevered beam with point load (warm up)

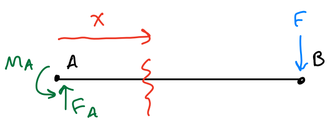


→ inside the beam
- Shear force
- Bending moment



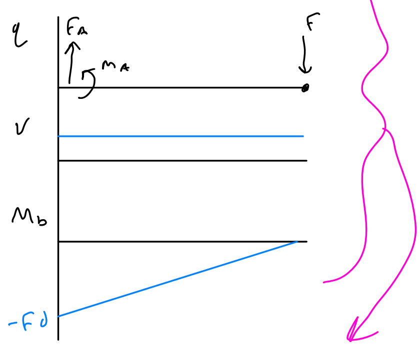
internal loads

① Calculate external reaction force / moment



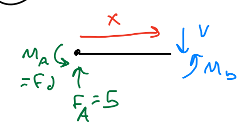
$$\sum F_y = 0 = F_A - F \longrightarrow \boxed{F_A = F}$$

$$\sum M_A = 0 = M_A - Fd \qquad \boxed{M_A = Fd}$$



Shear Force Diagram Bending Moment

② Method of sections to determine internal loading

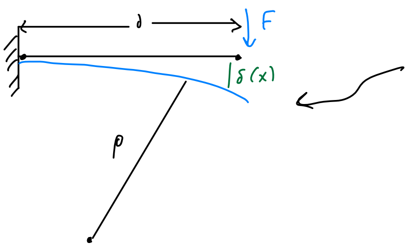


$$\sum F_y = 0 = F_A - V \longrightarrow \boxed{V = F_A = F}$$

$$\sum M_A = M_A - Vx + M_b$$

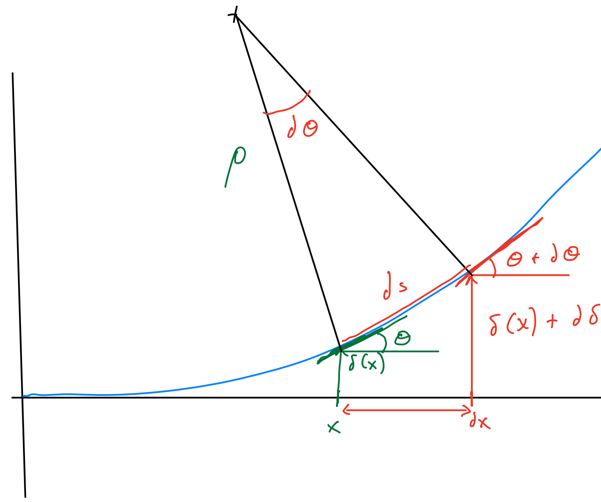
$$\boxed{M_b = Vx - Fd}$$

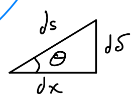
Moment curvature Relationship



negative bending moment in our example (bends downward)

Consider positive bending moment





$$\rho \, d\theta = ds$$

curvature $\kappa \equiv \dfrac{1}{\rho} = \dfrac{d\theta}{ds}$

$\dfrac{d\delta}{dx} = \tan\theta \;\Big|\; \cong \theta$ slope

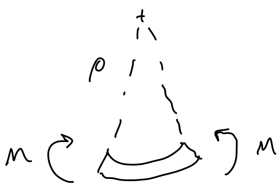
θ small

$dx = ds\cos\theta \;\Big|\; \longrightarrow dx \cong ds$

θ small

$$\kappa = \frac{1}{\rho} \cong \frac{d\theta}{dx} = \frac{d^2\delta}{dx^2}$$

From deform class, for beam element following Hooke's law:



$$\kappa = \frac{1}{\rho} = \frac{M}{EI}$$

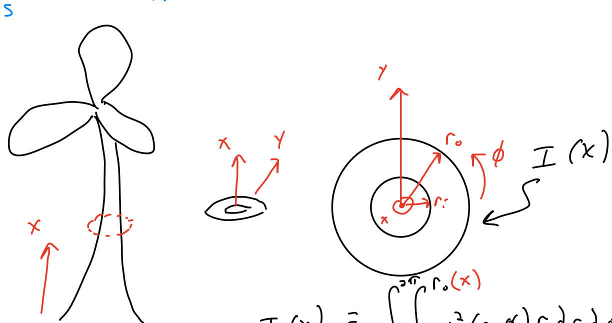
Young's Modulus

$I = \int_A y^2 \, dA$

2nd Area moment of inertia

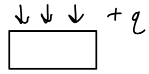
$$\boxed{\frac{d^2\delta}{dx^2} = \frac{M_b(x)}{EI(x)}}$$

tcrapz
or
ODE 45

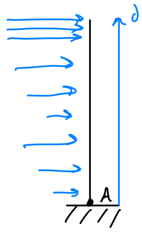


$$I(x) = \int_0^{2\pi}\int_{r_i(x)}^{r_o(x)} y^2(r,\phi)\, r\, dr\, d\phi$$

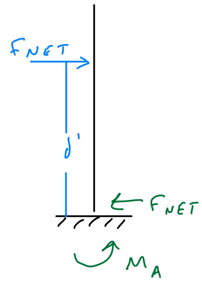
Distributed load



1) Find external reaction moment & force



We find equivalent point load



$$F_{NET} = \int_0^d q(x)\,dx \quad *$$

$$\sum M_A = 0 = M_A - \int_0^d q(x)\,x\,dx$$

$$M_A = \int_0^d q(x)\,x\,dx \quad *$$

$$F_{NET}\,d' = M_A$$

$$d' = \frac{M_A}{F_{NET}} \quad *$$

2) Method of integration (in place of Method of sections)

$$\frac{dV}{dx} = -q \longrightarrow V = -\int_0^x q\,dx + C_1$$ ← BC on $V\Big|_{x=0} = F_A$

$$\frac{dM}{dx} = V \longrightarrow M = \int_0^x V\,dx + C_2$$ ← BC on $M_b\Big|_{x=0} = M_A$

3) Deform

$$\frac{d\theta}{dx} = \frac{M_b}{EI} \longrightarrow \theta = \int \frac{M_b}{EI}\,dx + C_3$$ ← hint: $\theta(0) = 0$

$$\frac{d\delta}{dx} = \theta \qquad \delta = \int \theta\,dx + C_4$$ ← hint: $\delta(0) = 0$