CS 512: Formal Methods Spring 2016

Assignment 6: Linear Temporal Logic

Instructor: Assaf Kfoury Author: Patrick Gomes

1 Problem 1

a)

$$\varphi_1(x) \triangleq (1 < x) \ \wedge \forall y(\exists z(y \times z = x) \rightarrow (y = 1 \vee y = x))$$
$$\varphi_2(m,n) \triangleq (1 < m) \wedge (1 < n) \wedge \forall y(\exists z(y \times z = m) \rightarrow (\neg \exists v(y \times v = n)$$
$$\varphi_3(X) \triangleq \forall x(X(x) \rightarrow \exists y(y \times (1+1) = x)$$
$$\varphi_4(X) \triangleq \forall x(X(x) \rightarrow (x = 1 \vee \exists y(X(y) \wedge (x = y * (1+1))))$$

2 Problem 2

Formula: ¬p U (F r ∨ G ¬q → q W ¬r)

Subformulas:

p

$\neg p$

r

$F\ r$

q

$\neg q$

G ¬ q

F r ∨ G ¬ q

$\neg r$

q W ¬ r

F r ∨ G ¬ q → q W ¬ r

¬ p U F r ∨ G ¬ q → q W ¬ r

3 Problem 3

a) G a
$\pi = (q_3, q_4)^i$
$\mathcal{M}, q_3 \nvDash \phi$ A counter example: $\pi = q_3, q_2$. q_2 is $\neg a$

b) a U b
$\pi = q_3, q_2$
$\mathcal{M}, q_3 \not\models \phi$ A counter example: $\pi = q_3, q_1$. q_1 is $\neg a$ but b is not true.

c) a U X$(a \wedge \neg b)$
$\forall i \geq 1$ $\pi^i = q_3, q_4, q_3$
$\mathcal{M}, q_3 \not\models \phi$ A counter example: $\pi = q_3, q_2$. q_2 is $\neg a$ and there is no way to get to q_3 and satisfy the next clause.

d) X $\neg b \wedge$ G$(\neg a \vee \neg b)$
$\pi = q_3, q_1, q_2^i$
$\mathcal{M}, q_3 \not\models \phi$ A counter example: $\pi = q_3, q_4$. q_4 does not satisfy the next clause

e) X (a $\wedge$ b) $\wedge$ F$(\neg a \wedge \neg b)$
$\pi = q_3, q_4, q_3, q_1$
$\mathcal{M}, q_3 \not\models \phi$ A counter example: $\pi = q_3, q_2$. q_2 does not satisfy the next clause.

4 Problem 4

$$\pi \models (X\varphi)U(X\psi) \text{ iff (By expanding definition of U)}$$
$$\pi \models (\exists i \geq 1)(\pi^i \models X\psi \wedge (\forall j < i)(\pi^j \models X\varphi)) \text{ iff (By removal of X and representing next state, not current.)}$$
$$\pi^2 \models (\exists i \geq 1)(\pi^{i+1} \models \psi \wedge (\forall j < i)(\pi^{j+1} \models \varphi)) \text{ iff (By condensing definiton of U)}$$
$$\pi^2 \models \varphi U \psi \text{ iff (By reintroducing X to represent current state.)}$$
$$\pi \models X(\varphi U \psi)$$