INTERNATIONAL UNIVERSITY

DEPARTMENT OF INDUSTRIAL ENGINEERING AND MANAGEMENT

DETERMINISTIC MODELS IN OPERATION RESEARCH

Project case study group 7

Lecturer: Nguyễn Văn Hợp

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GROUP EVALUATION FORM

1. Timeline

Step	Objective	Duration
1	Group register	2 weeks
2	Problem Acknowledgement	1 week
3	Model development	2 weeks
4	Solve the problem with assumed data	5 weeks
5	Sensitivity Analysis	2 weeks
6	Propose Applications	1 week
7	Report Writing	1 week
8	Submission and presentation	1 week

2. Group evaluation (by lecturer)

	Criteria	Maximum mark	Your mark
1	Problem Identification	10	
2	Model Development	25	
3	Problem Solving	25	
4	Applications	15	
5	Writing report (grammar, readability, format, and flow)	10	
6	Oral Defense (giving clear presentation and satisfactory answers)	15	
	Total	100	

3. Policies

Penalty:

- Late submission 1 day: -10%
- Late submission 2 days: -20%
- Late submission > 2 days: Submission is not accepted!
- Absent from presentation: 20%

4. Group Individual Assessment

Full Name	Student ID	Contribution
Phạm Ngọc Huy	IELSIU19166	100%
Ngô Trọng Gia	IELSIU19142	100%
Lê Thị Thùy Trang	IEIEIU19073	100%
Nguyễn Hải Nam	IEIEIU19008	100%

INTRODUCTION

1. Background of study

Operation Research (OR) is the study of applying mathematics to help answering the business's questions and be their helpful information in decisions making. OR can be applied to a variety of use cases, including: Scheduling and time management, urban and agricultural planning, enterprise resource planning and supply chain management, inventory management, network optimization and engineering, packet routing optimization, risk management.

2. Introduction to Linear Programming (LP)

Linear Programming (LP) is a mathematical method that help the users to make decisions related with Resources Allocation. In addition, the goal for LP is maximizing or minimizing a linear function which is subjected to linear constraints (equality or inequality).

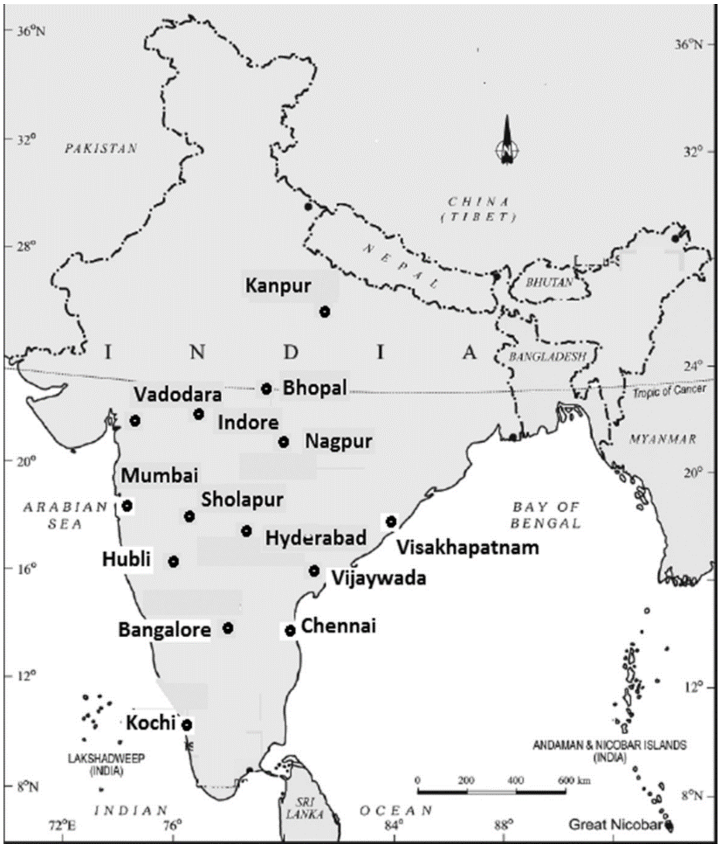
In short, Linear Programming is the method of evaluating different inequalities relevant to a situation such as Resources Limit and optimize the solution in order to obtain the highest (**maximization**) or lowest (**minimization**) value for the problem.
Components of Linear Programming problem:

- **Variables:** These parameters may be numerical (real numbers) as in the number or quantity of products needed to produce or binary for a Yes/No decision on whether a distribution center is open or not. Linear programming determines the optimal value for such variables.
- **Objective Functions:** such as maximum profit function, are a linear function of an optimization's variables. The LP problem will give us the optimal value of the problem by bounding between the parameters and variables to find that value.
- **Constraints:** are also linear functions of an optimization's variables and are used to restrict the values an optimization can return for a variable. Constraints must be Boolean linear functions. Examples of constraints could be a specified ratio of budget allocation or the total number of items a factory can produce.

3. Problem statement

Problem 11:

The company produces five commodities CA, CB, CC, CD, and CE in four plants located in Vadodara, Visakhapatnam, Nagpur, and Kochi as shown in Figure 11.1. Each plant has a certain maximum production capacity for each commodity, as given in Table 11.1. It is assumed that the cost of production of these commodities is the same at all the plants.



	Vadodara	Visakhapatnam	Nagpur	Kochi
CA	200	400	800	400
CB	400	600	400	600
CC	800	200	600	800
CD	400	400	200	200
CE	600	800	400	400

Table 11.1

	Mumbai	Chennai	Bangalore	Hyderabad	Bhopal	Kanpur
CA	50	50	100	400	50	160
CB	100	200	150	300	100	100
CC	400	100	200	100	100	100
CD	50	100	200	50	200	50
CE	100	500	50	50	50	300

Table 11.2

The customers for these five commodities are located at different parts of the country around the cities of Mumbai, Chennai, Bangalore, Hyderabad, Bhopal, and Kanpur shown in Figure 11.1. The demand of the customers for each of the commodities is known and given in Table 11.2. The company is planning to set up at most three distribution centers to streamline the supply from the plants to the customers.1 Commodities will be transported from the production plants to the distribution centers, from where they will be sent to the customers. Each customer will be served by a single distribution center. No commodity will be transported directly from any plant to any customer. The company has identified four candidate locations at Sholapur, Indore, Vijayawada, and Hubli (as shown in Figure 11.1) for the planned distribution centers. The distances (in kilometers) from the plants and customers to the candidate locations are given in Table 11.3. The company wishes to select the distribution center locations such that the transportation cost from the plants to the distribution centers and the distribution centers to the customers and the cost of the distribution centers is minimized. In the process, the allocation of the customers to distribution centers will also be determined.

		Solapur	Indore	Vijayawada	Hubli
Plants	Vadodara	791	347	1386	972
	Visakhapatnam	949	1287	349	1127
	Nagpur	587	510	709	884
	Kochi	1173	1973	1090	964
Customers	Mumbai	402	585	997	583
	Chennai	956	1455	458	763
	Bangalore	618	1418	660	409
	Hyderabad	310	829	272	511
	Bhopal	844	196	1061	1202
	Kanpur	1331	703	1452	1709

Table 11.3

	CA	CB	CC	CD	CE
Transportation cost per unit per kilometer	1	0,2	5	2	10

Table 11.4

	Solapur	Indore	Vijayawada	Hubli
Fixed cost	2000	5000	3000	1000
Variable cost	2	3	4	1
Minimum annual throughput	200	100	500	200
Maximum annual throughput	1000	1100	2000	2000

Table 11.5

We assume that transportation cost is directly proportional to the distance traversed. The transportation cost per unit per kilometer for the five commodities CA, CB, CC, CD, and CE are given in Table 11.4. Each distribution center location has different fixed costs and variable costs as given in Table 11.5. The variable cost of each location is directly proportional to the annual throughput. The minimum and maximum annual throughput of the distribution centers are also given in the Table 11.5.

1. Select the distribution center locations such that the transportation cost from the plants to the distribution centers and the distribution centers to the customers and the cost of the distribution centers is minimized?
2. Formulate the model to solve the optimization problem according to the case described below following the objective and constraints required?
3. Solve the model by using LINGO or CPLEX and display the solution clearly?
4. Do and propose the sensitivity analysis?

4. Problem description

- The customers for these five commodities are located at different parts of the country around the cities of Mumbai, Chennai, Bangalore, Hyderabad, Bhopal, and Kanpur (Figure 11.1).
- Commodities will be transported from the production plants to the distribution centers, from where they will be sent to the customers.
- The company has identified four candidate locations at Sholapur, Indore, Vijayawada, and Hubli for the planned distribution centers. (Figure 11.1).
- The distances (in kilometers) from the plants and customers to the candidate locations are given in (Table 11.3).
- The transportation cost per unit per kilometer for the five commodities CA, CB, CC, CD, and CE are given in (Table 11.4).
- Each distribution center location has different fixed costs and variable costs, the minimum and maximum annual throughput of the distribution centers as given in (Table 11.5).

The company wishes to select the distribution center locations such that the transportation cost from the plants to the distribution centers and the distribution centers to the customers and the cost of the distribution centers is minimized.

5. Problem objective

Based on the information above, the objective of this project is:

- First, to formulate a linear program (setting up objective function, constraint, decision variables) and then solve it using the LINGO or CPLEX application to determine the distribution center locations that minimized the transportation cost.
- Second, to use Simplex Method in order to apply Sensitivity Analysis and determine the optimal range for each type of products which constrained by the resources or the rate of change of the resource constraints and determine the Shadow Price and/or Reduced Cost appeared in this problem.
- Finally, propose some real life practical applications of LP models.

MATHEMATICAL MODEL/ ALGORITHMS

A. Summary

Family Thrillseeker:

- Structure: 4 doors
- Profit: \$3,600/ car
- Demand: no limit
- Manufacture: 6 labor-hour/ car

Classy Cruiser:

- Structure: 2 doors
- Profit: \$5,400/ car
- Demand: 3,500 cars (limit)
- Manufacture: 10.5 labor-hour/ car

Total capacity of factory: 48,000 labor-hour/ month

Total doors received from supplier: 20,000 doors.

B. Problem solving

❖ Decision variables

x: The number of Family Thrillseeker should be assemble

$x \geq 0$

y: The number of Classy Cruiser should be assemble

$0 \leq y \leq 3,500$ (limited demand)

❖ Objective function

Maximize: $Z = 3600x + 5400y$

Z: total profit

❖ Constraints

The total capacity of the factory is 48,000 labor-hour/ month while Family Thrillseeker need 6 labor-hour and Classy Cruiser need 10.5 labor-hour to produced, then we have:

$6x + 10y \leq 48000$

The total number of doors received from supplier is 20,000 doors (10,000 left door and 10,000 right door for both two type of cars). Family Thrillseeker has 4 door/ car and Classy Cruiser has 2 door/ car, then we have:

$4x + 2y \leq 20000$

❖ Full mathematical model

Maximize Z = \$3600x + \$5400y

$$\text{Subject to:}\begin{cases} 6x + 10.5y \leq 48{,}000 \\ 4x + \quad 2y \leq 20{,}000 \\ \qquad\quad y \leq 3{,}500 \\ \qquad\quad x \geq 0 \\ \qquad\quad y \geq 0 \end{cases}$$

LINGO CODE

```
max = 3600*x + 5400*y;
!constraints;
6*x+10.5*y<=48000;
4*x+2*y<=20000;
y<=3500;
x>=0;
y>=0;
@GIN(x);
@GIN(y);
```

LINGO RESULT

```
Global optimal solution found.
Objective value:                              0.2664000E+08
Objective bound:                              0.2664000E+08
Infeasibilities:                               0.000000
Extended solver steps:                                0
Total solver iterations:                              2
Elapsed runtime seconds:                           0.08

Model Class:                                       PILP

Total variables:                      2
Nonlinear variables:                  0
Integer variables:                    2

Total constraints:                    6
Nonlinear constraints:                0

Total nonzeros:                       9
Nonlinear nonzeros:                   0


                              Variable           Value        Reduced Cost
                                     X        3800.000           -3600.000
                                     Y        2400.000           -5400.000

                                   Row    Slack or Surplus      Dual Price
                                     1        0.2664000E+08        1.000000
                                     2         0.000000            0.000000
```

The number of Family Thrillseeker should be assemble is 3800 units

The number of Classy Cruiser should be assemble is 2400 units

Profit per labor of each types of product is calculated as followed:

For FT: $\frac{(\$3600\ of\ profit\ per\ unit)*3800\ units}{6\ labors\ needed}$ = $2,280,000 / unit of labor

For CC: $\frac{(\$5400\ of\ profit\ per\ unit)*2400\ units}{10.5\ labors\ needed}$ = $1,234,285.71 / unit of labor

Hence, the Family Thrillseeker generates a larger profit than the Classy Cruiser.

MANNUAL SOLVING AND SENSITIVITY ANALYSIS

Objective coefficient ranges, and Right hand side ranges, Slack or surplus, reduced cost and dual cost are required for sensitivity analysis.

1. Simplex method and sensitivity analysis preparation

We have: $Z - 3{,}600 * x - 5{,}400 * y = 0$, and

$$\begin{cases} 6x + 10.5y + s_1 = 48{,}000 \\ 4x + \quad 2y + s_2 = 20{,}000 \\ \qquad\quad y + s_3 = 3{,}500 \end{cases} \text{ and } \begin{cases} x, y \geq 0 \\ s_1,\ s_2,\ s_3 \geq 0 \end{cases}$$

Simplex Tableau:

Iteration	Basic variables	x	y	s_1	s_2	s_3	RHS	Ratio
0	**Z**	−3600	**−5400**	0	0	0	0	
	s_1	6	10.5	1	0	0	48000	4571
	s_2	4	2	0	1	0	20000	10000
	s_3	0	**1**	0	0	1	3500	**3500**

Iteration	Basic variables	x	y	s_1	s_2	s_3	RHS	Ratio
1	**Z**	**−3600**	0	0	0	5400	18900000	
	s_1	**6**	0	1	0	−10.5	11250	**1875**
	s_2	4	0	0	1	−2	13000	3250
	y	0	1	0	0	1	3500	

Iteration	Basic variables	x	y	s_1	s_2	s_3	RHS	Ratio
2	**Z**	0	0	600	0	**−900**	25650000	
	x	1	0	0.167	0	−1.75	1875	
	s_2	0	0	−0.67	1	**5**	5500	**1100**
	y	0	1	0	0	1	3500	3500

Iteration	Basic variables	x	y	s_1	s_2	s_3	RHS	Ratio
3	**Z**	0	0	480	180	0	26640000	
	x	1	0	−0.067	0.35	0	3800	
	s_3	0	0	−0.133	0.2	1	1100	
	y	0	1	0.133	−0.2	0	2400	

⇨ Base on the final tableau, we can start evaluating sensitivity analysis

2. Sensitivity analysis evaluating

The objective function coefficients range:

Profit of Family Thrillseeker per unit: in what range of x that makes the Z stay constant?

Profit of Classy Cruiser per unit: in what range of y that makes the Z stay constant?

Row 0: [0 0 480 180 0]

If $\mathbf{c_1}$ change to $\overline{\mathbf{c_1}} = \mathbf{3600} + \Delta\mathbf{c_1}$

New row 0 : [$0-\Delta c_1$ 0 480 180 0]

For x to stay a basic, $z_1^* - \overline{c_1}$ *must be equal to* 0.

Gaussian elimination:

Row 0:	[$0-\Delta c_1$	0	480	180	0]
$+ \Delta c_1 \times Row$ 1:	[Δc_1	0	$-0.067\Delta c_1$	$0.35\Delta c_1$	0]
	= [0	0	$480-0.067\Delta c_1$	$180+0.35\Delta c_1$	0]

The optimal does not change if $\begin{cases} 480 - 0.067\Delta c_1 \geq 0 \\ 180 + 0.35\Delta c_1 \geq 0 \end{cases} \leftrightarrow \begin{cases} \Delta c_1 \leq 7200 \\ \Delta c_1 \geq -514.286 \end{cases}$

$\rightarrow -514.286 \leq \Delta c_1 \leq 7200$

$\rightarrow 3085.714 \leq \overline{c_1} \leq 10800$

If $\mathbf{c_2}$ change to $\overline{\mathbf{c_2}} = \mathbf{5400} + \Delta\mathbf{c_2}$

New row 0 : [0 $0-\Delta c_2$ 480 180 0]

For y to stay a basic, $z_2^* - \overline{c_2}$ *must be equal to* 0.

Gaussian elimination:

Row 0:	[0	$0-\Delta c_2$	480	180	0]
$+ \Delta c_2 \times Row$ 3:	[0	Δc_2	$0.133\Delta c_2$	$-0.2\Delta c_2$	0]
	= [0	0	$480+0.133\Delta c_2$	$180-0.2\Delta c_2$	0]

The optimal does not change if $\begin{cases} 480 + 0.133\Delta c_2 \geq 0 \\ 180 - 0.2\Delta c_2 \geq 0 \end{cases} \leftrightarrow \begin{cases} \Delta c_2 \geq -3600 \\ \Delta c_2 \leq 900 \end{cases}$

$\rightarrow -3600 \leq \Delta c_2 \leq 900$

$\rightarrow 1800 \le \bar{c_2} \le 6300$

The right-hand-side values range:

$$\begin{pmatrix} b_1^* \\ b_2^* \\ b_3^* \end{pmatrix} = S^*b' = S^*b + S^*(b'-b) = \begin{pmatrix} 3800 \\ 1100 \\ 2400 \end{pmatrix} + \begin{pmatrix} -0.067 & 0.35 & 0 \\ -0.133 & 0.2 & 1 \\ 0.133 & -0.2 & 0 \end{pmatrix} \begin{pmatrix} \Delta b_1 \\ \Delta b_2 \\ \Delta b_3 \end{pmatrix}$$

<u>If $\mathbf{b_1}$ change to $\mathbf{b_1' = b_1 + \Delta b_1}$</u>

$$S^*b' = \begin{pmatrix} 3800 \\ 1100 \\ 2400 \end{pmatrix} + \begin{pmatrix} -0.067 & 0.35 & 0 \\ -0.133 & 0.2 & 1 \\ 0.133 & -0.2 & 0 \end{pmatrix} \begin{pmatrix} \Delta b_1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3800 - 0.067\Delta b_1 \\ 1100 - 0.133\Delta b_1 \\ 2400 + 0.133\Delta b_1 \end{pmatrix}$$

$$\begin{cases} 3800 - 0.067\Delta b_1 \ge 0 \\ 1100 - 0.133\Delta b_1 \ge 0 \\ 2400 + 0.133\Delta b_1 \ge 0 \end{cases} \leftrightarrow \begin{cases} \Delta b_1 \le 57000 \\ \Delta b_1 \le 8250 \\ \Delta b_1 \ge -18000 \end{cases} \leftrightarrow \begin{cases} \Delta b_1 \le 8250 \\ \Delta b_1 \ge -18000 \end{cases}$$

$\rightarrow -18000 \le \Delta b_1 \le 8250$

$\rightarrow -18000 \le b_1' - 48000 \le 8250$

$\rightarrow 30000 \le b_1' \le 56250$

<u>If $\mathbf{b_2}$ change to $\mathbf{b_2' = b_2 + \Delta b_2}$</u>

$$S^*b' = \begin{pmatrix} 3800 \\ 1100 \\ 2400 \end{pmatrix} + \begin{pmatrix} -0.067 & 0.35 & 0 \\ -0.133 & 0.2 & 1 \\ 0.133 & -0.2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \Delta b_2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3800 + 0.35\Delta b_2 \\ 1100 + 0.2\Delta b_2 \\ 2400 - 0.2\Delta b_2 \end{pmatrix}$$

$$\begin{cases} 3800 + 0.35\Delta b_2 \ge 0 \\ 1100 + 0.2\Delta b_2 \ge 0 \\ 2400 - 0.2\Delta b_2 \ge 0 \end{cases} \leftrightarrow \begin{cases} \Delta b_2 \ge -10857.14 \\ \Delta b_3 \ge -5500 \\ \Delta b_3 \le 12000 \end{cases} \leftrightarrow \begin{cases} \Delta b_2 \le 12000 \\ \Delta b_2 \ge -5500 \end{cases}$$

$\rightarrow -5500 \le \Delta b_2 \le 12000$

$\rightarrow -5500 \le b_2' - 20000 \le 12000$

$\rightarrow 14500 \le b_2' \le 32000$

<u>If $\mathbf{b_3}$ change to $\mathbf{b_3' = b_3 + \Delta b_3}$</u>

$$S^*b' = \begin{pmatrix} 3800 \\ 1100 \\ 2400 \end{pmatrix} + \begin{pmatrix} -0.067 & 0.35 & 0 \\ -0.133 & 0.2 & 1 \\ 0.133 & -0.2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \Delta b_3 \end{pmatrix} = \begin{pmatrix} 3800 \\ 1100 + \Delta b_3 \\ 2400 \end{pmatrix}$$

$\rightarrow 1100 + \Delta b_3 \ge 0$

$\rightarrow b_3' - 20000 \geq -1100$

$\rightarrow b_3' \geq 18900$

3. Slack or surplus

Row	Slack or Surplus
1	0.2664000E+08
2	0.000000
3	0.000000
4	1100.000
5	3800.000
6	2400.000

If the equation is:

Less than or equal to -> slack

Greater than or equal to -> surplus

If a constraint is exactly satisfied as equality, the slack or surplus value will be zero

If a constraint is violated, as in an infeasible solution, the slack or surplus value will be negative.

Base on the figure, row 2 and 3 are two binded constraint that make its slack equal to 0

1100 in row 4 can understand as un-ultilize source

2400 and 3800 in row 5 and 6 respectively are the extra amount which are being produced.

4. Dual price (shadow price)

Dual Price
1.000000
0.000000
0.000000
0.000000
0.000000
0.000000

The dual price gives the improvement in the objective function if the constraint is relaxed by one unit.

In the case of a less-than-or-equal constraint, such as a resource constraint, the dual price gives the value of having one more unit of the resource represented by that constraint.

In the case of a greater-than-or-equal constraint, such as a minimum production level constraint, the dual price gives the cost of meeting the last unit of the minimum production target.

5. Reduced cost (Opportunity cost)

Variable	Value	Reduced Cost
X	3800.000	-3600.000
Y	2400.000	-5400.000

Reduced cost value indicates how much the objective function coefficient on the corresponding variable must be improved before the value of the variable will be positive in the optimal solution.

In the maximum problem, reduced cost is the amount that the variables have to increase before they become positive in the optimal solution

APPLICATION

For introduction, Giao Liem village at Son Dong district in Bac Giang province is a poor village in the North of VietNam. They are very rich in lands but the fact that they use their lands very unwise. Now the government in Giao Liem village decide to reconstruct the structure of land using in order to maximize the profit earned by argriculture. Therefore, the objective of the construction is to determine the number of hectares of land to be used for maximizing the profit, along with some conditions constrained to the problem. This study contributed to the application of Linear Programming method in problems of production planning and analysis.

- **Known that the current net revenues of the current land using model is: 3698.54 millions VND**
- **Known that the current profit of the current land using model is: 2872.16 millions VND**

The main data for this study is a data obtained from the government of the Giao Liem village includes: The revenue per ha of land, the profit per ha of lands, the limitations of factors associated with the problem, tons of food can be produced per ha of land, man power, investment cost.
The analysis is carried out using Linear Programming (LP) with the objective function as followed

$$\text{Max } Z=\sum C_jX_j$$

, and subject to constraints (raw materials, processing time, and demand) :

$$\sum A_{ij}X_j \leq B_j, \text{ where } X_j \geq 0$$

Then initially begin with the problem solving.
First we have the Variables notation:

Description	X
Hectares of land using for producing spring rice	1
Hectares of land using for producing seasonal rice	2
Land using for producing cassava	3
Land using for producing spring peanut	4
Hectares of land using for producing spring corn	5
Hectares of land using for producing winter corn	6
Hectares of land using for producing seasonal corn	7
Hectares of land using for producing bean	8
Hectares of land using for producing seasonal sweet potato	9
Hectares of land using for producing spring sweet potato	10
Hectares of land using for producing winter vegetable	11
Hectares of land using for producing spring vegetable	12
Hectares of land using for producing lychee	13
Numbers of pigs associated with land using	14
Amount of money loaned from the bank for investment	15

Secondly, an overview of the revenues and profits produced by aboves category, along with the limitations for each types of land, tons of food can be produced on 1 hectare of land, man power needs on specific type of land and the cost for investment.

Description	**X**	Net Revenue (VND/ha)	Profit (VND/ha)	Limitation of land can be used (ha)	Tons of food per ha land can produce (tons)	Man Power needed per ha (labor in shift)	Cost to invest (VND)
Hectares of land using for producing **spring** rice	1	5337	3313	≤45	5	190	1625.8
Hectares of land using for producing **seasonal** rice	2	5247	3008	≤99.17	5.02	212	1699
Land using for producing cassava	3	2464	1040	≤10	3	142	425.5
Land using for producing **spring** peanut	4	6974	5076	≤25	4.05	190	1396.6
Hectares of land using for producing **spring** corn	5	6770	4912	≤30	4.84	186	884.5

Hectares of land using for producing **winter** corn	6	6770	4912	≤30	4.84	186	884.5
Hectares of land using for producing **seasonal** corn	7	6770	4912	≤20	4.84	186	884.5
Hectares of land using for producing bean	8	5087.9	4653.4	15≤ x ≤45	3.12	115.5	392.9
Hectares of land using for producing **seasonal** sweet potato	9	3532	1532	No	4.89	200	635
Hectares of land using for producing **spring** sweet potato	10	3532	1532	No	4.89	200	635
Hectares of land using for producing **winter** vegetable	11	7736	5156	≤30	6.03	258	872.3
Hectares of land using for producing **spring** vegetable	12	7736	5156	≤30	6.03	258	872.3
Hectares of land using for producing lychee	13	4536	3959	≤300	1.6	58	7143
Numbers of pigs associated with land using	14	900	795	≤4000	12.06	11	592.7
Amount of money loaned from the bank for investment	15						

Furthermore, there are some policies to ensure the sufficient food needs or investment limitation, or ensure healthy working condition as followed:

- To ensure the basic food needs for 3476 people in the village, the foods (in tons) for spring rice, seasonal rice, spring corn, winter corn, seasonal corn must be at least 1043 tons.
- To ensure the healthy working conditions, annual working shift for all specific types of land must be less or equal to 420420 working shift.

- Net investment for all types of land is included the equity capital (known amout: 3.105.732 thousand VND) and the amount of money loaned from the bank for investment.
- In spring, there are some specific demand for spring foods so hectares of land for spring foods must be ensure to be no more than 89.57 hectares.
- In winter, there are some specific demand for winter foods so hectares of land for winter foods must be ensure to be no more than 134.5 hectares.
- In seasonal overview, there are some specific demand for seasonal foods so hectares of land for seasonal foods must be ensure to be no more than 35.4 hectares.

As all the information has been given above here we will have the objectives and constraints to formulate a model as followed:

First objective is to maximize the net revenues of the land using model:

- Max $Z_1 = 5337X_1 + 5247X_2 + 2464X_3 + 6974X_4 + 6770X_5 + 6770X_6 + 6770X_7 + 5087{,}9X_8 + 3532X_9 + 3532X_{10} + 7736X_{11} + 7736X_{12} + 4536X_{13} + 900X_{14}$

Second objective is to maximize the profit of the land using model:

- Max $Z_2 = 3313X_1 + 3008X_2 + 1040X_3 + 5076X_4 + 4912X_5 + 4912X_6 + 4912X_7 + 4653{,}4X_8 + 1532X_9 + 1532X_{10} + 5156X_{11} + 5156X_{12} + 3959X_{13} + 795X_{14}$

Subject to:

The constraint for limitation of lands can be used:

- $X_1 \leq 45$, $X_2 \leq 99.17$, $X_3 \leq 10$, $X_4 \leq 25$, $X_5 \leq 30$, $X_6 \leq 30$, $X_7 \leq 20$, $X_8 \leq 45$, $X_8 \geq 15$, $X_{11} \leq 30$, $X_{12} \leq 30$, $X_{13} \leq 300$, $X_{14} \leq 4000$

The constraint for ensuring the basic food needs which is 1043 tons of spring rice, seasonal rice, spring corn, winter corn, seasonal corn:

- $5X_1 + 5{,}05X_2 + 4{,}84X_5 + 4{,}84X_6 + 4{,}84X_7 \geq 1043$

The constraint for the sufficient labor and healthy working conditions:

- $190X_1 + 212X_2 + 142X_3 + 190X_4 + 186X_5 + 186X_6 + 186X_7 + 115.5X_8 + 200X_9 + 200X_{10} + 258X_{11} + 258X_{12} + 58X_{13} + 11X_{14} \leq 420420$

The constraint for investment conditions which is net investment for all types of land is included the equity capital (known amout: 3.105.732 thousand VND) and the amount of money loaned from the bank for investment:

- $1625{,}8\ X_1 + 1699X_2 + 425{,}5X_3 + 1396{,}6X_4 + 884{,}5X_5 + 884{,}5X_6 + 884{,}5X_7 + 392.9X_8 + 635X_9 + 635X_{10} + 872{,}3X_{11} + 872{,}3X_{12} + 7143X_{13}$

$+ 592{,}7X_{14} - X_{15} \leq 3.202.077$

The constraint for satisfying the specific demand of spring foods:

- $X_3 + X_4 + X_5 + X_8 + X_{10} + X_{12} \leq 89{,}57$

The constraint for satisfying the specific demand of winter foods:

- $X_3 + X_6 + X_{11} \leq 134{,}5$

The constraint for satisfying the specific demand of spring foods:

- $X_3 + X_7 + X_9 \leq 35{,}4$

The constraint for ensuring all decision variables are non-negative:

- $X_j \geq 0$ with j = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.

After solving

No.	**Description**	**X**	Parameter	**Results**
1	Hectares of land using for producing **spring** rice	1	Ha	45,00
2	Hectares of land using for producing **seasonal** rice	2	Ha	99,17
3	Land using for producing cassava	3	Ha	0
4	Land using for producing **spring** peanut	4	Ha	0
5	Hectares of land using for producing **spring** corn	5	Ha	15,50
6	Hectares of land using for producing **winter** corn	6	Ha	30,00
7	Hectares of land using for producing **seasonal** corn	7	Ha	20,00
8	Hectares of land using for producing bean	8	Ha	44,00
9	Hectares of land using for producing **seasonal** sweet potato	9	Ha	15,40

10	Hectares of land using for producing **spring** sweet potato	10	Ha	0
11	Hectares of land using for producing **winter** vegetable	11	Ha	30,00
12	Hectares of land using for producing **spring** vegetable	12	Ha	30,00
13	Hectares of land using for producing lychee	13	Ha	248,00
14	Numbers of pigs associated with land using	14	Ha	3000
15	Amount of money loaned from the bank for investment	15	x1000VND	823.296,2
	Net Revenue		Millions VND	**5.770,692**
	Profit		Millions VND	**4.675,347**

The result show that after applying the maximization model the net revenues increase to **5.770,692 millions VND** according to the old net revenues is **3698.54 millions VND**, then **the increase of 56.02%** in the net revenue is confirmed. The profit increases to **4.675,347 millions VND** according to the old profit is **2872.16 millions VND**, then the according to the old net revenues is **3698.54 millions VND** in the profit is confirmed.

CONCLUSION

For the Automobile Alliance's problem, Rachel should manufacture 3800 Family Thrillseekers and 2400 Classy Cruisers. With those setting, the profit will be $26,640,000.

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THE END.