Exercise 1: Entropy

A fair dice is rolled at the same time as a fair coin is tossed. Let A be the number on the upper surface of the dice and let B describe the outcome of the coin toss, where

$$B = \begin{cases} 1, & \text{head}\,, \\ 0, & \text{tail}\,. \end{cases}$$

Two random variables X and Y are given by $X = A + B$ and $Y = A - B$, respectively.

(a) Calculate the entropies $H(X)$ and $H(Y)$, the conditional entropies $H(Y|X)$ and $H(X|Y)$, the joint entropy $H(X, Y)$ and the mutual information $I(X; Y)$.

(b) Show that, for independent discrete random variables X and Y,

$$I(X; X + Y) - I(Y; X + Y) = H(X) - H(Y)$$