

# Introduction to Machine Learning

## Evaluation: Multi-Class AUC

$AUC(pos|neg) = AUC(1|3)$

	Y	$\hat{\pi}_1$	$\hat{\pi}_2$	$\hat{\pi}_3$
pos	1	0.7	0.2	0.1
pos	1	0.5	0.3	0.2
	2	0.3	0.5	0.2
	2	0.4	0.5	0.1
neg	3	0.6	0.1	0.3
neg	3	0.1	0.1	0.8

$AUC(pos|neg) = AUC(3|1)$

	Y	$\hat{\pi}_1$	$\hat{\pi}_2$	$\hat{\pi}_3$
neg	1	0.7	0.2	0.1
neg	1	0.5	0.3	0.2
	2	0.3	0.5	0.2
	2	0.4	0.5	0.1
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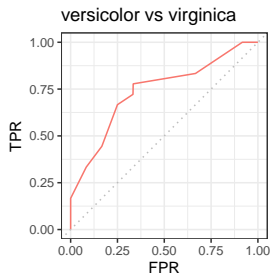
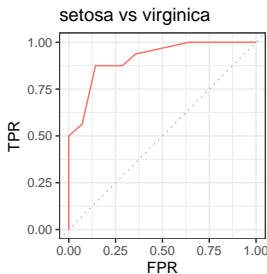
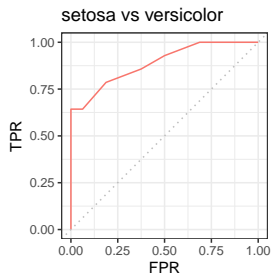
### Learning goals

- Understand that generalizing AUC to multi-class is not trivial
- Learn how multi-class AUC can be derived

# MULTI-CLASS AUC

- AUC and other ROC metrics for binary classification
- Different ways to estimate **multi-class AUC**
- Often based on aggregated binary AUCs:  
e.g. 1-vs-1 or 1-vs-rest

## Example: 1-vs-1 on iris



# MULTI-CLASS AUC

- Def  $\text{AUC}(k | \ell)$  for classes  $k$  (pos) and  $\ell$  (neg)
- Compute AUC: Subset preds to rows of true  $k$  and  $\ell$ , use  $\hat{\pi}_k$
- Interpret: Prob that random member of  $\ell$  has a lower prob to belong to class  $k$  than random member of class  $k$ .

**Example:**  $\text{AUC}(3|1)$  with  $g = 3$  classes

$\text{AUC}(\text{pos}|\text{neg}) = \text{AUC}(3|1)$

	Y	$\hat{\pi}_1$	$\hat{\pi}_2$	$\hat{\pi}_3$
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- 1 Subset pred rows to true classes 1 and 3
- 2 Use  $k = 3$  as pos and  $\ell = 1$  as neg class
- 3 Compute standard AUC with  $\hat{\pi}_3$  as scores
- 4  $\text{AUC}(3|1) = 1$ :  
all pos have higher  $\hat{\pi}_3$  than negs

# MULTI-CLASS AUC

- For binary classes: always  $AUC(1|0) = AUC(0|1)$
- For multi-class usually:  $AUC(k | \ell) \neq AUC(\ell | k)$
- **Example** with  $g = 3$  where  $AUC(1|3) \neq AUC(3|1)$ :
  - $AUC(3|1) = 1$  (RHS) as before
  - $AUC(1|3) \neq 1$  (LHS)

$AUC(pos|neg) = AUC(1|3)$

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# MULTI-CLASS AUC

Hand and Till (2001) proposed to avg AUC via **1-vs-1**:

- For all class pairs, compute  $\text{AUC}(k | \ell)$ .

$$\text{AUC}_{MC} = \frac{1}{g(g-1)} \sum_{k \neq \ell} \text{AUC}(k | \ell) \in [0, 1].$$

## Comments:

- Other defs use **1-vs-rest** and need to avg only  $g$  AUC values
- 1-vs-rest creates imbal classes even if orig classes are balanced
- Imbalanced classes can be considered by weighting individual AUC values with class priors [Ferri et al. (2003)]