

Exercise 1: Risk Minimizers for the L2-Loss

Assume that the feature space is $\mathcal{X} = [-10, 10]$ and the label space is $\mathcal{Y} = \mathbb{R}$. We consider the L2-loss:

$$L(y, f(\mathbf{x})) = (y - f(\mathbf{x}))^2.$$

Assume that $p_x \sim \text{Unif}(-10, 10)$ and that $p_{y|x=x} \sim \mathcal{N}(a + bx, 1)$ for some $a, b \in \mathbb{R}$.

(i) What is f^* ?

(ii) What is its risk?

(iii) What is \bar{f} (the optimal constant model in terms of the theoretical risk)?

(iv) What is its risk? (Hint: $\mathbb{E}_x(x^2) = 100/3$.)