

### Exercise 1: Kernels

A **(Mercer) kernel** on a space  $\mathcal{X}$  is a continuous function

$$k : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}$$

of two arguments with the properties

- Symmetry:  $k(\mathbf{x}, \tilde{\mathbf{x}}) = k(\tilde{\mathbf{x}}, \mathbf{x})$  for all  $\mathbf{x}, \tilde{\mathbf{x}} \in \mathcal{X}$ .
- Positive definiteness: For each finite subset  $\{\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(n)}\}$  the **kernel Gram matrix**  $K \in \mathbb{R}^{n \times n}$  with entries  $K_{ij} = k(\mathbf{x}^{(i)}, \mathbf{x}^{(j)})$  is positive semi-definite.

Which of the following functions are kernels? Justify your answer in each case.

(a)  $k(\mathbf{x}, \tilde{\mathbf{x}}) = \mathbf{x}^\top \tilde{\mathbf{x}}$ , where  $\mathcal{X} \subset \mathbb{R}^p$ .

(b)  $k(x, \tilde{x}) = \cos(x + \tilde{x})$ , where  $\mathcal{X} \subset \mathbb{R}$ .

(c)  $k(x, \tilde{x}) = \max(x, \tilde{x})$ , where  $\mathcal{X} \subset \mathbb{R}_+$ .

(d)  $k(\mathbf{x}, \tilde{\mathbf{x}}) = \alpha k_1(\mathbf{x}, \tilde{\mathbf{x}}) + \beta k_2(\mathbf{x}, \tilde{\mathbf{x}})$ , where  $k_1$  and  $k_2$  are kernels and  $\alpha, \beta \geq 0$  as well as  $\mathcal{X} \subset \mathbb{R}^p$ .

- (e) Quiz time: Log in to Particify (<https://partici.fi/63221686>) and try to answer the questions for **Week 11**.