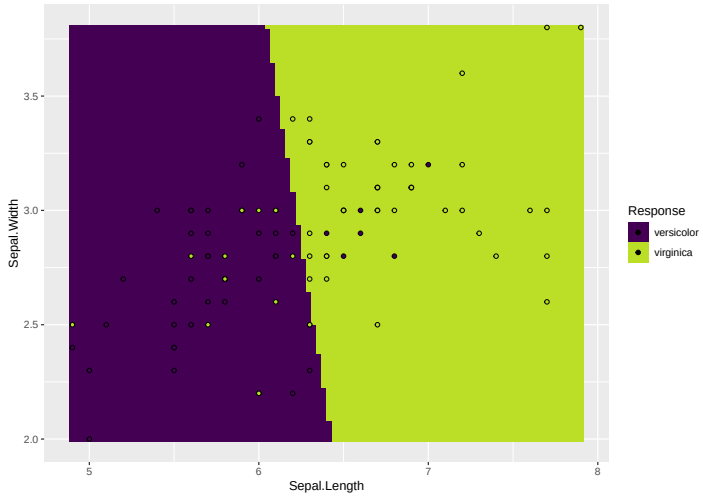
Introduction to Machine Learning

Classification: Linear Classifiers



Learning goals

- Know the definition of a linear classifier

LINEAR CLASSIFIERS

Linear classifiers are an important subclass of classification models. If the discriminant function(s) $f_k(\mathbf{x})$ can be specified as linear function(s) (possibly through a rank-preserving, monotone transformation $g : \mathbb{R} \to \mathbb{R}$), i. e.

$$g(f_k(\mathbf{x})) = \boldsymbol{w}_k^\top \mathbf{x} + b_k,$$

we will call the classifier a **linear classifier**.

NB: $\boldsymbol{w}_k$ and b_k do not directly refer to the parameters $\boldsymbol{\theta}_k$ of k-th scoring function f_k but the transformed version.

LINEAR CLASSIFIERS

We can also easily show that the decision boundary between classes i and j is a hyperplane. For every $\mathbf{x}$ where there is a tie in scores:

$$\begin{aligned} f_i(\mathbf{x}) &= f_j(\mathbf{x}) \\ g(f_i(\mathbf{x})) &= g(f_j(\mathbf{x})) \\ \boldsymbol{w}_i^\top \mathbf{x} + b_i &= \boldsymbol{w}_j^\top \mathbf{x} + b_j \\ (\boldsymbol{w}_i - \boldsymbol{w}_j)^\top \mathbf{x} + (b_i - b_j) &= 0 \end{aligned}$$

This is a **hyperplane** separating two classes.

LINEAR VS NONLINEAR DECISION BOUNDARY