Exercise 1

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Homework:

1. To evaluate a new test for detecting Hansen's disease, a group of people 5% of which are known to have Hansen's disease are tested. The test finds Hansen's disease among 98% of those with the disease and 3% of those who don't. What is the probability that someone testing positive for Hansen's disease under this new test actually has it?

2. Proof the following distributions are normalized then calculate the mean and standard deviation of these distribution:

   (a) Uni-variate normal distribution.

   (b) (Optional) Multivariate normal distribution.

Task:

1. The probability that someone testing positive for Hansen's disease is:

$$P = \frac{0,98.0,05}{0,98.0,05 + 0,03.(1 - 0,05)} \approx 0,632$$

2.

   (a) Uni-variate normal distribution:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}.e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

+)Proof the Uni-variate is normalize:

$$\Leftrightarrow \int_{-\infty}^{\infty} f(x)d(x) = 1$$

$$\Leftrightarrow \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{\frac{(x-\mu)^2}{2\sigma^2}} dx = 1$$

$$\Leftrightarrow \int_{-\infty}^{\infty} e^{\frac{(x-\mu)^2}{2\sigma^2}} dx = \sqrt{2\pi\sigma^2}(1)dx$$

Suppose:

$$y = \frac{x-\mu}{\sqrt{2}\sigma}$$

$$(1) \rightarrow \frac{\sqrt{2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-y^2} dy$$

$$\Leftrightarrow \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-y^2} dy$$

Then, using the Gaussian integral identity:

$$\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$$

$$\Leftrightarrow \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-y^2} dy = 1$$

$$\Leftrightarrow Proofed:$$

+)Mean:
Let:

$$Z = \frac{X-\mu}{\sigma}$$

We have:

$$E(Z) = \int_{-\infty}^{\infty} x f_Z(x) dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{\frac{-x^2}{2} dx} = 0$$

Because:

$$X = \mu + \sigma Z$$

$$\Rightarrow E(X) = \mu + E(\mu)E(Z)$$

$$\Rightarrow E(X) = \mu$$

+)Variance:

$$Var(Z) = E(Z^2) - (E(Z))^2$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{\frac{-x^2}{2} dx} = 1$$

Because:

$$X = \mu + \sigma Z$$

$$\Rightarrow Var(X) = Var(\mu) + \sigma^2 Var(Z)$$

$$\Rightarrow Var(X) = 0 + \sigma^2 1 = \sigma^2$$

(b) Multivariate normal distribution: