1. Prime numbers

Definition 1.1 (Prime numbers). A natural number is called a *prime number* if it is greater than 1 and cannot be written as the product of two smaller natural numbers.

Theorem 1.1. *Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do.*

Theorem 1.2 (Euclid). *There are infinitely many primes.*

Corollary 1.2.1. *There is no largest prime number.*

Corollary 1.2.2. *There are infinitely many composite numbers.*

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Theorem 1.3. *There are arbitrarily long stretches of composite numbers.*

Proof. For any $n > 2$, consider

$$n! + 2, \quad n! + 3, \quad \dots, \quad n! + n.$$

□

2. Restated or deferred

Theorem 1.2 (Euclid). *There are infinitely many primes.*

Proof. Suppose to the contrary that $p_1, p_2, \dots, p_n$ is a finite enumeration of all primes. Set $P = p_1 p_2 \dots p_n$. Since $P + 1$ is not in our list, it cannot be prime. Thus, some prime factor p_j divides $P + 1$. Since p_j also divides P, it must divide the difference $(P + 1) - P = 1$, a contradiction. □

Corollary 1.2.2. *There are infinitely many composite numbers.*

Theorem 1.3. *There are arbitrarily long stretches of composite numbers.*

Proof. For any $n > 2$, consider

$$n! + 2, \quad n! + 3, \quad \dots, \quad n! + n.$$

□

3. Only Theorems and Corollaries

Theorem 1.2 (Euclid). *There are infinitely many primes.*

Corollary 1.2.2. *There are infinitely many composite numbers.*

Theorem 1.3. *There are arbitrarily long stretches of composite numbers.*