CS 512: Formal Methods Spring 2016

Assignment 9: Modeling with CTL

Instructor: Assaf Kfoury Author: Patrick Gomes

1 Problem 1

e) Yes it is satisfiable with a δ of 0.001, the answer is x = y = 0.0008217490086014628 given by dReal
f) No it is no longer satisfiable.
g) If you allow a δ of 0.2 then the solution is satisfiable. dReal has given back: x = 0.3622065017492991 and y = 0.3543384320284063 which are within our delta bounds.

2 Problem 2

Defining 'right' to be equivalent to i + 1 unless i = 5 in which case right is i = 1.

a) $AG\ \neg(w_1 \wedge w_2 \wedge w_3 \wedge w_4 \wedge w_5)$
From now onwards, it is never the case that all philosophers are waiting.

b) $AG(AFe_1 \wedge AFe_2 \wedge AFe_3 \wedge AFe_4 \wedge AFe_5)$
From now onwards, all philosophers will eventually eat.

b) $AG(AFe_1 \wedge AFe_2 \wedge AFe_3 \wedge AFe_4 \wedge AFe_5)$
From now onwards, all philosophers will eventually eat (i.e. not starve to death).

d) $AG((e_1 \rightarrow AFe_2) \wedge (e_2 \rightarrow AFe_3) \wedge (e_3 \rightarrow AFe_4) \wedge (e_4 \rightarrow AFe_5) \wedge (e_5 \rightarrow AFe_1)$
If a philosopher can eat, then at some point in the future the philosopher to his right will be able to eat.

e) $AG(AFe_1 \vee AFe_2 \vee AFe_3 \vee AFe_4 \vee AFe_5)$
From now onwards, some philosopher will be able to eat in the future.

f) $AG(AFe_1 \wedge AFe_2 \wedge AFe_3 \wedge AFe_4 \wedge AFe_5)$
From now onwards, all philosophers will inevitably eat in the future.

g) $AG((e_1 \rightarrow \neg(e_2 \wedge e_5)) \wedge (e_2 \rightarrow \neg(e_1 \wedge e_3)) \wedge (e_3 \rightarrow \neg(e_2 \wedge e_4)) \wedge (e_4 \rightarrow \neg(e_3 \wedge e_5)) \wedge (e_5 \rightarrow \neg(e_4 \wedge e_1)))$
From now onwards, if a philosopher is eating, the philosophers to his left and right can't be eating.

h) $AG(e_1 \vee e_2 \vee e_3 \vee e_4 \vee e_5)$
From now onwards, at least one philosopher is always eating.

3 Problem 3

a)
All available states:

$$\begin{aligned}
s_0 &- (f, f, f, f, f) \\
s_1 &- (t, f, f, f, f) \\
s_2 &- (t, f, t, f, f) \\
s_3 &- (t, f, f, t, f) \\
s_4 &- (f, t, f, f, f) \\
s_5 &- (f, t, f, t, f) \\
s_6 &- (f, t, f, f, t) \\
s_7 &- (f, f, t, f, f) \\
s_8 &- (f, f, t, f, t) \\
s_9 &- (f, f, f, t, f) \\
s_{10} &- (f, f, f, f, t)
\end{aligned}$$

State transitions for each state:

$$\begin{aligned}
s_0 - &> \text{all states} \\
s_1 - &> \{s_0, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}\} \\
s_2 - &> \{s_0, s_4, s_5, s_6, s_9, s_{10}\} \\
s_3 - &> \{s_0, s_4, s_6, s_7, s_8, s_9, s_{10}\} \\
s_4 - &> \{s_0, s_1, s_2, s_3, s_7, s_8, s_9, s_{10}\} \\
s_5 - &> \{s_0, s_1, s_2, s_7, s_8, s_9, s_{10}\} \\
s_6 - &> \{s_0, s_1, s_2, s_3, s_7, s_9\} \\
s_7 - &> \{s_0, s_1, s_3, s_4, s_5, s_6, s_9, s_{10}\} \\
s_8 - &> \{s_0, s_1, s_3, s_4, s_5, s_9\} \\
s_9 - &> \{s_0, s_1, s_2, s_4, s_6, s_7, s_8, s_{10}\} \\
s_{10} - &> \{s_0, s_1, s_2, s_3, s_4, s_5, s_7, s_9\}
\end{aligned}$$

b) The first CTL is not satisfied because the path s_0, s_3 violates it.
The second CTL is not satisfied because the paths_0, s_1 violates it.
The third CTL is satisfied!

c) All you need to do is add an intermediate set of states such that after eating you go to a state where your value for f is true and then it resets back to waiting to eat.