



## Data Structures

### Chapter 5 Tree

1. introduction
2. Binary tree
  - Definition and Properties
  - Traversal
  - **Coding**
3. Binary search tree
4. Tree balancing

사람아  
주께서 산한 것이 무엇임을  
네게 보이셨나니

여호와께서 네게 구하시는 것이  
오직 공의를 행하며 인자를 사랑하며  
겸손히 네 하나님과 함께 행하는 것이 아니냐

미  
가  
6  
장  
8  
절

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절

**He has showed you, O man, what is good. And what does the LORD require of you?  
To act justly and to love mercy and to walk humbly with your God. Micah 6:8**

**하나님이 우리를 구원하사 기록하신 소명으로 부르심은 우리의 행위대로 하심이 아니요 오직  
자기의 뜻과 영원 전부터 그리스도 예수 안에서 우리에게 주신 은혜대로 하심이라 (딤후1:9)**

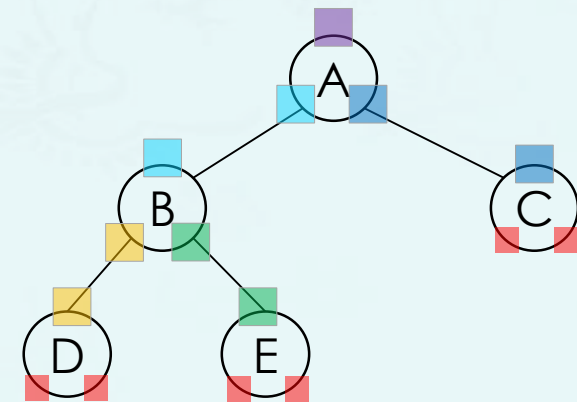
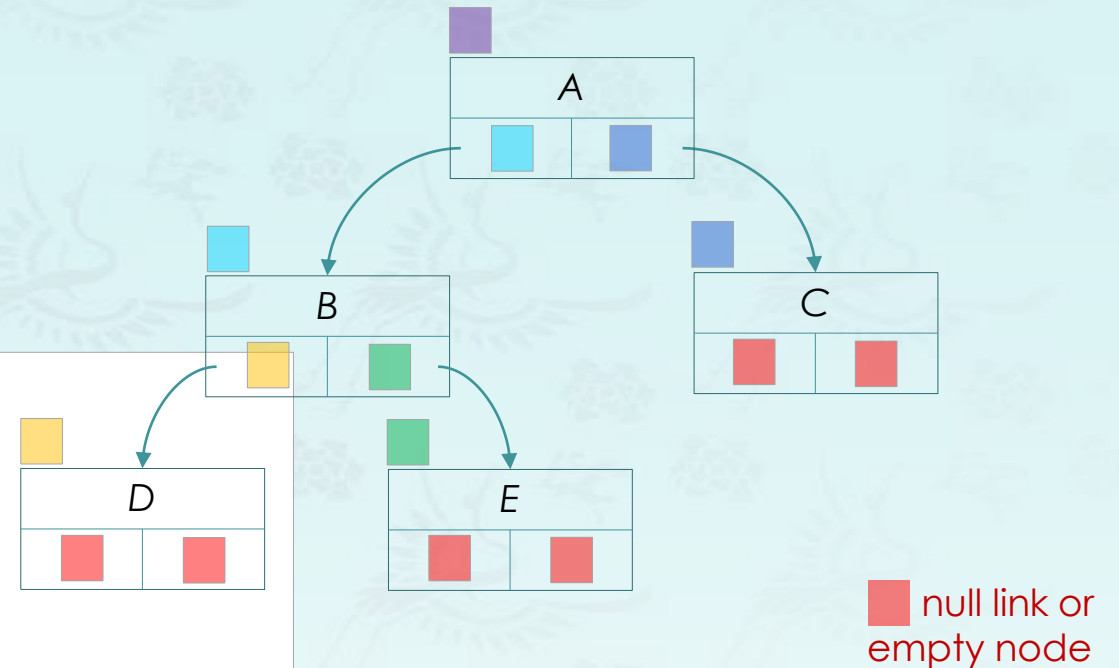
# Recursion & Tree Structure

```
struct TreeNode{
    int      key;
    TreeNode* left;
    TreeNode* right;
};
using tree = TreeNode*;
```

```
struct TreeNode{
    int      key;
    TreeNode* left;
    TreeNode* right;

    TreeNode(int k, TreeNode* l, TreeNode* r) {
        key = k; left = l; right = r;
    }
    TreeNode(int k) : key(k), left(nullptr), right(nullptr) {}

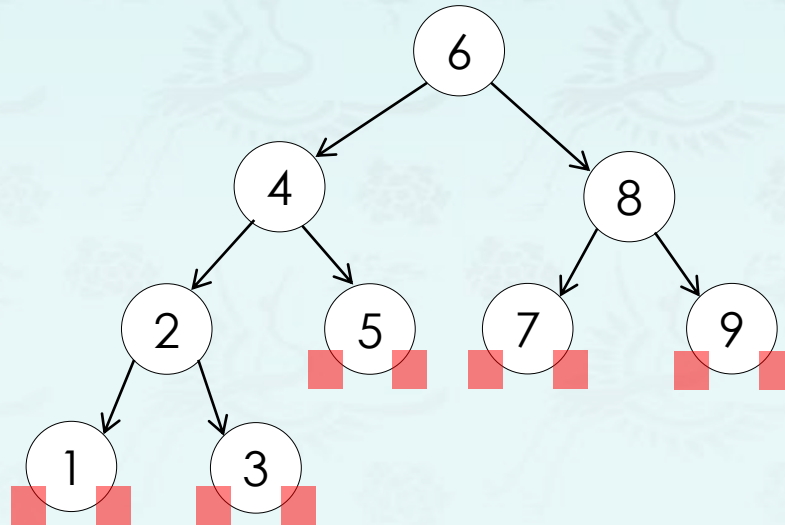
    ~TreeNode(){}
};
using tree = TreeNode*;
```



## Operations: inorder()

```
// Given a binary tree, its node values in inorder are passed  
// back through the argument v which is passed by reference.  
void inorder(tree node, vector<int>& v) {  
    if (empty(node)) return;  
  
    inorder(node->left, v);  
    v.push_back(node->key);  
    inorder(node->right, v);  
}
```

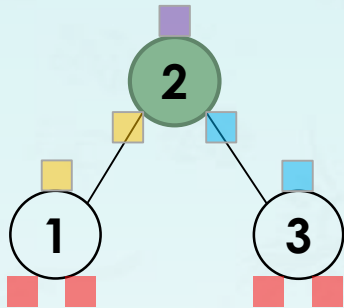
```
// pinorder() and its result  
vec.clear();  
inorder(root, vec);  
cout << "inorder: ";  
for (auto i : vec)  
    cout << i << " ";  
cout endl;
```



# Binary tree traversals

## ■ **Example: Inorder traversal(LVR)**

- Step 1 – Recursively traverse left subtree.
- Step 2 – Visit root node. (print or save it.)
- Step 3 – Recursively traverse right subtree.



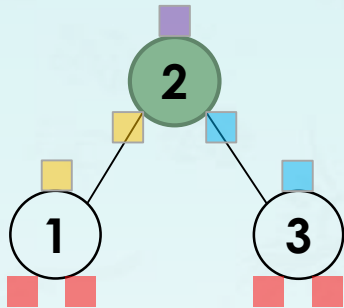
```
void inorder(tree root) {  
    if (root == nullptr) return;  
  
    inorder(root->left);    L  
    cout << root->key;      V  
    inorder(root->right);   R  
}
```



# Binary tree traversals

## ■ **Example: Inorder traversal(LVR)**

- Step 1 – Recursively traverse left subtree.
- Step 2 – Visit root node. (print or save it.)
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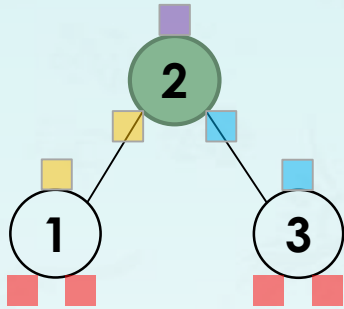
```
void inorder(tree root) {  
    if (root == nullptr) return;  
  
    inorder(root->left);    L  
    cout << root->key;    V  
    inorder(root->right);  R  
}
```

```
int main() {  
  
}
```

# Binary tree traversals

## ■ Example: Inorder traversal(LVR)

- Step 1 – Recursively traverse left subtree.
- Step 2 – Visit root node. (print or save it.)
- Step 3 – Recursively traverse right subtree.



```
void inorder(tree root) {  
    if (root == nullptr) return;  
  
    inorder(root->left);    L  
    cout << root->key;    V  
    inorder(root->right);  R  
}
```

```
int main() {  
    tree l = new TreeNode(1);  
    tree r = new TreeNode(3);  
    tree root = new TreeNode(2, l, r);  
    inorder(root);  
}
```

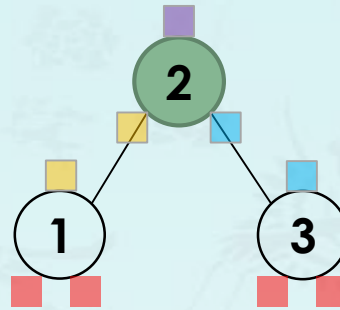


# Binary tree traversals - Inorder traversal(LVR)

(1) (2) (3) (5)

```
void inorder(tree root) {  
    if (root == nullptr) return;  
    push → inorder(root->left);  
    pop → cout << root->key;  
    push → inorder(root->right);  
    pop →  
}
```

push pop  
system stack

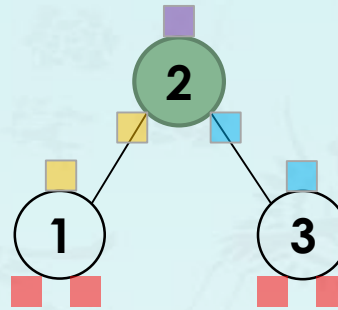
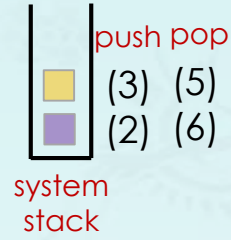


# Binary tree traversals - Inorder traversal(LVR)

(1) (2) (3) (5)

```
void inorder(tree root) {
    if (root == nullptr) return;
    inorder(root->left);
    cout << root->key;
    inorder(root->right);
}
```

push  
pop  
push  
pop



- (1) The initial call `inorder(purple, key=2)` is made.
- (2) The purple calls `inorder( )` with left (or orange).  
The purple FC (the initial call) stops and stacked.
- (3) The orange calls `inorder( )` with left (or pink)  
The orange FC stops and stacked.  
The pink FC returns immediately since it is null.  
The pink FC is over 1<sup>st</sup> time. Then, stack pops. ■
- (4) The orange pops from stack & continues where it left.  
■ `cout << root->key;` ➡ 1
- (5) The orange calls `inorder( )` with right(or pink).  
The orange FC stops and stacked. ←  
The pink FC returns immediately since it is null.  
The orange FC finishes its job here. Stack pops. ■
- (6) The purple pops from stack & continues where it left. ■  
■ `cout << root->key;` ➡ 2 ←

how many calls in stack?

how many calls in stack?

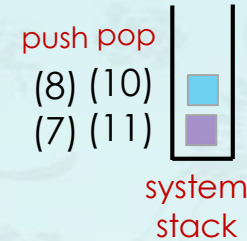
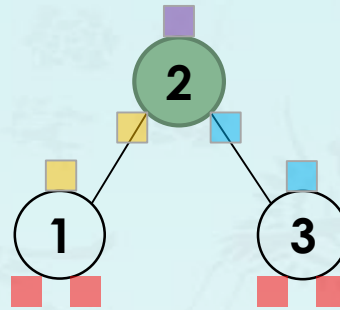
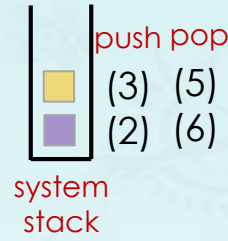
❖ FC stands for function call.

# Binary tree traversals - Inorder traversal(LVR)

(1) (2) (3) (5)

```
void inorder(tree root) {
    if (root == nullptr) return;
    inorder(root->left);
    cout << root->key;
    inorder(root->right);
}
```

push  
pop  
push  
pop



(7) (8) (10)

```
void inorder(tree root) {
    if (root == nullptr) return;
    inorder(root->left);
    cout << root->key;
    inorder(root->right);
}
```

- (1) The initial call `inorder(purple, key=2)` is made.
- (2) The purple calls `inorder(orange)` with left (or orange). The purple FC (the initial call) stops and stacked.
- (3) The orange calls `inorder(pink)` with left (or pink). The orange FC stops and stacked. The pink FC returns immediately since it is null. The pink FC is over 1<sup>st</sup> time. Then, stack pops.
- (4) The orange pops from stack & continues where it left.  
`cout << root->key;`    ➡    1
- (5) The orange calls `inorder(pink)` with right (or pink). The orange FC stops and stacked. The pink FC returns immediately since it is null. The orange FC finishes its job here. Stack pops.
- (6) The purple pops from stack & continues where it left.  
`cout << root->key;`    ➡    2

- (7) The purple call `inorder(blue)` with right (or blue). The purple FC stops and stacked.
- (8) The blue calls `inorder(pink)` with left (or pink). The blue FC stops and stacked. The pink FC returns immediately since it is null. The pink FC is over. Then, stack pops.
- (9) The blue pops from stack & continues where it left.  
`cout << root->key;`    ➡    3
- (10) The blue calls `inorder(pink)` with right (or pink). The blue FC stops and stacked. The pink FC returns immediately since it is null. The blue FC finishes its job here. Stack pops.
- (11) The purple pops from stack & continues where it left. The purple finishes its job and returns to the caller(main).

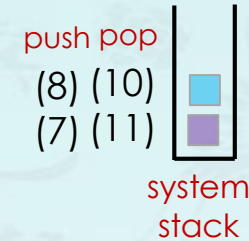
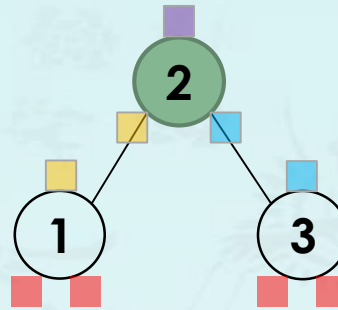
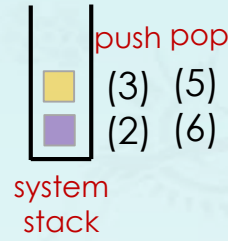
❖ FC stands for function call.

# Binary tree traversals - Inorder traversal(LVR)

(1) (2) (3) (5)

```
void inorder(tree root) {
    if (root == nullptr) return;
    inorder(root->left);
    cout << root->key;
    inorder(root->right);
}
```

push  
pop  
push  
pop



(7) (8) (10)

```
void inorder(tree root) {
    if (root == nullptr) return;
    inorder(root->left);
    cout << root->key;
    inorder(root->right);
}
```

- (1) The initial call `inorder(purple, key=2)` is made.
- (2) The purple calls `inorder(orange)` with left (or orange). The purple FC (the initial call) stops and stacked.
- (3) The orange calls `inorder(pink)` with left (or pink). The orange FC stops and stacked. The pink FC starts.
- (4) The orange pops from stack & continues where it left. `cout << root->key;` The pink FC finishes its job here. Stack pops.
- (5) The orange pops from stack & continues where it left. The orange FC stops and stacked. The pink FC returns immediately since it is null. The orange FC finishes its job here. Stack pops.
- (6) The purple pops from stack & continues where it left. `cout << root->key;` The purple FC stops and stacked.
- (7) The purple calls `inorder(blue)` with right (or blue). The purple FC stops and stacked. The blue FC starts.
- (8) The blue calls `inorder(pink)` with left (or pink). The blue FC stops and stacked. The pink FC starts.
- (9) The blue pops from stack & continues where it left. `cout << root->key;` The pink FC finishes its job here. Stack pops.
- (10) The blue pops from stack & continues where it left. The blue FC stops and stacked. The pink FC returns immediately since it is null. The blue FC finishes its job here. Stack pops.
- (11) The purple pops from stack & continues where it left. The purple finishes its job and returns to the caller(main).

- **The final output:**
- **The number of times of `inorder()` invoked:**
- **The number of times of the first line return executed:**
- **The number of times of the hidden return executed:**
- **List root's keys passed as an argument and its count:**

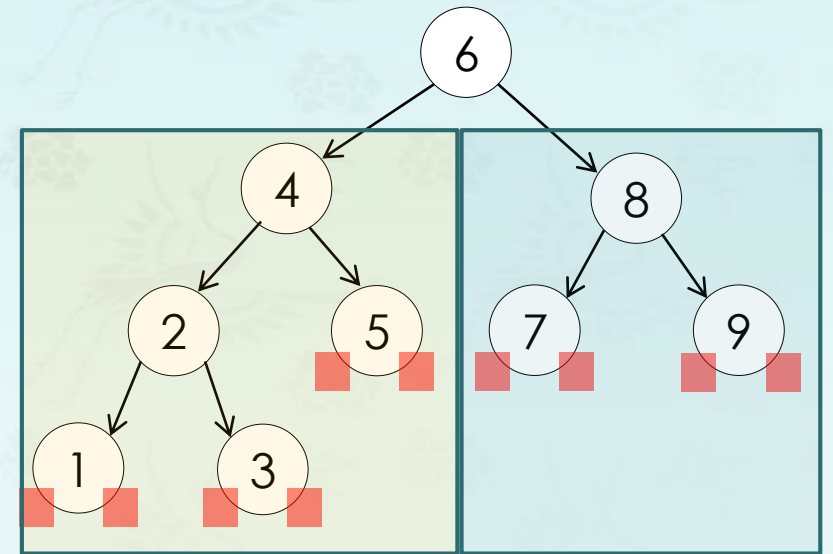
(6) The purple pops from stack & continues where it left.  
 purple `cout << root->key;` ➡ 2

(11) The purple pops from stack & continues where it left.  
 The purple finishes its job and returns to the caller(main).

❖ FC stands for function call.

## Operations: size()

```
// returns the number of nodes in the binary tree
int size(tree node) {
    if (empty(node)) return 0;
    return size(node->left) + size(node->right) + 1;
}
```

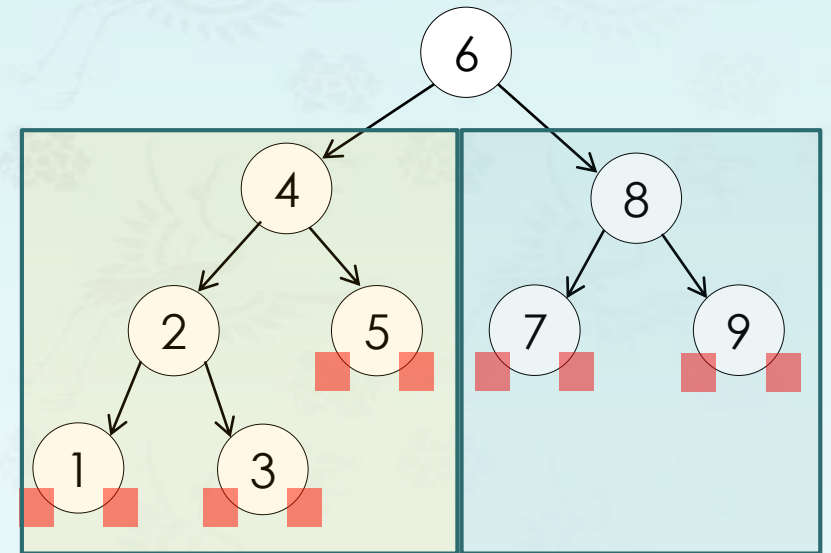


# Operations: size()

```
// returns the number of nodes in the binary tree
int size(tree node) {
    if (empty(node)) return 0;
    return size(node->left) + size(node->right) + 1;
}
```

- Q1. What is the total number of the function calls to complete with the tree and how many returns each?

```
// returns the number of nodes in the binary tree
int size(tree node) {
    if (empty(node)) return 0;
    int left = size(node->left);
    int right = size(node->right);
    return left + right + 1;
} // debug & trace friendly version
```

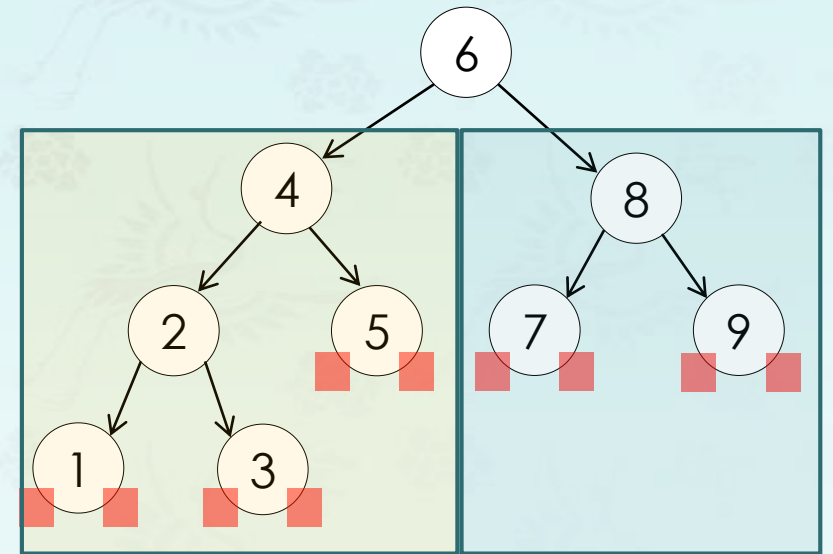




## Operations: size()

```
// returns the number of nodes in the binary tree
int size(tree node) {
    if (empty(node)) return 0;
    return size(node->left) + size(node->right) + 1;
}
```

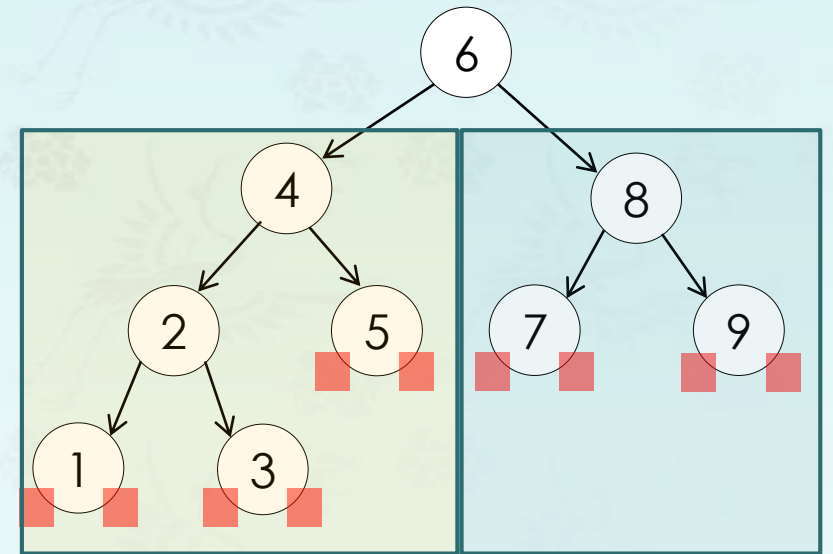
- Q1. What is the total number of the function calls to complete with the tree and how many returns each?
- Q2. Which node invokes the last function call?
- Q3. Which node finishes its size function call and returns size = 1 for the first time?



# Operations: height()

```
// returns the max depth of a tree.  
// height = -1 for empty tree, 0 for root only tree  
int height(tree node) {  
    if (empty(node)) return -1;  
    int left  = height(node->left);  
    int right = height(node->right);  
    return max(left, right) + 1;  
}
```

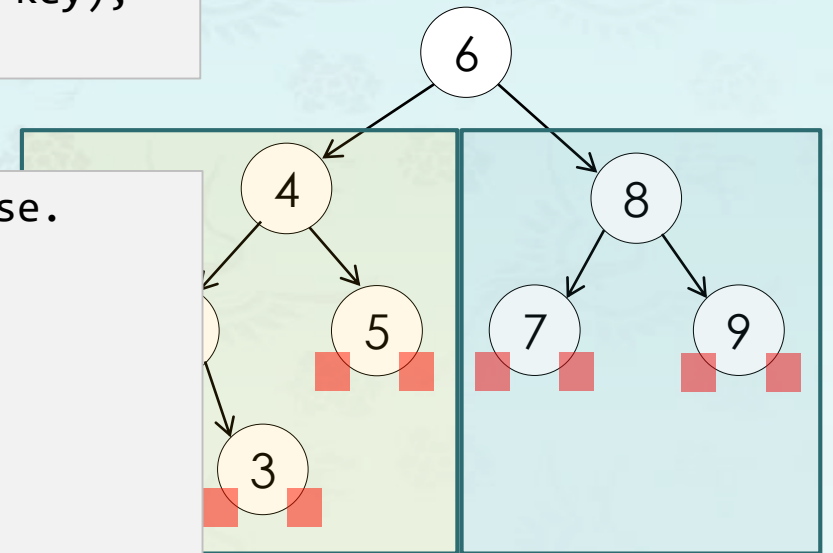
- Q1. What is the total number of the function call to complete with the tree below?
- Q2. What is the return value of the 10<sup>th</sup> and 12<sup>th</sup> function call?
- Q3. What is the return value of the node 2?



## Operations: containsBT(), findBT()

```
// returns true if key is in a given binary tree, false otherwise.  
bool containsBT(tree root, int key) {  
    if (empty(root)) return false;  
    if (key == root->key) return true;  
  
    return containsBT(root->left, key) || containsBT(root->right, key);  
}
```

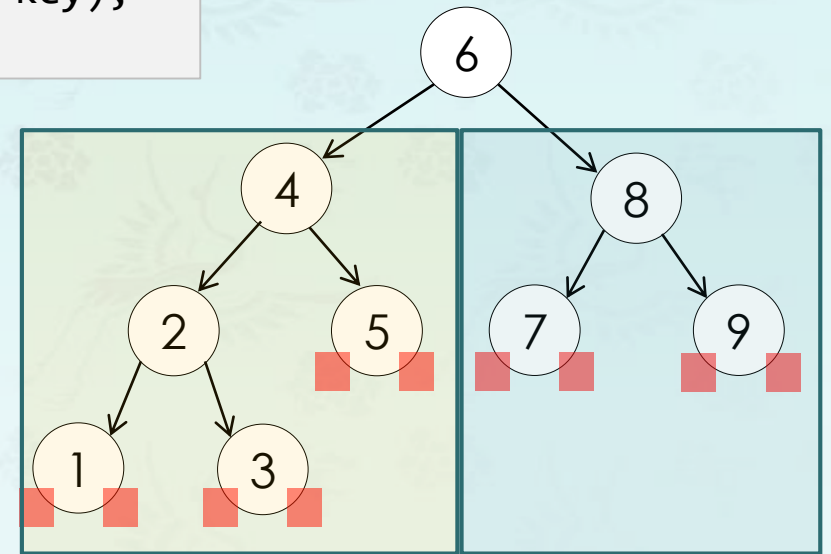
```
// returns true if key is in a given binary tree, false otherwise.  
bool containsBT(tree root, int key) {  
    if (empty(root)) return false;  
    if (key == root->key) return true;  
  
    if (containsBT(root->left, key)  
        return true;  
    if (containsBT(root->right, key)  
        return true;  
    return false;  
} // debug & trace friendly version
```



## Operations: containsBT(), findBT()

```
// returns true if key is in a given binary tree, false otherwise.  
bool containsBT(tree root, int key) {  
    if (empty(root)) return false;  
    if (key == root->key) return true;  
  
    return containsBT(root->left, key) || containsBT(root->right, key);  
}
```

- Q1: Which node invokes **containsBT(root->right, key)** for the first time?
- Q2: Which node will invoke **return false** for the first time?
- Q3: How many function calls are made to reach the node **key=5**?
- Q4: How many function calls still remains in the system stack to finish after key=5 is found and what are they?



# Data Structures

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