Problem 1
Re-transform the logistic regression algorithm

Solution
The model logistic regression:

$$p(C1 \mid \phi) = y(\phi) = \sigma(w^T \phi)$$
$$p(C2 \mid \phi) = 1 - p(C1 \mid \phi)$$

For a data set ϕ_n, t_n, where $t_n \in 0, 1$ and $\phi_n = \phi(x_n)$ with n=1,...,N, the livelihood function can be written:

$$p(t \mid W) = \prod_{n=1}^{N} y_n^{t_n}(1 - y_n)^{1-t_n}$$

where $t_n = (t_1, ..., tN)^T$ and $y_n = p(C1 \mid \phi_n)$
Taking negative logarithm, we have:

$$L = -logp(t \mid w) = -\int_{n=1}^{N} t_n log y_n + ((1 - t_n)log(1 - y_n)$$

where $y_n = \sigma(a_n) and a_n = w^T \phi_n$
We have:

$$\frac{\partial L}{\partial w} = \sum_{n=1}^{N} (\frac{\partial L}{\partial y_n} . \frac{\partial y_n}{\partial a_n} . \frac{\partial a_n}{\partial w})$$
$$\frac{\partial L}{\partial w} = \frac{y_n - t_n}{y_n(1 - y_n)}$$
$$\frac{\partial y_n}{\partial a_n} = y_n(1 - y_n)$$
$$\frac{\partial a_n}{\partial w} = \phi_n$$

So:

$$\frac{\partial L}{\partial w} = \sum_{n=1}^{N} (\frac{\partial L}{\partial y_n} . \frac{\partial y_n}{\partial a_n} . \frac{\partial a_n}{\partial w})$$
$$= \sum_{n=1}^{N} (y_n - t_n)\phi_n$$

Problem 2
Find the function: $f'(x) = f(x)(1 - f(x))$

Solution

$$f'(x) = f(x)(1 - f(x)$$
$$\Leftrightarrow \frac{d(f(x))}{dx} = f(x)(1 - f(x)$$
$$\Rightarrow \frac{d(f(x))}{f(x)(1 - f(x)} = dx$$
$$\Rightarrow \int \frac{d(f(x))}{f(x)(1 - f(x)} = \int dx$$

$$\Rightarrow \int (\frac{1}{f(x)} + \frac{1}{1-f(x)} d(f(x)) = \int dx$$

$$\Leftrightarrow ln(f(x)) - ln(1-f(x)) = x$$

$$\Leftrightarrow \frac{f(x)}{1-f(x)} = e^x$$

$$\Leftrightarrow f(x) = e^x - e^x f(x)$$

$$\Leftrightarrow f(x) = \frac{e^x}{1+e^x} = \sigma(x)$$