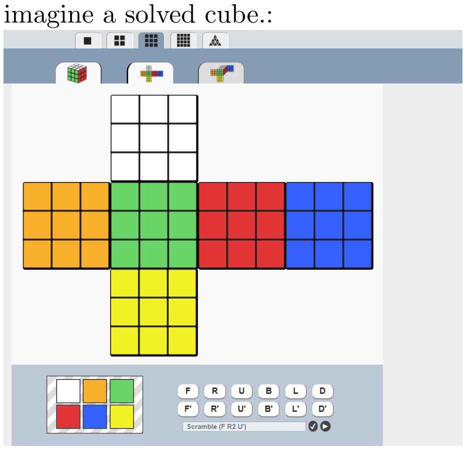
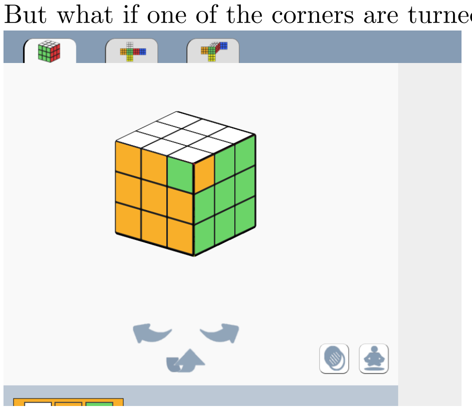
PART 1 AND 2 are completely wrong and were scrapped. Use Section 3 for the numbers used in the hashmaps.

1 My idea

Let's consider the 20 individual pieces which are all unique, and if we assume that there is only one "orientation" for the cube to actually be valid (idk if this is true), then we don't even need to consider the orientation and we can just use a list. What I mean by this:
Consider the solved cube is in the form $[p_1, p_2 \ldots p_{20}]$ In terms of the cube, imagine a solved cube.:



But what if one of the corners are turned the wrong way?



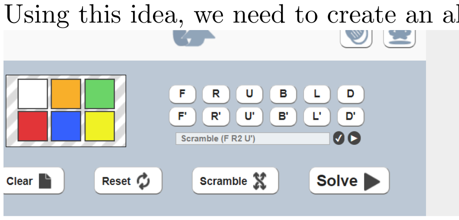
I am considering this to be impossible. I am basically assuming that there is only one correct "orientation" for each cube. So this means if we just order the list properly, the orientation should solve itself.

2 Implementation:

Consider the list $[p_1, p_2, \ldots, p_20]$ to be the "solved" version. Each p_i will be a unique piece. For example, p_2 is the orange-white edge piece. There is clearly only one of each piece, so the uniqueness of each element exists. If we have a scrambled cube, the configuration of the cube can be stored as a list of 20 items, where each p_i represents a piece. So, a scrambled list would look like:

$$[p_1, p_{15}, p_{17} ..., p_4, p_5]$$

So now we need to sort this list and make it like $[p_1, p_2, \ldots, p_20]$. But we are not allowed to sort the list using the common sorting methods. This is where we have to create rules. When we turn the cube, the list will change. Using the different moves of the cube multiple times, we are guaranteed to solve the cube. Using this idea, we need to create an algorithm for each move. The moves are:



So our first step is to create functions for reach move.
For example, if we do the move F:
F: $[p_1, p_2, \ldots, p_{20}] \implies [p_7, p_4, p_1, p_8, p_5, p_2, p_9, p_6, \ldots, p_{20}]$
(do not use this for the F implementation I might have done it wrong)
So each "move" will alter the list in some way
Then we will feed all the information into an LLM and train it to solve it.

3 Cube Orientation:

Consider the following:

$$p_1 = \text{yob corner} \tag{1}$$
$$p_2 = \text{ yb edge} \tag{2}$$
$$p_3 = \text{ybr corner} \tag{3}$$
$$p_4 = \text{ob edge} \tag{4}$$
$$p_5 = \text{br edge} \tag{5}$$
$$p_6 = \text{wbo corner} \tag{6}$$
$$p_7 = \text{wb edge} \tag{7}$$
$$p_8 = \text{wbr corner} \tag{8}$$
$$p_9 = \text{yo edge} \tag{9}$$
$$p_{10} = \text{yr edge} \tag{10}$$
$$p_{11} = \text{wr edge} \tag{11}$$
$$p_{12} = \text{wo edge} \tag{12}$$
$$p_{13} = \text{yog corner} \tag{13}$$
$$p_{14} = \text{yg edge} \tag{14}$$
$$p_{15} = \text{ygr corner} \tag{15}$$
$$p_{16} = \text{og edge} \tag{16}$$
$$p_{17} = \text{gr edge} \tag{17}$$
$$p_{18} = \text{owg corner} \tag{18}$$
$$p_{19} = \text{gw edge} \tag{19}$$
$$p_{20} = \text{grw corner} \tag{20}$$
$$\tag{21}$$

In case you couldn't already tell, I am not including the centre pieces because they cannot move.

4 Moves:

Consider when the cube is in the correct position:

$Cube = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}\}$
$R : \{p_1, p_2, p_8, p_4, p_{11}, p_6, p_7, p_{20}, p_9, p_5, p_{17}, p_{12}, p_{13}, p_{14}, p_3, p_{16}, p_{10}, p_{18}, p_{19}, p_{15}\}$
$L : \{p_{13}, p_2, p_3, p_9, p_5, p_1, p_7, p_8, p_{16}, p_{10}, p_{11}, p_4, p_{18}, p_{14}, p_{15}, p_{12}, p_{17}, p_6, p_{19}, p_{20}\}$
$F : \{p_6, p_4, p_1, p_7, p_2, p_8, p_5, p_3, p_9, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}\}$
$U : \{p_3, p_{10}, p_{15}, p_4, p_5, p_6, p_7, p_8, p_2, p_{14}, p_{11}, p_{12}, p_1, p_9, p_{13}, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}\}$
$D : \{p_1, p_2, p_3, p_4, p_5, p_{18}, p_{12}, p_6, p_9, p_{10}, p_7, p_{19}, p_{13}, p_{14}, p_{15}, p_{16}, p_{17}, p_{20}, p_{11}, p_8\}$
$B : \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{15}, p_{17}, p_{20}, p_{14}, p_{19}, p_{13}, p_{16}, p_{18}\}$
$R' : \{p_1, p_2, p_{15}, p_4, p_{10}, p_6, p_7, p_3, p_9, p_{17}, p_5, p_{12}, p_{13}, p_{14}, p_{20}, p_{16}, p_{11}, p_{18}, p_{19}, p_8\}$
$L' : \{p_6, p_2, p_3, p_{12}, p_5, p_{18}, p_7, p_8, p_4, p_{10}, p_{11}, p_{16}, p_1, p_{14}, p_{15}, p_9, p_{17}, p_{13}, p_{19}, p_{20}\}$
$F' = \{p_3, p_5, p_8, p_2, p_7, p_1, p_4, p_6, p_9, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}\}$
$U' : \{p_{13}, p_9, p_1, p_4, p_5, p_6, p_7, p_8, p_{14}, p_2, p_{11}, p_{12}, p_{15}, p_{10}, p_3, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}\}$
$D' : \{p_1, p_2, p_3, p_4, p_5, p_8, p_{11}, p_{20}, p_9, p_{10}, p_{19}, p_7, p_{13}, p_{14}, p_{15}, p_{16}, p_{17}, p_6, p_{12}, p_{18}\}$
$B' : \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{18}, p_{16}, p_{13}, p_{19}, p_{14}, p_{20}, p_{17}, p_{15}\}$