

Assignment - I

1) Show that electron does not exist inside the nucleus of the atom.

According to theory of relativity, the energy E of particle is expressed as.

$$E = mc^2 \rightarrow \textcircled{1}$$

where m is the relativistic mass of a particle moving with a velocity v and the expression for it in terms of rest mass can be written as

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow \textcircled{2}$$

If p is a momentum of the particle, then the expression for kinetic energy of the particle in terms of momentum is given

$$E = \frac{p^2}{2m} \rightarrow \textcircled{3}$$

According to Heisenberg's uncertainty principle, we have

$$\Delta x \Delta p_x \geq \frac{\hbar}{4\pi} \quad \text{or} \quad \Delta p_x \geq \frac{\hbar}{4\pi \Delta x} \rightarrow \textcircled{4}$$

We know that the diameter of the nucleus is of the order of 10^{-14} m . If an electron is to exist inside the nucleus, then the uncertainty in its position Δx must not exceed the size of the nucleus. i.e. $\Delta x \leq 10^{-14} \text{ m}$

Using the Δx in equation $\textcircled{4}$ we have

$$\Delta p_x \geq \frac{\hbar}{4\pi \Delta x} \geq \frac{6.63 \times 10^{-34}}{4\pi \times 10^{-14}} \geq 0.5 \times 10^{-20} \text{ Ns}$$

$$\therefore p_x \geq 0.5 \times 10^{-20} \text{ Ns}$$

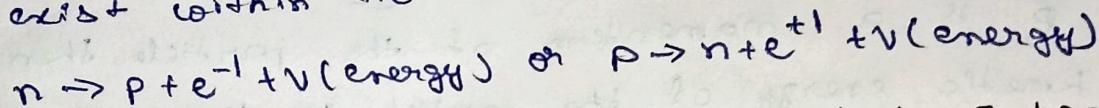
This is the uncertainty in momentum of an electron and it is equal to the momentum of the electron inside the nucleus.

Using momentum value P_x in an equation (3) we get

$$E = \frac{P^2}{2m} = \frac{(0.2 \times 10^{-20})^2}{2 \times 9.1 \times 10^{-31}} = 1.8 \times 10^{12} \text{ J}$$

$$E = \frac{1.8 \times 10^{12}}{1.602 \times 10^{-19}} = 9.36 \times 10^6 \text{ eV} \approx 9.4 \text{ MeV}$$

An electron may exist inside the nucleus if its energy is equal to or greater than 9.4 MeV. But the experimental investigations on β -decay say that kinetic energy of the β particle is 3 to 4 MeV. This clearly indicates that electrons cannot exist within the nucleus.



b) Obtain the expression for time independent Schrödinger's wave equation

By de-Broglie idea of matter waves

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

p = momentum of the particle

Let the wave function

$$\Psi = A e^{i(kx - \omega t)} \quad \rightarrow ①$$

where Ψ is total wave function

Let differentiate Ψ by twice with respect to x .

$$\frac{d\Psi}{dx} = A(ik)e^{i(kx - \omega t)}$$

$$\frac{d^2\Psi}{dx^2} = A(i^2k^2)e^{i(kx - \omega t)}$$

$$\frac{d^2\Psi}{dx^2} = -k^2\Psi \text{ or } \frac{d^2\Psi}{dx^2} + k^2\Psi = 0 \quad \rightarrow ② \quad \therefore i^2 = -1$$

$$\text{But } k = \frac{2\pi}{\lambda} \text{ and } \lambda = \frac{h}{mv}$$

$$k = \frac{2\pi mv}{h} \text{ and } k^2 = \frac{4\pi^2 m^2 v^2}{h^2}$$

eq ② becomes

$$\frac{d^2\psi}{dx^2} + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0 \rightarrow ③$$

Total energy

$$\therefore E = T + V$$

$$T = \frac{1}{2}mv^2 \therefore \frac{1}{2}mv^2 = mv^2. \text{ or } mv^2 = 2(E - V)$$

eq ③ we get

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0 \rightarrow ④$$

This is time independent 1-dimensional Schrodinger equation

④ extended for 3-dimension

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0 \rightarrow ⑤$$

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0 \rightarrow ⑥$$

where

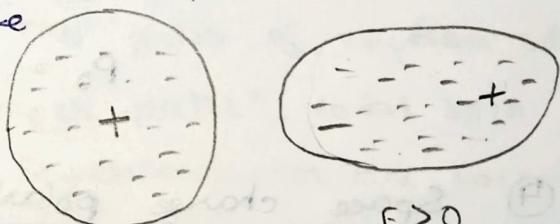
$$\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}$$

where Ψ is $\Psi(x, y, z)$

2
a) Describe in brief the various types of mechanism of polarization.

i) Electonic Induced polarization:

It occurs in mono atomic gases like Hg, Ne, Ar etc. Before the application of EF the '+ve' charge will be surrounded by '-ve' charge



After the Application of EF the '+ve' charge move in the direction of applied EF. so they create dipole moment they are independent of temp. the time taken for completion

t_p is 10^{-15} sec.

$$P = N \alpha \epsilon \quad P = \epsilon_0 (\epsilon_r - 1) E \quad \alpha_e = \frac{P_e}{N E} = \frac{\epsilon_0 (\epsilon_r - 1)}{N \beta} \propto -\frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

② Ionic polarization

It occurs in the dielectric material which possess ionic bond such as NaCl. Before EF the +ve & -ve charge are mixed together. After EF the -ve charge is E = 0 and the +ve charge moves in the direction of EF & opp to the EF. It doesn't depend on temp & time to polarization 10^{-15} to 10^{-12} s.

$$P_e = N \alpha_e E \quad \alpha_e = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

③ Orientational polarization

It occurs in DM's which has a molecules with permanent dipole movement. Before it molecules moving in random direction because of thermal energy. After EF molecules with obtain E ≠ 0 & movements with the direction of EF and the dipole will get aligned therefore net dipole movement is zero temp dependent. Time to polarization is 10^{-12} s.

$$P_o = N \alpha_o E \quad \alpha_o = \frac{\mu^2}{3kT}$$

$$P_o = \frac{Nm^2 E}{3kT}$$

④ Space charge polarization

It takes place in multiphase DM's in which the resistivity changes for different phases. before EF

+ve & -ve charge will be mixed together. After EF high temperature E = 0

+ve & -ve charge will occupies in off part of at

low resistivity phase. It is strongly dependent on high temperature

Explain in brief the BCS theory of Superconductivity.
BCS theory is based upon the formation of Cooper pairs. During the flow of current in a superconductor, when an electron comes near a positive ion core of the lattice it experiences an attractive force because of the opposite charge polarity between electron and ion core, the ion core will be displaced from its position due to this interaction which is called 'lattice distortion'.

Now, an electron which comes near that place will also interact with the distorted lattice which tends to reduce the energy of electron. The interaction is called 'electron-lattice-electron' interaction via the phonon field. Because of the reduction of energy during the interaction, it is treated as equivalent to establishing an attractive force between the two electrons, becomes Cooper.

Below critical temperature attraction reaches a maximum value of any two electrons of equal and opposite spins & opposite momentum. This force of attraction exceeds the Columbian force for repulsion between two electrons stick together and move as pairs. The pairs of electrons of opposite moment are called 'cooper pairs'. Total spin of zero, the electron pairs in a superconductor are bosons.

$T < T_c$ lattice electron interaction is stronger of coulomb pair encompasses as many as 10^6 other pairs. When the electron flows in the form of cooper pairs in materials, they do not encounter any scattering and thereby resistance reduces to zero or conductivity becomes infinity which is known as 'superconductivity'.

3

2)

Derive the expression for density of states of metal

$g(E) dE = \frac{\text{No. of energy states available b/w } E \text{ and } E + dE}{\text{Volume of that metal}}$

$$\text{Energy } E_n = \frac{n^2 h^2}{8mL^2} \rightarrow ①$$

In three dimension

$$E = \frac{h^2}{8mL^2} (n_x^2 + n_y^2 + n_z^2). \rightarrow ②$$

$n_x, n_y \& n_z$ are non zero positive integers

$$E_0 = \frac{h^2}{8mL^2} + R^2 = n_x^2 + n_y^2 + n_z^2$$

$$E_0 = E_0 R^2$$

The number of allowed energy values $N(E) dE$

$$N(E) dE = \frac{1}{8} \times 4\pi R^2 dR \rightarrow ③$$

$$N(E) dE = \frac{1}{2} \pi R^2 dR \rightarrow ④$$

$$Z(E) dE = 2 \times \frac{1}{2} \pi R^2 dR = \pi R^2 dR \rightarrow ⑤$$

$$E = E_0 R^2$$

$$R = \sqrt{\frac{E}{E_0}} \quad \& \quad dE = E_0 2R dR \quad \text{or} \quad R dR = \frac{dE}{2E_0}$$

$$R^2 dR = R R dR = \sqrt{\frac{E}{E_0}} \frac{dE}{2E_0} = \frac{1}{2} \frac{\sqrt{E} dE}{\sqrt{E_0^3}} \rightarrow ⑥$$

$R^2 dR$ in eqn ⑤

$$Z(E) dE = \frac{\pi}{2} \frac{\sqrt{E} dE}{\sqrt{E_0^3}}$$

$$\text{But } E_0 = \frac{h^2}{8mL^2} \rightarrow ⑦$$

$$\therefore Z(E) dE = \frac{\pi}{2} \frac{\sqrt{E} dE}{\left(\frac{h^2}{8mL^2}\right)^{3/2}} = \frac{\pi}{2} \left(\frac{8mL^2}{h^2}\right)^{3/2} E^{1/2} dE$$

$$Z(E) dE = \frac{\pi}{2} \left(\frac{8m}{\hbar^2} \right)^{3/2} L^3 E^{1/2} dE \rightarrow ⑧$$

$$\therefore g(E) dE = \frac{Z(E) dE}{\text{Volume } (L^3)}$$

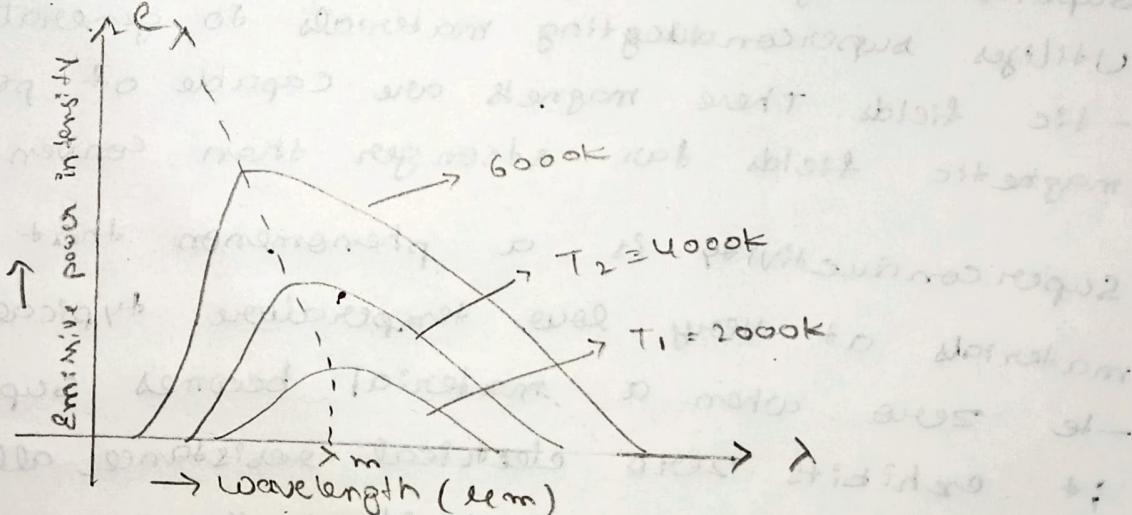
$$\text{Hence } g(E) dE = \frac{\pi}{2} \left(\frac{8m}{\hbar^2} \right)^{3/2} L^3 E^{1/2} dE \rightarrow ⑨$$

$$g(E) dE = \frac{\pi}{2} \frac{8\sqrt{2}m^{3/2}}{\hbar^3} E^{1/2} dE \rightarrow ⑩$$

equation 9 & 10 represent density of energy states

- a) Explain the energy distribution in the spectrum of a black body.

The curve in figure shows the variation of intensity of radiation with wavelength for different temperature of the blackbody ($T_1, 2T_2, 3T_3$).



- 1) At a given temperature, the energy distribution is not uniform in the spectrum of blackbody radiation
- 2) At a given temperature, blackbody emits continuously range of wavelength of radiation. As the wavelength

increases the intensity of radiation increases reaches a maximum value at a particular wavelength λ_m and then decreases with further increases in wavelength

- ③ As temperature increases, wavelength λ_m maximum energy density decreases and vice versa

$$\lambda_m \propto 1/T$$

- ④ For all wavelength, an increase in temperature cause an increase in energy emission or intensity

- ⑤ the area under curve represent the total energy emitted for the complete spectrum. area under the curve increases with increase in temperature

$$E \propto T^4$$

- b) Briefly explain about Superconducting magnets
Superconducting magnets are a type of electromagnet that utilizes superconducting materials to generate strong magnetic fields. These magnets are capable of producing intense magnetic fields far stronger than conventional magnets.

Superconductivity is a phenomenon that occurs in certain materials at very low temperature, typically near absolute zero. When a material becomes superconducting, it exhibits zero electrical resistance, allowing electric current to flow through it without any energy loss. This unique property makes superconductors ideal for creating powerful magnets.

To create a superconducting magnet, the superconduc-

-ting wire is wound into a coil or solenoid configuration
-n . when a direct current (DC) is passed through the
coil , a strong magnetic field is generated . the magnetic
field strength of a superconducting magnet is directly
proportional to the current flowing through the coil