Robotics
Exercise 1

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1 Matrix equations (4 points)

a) Let X, A be arbitrary matrices, A invertible. Solve for X:

$$XA + A^\top = \mathbf{I}$$

b) Let X, A, B be arbitrary matrices, $(C - 2A^\top)$ invertible. Solve for X:

$$X^\top C = [2A(X + B)]^\top$$

c) Let $x \in \mathbb{R}^n, y \in \mathbb{R}^d, A \in \mathbb{R}^{d\times n}$. A obviously *not* invertible, but let $A^\top A$ be invertible. Solve for x:

$$(Ax - y)^\top A = \mathbf{0}_n^\top$$

d) As above, additionally $B \in \mathbb{R}^{n\times n}$, B positive-definite. Solve for x:

$$(Ax - y)^\top A + x^\top B = \mathbf{0}_n^\top$$

2 Vector derivatives (5 points)

Let $x \in \mathbb{R}^n,\ y \in \mathbb{R}^d,\ f, g : \mathbb{R}^n \to \mathbb{R}^d,\ A \in \mathbb{R}^{d\times n},\ C \in \mathbb{R}^{d\times d}$. (Also provide the dimensionality of the results.)
a) What is $\frac{\partial}{\partial x} x$?
b) What is $\frac{\partial}{\partial x}[x^\top x]$?
c) What is $\frac{\partial}{\partial x}[f(x)^\top f(x)]$?
d) What is $\frac{\partial}{\partial x}[f(x)^\top C g(x)]$?
e) Let B and C be symmetric (and pos.def.). What is the minimum of $(Ax - y)^\top C(Ax - y) + x^\top Bx$ w.r.t. x ?

3 Optimization (3 points)

Given $x \in \mathbb{R}^n,\ f :\ \mathbb{R}^n \to \mathbb{R}$, we want to find $\text{argmin}_x f(x)$. (We assume f is uni-modal.)
a) What 1st-order optimization methods (querying $f(x), \nabla f(x)$ in each iteration) do you know?
b) What 2nd-order optimization methods (querying $f(x), \nabla f(x), \nabla^2 f(x)$ in each iteration) do you know?
c) What is backtracking line search?