Laboration 3

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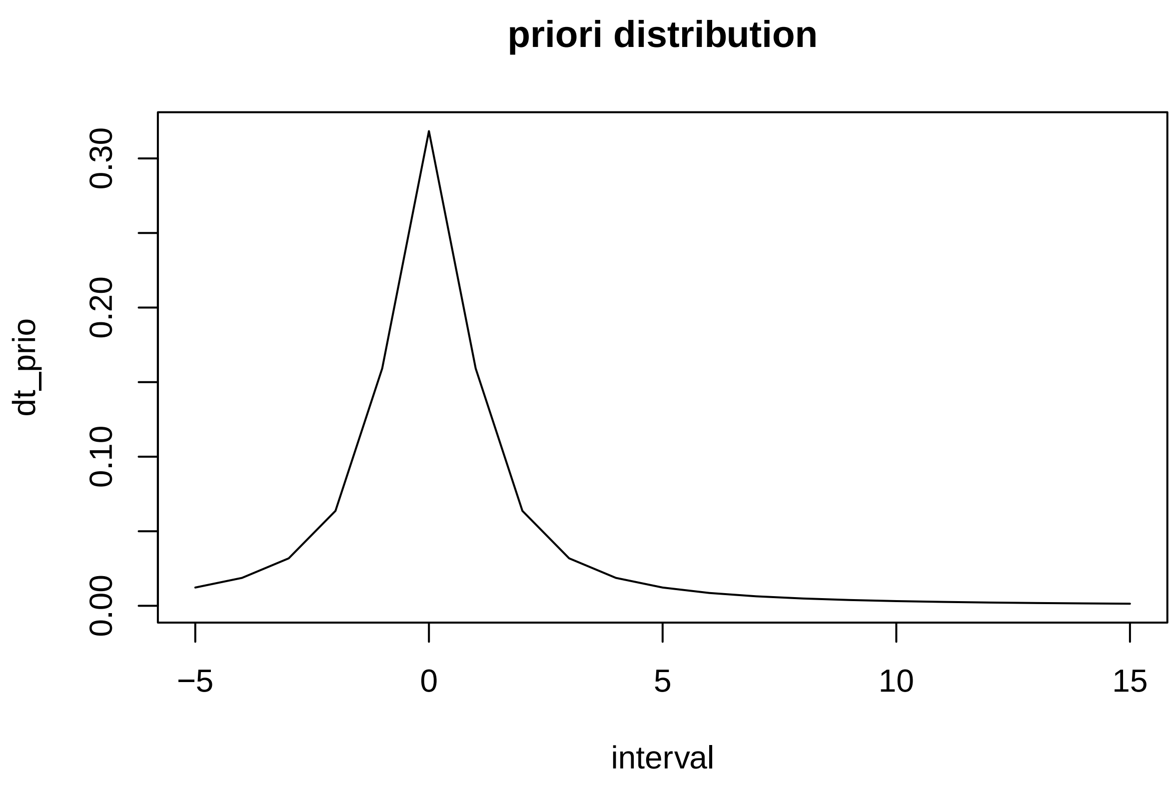
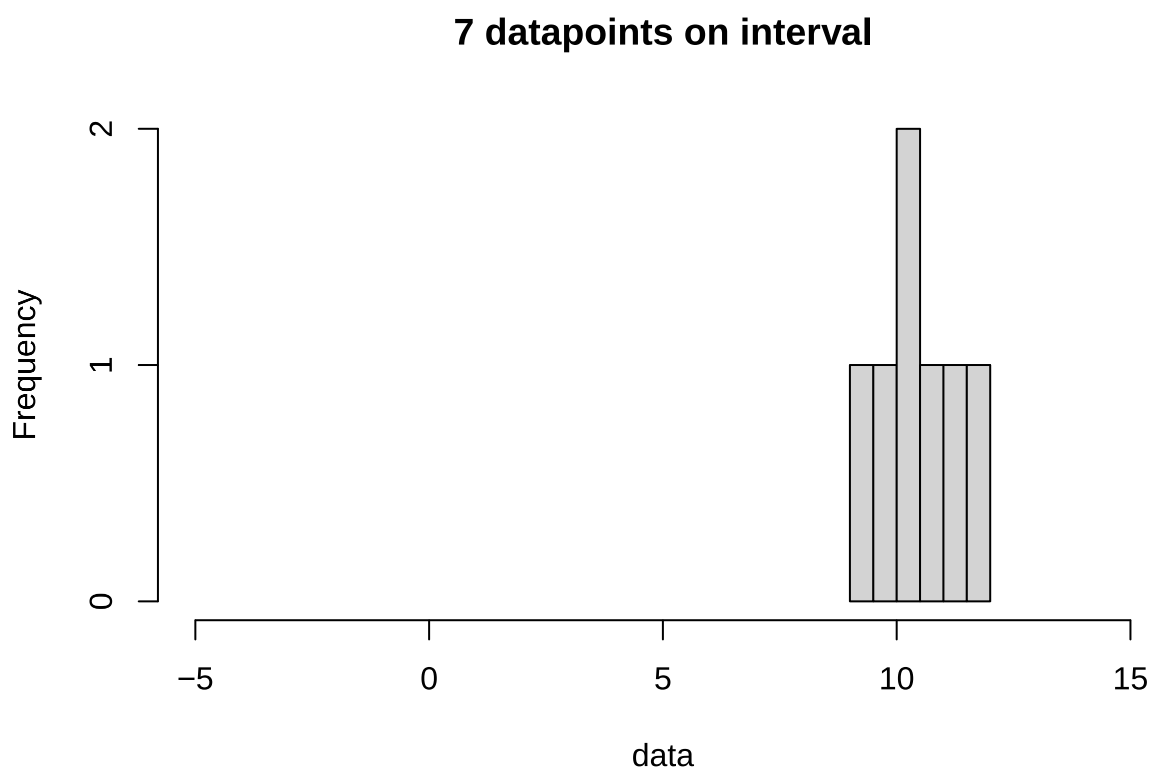
Uppgift 3.1.1

```
# (1)
interval <- seq(-5, 15, 1)
dt_prio <- dt(interval, 1)
plot(interval, dt_prio, type="l", main="priori distribution")
```



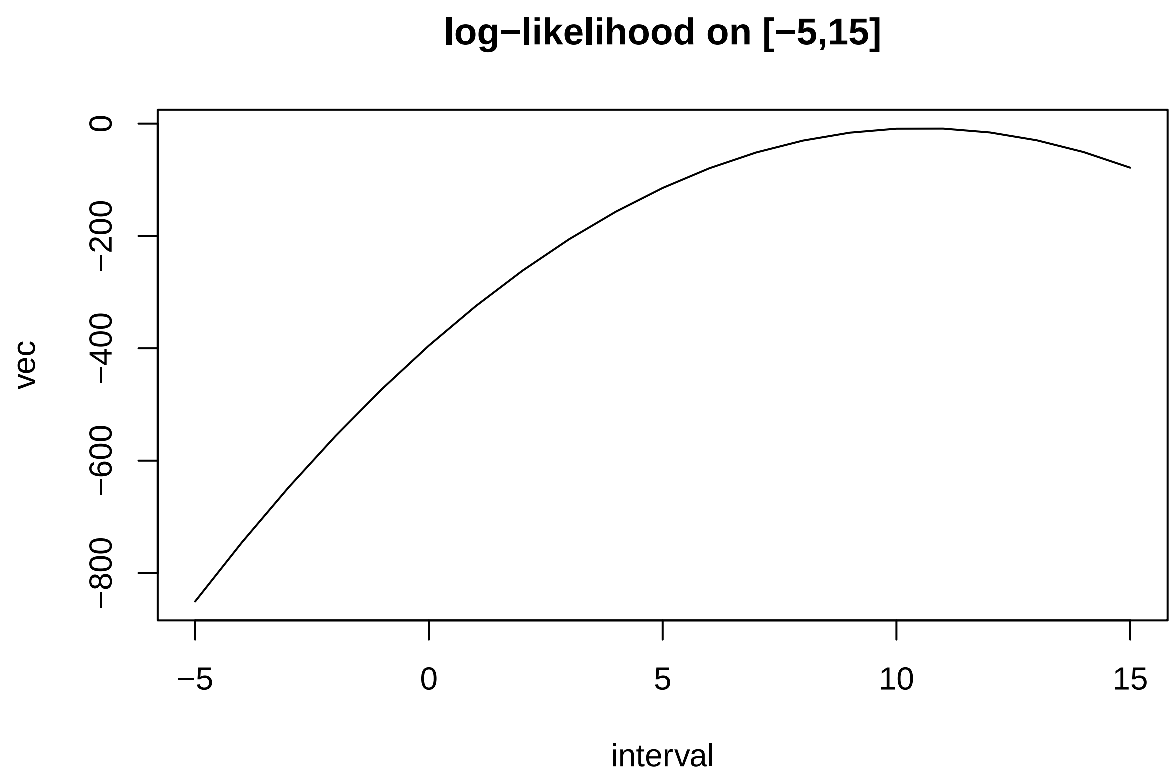

```
#lines(interval, dt_prio, col="blue", lwd=2)

# (2)

data <- c(11.3710, 9.4353, 10.3631, 10.6329, 10.4043, 9.8939, 11.5115)
hist(data, xlim= c(-5, 15), main="7 datapoints on interval")
```




```
# (3)

normal_log_likelihood <- function(mu, data){
  -(length(data)/2)*log(2*pi)-(length(data)/2)*log(1)-(1/(2*1))*sum((data-mu)*(data-mu))
}
llik <- normal_log_likelihood(-5, data)
round(llik, 1)
```

```
## [1] -850.7
```

```
vec <- c(length=length(interval))

for (i in seq_along(interval)) {
  vec[i] <- normal_log_likelihood(interval[i], data)
}

plot(interval, vec, type="l", main="log-likelihood on [-5,15]")
```




```
# (4)
```

$f(\theta \mid y) \propto f(y \mid \theta) * f(\theta)$

We change θ into μ and we get:

$f(\mu \mid y) \propto f(y \mid \mu) * f(\mu)$

The likelihood function is as follows:

$f(y \mid \mu) = (2\pi\sigma^2)^{-n/2} exp\left(-\frac{1}{2\sigma^2}\sum_{j=1}^{n}(y_j - \mu)^2\right)$

Since we are given $\sigma = 1$, just change to 1 where its present:

$f(y \mid \mu) = (2\pi)^{-n/2} exp\left(-\frac{1}{2}\sum_{j=1}^{n}(y_j - \mu)^2\right) = (2\pi)^{-n/2} exp\left(-\frac{1}{2}\sum_{j=1}^{n}(y_j - \mu)^2\right)$

Next step we remove terms without μ in them, as the tip suggested in the lab document:

$f(y \mid \mu) = exp\left(-\frac{1}{2}\sum_{j=1}^{n}(y_j - \mu)^2\right)$

Now the likelihood function part is done, next up is the Priori.
The Priori is a t-distribution due to its degrees of freedom. The PDF for the distribution is as follows:

$f(\mu) = \frac{\Gamma\frac{v+1}{2}}{\sqrt{v*\pi}*\Gamma\frac{v}{2}} * \left(1 + \frac{\mu^2}{v}\right)^{-\frac{v+1}{2}}$

As with σ, we are given the value of v for this distribution. Plug in 1 where v is:
Also, $\Gamma(1)$ is $0! = 1$.

$$f(\mu) = \frac{\Gamma\frac{1+1}{2}}{\sqrt{1*\pi*}\Gamma\frac{1}{2}} * \left(1 + \frac{\mu^2}{1}\right)^{-\frac{1+1}{2}} = \frac{\Gamma\frac{2}{2}}{\sqrt{\pi*}\Gamma\frac{1}{2}} * \left(1 + \mu^2\right)^{-\frac{2}{2}} = \frac{\Gamma(1)}{\sqrt{\pi*}\Gamma\frac{1}{2}} * \frac{1}{(1+\mu^2)} = \frac{1}{\sqrt{\pi*}\Gamma\frac{1}{2}*(1+\mu^2)}$$

Multiply the likelihood with the prior to get the posterior:

$$f(\mu \mid y) = exp\left(-\frac{1}{2}\sum_{j=1}^{n}(y_j - \mu)^2\right) * \frac{1}{\sqrt{\pi*}\Gamma\frac{1}{2}*(1+\mu^2)} = \frac{exp\left(-\frac{1}{2}\sum_{j=1}^{n}(y_j-\mu)^2\right)}{\sqrt{\pi*}\Gamma\frac{1}{2}*(1+\mu^2)}$$

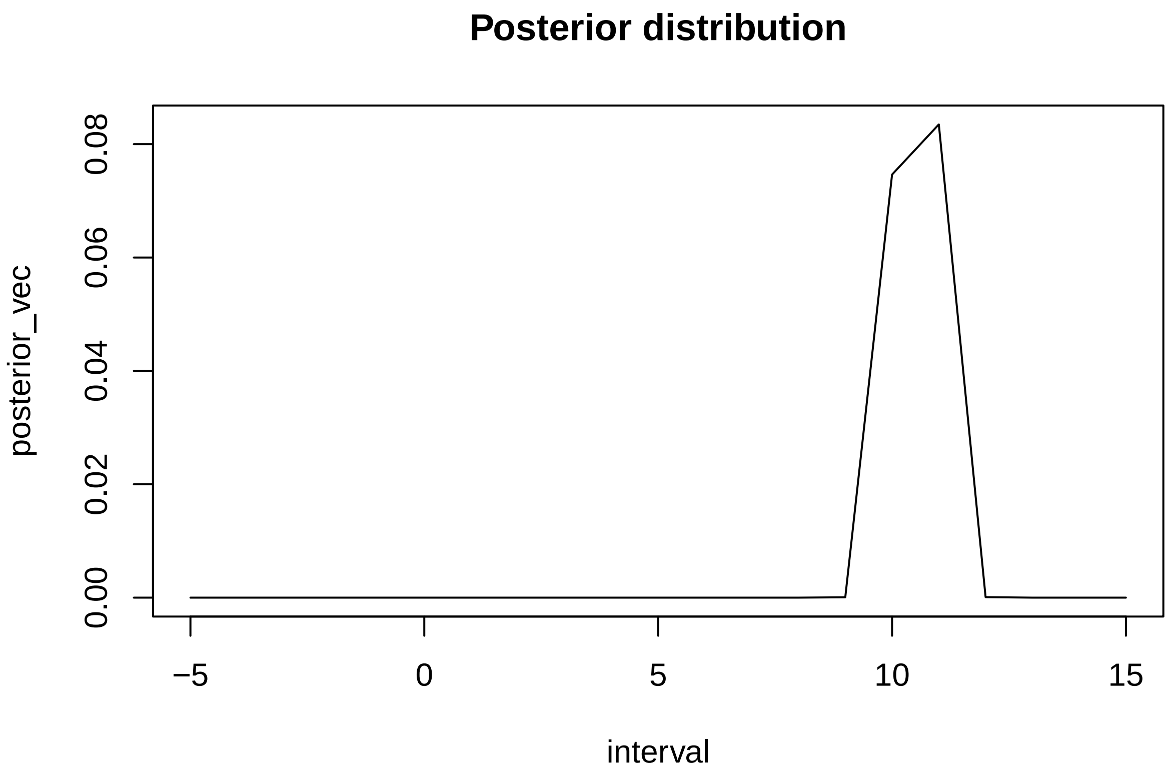
Note, $\Gamma\left(\frac{1}{2}\right) = \sqrt{(\pi)}$.

$$= \frac{1}{\sqrt{(\pi)}\sqrt{(\pi)}} * \frac{exp\left(-\frac{1}{2}\sum_{j=1}^{n}(y_j-\mu)^2\right)}{1+\mu^2}$$

Again we aim to remove terms without μ in them, and we get:

$$f(\mu \mid y) = \frac{exp\left(-\frac{1}{2}\sum_{j=1}^{n}(y_j-\mu)^2\right)}{1+\mu^2}$$

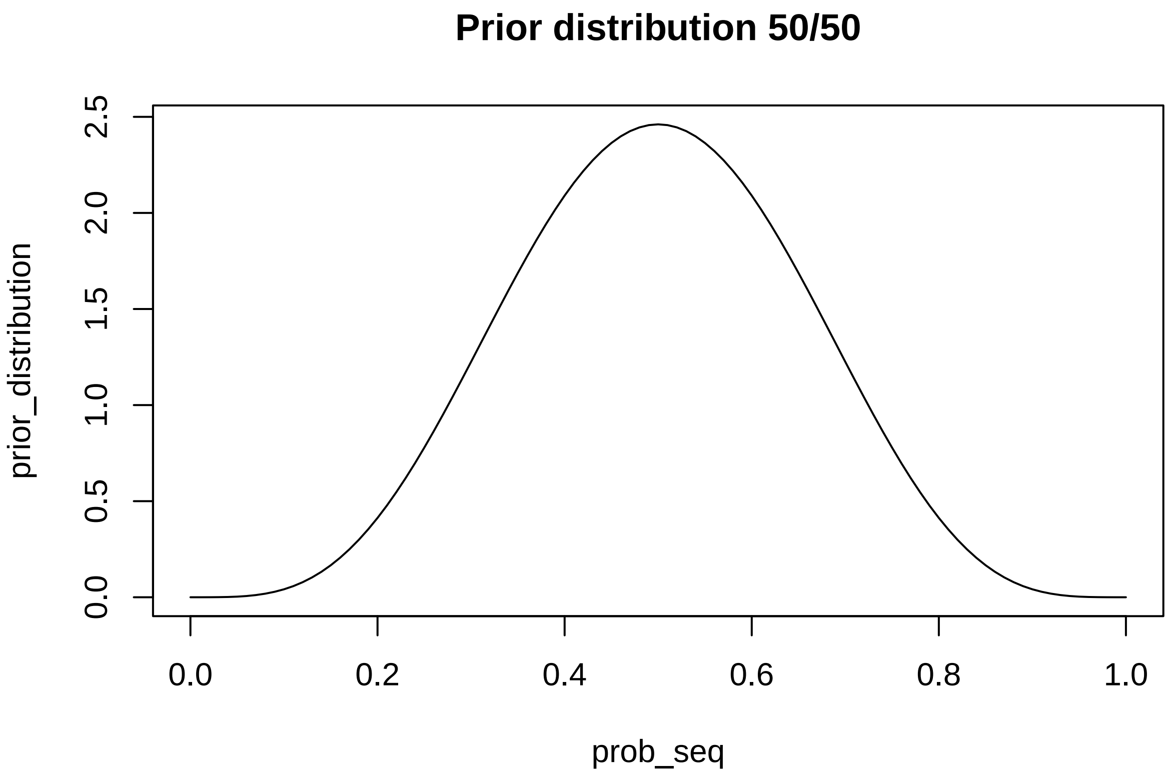
```
# (5)

interval <- seq(-5, 15, 1)
data <- c(11.3710, 9.4353, 10.3631, 10.6329, 10.4043, 9.8939, 11.5115)

posterior <- function(mu, data){
  return (exp((-1/2)*sum((data-mu)*(data-mu))))/(1+mu*mu)
}

posterior_vec <- c(length=length(interval))

for (i in seq_along(interval)) {
  posterior_vec[i] <- posterior(interval[i], data)
}

plot(interval, posterior_vec, type="l", main="Posterior distribution")
```



Uppgift 3.2.1

We have as much information about product A as product B, so their prior-values will be identical. However, we do know that the project is in prototype state, so we assume that its somewhat thoroughly worked on to not be in an unknown idea state - there is an actual product. Based on this, we believe that the interest for these products are 50/50. Hence, alpha = 5 and beta = 5.

```
# (1)

alpha = 5
beta = 5
prob_seq <- seq(0, 1, 0.01)
prior_distribution <- dbeta(prob_seq, alpha, beta)
plot(prob_seq, prior_distribution, type="l", main="Prior distribution 50/50")
```



(2)

Like in the assignment 3.1.1.(4), we find the posterior by multiplying the likelihood with the prior.

Likelihood of binomial is as follows:

$f(x \mid p) = \binom{n}{k} * p^k * (1-p)^{n-k}$

The prior function is as follows:

$f(p) = \frac{p^{\alpha-1} * (1-p)^{\beta-1}}{B(\alpha,\beta)}$

Again, we multiply the likelihood with the prior. We ignore the integral in the denominator since it will end up as a constant and being removed anyway. We aim to remove all terms that does not include p.

$f(p \mid x) = \binom{n}{k} * p^k * (1-p)^{n-k} * \frac{p^{\alpha-1} * (1-p)^{\beta-1}}{B(\alpha,\beta)}$

We have B(α, β) in the denominator. Exclude this as well as it is a constant. We are left with:

$f(p \mid x) = \binom{n}{k} * p^k * (1-p)^{n-k} * p^{\alpha-1} * (1-p)^{\beta-1}$

$\binom{n}{k}$ is also a constant. Exclude it. Final form:

$f(p \mid x) = p^k * (1-p)^{n-k} * p^{\alpha-1} * (1-p)^{\beta-1} = p^{k+\alpha-1} * (1-p)^{n-k+\beta-1}$

Expected Value for beta distribution is as follows:

$E(X) = \frac{\alpha}{\alpha+\beta}$

Add our known values to this function:

Product A:

$E(Xa) = \frac{8+5}{8+5+13-8+5} = \frac{13}{23} \approx 0.57$

Product B:

$E(Xb) = \frac{2+5}{2+5+3-2+5} = \frac{7}{13} \approx 0.54$

Product A has the highest expected value.

3.2.1 (3)

```
simulationA <- rbeta(1000, 8+5, 13-8+5)
simulationB <- rbeta(1000, 2+5, 3-2+5)
mean(simulationA)
```

```
## [1] 0.5665198
```

```
mean(simulationB)
```

```
## [1] 0.5340097
```

```
binsimulA <- rbinom(1000, 87, simulationA)
binsimulB <- rbinom(1000, 87, simulationB)


hist(binsimulA, main="Simulation product A binom")
```


```
hist(binsimulB, main="Simulation product B binom")
```


```
# (a)


probAover40 <- sum(binsimulA>40)/1000
probBover40 <- sum(binsimulB>40)/1000

probAover40
```

```
## [1] 0.807
```

```
probBover40
```

```
## [1] 0.67
```

```
# (b)

expectedA <- 87*mean(simulationA)
expectedB <- 87*mean(simulationB)
expectedA
```

```
## [1] 49.28722
```

```r
expectedB
```

```
## [1] 46.45884
```