Assignment 7: Modeling with LTL

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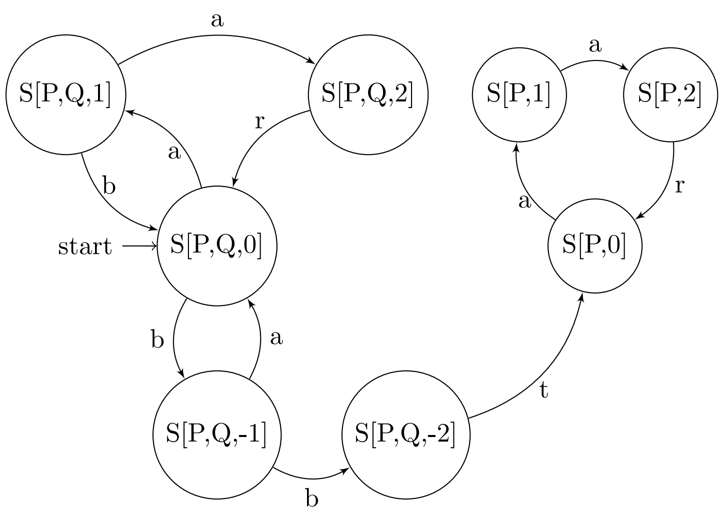
1 Problem 1

a)



b)

E{P,Q,1} = ab + aar
E{P,Q,2} = aar
E{P,Q,-1} = ba
Combining these all gives you E = (ab + aar + ba)*

c)

The path (bbt) brings us to S[P,0] and then repeat (aar) infinitely.
Thus F = E + bbt + $(\text{aar})^w$

d)

$$
\begin{aligned}
L(S[P,Q,0]) &= \{a \vee b \vee r\} \\
L(S[P,Q,1]) &= \{a\} \\
L(S[P,Q,2]) &= \{a\} \\
L(S[P,Q,-1]) &= \{b\} \\
L(S[P,Q,-2]) &= \{b\} \\
L(S[P,0]) &= \{r \vee t\} \\
L(S[P,1]) &= \{a\} \\
L(S[P,2]) &= \{a\}
\end{aligned}
$$

e)

$\varphi_1 \triangleq G\ F\ r$

Path for $\varphi_1 = b, b, t, (a, a, r)^*$

$\mathcal{M} \nvDash \varphi_1$, an infinite path (a,b)* will never make r true at any point.

$\varphi_2 \triangleq G((r \vee t) \rightarrow (Xa \wedge XXa))$

Path for $\varphi_2 = b, b, t, (a, a, r)^*$

$\mathcal{M} \nvDash \varphi_2$, an infinite path (a,a,r,b,a)* will not satisfy the conditional statement ever.

$\varphi_3 \triangleq ((a \vee b \vee r)\ U\ t)$

Path for $\varphi_3 = b, b, t, (a, a, r)^*$

$\mathcal{M} \nvDash \varphi_3$, an infinite path (a,b)* will never make t true at any point, so the strong until is never met.

2 Problem 2

$$\begin{aligned}
&\text{The conditions from the book:}\\
&\phi\ is\ a\ literal : return\ \phi\\
&\phi\ is\ \neg\neg\phi_1 : return\ \mathrm{NNF}(\phi_1)\\
&\phi\ is\ \phi_1 \wedge \phi_2 : return\ \mathrm{NNF}(\phi_1) \wedge \mathrm{NNF}(\phi_2)\\
&\phi\ is\ \phi_1 \vee \phi_2 : return\ \mathrm{NNF}(\phi_1) \vee \mathrm{NNF}(\phi_2)\\
&\phi\ is\ \neg(\phi_1 \wedge \phi_2) : return\ \mathrm{NNF}(\neg\phi_1) \vee \mathrm{NNF}(\neg\phi_2)\\
&\phi\ is\ \neg(\phi_1 \vee \phi_2) : return\ \mathrm{NNF}(\neg\phi_1) \wedge \mathrm{NNF}(\neg\phi_2)\\
&\text{On top of all of these conditions, you need to add the following conditions:}\\
&\phi\ is\ X\phi_1 : return\ X\mathrm{NNF}(\phi_1)\\
&\phi\ is\ \neg X\phi_1 : return\ \neg X\ \mathrm{NNF}(\phi_1)\\
&\phi\ is\ F\phi_1 : return\ F\ \mathrm{NNF}(\phi_1)\\
&\phi\ is\ \neg F\phi_1 : return\ G\ \mathrm{NNF}(\neg\phi_1)\\
&\phi\ is\ G\phi_1 : return\ G\ \mathrm{NNF}(\phi_1)\\
&\phi\ is\ \neg G\phi_1 : return\ F\ \mathrm{NNF}(\neg\phi_1)\\
&\phi\ is\ \phi_1 U\phi_2 : return\ \mathrm{NNF}(\phi_1)\ U\ \mathrm{NNF}(\phi_2)\\
&\phi\ is\ \neg(\phi_1 U\phi_2) : return\ \mathrm{NNF}(\neg\phi_1)\ R\ \mathrm{NNF}(\neg\phi_2)\\
&\phi\ is\ \phi_1 R\phi_2 : return\ \mathrm{NNF}(\phi_1)\ R\ \mathrm{NNF}(\phi_2)\\
&\phi\ is\ \neg(\phi_1 R\phi_2) : return\ \mathrm{NNF}(\neg\phi_1)\ U\ \mathrm{NNF}(\neg\phi_2)\\
&\phi\ is\ \phi_1 W\phi_2 : return\ \mathrm{NNF}(\phi_1)\ W\ \mathrm{NNF}(\phi_2)\\
&\phi\ is\ \neg(\phi_1 R\phi_2) : return\ \mathrm{NNF}(\neg(\phi_2\ R\ (\phi_1 \vee \phi_2)))
\end{aligned}$$