Digitise Optimise Visualise – Optimisation Theory and Practice

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Course objective

The aim of this coure is to give students a working knowledge of optimisation and its applications in Economics and Finance. I will put emphasis on formulating and implementing problems in a programming environment, with applications to real datasets.

Course description

The course will give an application-based overview of optimisation methods, with a focus on convex optimisation and duality relationships, which are the cornerstone of a modern approach to data science in Finance and Economics. At every step of the theoretical developments, we will be leaning on geometric visualisations in order to help better understand the problem. We will start with the theory of convex sets, and how to describe them with linear inequalities in linear vector spaces. With this foundation, we will introduce the notion of cones, which we will then use for defining a generalized notion of an inequality, and thus we will be able to define convex sets in more complicated spaces. Subsequently, we will move to the theory of convex functions on the multidimensional real vector space, and we will examine and interpret the notion of conjugate functions and dual spaces, with a focus on dual cones. Armed with these notions, we will move to formulating and solving optimisation problems. At each step – in linear, quadratic, conic problemc etc. – we will present relevant examples from economics and finance, that you will be able to experiment with based on real datasets. Some of these problems will turn our to be related to machine learning techniques. Finally, we will study general convex and non-convex optimisation problems and duality relationships. Throughout the course, we will be using Python and the `cvxopt` library to work with examples.

Material

The course will be based on Chapters 2-5, and excerpts from Chapters 6, 7 and 8 of "Convex Optimization" by Boyd and Vanderberghe. All concepts and exercises will be presented in the form of Jupyter notebooks with Python code illustrating the concepts. We will supplement these with relevant scientific literature on which the applications are based.

Grading

Grading will be based on an individual solution to a take-home exam.

Detailed contents

1. A glimpse into your nearest future
   - An example optimisation problem
     - geometric intuition of solution
   - Convex and non-convex problems
     - Convex problems are easy to solve
   - How to learn whether a solution exists?
   - How to give a bound on the optimal value?
     - Assessment of solutions to non-convex problems
2. Convex sets
   - Definition and geometric intuition
   - Operations preserving convexity
   - Cones and proper cones
   - Separating and supporting hyperplanes
   - Dual cones and generalized inequalities
3. Convex functions
   - Definition of convex/concave function
     - First- and second- order conditions for convexity
     - Examples: utility functions are concave
   - Sublevel sets and epigraphs
   - The conjugate function
     - Geometric interpretation
     - Examples
     - Fenchel's inequality
   - Convexity with respect to generalized inequalities
4. Optimisation problems
   - Standard form
     - Local and global optima
   - Convex (concave) optimisation problems
   - Optimality criteria
   - Linear programming
   - Quadratic programming
   - Second-order cone programming
     - Uncertainty and the mean-variance frontier
5. Duality and Karush-Kuhn-Tucker conditions
   - The Lagrangian and the Lagrange dual function
     - A lower bound for the minimum
     - Examples

- Lagrange dual function and conjugate functions
  - Polyhedral feasible sets and linear equalities
  - Examples (minimum entropy solutions to linear equation systems)
- Lagrange dual optimisation problem
  - Weak and strong duality, constraing qualification conditions
  - Geometric interpretations
- Karun-Kush-Tucker and complementary slackness conditions
- Primal solutions via the dual problem

6. Applications
   - Linear Programming:
     - Conditional Value-at-Risk (Expected Shortfall) and portfolio optimisation (Krokhmal and Uryasev, 2002)
   - Quadratic Programming:
     - Penalised regression: Lasso and application to Return Predictability (Zou, 2006; Goyal and Welch, 2008; Lee et al., 2018)
     - Markowitz portfolio optimisation: the mean-variance frontier (Markowitz, 1952; Cochrane, 2009; Back, 2014, or any other graduate Finance text; look up the original source only for a historical perspective)
     - From penalised regression to Support Vector Machines for classification
   - Conic Programming:
     - Markowitz portfolio selection under uncertainty about the parameters (Tütüncü and Koenig, 2004)

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