Quantum repeaters based on atomic ensembles and linear optics

Nicolas Sangouard

Group of Applied Physics, University of Geneva, CH-1211 Geneva 4, Switzerland

Laboratoire Matériaux et Phénomènes Quantiques, UMR CNRS 7162, Université Paris 7, 75013 Paris, France

Entanglement Swapping

Quantum Channel: optical fibers, free space transmission…

Rate of transmitted photons exponentially low

classical telecommunications: amplifier

quantum telecommunications: no-cloning theorem
Noiseless amplification is impossible unless one restricts oneself to sets of orthogonal states
quantum nature of protocols such as QKD arises precisely from the existence of nonorthogonal state

-> quantum repeater approach

Entanglement Swapping

Bell's theorem states that entangled states cannot be simulated by local hidden variables, thus showing that entanglement lies at the heart of quantum nonlocality.

joint measurement of systems B and C in a basis of entangled states, followed by classical communication of the result to the location of system A and/or D.

N: nesting level

One essential requirement: establish entanglement for the elementary links in a ''heralded'' way;
one has to know when the entanglement has been successfully established

Heralded Entanglement

create entanglement between two systems locally and then send one of the two systems (e.g., a photon) to the distant location:
require the ability to measure that the photon has arrived without destroying the entanglement, which is very difficult in practice

better approach is to create the entanglement ‘‘at a distance.’’
For example, entanglement between one atom in A and another atom in B can be created via the detection of a photon that could have been emitted by either atom, provided that the measurement of the photon is performed in such a way that all ‘‘which-way’’ information is erased

detection of the photon: heralding event for the creation of the entanglement between the two atoms.

Quantum Memories

Another essential requirement:
one has to be able to store the created elementary entanglement until entanglement has been established in the neighboring link as well, in order to then be able to perform the required entanglement swapping operation

Thus quantum repeaters require the existence of ‘‘quantum memories’’,
If such memories are not available, the only solution is to create entanglement in all links simultaneously

Entangling Measurements

one has to be able to perform the required entanglement swapping operations between the quantum memories, i.e., to perform local joint measurements projecting onto entangled states between two memories

General quantum gates (e.g., CNOT gates) between neighboring memories.
generally a difficult task

Dedicated, simpler solutions:
entangling measurements that work only with a certain probability;

Original Protocol

entanglement purification steps that allow one in principle to purify the effects of any kind of decoherence

requires the preparation of at least two initial pairs for every purified pair at any given nesting level for which purification is implemented, leading to significant overheads and thus to lower rates

makes it advantageous to forgo full entanglement purification for simple architectures of just a few links, where it is not necessary for small, but realistic error probabilities per operation. In the present review our focus will be on such simple architectures, because they offer the most realistic chance in the short and medium terms of achieving the most immediate goal of a quantum repeater, namely, to outperform the quantum state distribution rate achievable by direct transmission

DLCZ Protocol

The authors showed how to meet all the above requirements using atomic ensembles as quantum memories, and linear optical techniques in combination with photon counting to perform all the required operations.

atomic ensembles rather than single quantum systems as memories collective effects related to the large number of atoms in the ensemble make it much easier to achieve a strong and controllable coupling between the memory and the photons that serve as long distance quantum information carriers.

Spontaneous Raman emission

The basic process at the heart of the DLCZ protocol is the spontaneous Raman emission of a photon, which simultaneously creates a collective spin excitation in the atomic ensemble

This correlation between emitted photons and atomic excitations in each ensemble forms the basis for the generation of entanglement between distant ensembles (for each elementary link), which is done via a single-photon detection that erases all which-way information, following the principle outlined above for the case of individual atoms.

The spin excitations can be efficiently reconverted into photons thanks to a collective interference effect. This forms the basis for the entanglement swapping operations, which are again done by detecting single photons while erasing which-way information.

Review of research

while pioneering, the DLCZ protocol does not yet allow one to outperform the direct transmission of photons in practice, several authors have proposed significant improvements to the protocol, while using the same or very similar experimental ingredients. These proposals have in turn spurred new experimental investigations. Here we review this area of research

Review of research

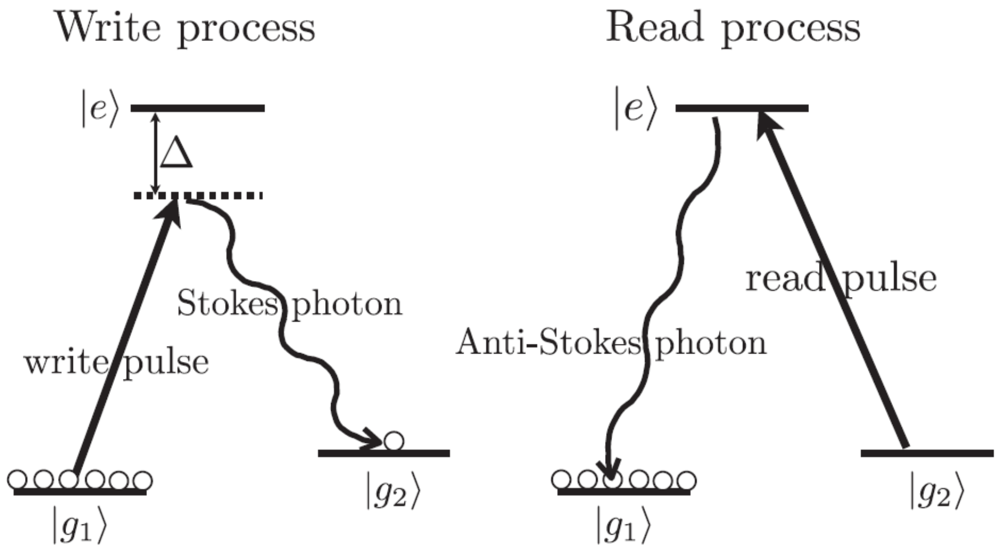
In the DLCZ protocol, both entanglement generation and swapping are based on one single-photon detection each. Section III. A describes a protocol where entanglement is swapped based on two-photon detections, leading to an improvement in the overall rate.

describes protocols where entanglement is generated based on two-photon detections, leading to enhanced robustness with respect to phase fluctuations in the channel

DLCZ protocol

Write process: All atoms start out in g1. A laser pulse off-resonantly excites the g1-e transition, making it possible for a photon to be emitted on the e-g2 transition (with small probability).

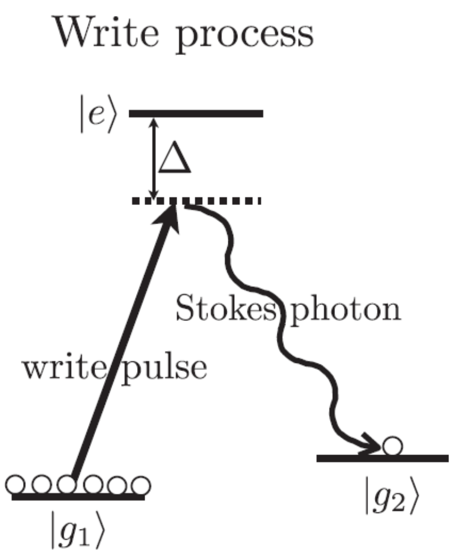
Read process: A resonant laser is applied on the g2-e transition, promoting the single atomic excitation from g2 back to e, followed by collective emission on the e-g1 transition of a Stokes photon in a well-defined direction.



DLCZ protocol

Detection of Strokes photon in the far field, such that no information is revealed about which atom it came from, creates an atomic state that is a coherent superposition of all the possible terms with N_A-1 atoms in g1 and one atom in g2,

$$\frac{1}{\sqrt{N_A}} \sum_{k=1}^{N_A} e^{i(\mathbf{k}_w - \mathbf{k}_S)\mathbf{x}_k} |g_1\rangle_1 |g_1\rangle_2 \cdots |g_2\rangle_k \cdots |g_1\rangle_{N_A},$$



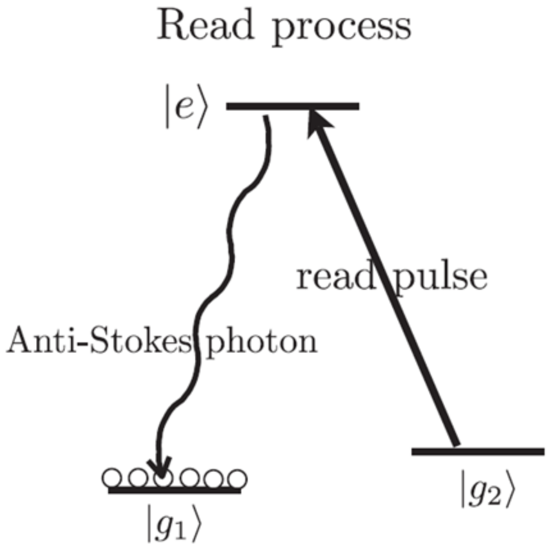
A remarkable feature of such collective excitations that are of great interest for practical applications is that they can be read out very efficiently by converting them into single photons that propagate in a well-defined direction, thanks to collective interference

DLCZ protocol

Resonant laser excitation of such a state on the g2-e transition (the read laser pulse) leads to an analogous state with NA-1 atoms in g1 and one delocalized excitation in e, but with supplementary phases eikrx0
k , where kr is the k vector of the read laser and x0 k is the position of the kth atom at the time of the readout (which may be different from its initial position xk if the atoms are moving).

All the terms in this state can decay to the initial state $|g_1\rangle^{\otimes N_A}$ while emitting a photon on the e-g1 transition (the anti-Stokes photon). The total amplitude for this process is then proportional to

$$\sum_{k=1}^{N_A} e^{i(\mathbf{k}_w - \mathbf{k}_S)\mathbf{x}_k} e^{i(\mathbf{k}_r - \mathbf{k}_{AS})\mathbf{x}'_k}.$$



Collective interference

The conditions for constructive interference of the NA terms in this sum depend on whether the atoms are moving during the storage.

If they are at rest ($\mathbf{x}_k = \mathbf{x}'_k$ for all k),

$$\mathbf{k}_w + \mathbf{k}_r - \mathbf{k}_S.$$

For atomic ensembles that contain sufficiently many atoms, emission in this one direction can completely dominate all other directions. This allows a very efficient collection of the anti-Stokes photon.

Collective interference

Note that there is no collective interference effect for the emission of the Stokes photon, since its emission by different atoms corresponds to orthogonal final states, e.g., the state $|g_2\rangle_1|g_1\rangle_2\cdots|g_1\rangle_{N_A}$ the Stokes photon was emitted by the first atom, etc. Full which-way information about the origin of the photon is thus stored in the atomic ensemble, making interference impossible.

As a consequence the total emission probability for the Stokes photon is simply given by the sum of the emission probabilities for each atom, and there is no preferred direction of emission.

Limiting factor

This possibility of creating multiple pairs of excitations, which becomes more significant as the probability of creating a single excitation is increased, is an important limiting factor in the quantum repeater protocols discussed in this review;

Entanglement creation for two remote atomic ensembles

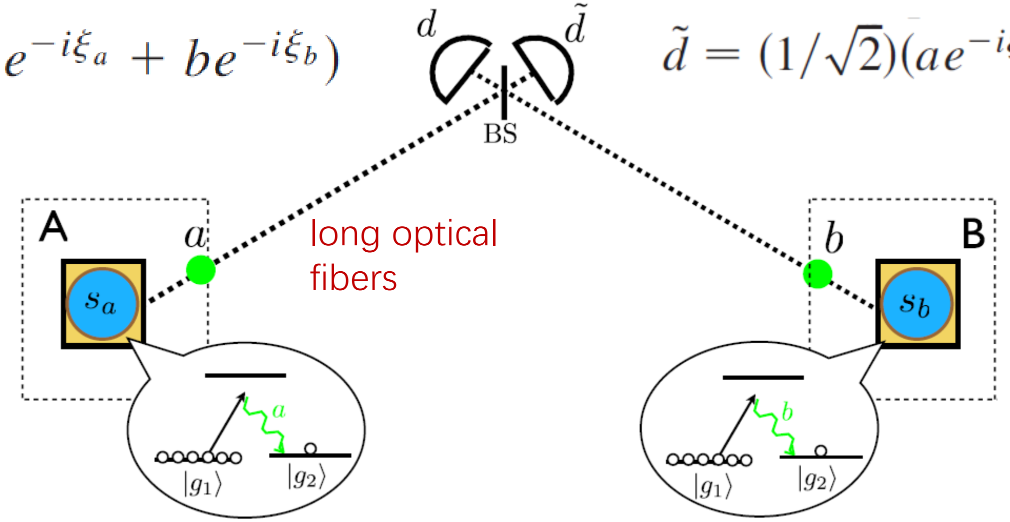
The two ensembles are simultaneously excited such that a single Stokes photon can be emitted with a small probability $p/2 = (\chi t)^2$,

$$\left(1 + \sqrt{\frac{p}{2}}(s_a^\dagger a^\dagger e^{i\phi_a} + s_b^\dagger b^\dagger e^{i\phi_b}) + O(p)\right)|0\rangle.$$

The Stokes photons are coupled into optical fibers and combined on a beam splitter at a central station between A and B.

$$d = (1/\sqrt{2}) \times (ae^{-i\xi_a} + be^{-i\xi_b})$$

$$\tilde{d} = (1/\sqrt{2})(ae^{-i\xi_a} - be^{-i\xi_b})$$



Creating entanglement by a single-photon detection

inspired by similar proposals for entangling two individual quantum systems rather than two ensembles (Bose et al., 1999; Cabrillo et al., 1999).

The detection of a single photon in d, for example, projects the state of the two atomic ensembles in

$$|\psi_{ab}\rangle = \frac{1}{\sqrt{2}}(s_a^{\dagger} e^{i(\phi_a+\xi_a)} + s_b^{\dagger} e^{i(\phi_b+\xi_b)})|0\rangle.$$

A single atomic excitation is thus delocalized between A and B. This corresponds to an entangled state that can be rewritten as

$$|\psi_{ab}\rangle = \frac{1}{\sqrt{2}}(|1_a\rangle|0_b\rangle + |0_a\rangle|1_b\rangle e^{i\theta_{ab}}),$$

an empty ensemble A (B),

the storage of a single atomic excitation

The detection of a single Stokes photon at the central station in mode d or ~d, which could have come from either location A or B, heralds the storage of a single excitation (sa or sb) in one of the two ensembles.

Entanglement connection between the elementary links

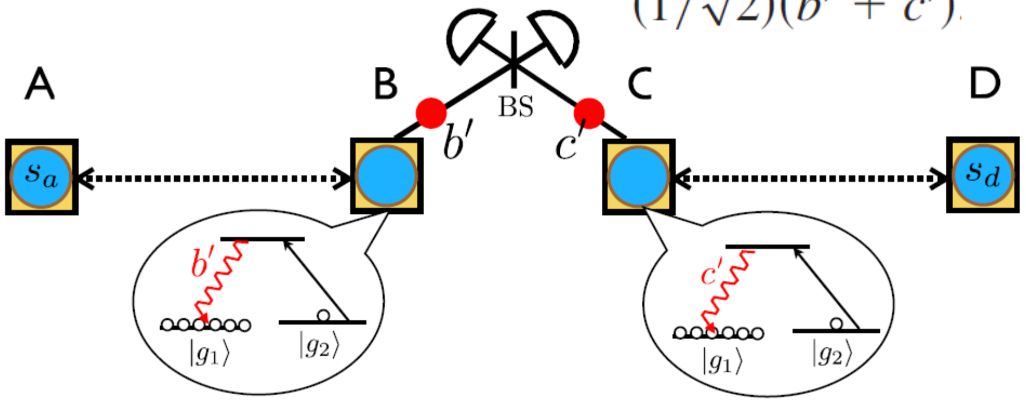
Once entanglement has been heralded within each elementary link, one wants to connect the links in order to extend the distance of entanglement
→ successive entanglement swapping

$$|\psi_{ab}\rangle \otimes |\psi_{cd}\rangle$$

A-B and C-D, respectively, are entangled by sharing a single excitation
The atomic excitations sb and sc that are probabilistically stored in the ensembles B and C are read out with a strong, resonant light pulse to be converted back into "anti-Stokes" photons associated with the mode b' and c'
Readout process efficient: collective interference → emission of the anti-Stokes photon in a well-defined mode

$$(1/\sqrt{2})(b' + c')$$

$$|\psi_{ad}\rangle = \frac{1}{\sqrt{2}}(s_a^\dagger + s_d^\dagger e^{i(\theta_{ab}+\theta_{cd})})|0\rangle.$$

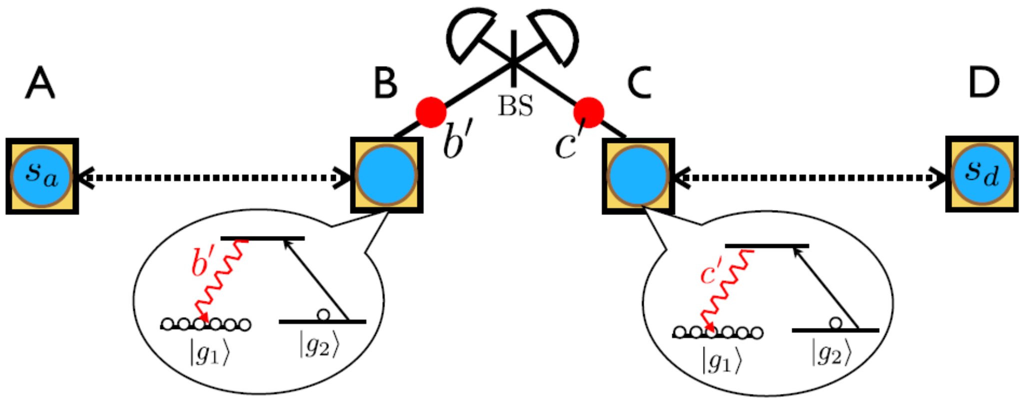


Entanglement connection between the elementary links

The two modes b0 and c0 are combined on a beam splitter, and the measurement of a single photon, e.g., in the mode $(1/\sqrt{2})(b' + c')$, will project the ensembles A and D into the entangled state

$$|\psi_{ad}\rangle = \frac{1}{\sqrt{2}}(s_a^\dagger + s_d^\dagger e^{i(\theta_{ab}+\theta_{cd})})|0\rangle.$$

The memories B and C are read out, and the resulting anti-Stokes photons are combined on a beam splitter. The detection of a single photon after the beam splitter, which could have come from either location B or C, heralds the storage of a single excitation (sa or sd) in the ensembles A and D and projects them into an entangled state $|\psi_{ad}\rangle$



Effect of nonunit efficiency

detector efficiency η_d: probability of detecting a photon when a single photon enters the detector
memory efficiency η_m: probability of converting a single atomic excitation into an anti-Stokes photon

The detectors can give the expected click when two photons are stored in the memories B and C, but only one is detected. In this case, the created state contains an additional vacuum component

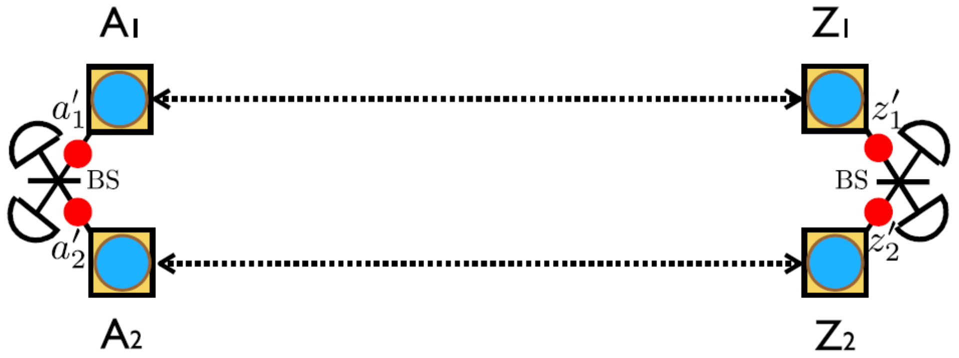
$$\rho_{ad} = \alpha_1 |\psi_{ad}\rangle\langle\psi_{ad}| + \beta_1 |0\rangle\langle 0|,$$

The success probability for the first swapping is given by $P_1 = \eta[1 - (\eta/2)]$. The relative weight of the vacuum component thus increases linearly with the number of elementary links $N = 2^n$ composing the quantum repeater.

Postselection of two-photon entanglement

The created entanglement, which consists of A and Z sharing a single delocalized excitation, is of limited use on its own, because it is difficult to perform measurements in any basis other than Fock states |0> and |1>. This is why in the DLCZ protocol the created single-excitation entanglement is now used as a building block for more directly useful two-photon entanglement;

The atomic excitations at the same location are read out and the emitted anti-Stokes photons are combined into a beam splitter and then counted. Measurements in arbitrary basis can be done by changing the beam-splitter transmission coefficients and phases.



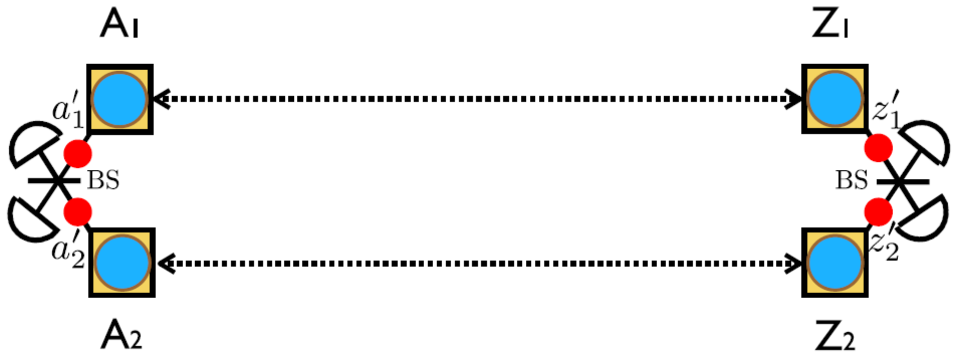
Postselection of two-photon entanglement

needs two ensembles at each location
Entanglements between A1 and Z1 and between A2 and Z2 have been established, such that we have the state $(1/2)(a_1'^\dagger + e^{i\theta_1} z_1'^\dagger)(a_2'^\dagger + e^{i\theta_2} z_2'^\dagger)|0\rangle$

The projection of this state onto the subspace with one photon in each location is

$$|\Psi_{az}\rangle = \frac{1}{\sqrt{2}}(a_1'^\dagger z_2'^\dagger + e^{i(\theta_2-\theta_1)} a_2'^\dagger z_1'^\dagger)|0\rangle,$$

which is analogous to conventional polarization or time-bin entangled states. The required projection can be performed postselectively by converting the atomic excitations back into anti-Stokes photons and counting the number of photons in each location.



Calculation of the entanglement distribution time

The general formula for calculating the time required for a successful distribution of an entangled state $|\Psi_{az}\rangle$s

Entanglement creation attempts for elementary links only succeed with a probability P_0. After every attempt, one has to wait to find out whether the attempt has succeeded (whether there was a photon detection in the central station). If not, the memory has to be emptied, and one tries again.

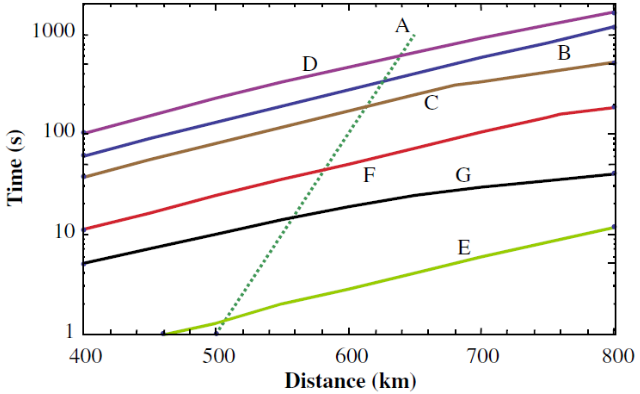
Furthermore, the total time is inversely proportional to the success probabilities at each level Pi and to the probability of successful postselection at the end, Pps.

Comparison to direct transmission

compare the entanglement distribution time for the DLCZ protocol to the time for quantum state distribution using direct transmission

The most important question in the short and medium terms is what distribution rates a given repeater protocol can achieve in that distance range

Even more importantly, for the repeater to work, the memory storage time has to be comparable to the mentioned 340 s. In particular, it has to be long enough for the final postselection to be possible, i.e., long enough to create two independent single-photon entangled states over the whole distance. This is extremely challenging.



Discussion—Limitations

There is a trade-off between high fidelity of the distributed state and high distribution rate. The errors due to multiple emissions from individual ensembles grow quadratically with the number of elementary links N.

The multiphoton errors grow only linearly with N, thanks to the use of entanglement swapping operations based on two-photon detections instead of a single-photon detection.

Over such long time scales, both the phases of the pump lasers and the fiber lengths are expected to fluctuate significantly. This problem has to be addressed in any practical implementation of the protocol, either through active stabilization of the fiber lengths, or possibly through the use of self-compensating Sagnac-type configurations.

Discussion—Limitations

in the DLCZ repeater protocol one is a priori limited to a single entanglement generation attempt per elementary link per time interval L0/c. this limitation can be overcome using memories that can store a large number of distinguishable modes;

For long communication distances to be realistic, the wavelength of the Stokes photons has to be in the optimal range for telecom fibers (about 1:5 um). this requirement can be overcome by separating entanglement generation and storage.

IMPROVEMENTS

Section III A & B: entanglement swap & generation using two-photon detection

in the DLCZ repeater protocol one is a priori limited to a single entanglement generation attempt per elementary link per time interval L0/c. this limitation can be overcome using memories that can store a large number of distinguishable modes;

Section III.C reviews the idea of using memories that can store multiple temporal modes. Such memories can be realized using inhomogeneously broadened atomic ensembles in certain solid-state systems. Their use in the present context is made possible by the realization that a DLCZ-type atomic ensemble can be emulated by combining a photon-pair source with a memory that can absorb and emit photons. This approach promises a great enhancement in the entanglement generation rate.

IMPROVEMENTS

Section III.D reviews work on spatial multiplexing, which would be even more powerful than the temporal variety.

Section III.E discusses a protocol based on single-photon sources, which can be effectively implemented with atomic ensembles, and which yields a significantly enhanced rate compared to the DLCZ protocol.

Section III.F describes protocols that are based on effectively approximating ideal photon-pair sources with atomic ensembles, leading to both enhanced rates and greatly enhanced robustness.

Entanglement swapping via two-photon detections

Entanglement generation via two-photon detections

Photon-pair sources and multimode memories

DLCZ-type atomic ensemble can be emulated by the combination of a photon-pair source and a quantum memory that can absorb and reemit photons.

Advantage, allowing greater wavelength flexibility for the memory compared to the DLCZ situation where the Stokes photon has to be emitted at a telecom wavelength

an approach toward multiplexing

Separation of entanglement generation and storage

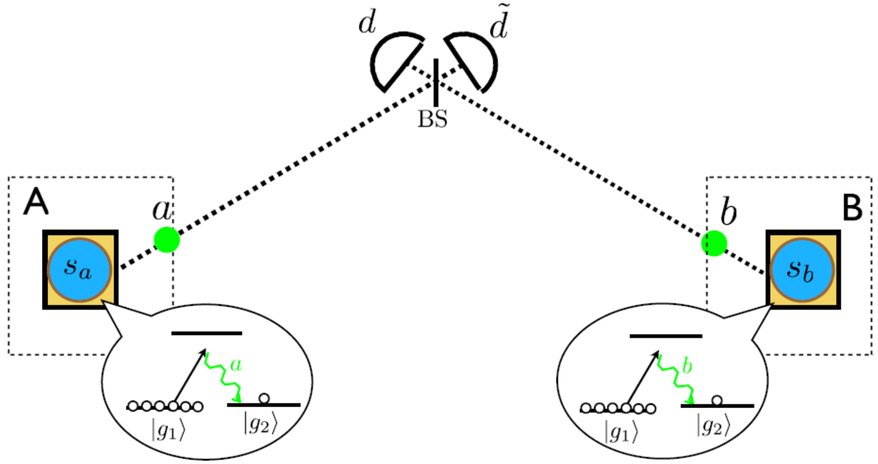
The basic element of all the protocols discussed so far is an ensemble of three-level atoms that is coherently excited in order to generate a Stokes photon by Raman scattering, heralding the storage of an atomic spin excitation that can later be reconverted into an anti-Stokes photon

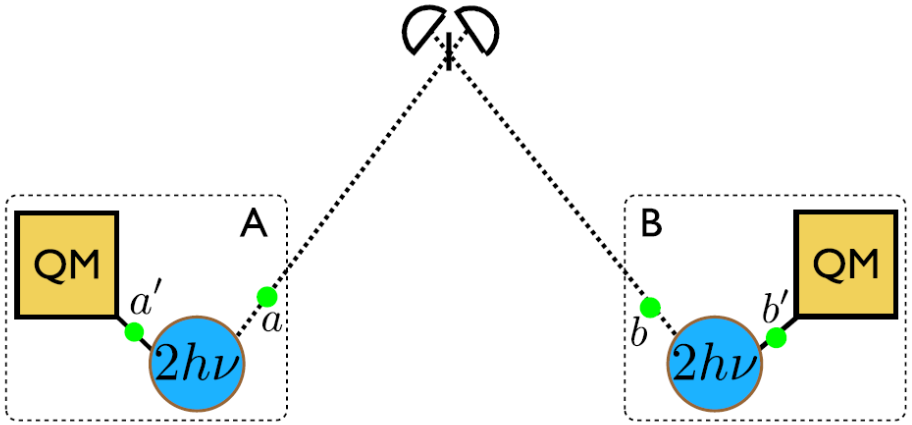
wavelength conversion… so far is not very efficient at the single-photon level
the use of erbium-doped crystals as quantum memories, where initial experimental and theoretical investigations have been performed, but implementation of the DLCZ protocol is still a distant and uncertain prospect.
(?)

Separation of entanglement generation and storage

A different approach for long-distance entanglement creation
It combines pair sources and absorptive memories to emulate the DLCZ protocol;

REI: weak oscillator strengths, which makes the off-resonant Raman excitation very challenging to implement



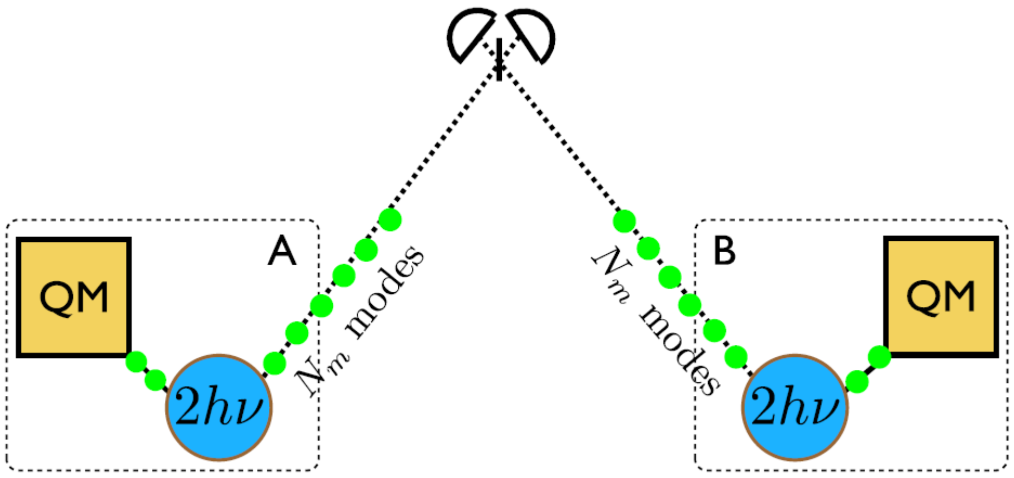


allowing greater wavelength flexibility for the memory compared to the DLCZ situation where the Stokes photon has to be emitted at a telecom wavelength

Spatially multiplexed memories

in the DLCZ protocol one is a priori limited to a single entanglement generation attempt per communication time interval L0/c, because the memories have to be emptied after every unsuccessful attempt.
temporal multiplexing, which overcomes this limitation

inhomogeneous transitions for the generation of temporally multi-mode spin-photon correlations ?



REI

All rare-earth elements have in common weak dipole moments on the relevant 4f - 4f transitions and large inhomogeneously broadened spectra due to the interaction with the host crystal. These two properties make the creation of Stokes – anti-Stokes pairs in rare-earth doped solids challenging.

REI

A clear motivation for the storage of telecom-wavelength single photons is for long-range entanglement distribution (201). The role of the memory in these schemes is to act as a temporal buffer to overcome the inherently probabilistic nature of the photon pair generation.

REI

A photon-pair source with controllable delay based on shaped inhomogeneous broadening of rare-earth doped solids

Pavel Sekatski,[1] Nicolas Sangouard,[1] Nicolas Gisin,[1] Hugues de Riedmatten,[1, 2, 3] and Mikael Afzelius[1]

[1] *Group of Applied Physics, University of Geneva, 1211 Geneva 4, Switzerland*
[2] *ICFO-Institute of Photonic Sciences, Mediterranean Technology Park, 08860 Castelldefels (Barcelona), Spain*
[3] *ICREA-Institució Catalana de Recerca i Estudis Avançats, 08015 Barcelona, Spain*

(Dated: August 29, 2018)

Spontaneous Raman emission in atomic gases provides an attractive source of photon pairs with a controllable delay. We show how this technique can be implemented in solid state systems by appropriately shaping the inhomogeneous broadening. Our proposal is eminently feasible with current technology and provides a realistic solution to entangle remote rare-earth doped solids in a heralded way.

REI

Multi-mode and long-lived quantum correlations between photons and spins in a crystal

Cyril Laplane, Pierre Jobez,* Jean Etesse, Nicolas Gisin, and Mikael Afzelius†
Groupe de Physique Appliquée, Université de Genève, CH-1211 Genève 4, Switzerland

The realization of quantum networks and quantum repeaters remains an outstanding challenge in quantum communication. These rely on entanglement of remote matter systems, which in turn requires creation of quantum correlations between a single photon and a matter system. A practical way to establish such correlations is via spontaneous Raman scattering in atomic ensembles, known as the DLCZ scheme. However, time multiplexing is inherently difficult using this method, which leads to low communication rates even in theory. Moreover, it is desirable to find solid-state ensembles where such matter-photon correlations could be generated. Here we demonstrate quantum correlations between a single photon and a spin excitation in up to 12 temporal modes, in a $^{151}Eu^{3+}$ doped Y_2SiO_5 crystal, using a novel DLCZ approach that is inherently multimode. After a storage time of 1 ms, the spin excitation is converted into a second photon. The quantum correlation of the generated photon pair is verified by violating a Cauchy - Schwarz inequality. Our results show that solid-state rare-earth crystals could be used to generate remote multi-mode entanglement, an important resource for future quantum networks.