Deliverable 5: Tower Deflection and Stress Analysis - Complete Methodology

Wind Turbine Analysis Project

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1 Overview

Deliverable 5 performs comprehensive tower deflection and stress analysis for a wind turbine tower, considering distributed wind loading, rotor thrust forces, and fatigue analysis. The analysis uses beam theory with proper boundary conditions and includes Mohr's circle and Goodman diagram analysis.

2 Input Parameters

The analysis requires the following inputs from previous deliverables:

- **Lambda (λ)**: Tip speed ratio from Deliverable 4
- **Pitch angle (θ)**: Blade pitch angle from Deliverable 4
- **Wind speed (V_{wind})**: 14.6 m/s (rated wind speed)

3 Step-by-Step Analysis Process

3.1 Step 1: Parameter Extraction and Setup

Function: `ParameterExtraction()`

Key Parameters Extracted:

- Tower specifications from `towerSpecs.csv`
- Material properties (steel: $E = 200$ GPa, $S_{ut} = 450$ MPa, $S_y = 345$ MPa)
- Air properties ($\rho_{air} = 1.1$ kg/m^3, $\mu = 1.8 \times 10^{-5}$ Pa·s)
- Turbine specifications (hub height = 80.4 m, rotor radius = 48 m)

Tower Geometry:

- Tower height: 77.7 m (from tower specifications)
- Hub height: 80.4 m
- Nacelle region: 77.7 m to 83.1 m (5.4 m total height)

3.2 Step 2: Rotor Thrust Force Calculation

Function: `calculateRotorThrust(data, V_wind, lambda, pitch_deg)`
Method: Blade Element Momentum (BEM) Theory
Process:

1. Extract blade profile data (chord, twist, airfoil assignments)
2. Calculate local tip speed ratio: $\lambda_r = \lambda \cdot r/R$
3. For each blade station:
   - Calculate induction factors: $a = 1/3$, $a' = -0.5 + 0.5\sqrt{1 + \frac{4}{\lambda_r^2}a(1-a)}$
   - Calculate inflow angle: $\phi = \tan^{-1}\left(\frac{1-a}{(1+a')\lambda_r}\right)$
   - Calculate angle of attack: $\alpha = \phi - (\theta_{twist} + \theta_{pitch})$
   - Interpolate airfoil coefficients: C_L, C_D from performance data
   - Calculate force coefficients: $C_n = C_L \cos\phi + C_D \sin\phi$, $C_t = C_L \sin\phi - C_D \cos\phi$
   - Calculate relative velocity: $V_{rel} = \sqrt{(V_{wind}(1-a))^2 + (\omega r(1+a'))^2}$
   - Calculate sectional thrust: $dT = 0.5\rho V_{rel}^2 c C_n$
4. Integrate thrust: $T = B\int_{r_{hub}}^{R} dT\,dr$

Output: Total rotor thrust force [N]

3.3 Step 3: Load Case Definition

Function: `defineLoadCases(V_wind, thrust_force, data, lambda, pitch_deg)`
Two Load Cases:

1. **Load Case 1 (Maximum Loading):**
   - Wind speed: $V_{wind} = 14.6$ m/s
   - Wind direction: 315° (from northwest)
   - Thrust force: Full calculated thrust
2. **Load Case 2 (Minimum Loading):**
   - Wind speed: $V_{wind} = 7.3$ m/s (50% of rated)
   - Wind direction: 157° (from southeast)
   - Thrust force: Recalculated for reduced wind speed

3.4 Step 4: Tower Section Properties Calculation

Function: `computeTowerSectionProperties(tower_specs)`
Geometric Properties:

- Outer diameter: D_{outer} [m]
- Wall thickness: t [m]

- Inner diameter: $D_{inner} = D_{outer} - 2t$ [m]
- Cross-sectional area: $A = \frac{\pi}{4}(D_{outer}^2 - D_{inner}^2)$ [m^2]
- Moment of inertia: $I = \frac{\pi}{64}(D_{outer}^4 - D_{inner}^4)$ [m^4]
- Section modulus: $Z = \frac{I}{c} = \frac{2I}{D_{outer}}$ [m^3]

Interpolation: Properties interpolated at each height using linear interpolation.

3.5 Step 5: Wind Loading Calculation

Function: `computeWindLoading(section_props, load_case, z, data)`

Atmospheric Boundary Layer:

$$V(z) = V_{ref}\left(\frac{z}{z_{ref}}\right)^{\epsilon} \tag{1}$$

where $\epsilon = 1/7$ (power law exponent), $z_{ref} = 80.4$ m (hub height).

Drag Force Calculation:

1. Calculate Reynolds number: $Re = \frac{\rho V D}{\mu}$
2. Calculate drag coefficient: $C_d = f(Re)$ using `cylinderCDlocal()`
3. Calculate distributed drag force: $F_d = 0.5\rho V^2 D C_d$ [N/m]

Nacelle Load Distribution:

$$q_{nacelle} = \frac{T_{thrust}}{h_{nacelle}} \quad \text{for } z \in [77.7, 83.1] \text{ m} \tag{2}$$

where $h_{nacelle} = 5.4$ m (nacelle region height).

3.6 Step 6: Tower Deflection Analysis

Function: `solveTowerDeflection(section_props, load_cases, E, data)`

Discretization:

- Number of elements: $n = 50$
- Height range: $z \in [0, 83.1]$ m
- Element size: $\Delta z = \frac{83.1}{50} = 1.662$ m

Moment Calculation: For each height z_i:

$$M(z_i) = \int_{z_i}^{z_{max}} F_d(\xi)(\xi - z_i)\, d\xi + \int_{z_i}^{z_{max}} q_{nacelle}(\xi)(\xi - z_i)\, d\xi \tag{3}$$

Deflection Calculation:

1. Calculate curvature: $\kappa = \frac{M}{EI}$
2. Set curvature to zero in nacelle region (rigid assumption)
3. First integration (slope): $\theta = \int_0^z \kappa\, dz$
4. Second integration (deflection): $\delta = \int_0^z \theta\, dz$
5. Apply boundary condition: $\delta(0) = 0$

3.7 Step 7: Stress Analysis

Function: `computeStressAnalysis(section_props, load_cases, deflection_results, E, data)`
Stress Components:

1. **Bending Stress:** $\sigma_b = \frac{Mc}{I}$ [Pa]
2. **Axial Stress:** $\sigma_a = \frac{T}{A}$ [Pa]
3. **Combined Stress:** $\sigma_{max} = \sigma_b + \sigma_a$ [Pa]

Directional Effects:

- Case 1: $\sigma_{b,eff} = \sigma_b \cos(315) = \sigma_b \cdot 0.707$
- Case 2: $\sigma_{b,eff} = \sigma_b \cos(157) = \sigma_b \cdot (-0.921)$

Fatigue Analysis:

- Mean stress: $\sigma_m = \frac{\sigma_{max,1}+\sigma_{max,2}}{2}$
- Alternating stress: $\sigma_a = \frac{|\sigma_{max,1}-\sigma_{max,2}|}{2}$
- Endurance limit: $S_e = 0.5S_{ut} \cdot C_L \cdot C_G \cdot C_S$
- Safety factor: $n_f = \frac{1}{\frac{\sigma_a}{S_e}+\frac{\sigma_m}{S_{ut}}}$

3.8 Step 8: Failure Analysis

Static Failure Theories:

1. **Maximum Normal Stress Theory (MNST):**

$$SF_{MNST} = \frac{S_y}{\sigma_{max}} \tag{4}$$

2. **Maximum Shear Stress Theory (MSST):**

$$\tau_{max} = \frac{\sigma_{max}}{2}, \quad SF_{MSST} = \frac{S_{sy}}{\tau_{max}} = \frac{S_y/2}{\tau_{max}} \tag{5}$$

3. **Distortion Energy Theory (DET):**

$$\sigma_{eq} = \sigma_{max}, \quad SF_{DET} = \frac{S_y}{\sigma_{eq}} \tag{6}$$

3.9 Step 9: Visualization

Functions:

- `createTowerAnalysisPlots()`: Comprehensive load distribution plots
- `createMohrCirclePlots()`: Mohr's circle analysis
- `createGoodmanDiagram()`: Fatigue analysis diagram

Key Plots:

1. Load distribution (wind drag + nacelle load)
2. Shear force distribution
3. Bending moment distribution
4. Slope distribution
5. Deflection distribution
6. Mohr's circles for stress states
7. Goodman diagram for fatigue analysis

4 Key Equations and Calculations

4.1 Beam Theory

$$\frac{d^2\delta}{dz^2} = \frac{M(z)}{E(z)I(z)} \tag{7}$$

$$M(z) = \int_z^{z_{max}} F(\xi)(\xi - z)\, d\xi \tag{8}$$

$$V(z) = -\frac{dM}{dz} \tag{9}$$

4.2 Material Properties

$$E = 200 \times 10^9 \text{ Pa} \tag{10}$$

$$S_{ut} = 450 \times 10^6 \text{ Pa} \tag{11}$$

$$S_y = 345 \times 10^6 \text{ Pa} \tag{12}$$

$$\rho_{air} = 1.1 \text{ kg/m}^3 \tag{13}$$

4.3 Fatigue Analysis

$$S_e = S_{ut} \cdot 0.5 \cdot C_L \cdot C_G \cdot C_S \tag{14}$$

$$C_L = 1.0 \text{ (load factor)} \tag{15}$$

$$C_G = 0.9 \text{ (gradient factor)} \tag{16}$$

$$C_S = 0.7 \text{ (surface factor)} \tag{17}$$

5 Output Results

The analysis provides:

- Maximum tip deflection [m]
- Maximum stress at tower base [MPa]

- Fatigue safety factor
- Static failure safety factors (MNST, MSST, DET)
- Comprehensive stress and deflection distributions
- Visualization plots for all analysis components

6 Verification Checklist

To verify the solution:

1. Check that thrust force calculation uses correct BEM parameters
2. Verify wind profile follows power law with $\epsilon = 1/7$
3. Confirm nacelle load is distributed over correct height range (77.7-83.1 m)
4. Validate that deflection calculation uses proper beam theory
5. Ensure stress analysis includes both bending and axial components
6. Verify fatigue analysis uses correct endurance limit calculation
7. Check that all safety factors are calculated correctly
8. Confirm plots show both wind drag and nacelle load lines