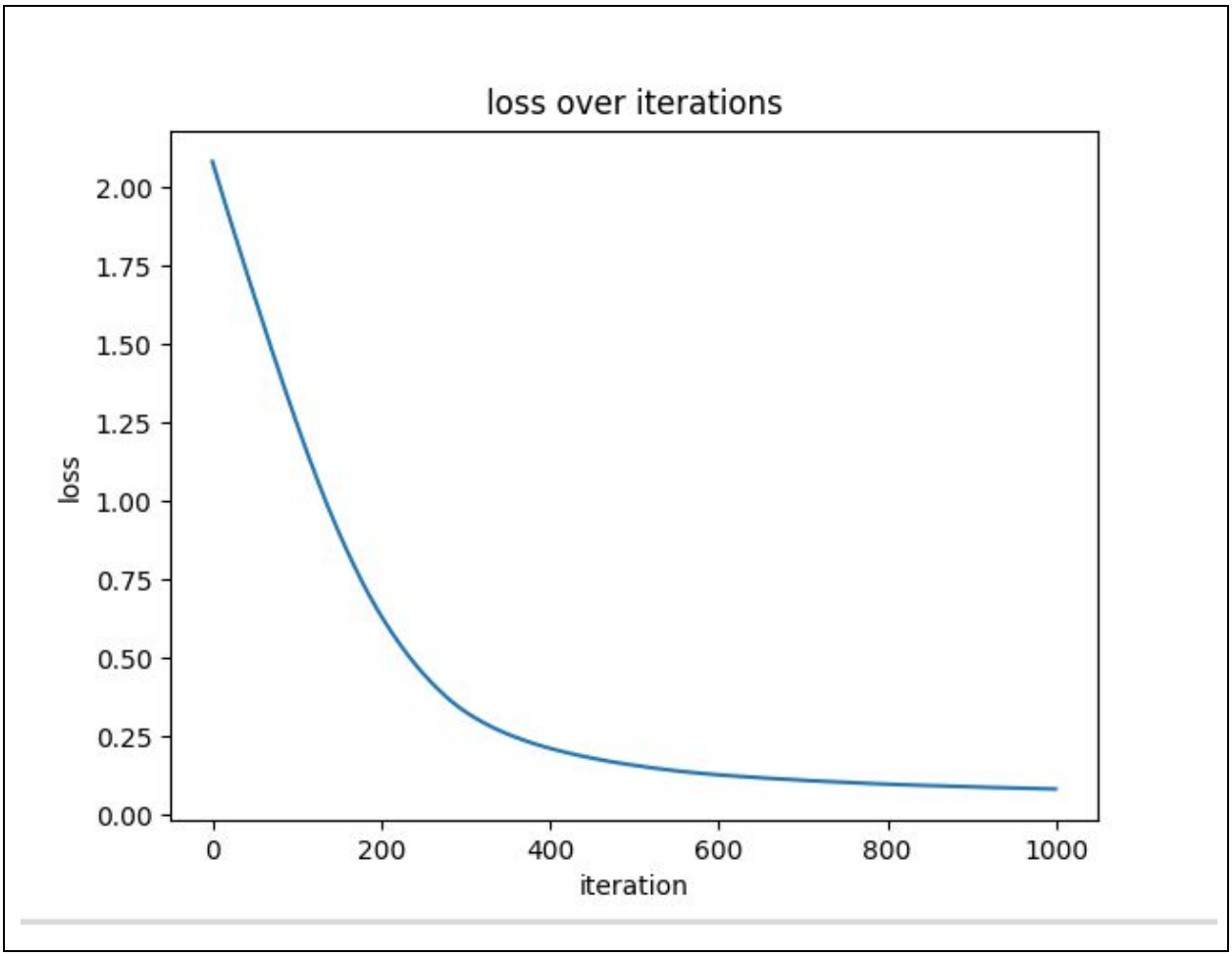
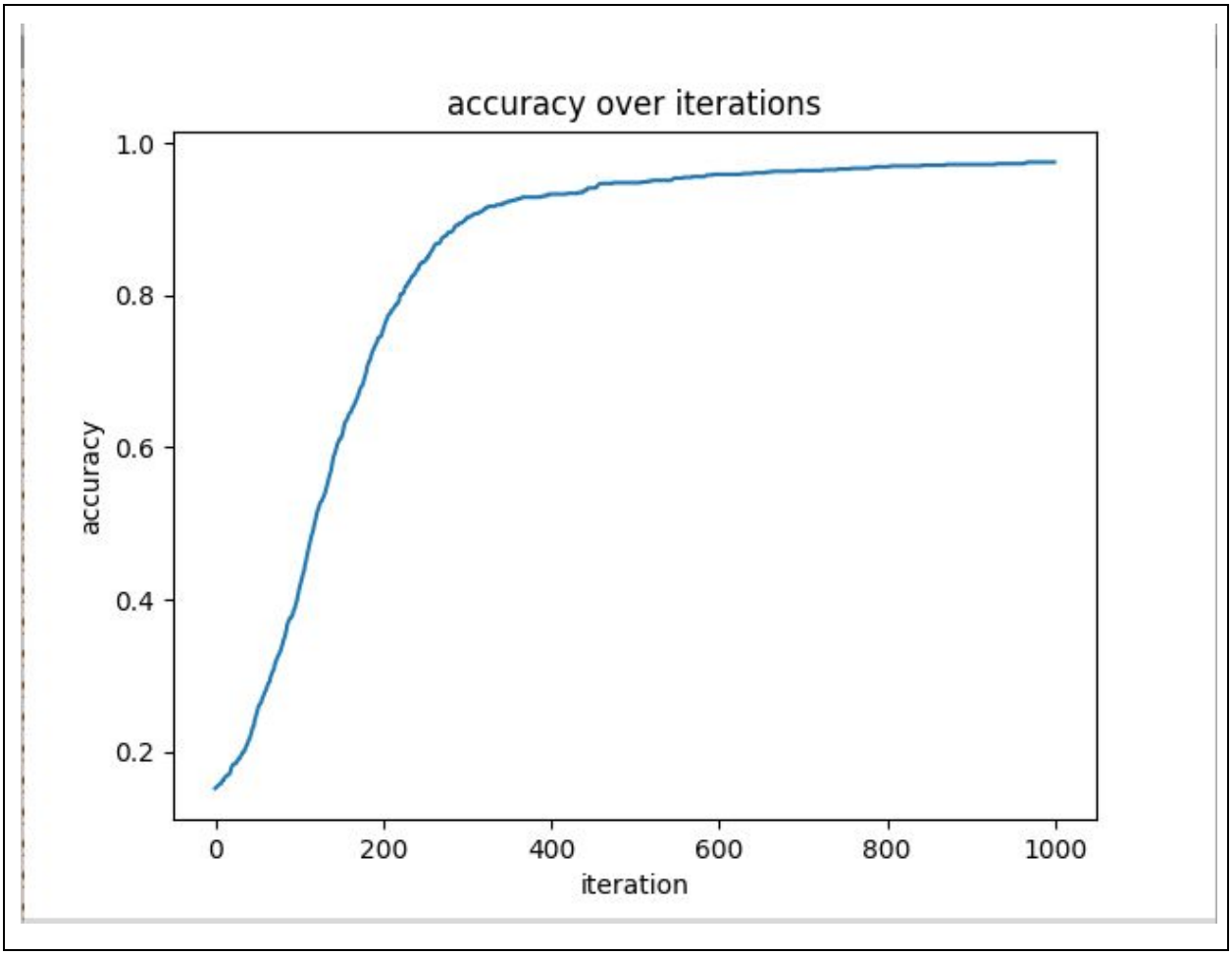
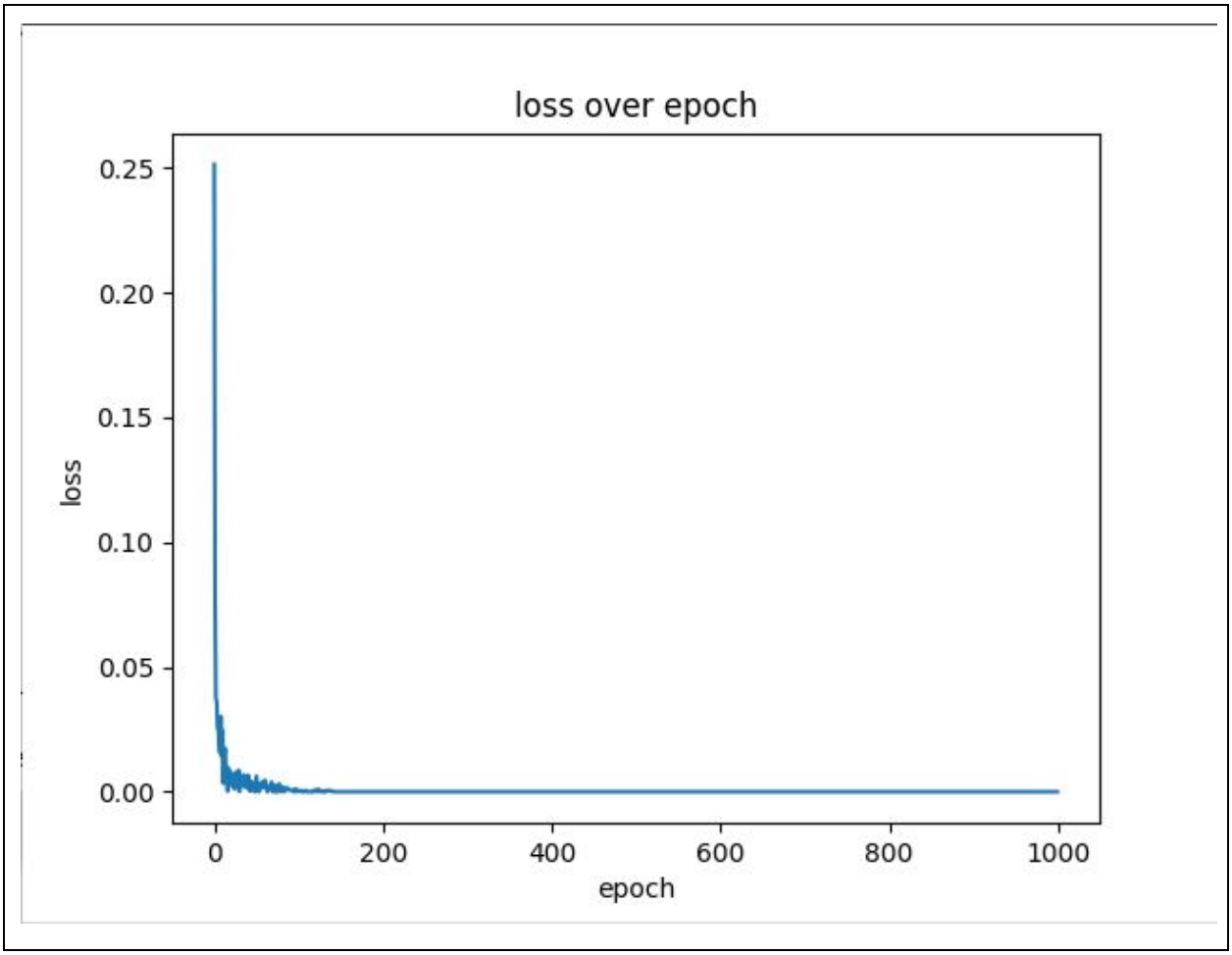
1. (1.25 points) Visualizing Gradient Descent

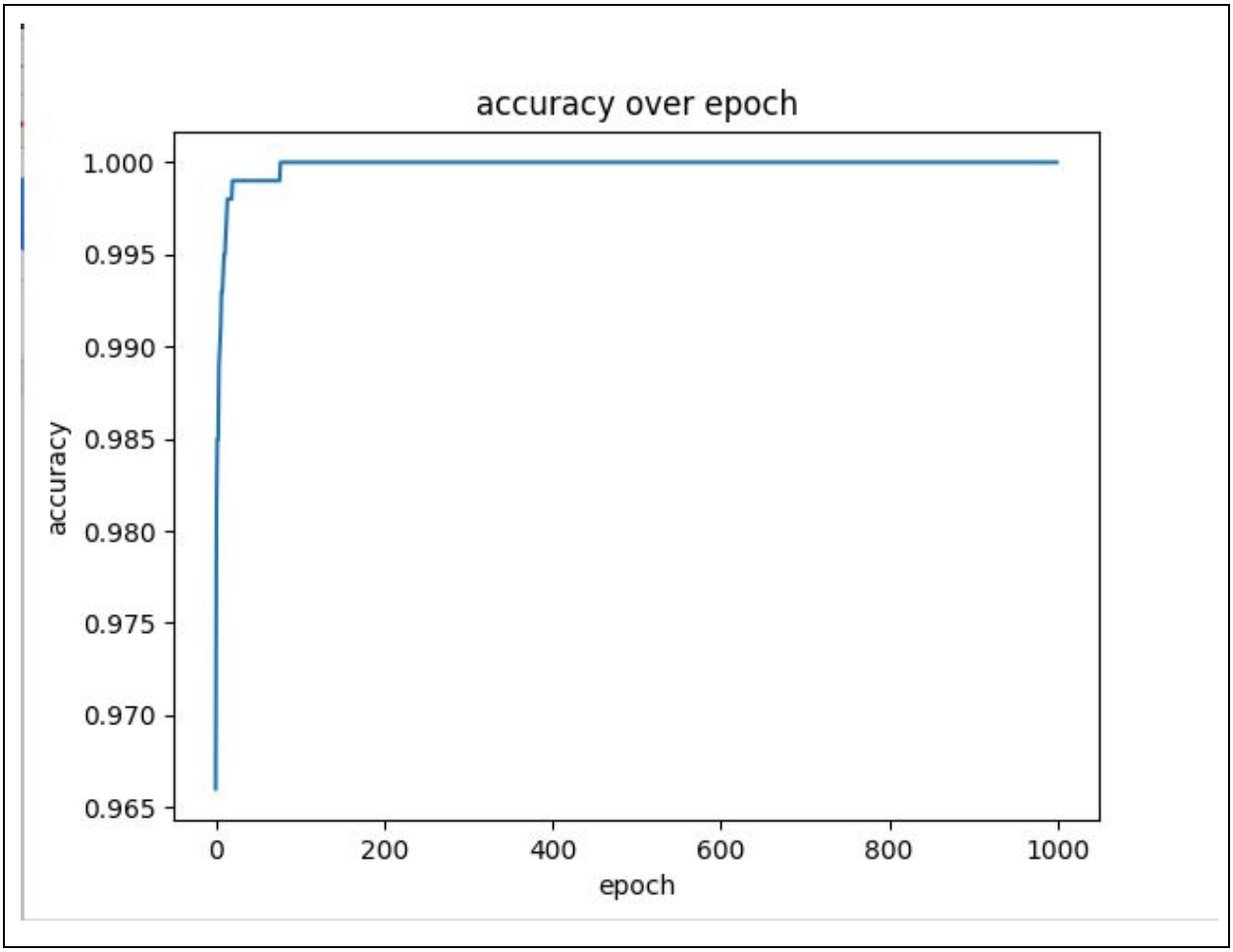
a. (0.5 points) Setup an experiment to classify the mnist-binary dataset (see load_data.py for more on that dataset) using the gradient descent learner. Use fraction=1 when calling load_data. For your loss function, use hinge loss and set the learning rate (i.e., lambda) to 1e-4. Keep all other gradient descent parameters as their defaults. After each training iteration through the dataset (using batch gradient descent on just the training data), compute both the loss and accuracy of the model on the full training dataset. Terminate after convergence or after **1000 iterations (i.e., max_iter = 1000)**. Construct and include **two (labeled) plots of the loss and accuracy at each iteration**. Note that when batch_size is not set, it trains on all available training data for each step.





b. (0.5 points) Repeat the previous experiment, but this time using stochastic gradient descent (i.e., we accomplish this by changing batch_size). Compute and plot the loss and accuracy over the entire dataset after each epoch. One epoch is one iteration through the entire dataset. Note that you will need to increase the value of max_iter to do this. For example, if we have 500 examples, setting max_iter=500 and **batch_size=1** would result in one epoch of stochastic gradient descent. Terminate after convergence or after **1000 epochs**. Construct and include the two (labeled) plots of the loss and accuracy at each epoch.


loss over epoch
0.25
0.20
0.15
0.10
0.05
0.00
loss
0
200
400
600
800
1000
epoch

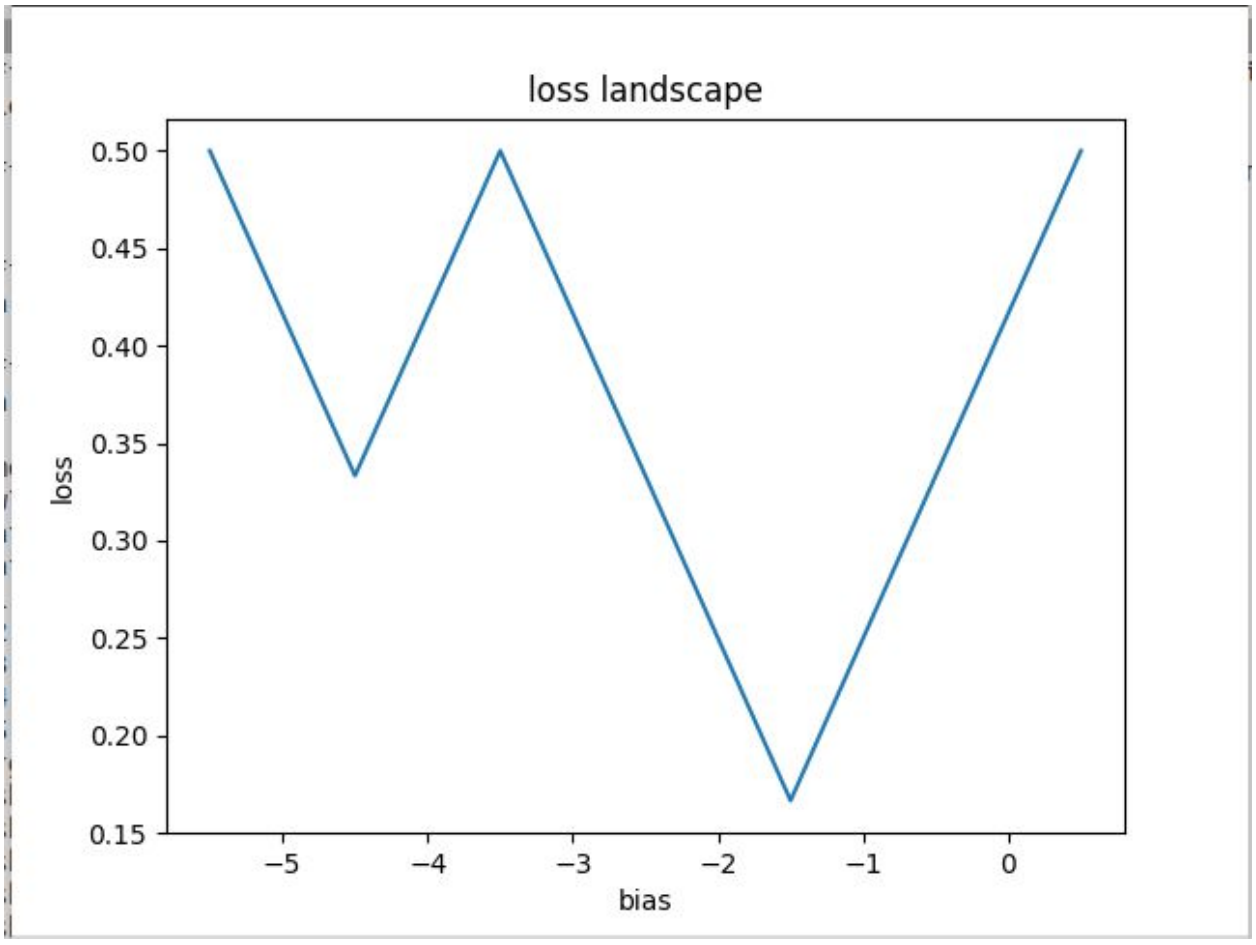


c. (0.25 points) Compare the plots generated from batch gradient descent and stochastic gradient descent. Which one seems to be faster to train? Which one converges to a lower average loss on the training data? Are there differences in the behavior of the loss and accuracy throughout training?

Answer: from the plots, it is easy to see that batch gradient descent is much faster to train as the loss and accuracy converge at a much higher rate. The behaviour difference is primarily the convergence rate in both loss and accuracy, and the oscillation in batch gradient descent when it is close to converge. That is due to the randomness in batch selection.

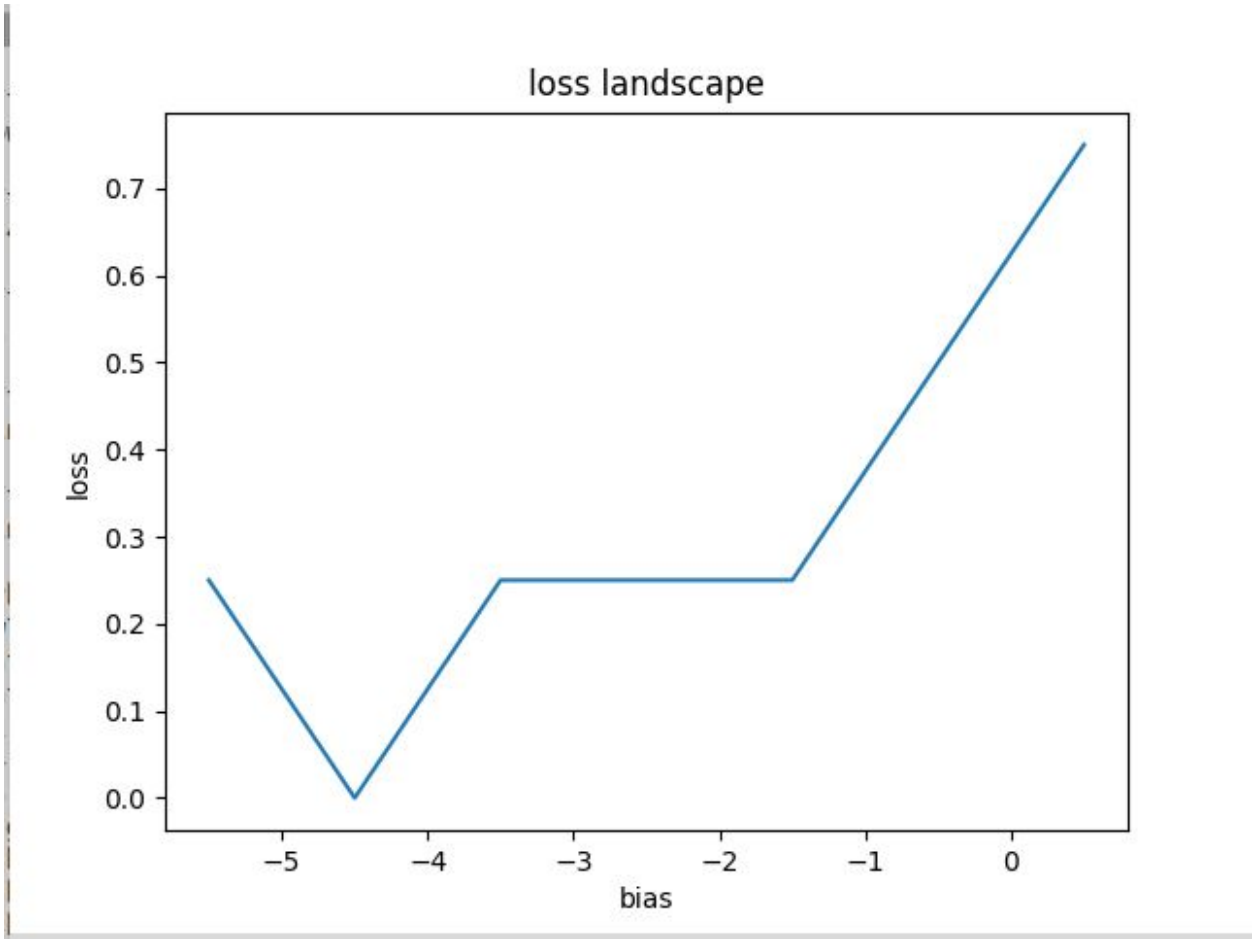
2. (1.25 points) Loss Landscapes and the Effects of Batching

a. (0.5 points) Here we will setup an experiment that will allow us to visualize the loss landscape of the synthetic dataset (see load_data.py for more on that dataset). Here, we define the "loss landscape" as the **loss at every value value of the model parameters with respect to our input data**. For more information about loss landscapes see the lectures slides for gradient descent. For this experiment, we will be looking at the loss landscape of a model with only bias terms, meaning all other **parameters have values of exactly 1**. Note that this model does not do any "learning" because we are setting the weights explicitly (this means you will not be using your fit() function). In your experiment, **first load the synthetic dataset (with fraction=1)**. Using only the bias term, determine **the 0-1 loss over the entire dataset** by explicitly setting your bias values to **0.5, -0.5, -1.5, -2.5, -3.5, -4.5, and -5.5**. Construct a plot of the loss landscape by **plotting the bias on the X axis and the loss on the Y axis. I**nclude your (labeled) plot and describe the minima of the loss landscape.



Answer: the minima is at -1.5. Therefore bias that is in (-3.5, 0.5) will go towards this minima. Otherwise, gradient descent will make the loss converge to a different minima value.

b. (0.5 points) Now we will examine how the loss landscape changes when you select data differently. By analyzing the synthetic dataset and the loss landscape plot you have generated, select **a set of 4 points from the dataset** that, if you optimize considering only that set, cause the **global minimum** of the loss landscape to shift. Repeat your experiment from part (a) on this set of 4 points, plot the resulting loss landscape and compare it to the loss landscape from part (a).



Answer: features with indices 0,1,4,5 were selected. When bias = -4.5, the model can predict equal number of features correctly. However in (a), only when bias = -1.5 can the model predict most features correctly. That is why the global minimas are different.

c. (0.25 points) Based on your answers from part (a) and (b), explain what effect batching can have on the loss landscape and the convergence of gradient descent.

Answer: Batching can change the loss landscape, and may potentially change the global minima gradient descent converges to.

3. (1 point) Multiclass Classification with Gradient Descent

a. (0.5 points) Setup an experiment to classify the **mnist-multiclass** dataset using the One-vs-All gradient descent learner (see load_data.py for more on that dataset). Use **fraction=0.75** when calling load_data. Use **squared loss** and **l1 regularization** with the default learning rate and regularization parameters. Train your learner on the training partition (**report what values you used for batch_size and max_iter**), and **run prediction on the testing partition**. Generate the **confusion matrix** of the testing partition (using the confusion_matrix function in metrics.py) and include the confusion matrix **as a table**. **Label the rows with the ground truth class and the columns with the predicted class**. Describe the **most common misclassifications** made by your system. Which numbers appear to be **most difficult** to distinguish from one another? Which appear to **be easiest** to distinguish?

Answer: Row is ground truth, column is the prediction (sorry this is probably not the best visualization :p). Batch size is 30, max_iter is 700.

pred\ GT	0	1	2	3	4
0	19	1	5	0	0
1	0	25	0	0	0
2	0	9	14	0	2
3	0	7	3	14	1
4	2	6	2	0	15

The most difficult digit to distinguish is digit 1, as it has the lowest recall value. The easiest digit to distinguish is 3, as the recall of 3 is 100%.

b. (0.25 points) In this assignment, we used One-vs-All (OVA) with linear decision boundaries to handle multiclass classification. For a dataset with c classes and d features, **how many linear binary classifiers are needed in OVA classification**? What is the **space complexity of the OVA model** (i.e., the set of learned decision boundaries)?

Answer: you need c number of linear classifiers. The space complexity is c*(d+1).

c. (0.25 points) An alternative to OVA classification is One-vs-One (OVO) classification, in which a binary classifier is trained to discriminate between each pair of classes, and the final prediction is decided via majority vote. For the same dataset considered in part (b), how many binary classifiers are need in OVO classification? What is the space complexity of our OVO model?

Answer: since there are 5 digits, you need C(5,2), which is 5*4/(2*1) = 10 number of OVO linear classifiers.

The space complexity is c*(c-1)/2 * (d+1)

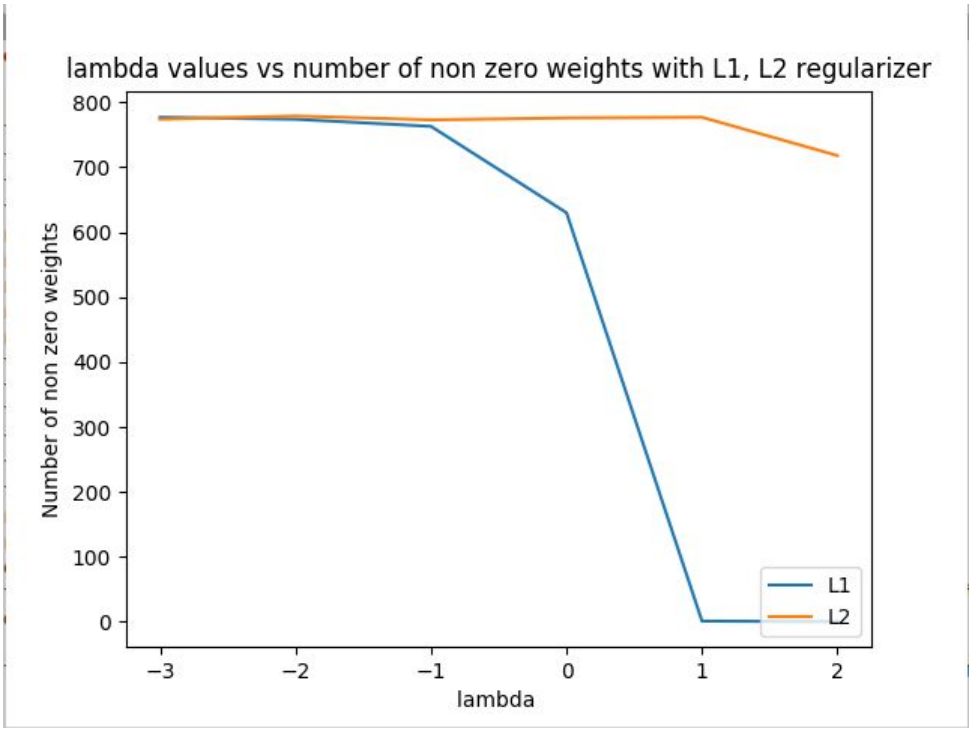
4. (1.5 points) Regularization and Feature Selection

a. (0.5 points) Here we will explore the use of regularization as a means of feature selection. Setup an experiment using the **mnist-binary** dataset. Use **fraction=1** when calling load_data. Run gradient descent on the mnist-binary training dataset using **squared loss**, using both **l1 and l2 regularization**. Set the step size, or learning_rate, to **1e-5** and the maximum number of iterations to **2000**. Report your values for **how many digits from each class you used**, what your **batch size** was, and **how many iterations** you used. For each regularizer, run the **algorithm once** for each of the following values for lambda: [1e-3, 1e-2, 1e-1, 1, 10, 100]. Plot the number of **non-zero model weights** from **the learned model for each value of lambda**. (Here we define non-zero values as values whose absolute value is greater than some epsilon, where **epsilon = 0.001**.) Plot both **regularizers on one plot**. Include your (labeled) plot and describe the trend in non-zero parameters for each regularizer.

Answer:
Number of 0: 500
Number of 1: 500
Batch size: 50

l1 iterations: [1643, 1829, 2000, 2000, 1053, 2000]
l2 iterations: [1643, 1829, 2000, 2000, 1053, 2000]

For L1, number of non zero weights do not significantly change until lambda is over 1. For L2, number of non zero weights drops faster than L1 and flattens after lambda gets larger than1

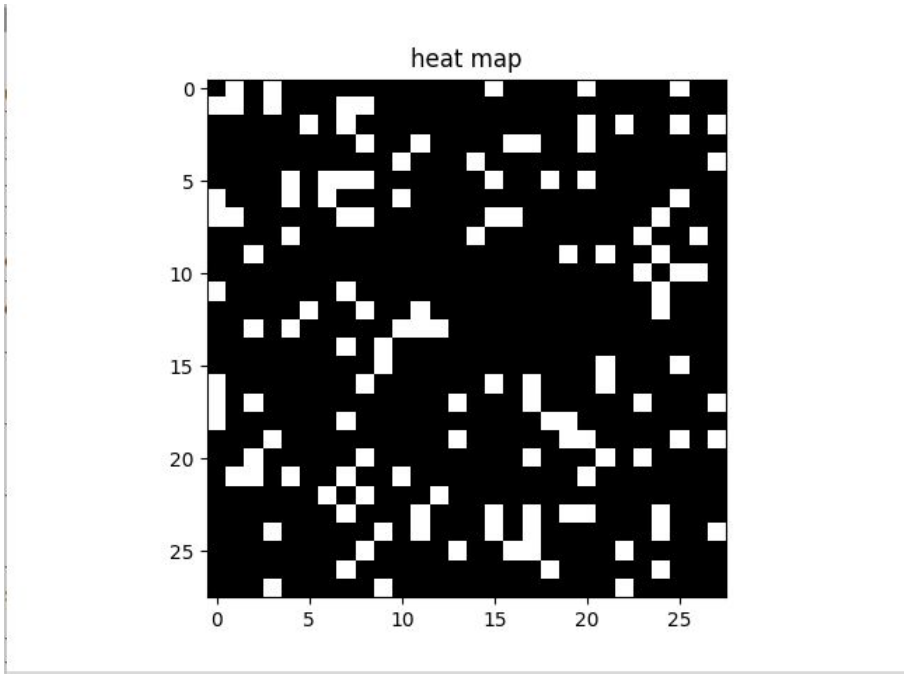


b. (0.5 points) Compared to the L2 regularizer, what property of the L1 regularizer allows it to promote **sparsity** in the model parameters? Describe a situation in which this sparsity is useful.

Answer:
Because L1 regularizer does not penalize high number of non-zero weights as much as L2, and L1 regularizer does not shift w towards zero as much as L2. This property is especially useful when features are "equally useful", i.e, the best fitting model should have a lot of features.

c. (0.5 points) Using **L1** regularization with **reg_param=1**, make a 2D heatmap that shows which **weights** in a trained model have non-zero values (as defined in problem 4a). Your heatmap should only have two colors--**one color for non-zero values and another color for zero values**. Make sure to clearly label which color corresponds to zero and non-zero values. This heatmap should have the same shape as your **input images**. Is there any discernible pattern as to which values are zero?

Answer: The black pixels are non zero terms, while the white ones are zero terms. Zero terms seem to be distributed quite randomly across columns, but there tend to be more consecutive zero terms across rows.


heat map
0
5
10
15
20
25
0
5
10
15
20
25