Problem 1
Transform back posterior on the class of latex, from $p(w \mid D) \rightarrow (w = X^T X + \alpha I)^{-1} X^T t$

Solution

$$p(w \mid x, t, \alpha, \beta) \propto p(t \mid x, w, \beta) p(w \mid \alpha)$$
$$\Rightarrow log(p(w \mid x, t, \alpha, \beta)) \propto log(p(t \mid x, w, \beta) p(w \mid \alpha))$$

We have:

$$p(t \mid x, w, \beta) = \prod_{n=1}^{N} N(t_n \mid y(x_n, w), \beta^{-1})$$
$$\Rightarrow p(t \mid x, w, \beta) = \prod_{n=1}^{N} \frac{1}{\sqrt{2\pi\beta^{-1}}} e^{\frac{-(t - y(x_n, w))^2}{2\beta^{-2}}}$$
$$log(p \mid x, w, \beta) = \frac{-\beta}{2} \sum_{n=1}^{N} ((t - y(x_n, w))^2) + noise$$

And:

$$p(w \mid a) = N(w \mid 0, \alpha^{-1} I)$$
$$= \frac{1}{(2\pi)^{D/2} \mid \sum \mid^{1/2}} e^{\frac{-(w-0)^T \sum^{-1} (w-0)}{2}}$$
$$log(p(w \mid a)) = \frac{-1}{2} w^T w + noise$$

So:

$$log(p(w \mid x, t, \alpha, \beta) \alpha \frac{-\beta}{2} \sum_{n=1}^{N} (t - y(x_n, w))^2 + \frac{-1}{2} w^T w$$

The maximum of the posterior is given by minimum of:

$$\frac{\beta}{2} \sum_{n=1}^{N} (t - y(x_n, w))^2 + \frac{-1}{2} w^T w$$

Or we minimize:

$$Q = \mid\mid Xw - t \mid\mid_2^2 + \lambda w^T w$$
$$\bigtriangledown Q_w = 2X^T (Xw - t) + 2\lambda w$$
$$\Rightarrow w = (X^T X + \lambda I)^{-1} X^T t$$