Quantum Eraser (量子擦除)

A

$$|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$

Coherent superposition:

Relative phase has distinguishable consequences

Could tell between + and -

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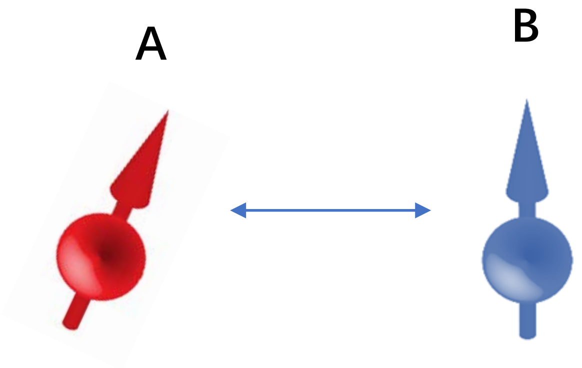


$$|\phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$$

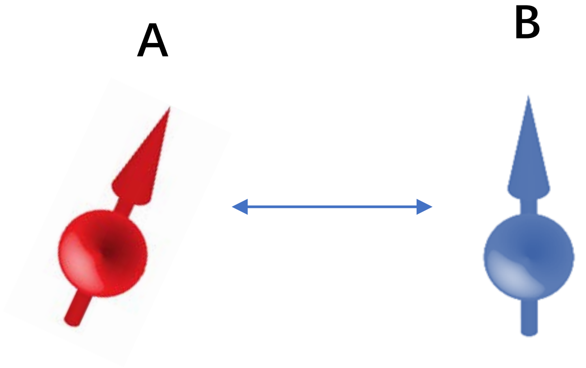
Entanglement causes decoherence:

Interference can occur only if there’s **no** possible way to

find out |0> or |1>

It becomes possible to determine that by measuring B

Quantum Eraser (量子擦除)



$$|\phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$$
$$= (|++\rangle + |--\rangle)/\sqrt{2}$$

Recover purity of A:

Measure B along **x** axis. No matter what the result is,

A is always in a coherent superposition of |0> and |1>

It is interesting:

If we do nothing, relative phase information of A is erased.

If we measure B in a special way, that information is recovered.

测量 B 的同时也破坏了一些信息（个人理解）。

Bell inequality

Alice, in Pasadena, has in her possession three coins laid out on a table, labeled 1, 2, 3. Each coin has either its heads (H) or tails (T) side facing up, but it is hidden under an opaque cover, so that Alice is not able to tell whether it is an H or a T. Alice can uncover any one of the three coins, and so learn its value (H or T). However, as soon as that one coin is uncovered, the other two covered coins instantly disappear in a puff of smoke, and Alice never gets an opportunity to uncover the other coins.

Alice **Bob**

Bell inequality

She has many copies of the three-coin set, and eventually she learns that, no matter which coin she exposes, she is just as likely to find an H as a T. Bob, in Chicago, has a similar set of coins, also labeled 1, 2, 3. He too finds that each one of his coins, when revealed, is as likely to be an H as a T.

Alice **Bob**

Bell inequality

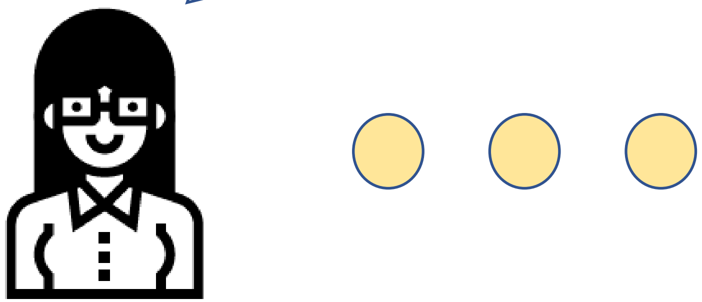
In fact, Alice and Bob have many identical copies of their shared set of coins, so they conduct an extensive series of experiments to investigate how their coin sets are correlated with one another. They quickly make a remarkable discovery: Whenever Alice and Bob uncover coins with the same label (whether 1, 2, or 3), they always find coins with the same value — either both are H or both are T. They conduct a million trials, just to be sure, and it works every single time! Their coin sets are perfectly correlated.

Alice **Bob**

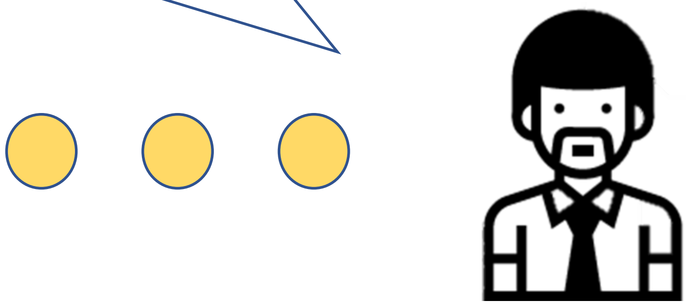
Bell inequality

You know, Bob, sometimes it’s hard for me to decide which of the three coins to uncover. I know that if I uncover coin 1, say, then coins 2 and 3 will disappear, and I’ll never have a chance to find out the values of those coins. Once, just once, I’d love to be able to uncover two of the three coins, and find out whether each is an H or a T. But I’ve tried and it just isn’t possible — there’s no way to look at one coin and prevent the other from going poof !

[Long pause] Hey . . . wait a minute Alice, I’ve got an idea . . . Look, I think there is a way for you to find the value of two of your coins, after all! Let’s say you would like to uncover coin 1 and coin 2. Well, I’ll uncover my coin 2 here in Chicago, and I’ll call you to tell you what I found, let’s say its an H. We know, then, that you are certain to find an H if you uncover your coin 2 also. There’s absolutely no doubt about that, because we’ve checked it a million times. Right?

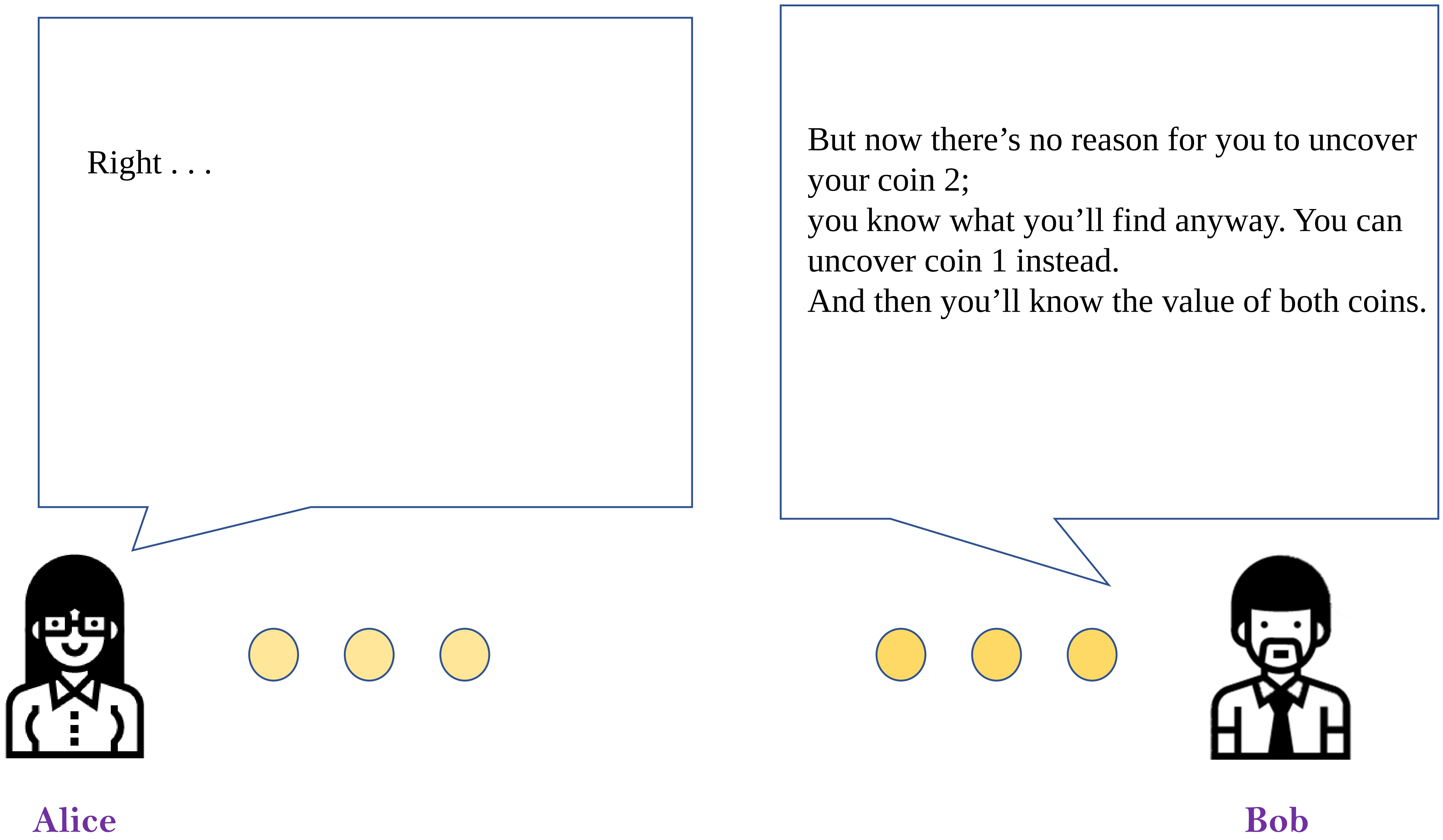


Alice



Bob

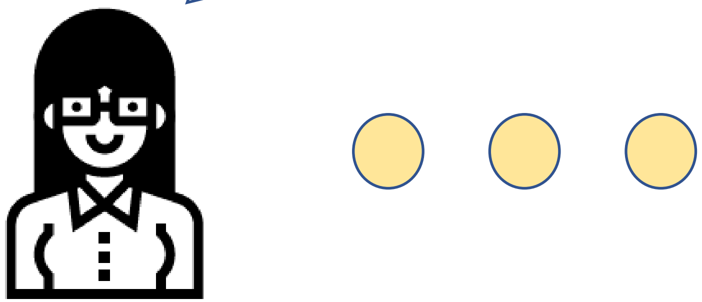
Bell inequality



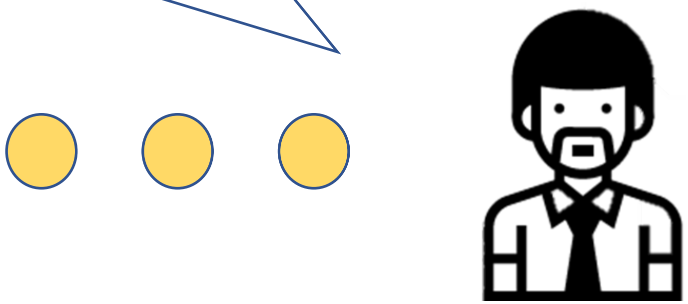
Bell inequality

Hmmm . . . yeah, maybe. But we won't be sure, will we? I mean,
yes, it always worked when we uncovered the same coin before,
but this time you uncovered your coin 2, and your coins 1 and 3 disappeared, and I uncovered my coin 1, and my coins 2 and 3 disappeared. There's no way we'll ever be able to check anymore what would have happened if we had both uncovered coin 2.

We don't have to check that anymore, Alice; we've already checked it a million times. Look, your coins are in Pasadena and mine are in Chicago. Clearly, there's just no way that my decision to uncover my coin 2 can have any influence on what you'll find when you uncover your coin 2. That's not what's happening. It's just that when I uncover my coin 2 we're collecting the information we need to predict with certainty what will happen when you uncover your coin 2. Since we're already certain about it, why bother to do it!

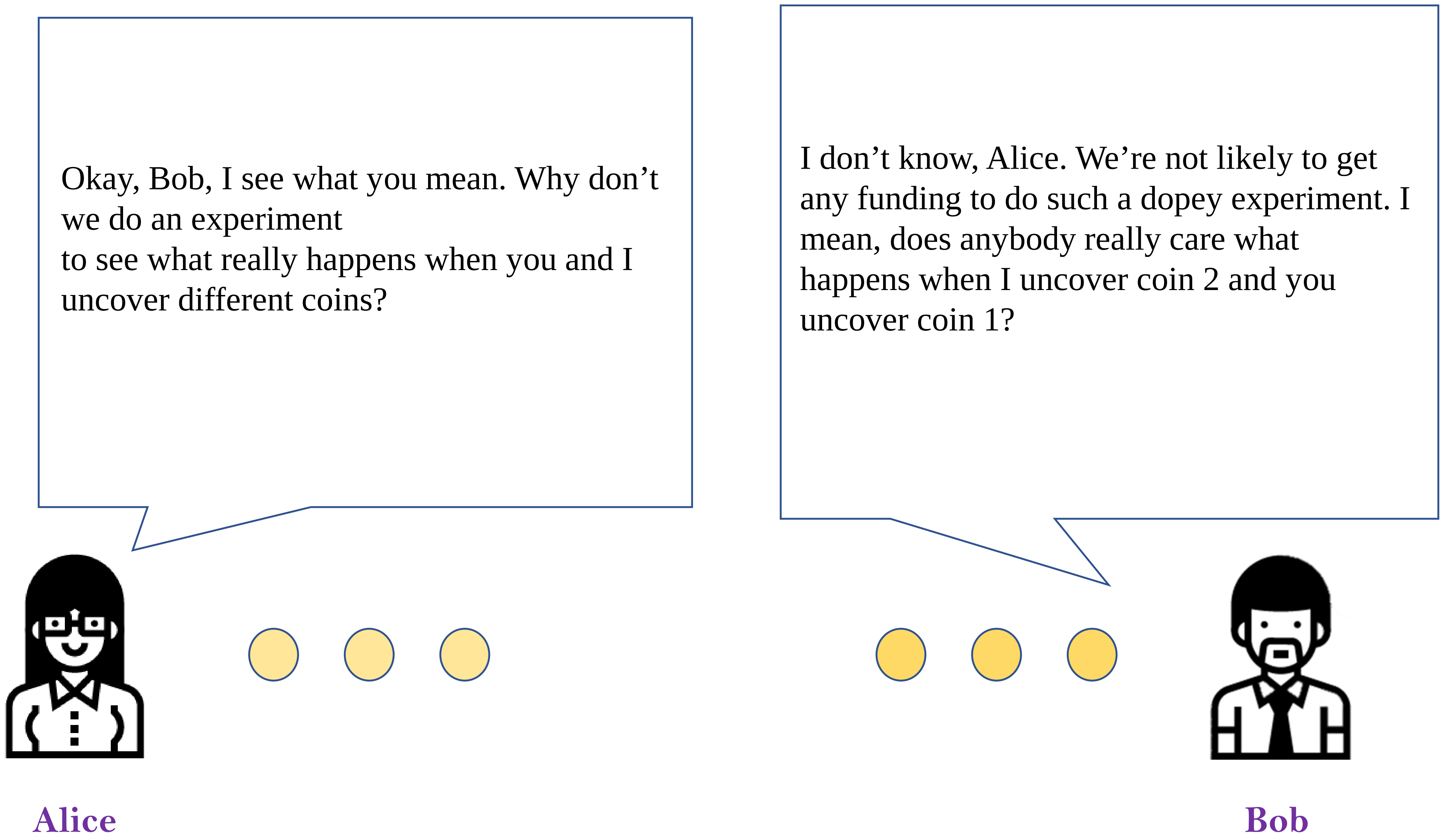


Alice



Bob

Bell inequality



Bell inequality

I’m not sure, Bob. But I’ve heard about a theorist named Bell.
They say that he has some interesting ideas about the coins. He might have a theory that makes a prediction about what we’ll find.
Maybe we should talk to him.

Good idea! And it doesn’t really matter whether his theory makes any sense or not. We can still propose an experiment to test his prediction, and they’ll probably fund us.

Alice **Bob**

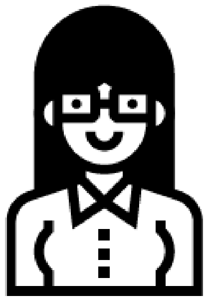
Bell inequality

So Alice and Bob travel to CERN to have a chat with Bell. They tell Bell about the experiment they propose to do. Bell listens closely, but for a long time he remains silent, with a faraway look in his eyes. Alice and Bob are not bothered by his silence, as they rarely understand anything that theorists say anyway. But finally Bell speaks.

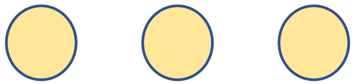
Alice **Bob**

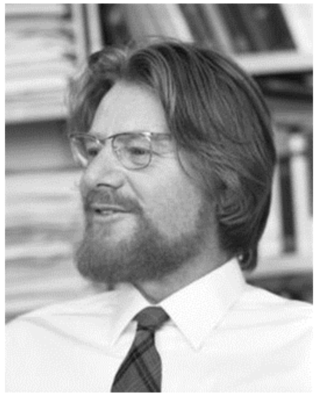
Bell inequality

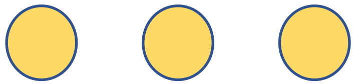
I think I have an idea . . . . When Bob uncovers his coin in Chicago, that can’t exert any influence on Alice’s coin in Pasadena. Instead, what Bob finds out by uncovering his coin reveals some information about what will happen when Alice uncovers her coin.



Alice



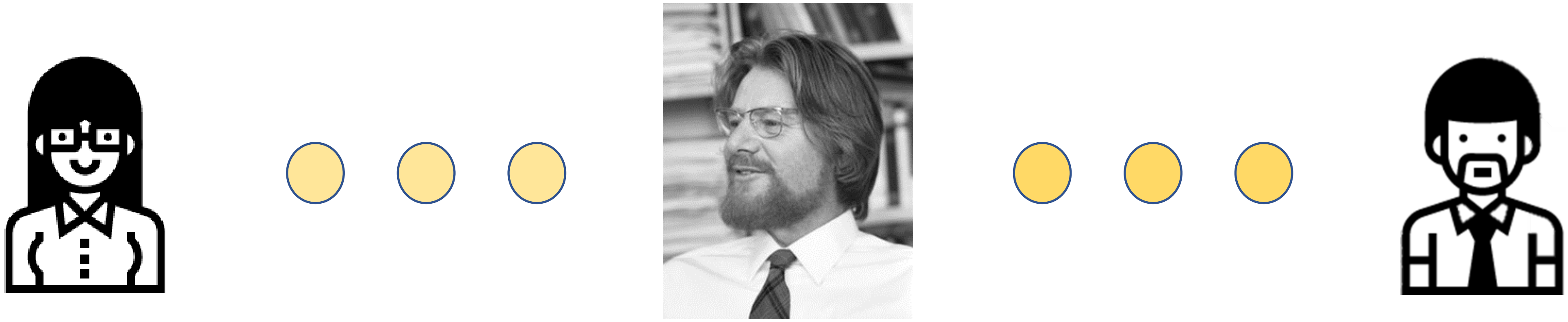




Bob

Bell inequality

Right. Sounds reasonable. So let's assume that Bob is right about that. Now Bob can uncover any one of his coins, and know for sure what Alice will find when she uncovers the corresponding coin. He isn't disturbing her coin in any way; he's just finding out about it. We're forced to conclude that there must be some **hidden variables** that specify the condition of Alice's coins. And if those variables are completely known, then the value of each of Alice's coins can be unambiguously predicted.



Alice

Bob

Bell inequality

When your correlated coin sets are prepared, the values of the hidden variables are not completely specified; that's why any one coin is as likely to be an H as a T. But there must be some probability distribution P(x, y, z) (with x, y, z ∈ {H, T}) that characterizes the preparation and governs Alice's three coins. These probabilities must be nonnegative, and they sum to one:

$$\sum_{x,y,z\in\{H,T\}} P(x,y,z) = 1$$

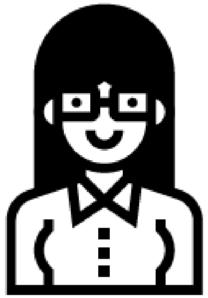
Alice

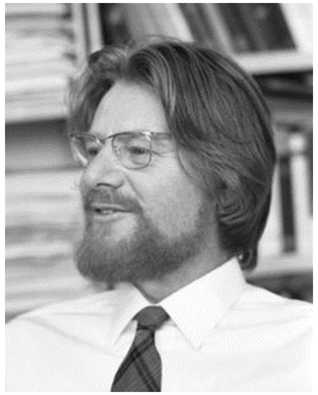
Bob

Bell inequality

Alice can't uncover all three of her coins, so she can't measure P(x, y, z) directly. But with Bob's help, she can in effect uncover any two coins of her choice. Let's denote with Psame(i, j), the probability that coins i and j (i, j = 1, 2, 3) have the same value, either both H or both T. Then we see that

$$\begin{aligned}
P_{\text{same}}(1,2) = &\quad P(HHH) + P(HHT) + P(TTH) + P(TTT) \\
P_{\text{same}}(2,3) = &\quad P(HHH) + P(THH) + P(HTT) + P(TTT) \\
P_{\text{same}}(1,3) = &\quad P(HHH) + P(HTH) + P(THT) + P(TTT)
\end{aligned}$$





Alice

Bob

Bell inequality

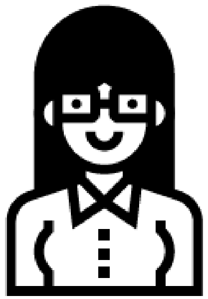
and it immediately follows from eq. (4.15) that

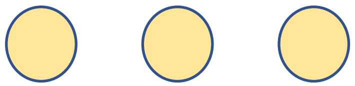
$$P_{\text{same}}(1,2) + P_{\text{same}}(2,3) + P_{\text{same}}(1,3)$$
$$= 1 + 2\ P(HHH) + 2\ P(TTT) \geq 1\ .$$

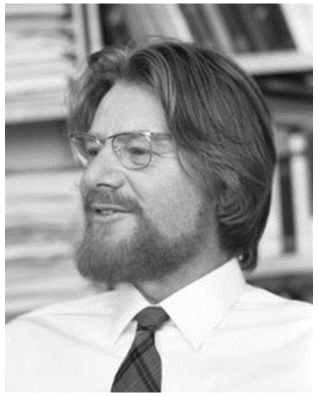
So that’s my prediction: Psame should obey the inequality

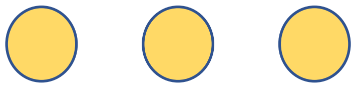
$$P_{\text{same}}(1,2) + P_{\text{same}}(2,3) + P_{\text{same}}(1,3) \geq 1\ .$$

You can test it my doing your experiment that “uncovers” two coins at a time.









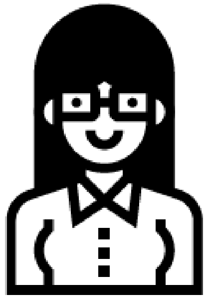
Alice

Bob

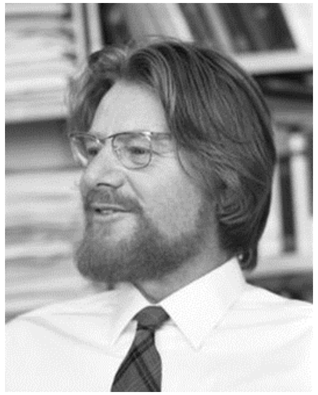
Bell inequality

I think I see . . . . Bell is saying that if there are three coins on a table, and each one is either an H or a T, then at least two of the three have to be the same, either both H or both T. Isn't that it, Bell?

Well, I guess the math looks right. But I don't really get it. Why does it work?.



Alice



Bob

Bell inequality

Bell stares at Alice, a surprised look on his face. His eyes glaze, and for a long time he is speechless. Finally, he speaks:

Bell inequality

Bell inequality

So Alice and Bob are amazed and delighted to find that Bell is that rarest of beasts— a theorist who makes sense. With Bell’s help, their proposal is approved and they do the experiment, only to obtain a shocking result. After many careful trials, they conclude, to very good statistical accuracy that

$$P_{\text{same}}(1,2) \simeq P_{\text{same}}(2,3) \simeq P_{\text{same}}(1,3) \simeq \frac{1}{4}$$

Alice **Bob**

Bell inequality

So Alice and Bob are amazed and delighted to find that Bell is that rarest of beasts— a theorist who makes sense. With Bell's help, their proposal is approved and they do the experiment, only to obtain a shocking result. After many careful trials, they conclude, to very good statistical accuracy that

$$P_{\text{same}}(1,2) \simeq P_{\text{same}}(2,3) \simeq P_{\text{same}}(1,3) \simeq \frac{1}{4}$$

And hence

$$P_{\text{same}}(1,2) + P_{\text{same}}(2,3) + P_{\text{same}}(1,3) \simeq 3 \cdot \frac{1}{4} = \frac{3}{4} < 1$$

Alice

Bob

Bell inequality

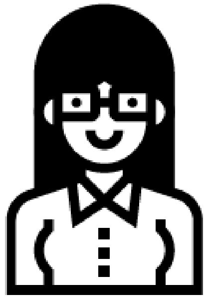
$$\boldsymbol{U} \otimes \boldsymbol{U}|\psi\rangle = |\psi\rangle$$

$$\left(\vec{\boldsymbol{\sigma}}^{(A)} + \vec{\boldsymbol{\sigma}}^{(B)}\right)|\psi^-\rangle = 0$$

$$|\psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

$$-\langle\psi^-|\left(\vec{\boldsymbol{\sigma}}^{(A)} \cdot \hat{a}\right)\left(\vec{\boldsymbol{\sigma}}^{(A)} \cdot \hat{b}\right)|\psi^-\rangle$$
$$= -a_i b_j \mathrm{tr}\left(\boldsymbol{\rho}_A \boldsymbol{\sigma}_i^{(A)} \boldsymbol{\sigma}_j^{(A)}\right) = -a_i b_j \delta_{ij} = -\hat{a} \cdot \hat{b} = -\cos\theta$$

?



Alice

three possible axes

?

Bob

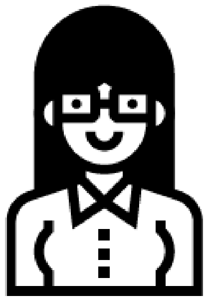
Bell inequality

$$\boldsymbol{E}(\hat{n}, \pm) = \tfrac{1}{2}(\boldsymbol{I} \pm \hat{n} \cdot \vec{\boldsymbol{\sigma}})$$

$$|\psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

$$P(++) = \langle\psi^-|\boldsymbol{E}^{(A)}(\hat{a},+)\boldsymbol{E}^{(B)}(\hat{b},+)|\psi^-\rangle = \frac{1}{4}(1-\cos\theta)$$
$$P(--) = \langle\psi^-|\boldsymbol{E}^{(A)}(\hat{a},-)\boldsymbol{E}^{(B)}(\hat{b},-)|\psi^-\rangle = \frac{1}{4}(1-\cos\theta)$$
$$P(+-) = \langle\psi^-|\boldsymbol{E}^{(A)}(\hat{a},+)\boldsymbol{E}^{(B)}(\hat{b},-)|\psi^-\rangle = \frac{1}{4}(1+\cos\theta)$$
$$P(-+) = \langle\psi^-|\boldsymbol{E}^{(A)}(\hat{a},-)\boldsymbol{E}^{(B)}(\hat{b},+)|\psi-\rangle = \frac{1}{4}(1+\cos\theta)$$

?



three possible axes

?

Alice **Bob**

Bell inequality

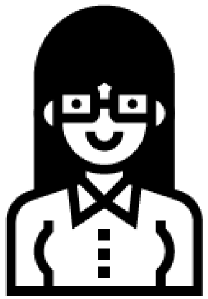
$$\boldsymbol{E}(\hat{n}, \pm) = \tfrac{1}{2}(\boldsymbol{I} \pm \hat{n} \cdot \vec{\boldsymbol{\sigma}})$$

$$|\psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

$$P_{\text{same}} = P(++) + P(--) = \frac{1}{2}(1 - \cos\theta)$$

$$P_{\text{opposite}} = P(+-) + P(-+) = \frac{1}{2}(1 + \cos\theta)$$

?



Alice

three possible axes

?

Bob

Bell inequality

First, Bell assumed that there is a joint probability distribution that governs the possible outcomes of all measurements that Alice and Bob might perform. This is the **hidden variable hypothesis**. He imagines that if the values of the hidden variables are exactly known, then the outcome of any measurement can be predicted with certainty — measurement outcomes are described probabilistically because the values of the hidden variables are drawn from an ensemble of possible values. Second, Bell assumed that Bob's decision about what to measure in Chicago has no effect on the hidden variables that govern Alice's measurement in Pasadena. This is the assumption that the **hidden variables are local**. If we accept these two assumptions, there is no escaping Bell's conclusion. We have found that the correlations predicted by quantum theory are incompatible with theses assumptions.

We have affirmed Bohr's **principle of complementary** — we are forbidden to consider simultaneously the possible outcomes of two mutually exclusive experiments.

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

Dense coding (超密编码)

$$|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)$$

1) $\boldsymbol{I}$ (she does nothing) ,

2) $\boldsymbol{\sigma}_1$ (180^o rotation about $\hat{x}$-axis) ,

3) $\boldsymbol{\sigma}_2$ (180^o rotation about $\hat{y}$-axis) ,

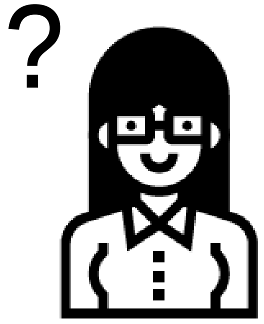
4) $\boldsymbol{\sigma}_3$ (180^o rotation about $\hat{z}$-axis) .

1) $|\phi^+\rangle_{AB}$,

2) $|\psi^+\rangle_{AB}$,

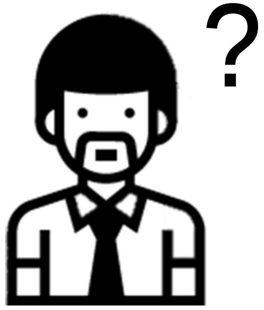
3) $|\psi^-\rangle_{AB}$ (up to a phase)

4) $|\phi^-\rangle_{AB}$.



Alice

each qubit by itself carries no information at all; all the information is encoded in the correlations between the qubits



Bob

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

Quantum teleportation(量子隐形传态)

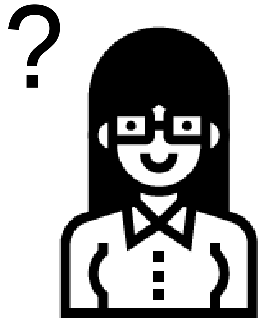
$$|\phi^{+}\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|00\rangle_{AB} + |11\rangle_{AB}\right)$$

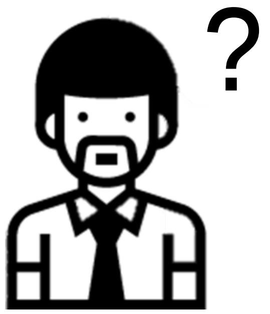
规则：

（1）Alice也不知道这个态

（2）通讯只能传递经典信息

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$





Alice **Bob**

Quantum teleportation(量子隐形传态)

$$|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_{AB} + |11\rangle_{AB})$$

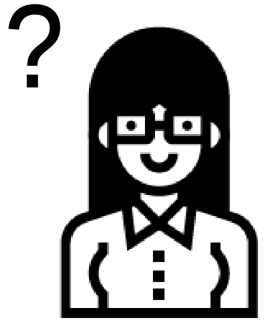
$$|\psi_0\rangle = |\psi\rangle|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)\right]$$

Quantum teleportation(量子隐形传态)

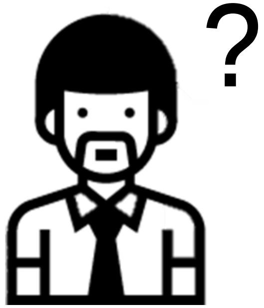
$$|\psi_0\rangle \xrightarrow{\textbf{CNOT}} |\psi_1\rangle = \frac{1}{\sqrt{2}}\left[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)\right]$$

$$|\psi_1\rangle \xrightarrow{\textbf{Hadamard}} |\psi_2\rangle = \frac{1}{2}\left[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\right]$$

$|\psi\rangle$



Alice



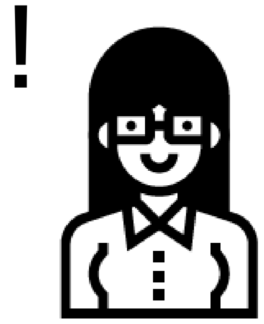
Bob

Quantum teleportation(量子隐形传态)

$$
\begin{aligned}
|\psi_2\rangle &= \frac{1}{2}\Big[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\Big] \\
&= \frac{1}{2}\Big[|00\rangle\left(\alpha|0\rangle + \beta|1\rangle\right) + |01\rangle\left(\alpha|1\rangle + \beta|0\rangle\right) \\
&\quad + |10\rangle\left(\alpha|0\rangle - \beta|1\rangle\right) + |11\rangle\left(\alpha|1\rangle - \beta|0\rangle\right)\Big]
\end{aligned}
$$

$|\psi\rangle$

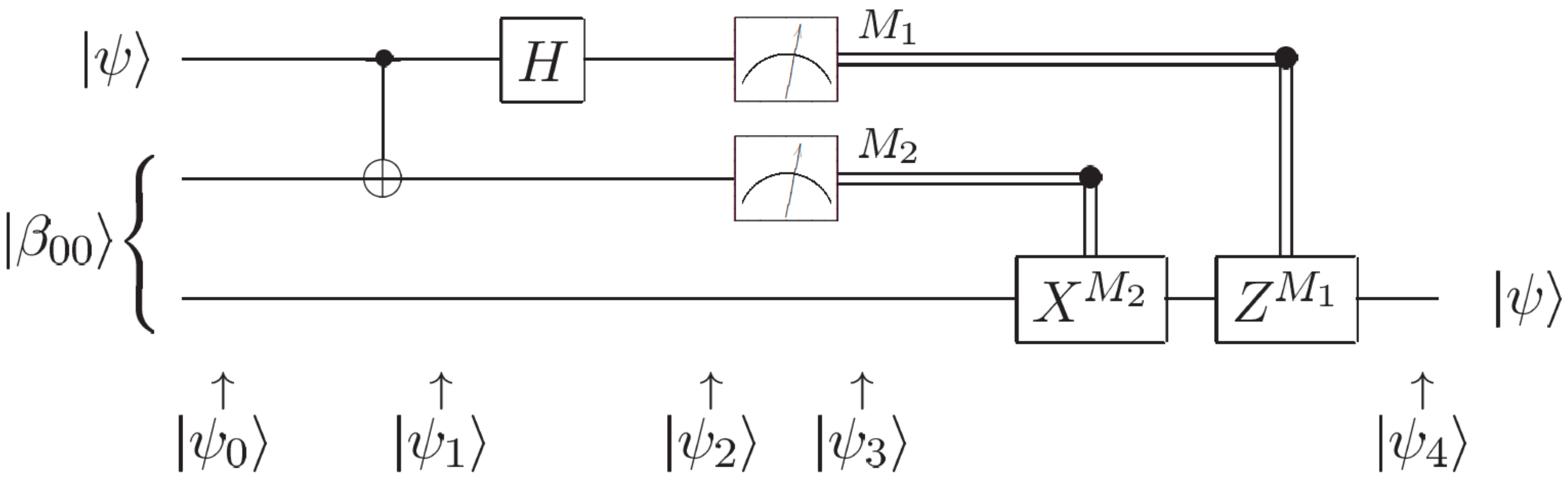
$$
\begin{aligned}
00 &\longmapsto |\psi_3(00)\rangle \equiv \big[\alpha|0\rangle + \beta|1\rangle\big] \\
01 &\longmapsto |\psi_3(01)\rangle \equiv \big[\alpha|1\rangle + \beta|0\rangle\big] \\
10 &\longmapsto |\psi_3(10)\rangle \equiv \big[\alpha|0\rangle - \beta|1\rangle\big] \\
11 &\longmapsto |\psi_3(11)\rangle \equiv \big[\alpha|1\rangle - \beta|0\rangle\big]
\end{aligned}
$$



Alice

Bob

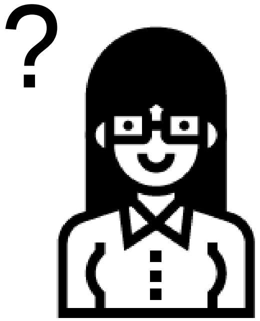
Quantum teleportation(量子隐形传态)



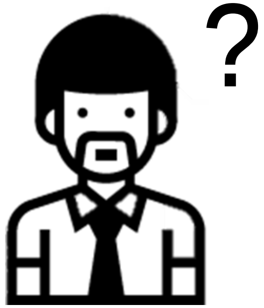
Quantum error correction(量子纠错)

规则：
（1）量子态不可克隆
（2）错误是连续的
（3）测量操作破坏量子信息

$$a|0\rangle + b|1\rangle \rightarrow a\underbrace{|000\rangle}_{\text{逻辑0}} + b\underbrace{|111\rangle}_{\text{逻辑1}}$$



Alice

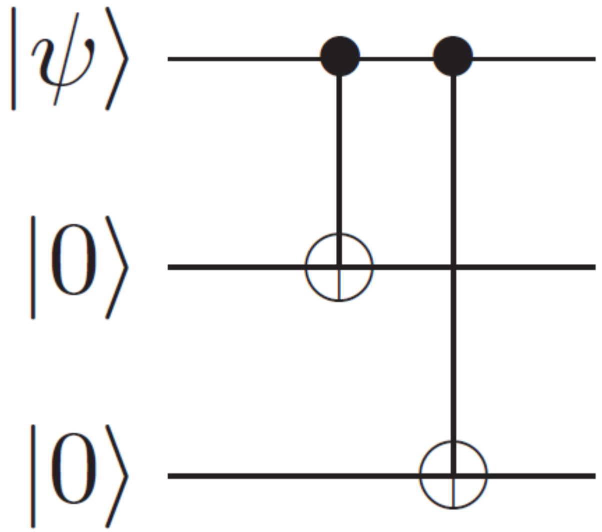


Bob

Quantum error correction(量子纠错)

$$a|0\rangle + b|1\rangle \quad \Longrightarrow \quad a|000\rangle + b|111\rangle$$

逻辑0　　逻辑1



$P_0 \equiv |000\rangle\langle 000| + |111\rangle\langle 111|$ no error

$P_1 \equiv |100\rangle\langle 100| + |011\rangle\langle 011|$ bit flip on qubit one

$P_2 \equiv |010\rangle\langle 010| + |101\rangle\langle 101|$ bit flip on qubit two

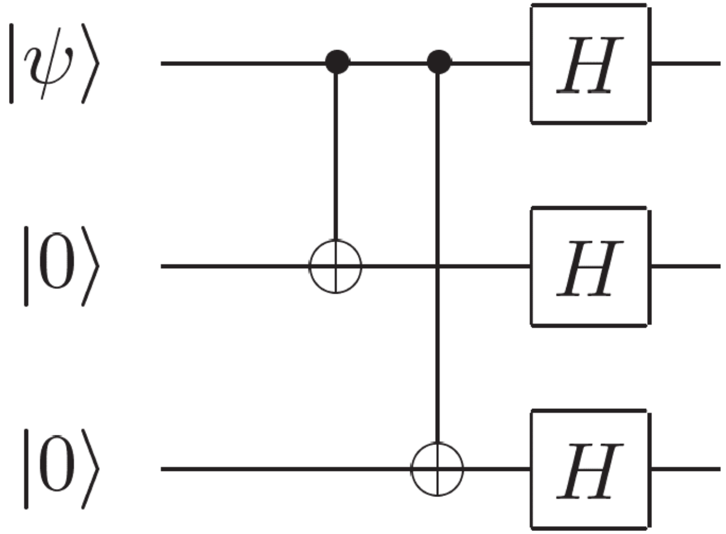
$P_3 \equiv |001\rangle\langle 001| + |110\rangle\langle 110|$ bit flip on qubit three.

Quantum error correction(量子纠错)

$$|+\rangle \equiv (|0\rangle + |1\rangle)/\sqrt{2},$$
$$|-\rangle \equiv (|0\rangle - |1\rangle)/\sqrt{2}$$

$$a|0\rangle + b|1\rangle \Rightarrow |0_L\rangle \equiv |+++\rangle \qquad |1_L\rangle \equiv |---\rangle$$

逻辑0 逻辑1

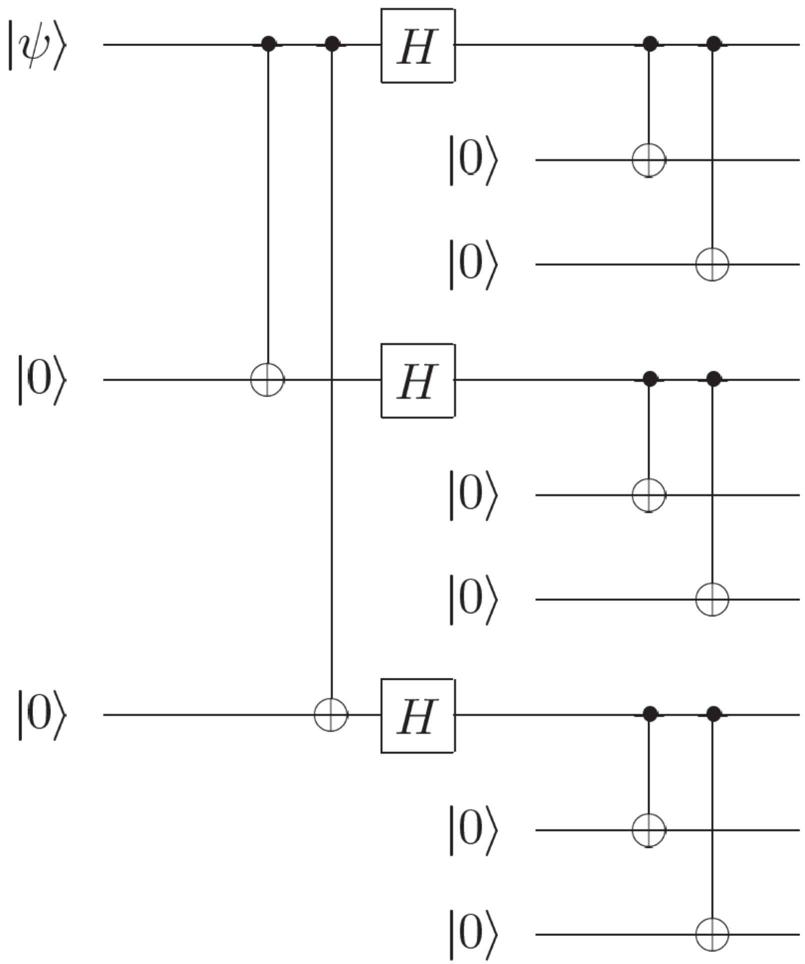


$$P_0 \equiv |000\rangle\langle 000| + |111\rangle\langle 111| \text{ no error}$$
$$P_1 \equiv |100\rangle\langle 100| + |011\rangle\langle 011| \text{ bit flip on qubit one}$$
$$P_2 \equiv |010\rangle\langle 010| + |101\rangle\langle 101| \text{ bit flip on qubit two}$$
$$P_3 \equiv |001\rangle\langle 001| + |110\rangle\langle 110| \text{ bit flip on qubit three.}$$

0,1 改为 +，-

Quantum error correction(量子纠错)

The Shor code



$$|0\rangle \rightarrow |0_L\rangle \equiv \frac{(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)}{2\sqrt{2}}$$

$$|1\rangle \rightarrow |1_L\rangle \equiv \frac{(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)}{2\sqrt{2}}$$

Shor编码可以纠正**任何**只作用在一个qubit上面的错误！（不提供证明）