CS 512: Formal Methods Spring 2016

Assignment 4: Formal Modeling in Propositional Logic and in QBF Logic

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1 Problem 1

P = {a, b, c} and S = {1, 2, 3}

a)

$$\bigwedge_{i \in P} (\bigvee_{j \in S} P_{i,j})$$

explanation: for each person i, $P_i, 1 \vee P_i, 2 \vee P_i, 3$ must be true, so they must have a seat.

b)

$$(\bigwedge_{i \in P} (\bigvee_{j \in S} (P_{i,j} \rightarrow \bigwedge_{j` \in S - \{j\}}))) \wedge (\bigwedge_{j \in S} (\bigvee_{i \in P} (P_{i,j}))))$$

explanation: first term is exclusive or on seating, only one person can be in a seat. second term is all seats need a person si

c)

1.

$$\neg P_{a,2} \wedge \neg P_{c,2} \wedge (P_{a,1} \rightarrow P_{c,3}) \wedge (P_{a,3} \rightarrow P_{c,1}) \wedge (P_{c,1} \rightarrow P_{a,3}) \wedge (P_{c,3} \rightarrow P_{a,1})$$

explanation: covers all possible cases of where a and c can be and where the other person should be. neither can be in 2.

2.

$$\neg P_{a,1}$$

3.

$$(P_{c,1} \wedge P_{b,3}) \wedge (P_{c,2} \wedge P_{b,1})$$

explanation: assume if c is in 1, b can't be in 2, else if c is in 2, b can't be in 3. covers all cases, if c is in 3, b can be anywh

4. No this isn't possible. a has to be in 3, since a can't be in 1 or 2 from (1) and (2). c can't sit next to a, so c sits in seat 1. b is left with seat 2 since all seats must be filled, and that violates (3), thus this isn't doable.

2 Problem 2

P = {0,. . . ,64} S = {0,. . . 32}
a)

$$\varphi_{m,n,s} \triangleq x_{m,n,s} \wedge (\bigwedge_{n‘\in P-\{n\}} \neg x_{m,n‘,s})$$

explanation: if m is at spot n, make sure m is nowhere else.

b)

$$\psi_{m,n,s} \triangleq \{\varphi_{m,n,s} \to \bigwedge_{m‘\in S-\{m\}} \neg\varphi_{m‘,n,s} \ \mid \ m \in S, n \in P-\{0\}\ \}$$

explanation: if a piece m is at spot n, make sure no piece m‘ is also in spot n, unless n is spot 0.
$\varphi_{m‘,n,s}$ is true only if m‘ is in spot n at time s.

c)

$$G_K \triangleq \{\bigvee_{n\in P-\{0\}} \varphi_{5,n,K}\} \wedge \neg\varphi_{29,0,K}$$

explanation: we need to check that the white king is alive on 1 space at step K,
and that the black king is off the board (spot 0) at step K

3 Problem 3

Not Transitive:

$$S = \{1,2,3\},\ R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)\},\ P(x,y)\ is\ true\ if\ (x,y) \in R$$

Reflexive and Symmetric are met for all $x \in S$ but $P((1,2)) \wedge P(2,3) \to P(1,3)$ is false because P(1,3) $\notin$ R

Not Symmetric:

P(x,y) is true if x $\geq$ y

Obviously Reflexive and Transitive, not Symmetric because 2 $\geq$ 1 is true but 1 $\geq$ 2 is false.

Not Reflexive:

P(x,y) is true if x $\neq$ y

Obviously not Reflexive because x $\neq$ x is always false. Symmetric because if x $\neq$ y then y $\neq$ x.

Transitive because x $\neq$ y $\wedge$ y $\neq$ z $\to$ x $\neq$ z