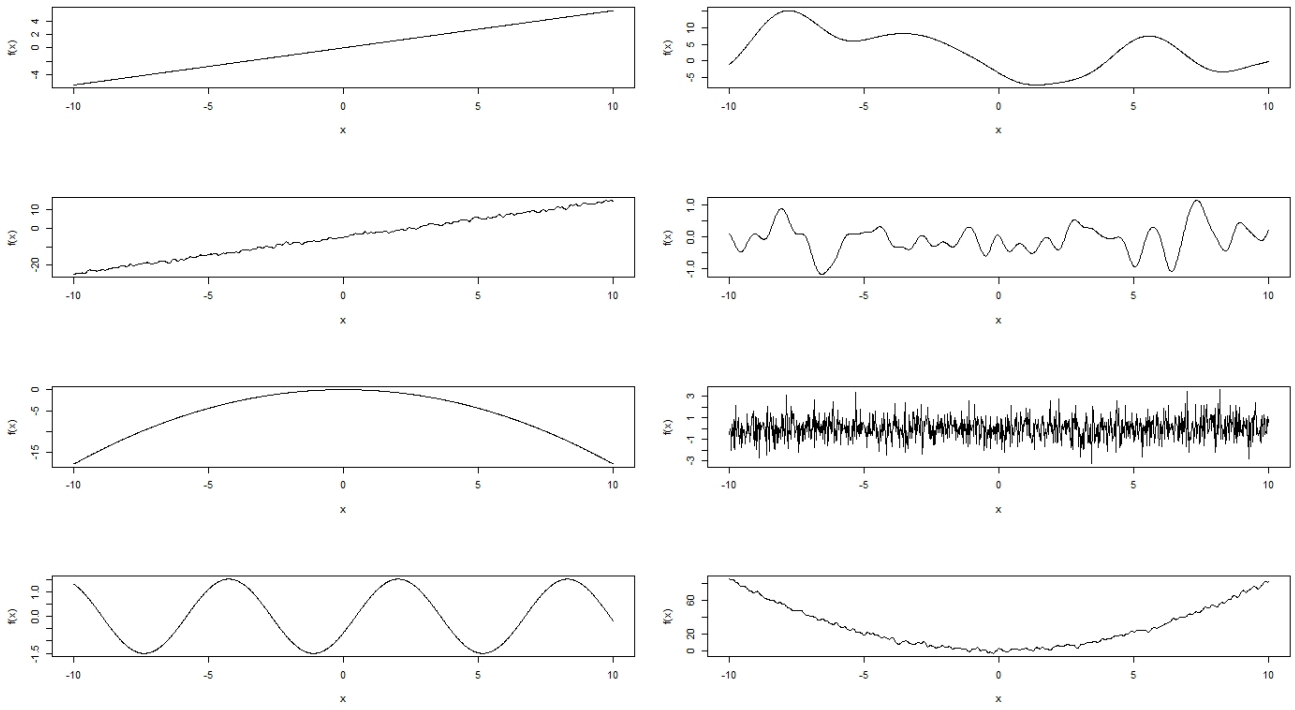
Exercise 1: Gaussian Processes - Covariance Function

Consider $\mathcal{X} = \mathbb{R}$. The following plot shows eight functions

$$f : \mathcal{X} \to \mathbb{R}$$
$$x \mapsto f(x)$$

randomly drawn from eight different Gaussian processes, each of which has a mean function of zero.



The covariance functions are one of those listed below. Indicate which of the functions above is most likely to have been drawn from the Gaussian process $\mathcal{GP}(0, k(x, x'))$ with that covariance function.

(a) $k(x, x') = \mathbb{1}_{[x=x']}$.

(b) $k(x, x') = x \cdot x'$.

(c) $k(x, x') = 0.5 \cdot x^2 \cdot (x')^2$.

(d) $k(x, x') = 0.5^2 \cdot \exp\left(-\frac{(x-x')^2}{0.5^2}\right)$.

(e) $k(x, x') = \cos(x - x')$.

(f) $k(x, x') = 8^2 \cdot \exp\left(-\frac{(x-x')^2}{5}\right)$.

(g) $k(x, x') = 25 + 25 \cdot x \cdot x' + 0.25 \cdot \exp\left(-\frac{(x-x')^2}{0.1^2}\right)$.

(h) $k(x, x') = 2 \cdot x^2 \cdot (x')^2 + 2 \cdot \exp\left(-\frac{(x-x')^2}{0.1^2}\right)$.