Discrete Mathematics

Chapter 5: Counting

Department of Mathematics
The FPT university

Chapter 5: Introduction

Topics covered:

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