Discrete Mathematics

Chapter 1: The Foundations: Logic and Proofs

Department of Mathematics
The FPT university

0. Course Introduciton

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Course name: Discrete Mathematics (MAD101)

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Textbook: *Discrete Mathematics and its applications*, 6th edition, K. Rosen

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- Chapter 2: Sets, Functions, Sequences, and Sums

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- Chapter 9: Graphs

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- Chapter 4: Induction and Recursion
- Chapter 5 + 7: Counting
- Chapter 9: Graphs
- Chapter 10: Trees

Chapter 1: Introduction

Topics covered:

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1.1 Propositional Logic

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1.2 Propositional Equivalences

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1.3 Predicates and Quantifiers

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1.1 Propositional Logic
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1.4 Nested Quantifiers
1.5 Rules of Inference
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1.7 Proof Methods and Strategy

1.1 Propositional Logic

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Logic and Bit Operations

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Note. Other notation for $\wedge, \vee, \oplus$ are AND, OR, XOR.

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Some
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Double negation law	$\neg(\neg p) \equiv p$
Identity laws	$p \wedge T \equiv p$ $p \vee F \equiv p$
Domination laws	$p \vee T \equiv T$ $p \wedge F \equiv F$
Negation laws	$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$
Idempotent laws	$p \vee p \equiv p$ $p \wedge p \equiv p$
Commutative laws	$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
Associative laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
De Morgan's laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Note:

$$p \to q \equiv \neg p \vee q.$$

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$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

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$$p \oplus q \equiv \neg(p \leftrightarrow q)$$

Example 1. Prove that $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

Example 2. Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

1.3 Predicates and Quantifiers

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(c) $\forall x((x > 0) \vee (x^2 \geq x))$

(d) $\exists x((x > 0) \to (x^2 \geq x))$

Quantifiers $\forall$, $\exists$.

Let $P(x)$ be a propositional function where x gets values in a particular domain.

The universal quantification $\forall x P(x)$= For all values of x in the domain, $P(x)$ is true

The existential quantification $\exists x P(x)$= There is at least a value of x in the domain such that $P(x)$ is true.

Example. Let x represent a real number. Determine the truth value of the following propositions

(a) $\forall x((x > 0) \to (x^2 \geq x))$

(b) $\forall x((x > 0) \wedge (x^2 \geq x))$

(c) $\forall x((x > 0) \vee (x^2 \geq x))$

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(c) $\forall x((x > 0) \lor (x^2 \geq x))$
(d) $\exists x((x > 0) \to (x^2 \geq x))$
(e) $\exists x((x > 0) \land (x^2 \geq x))$
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Negating Quantified Expressions

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Example. Rewrite the expression

$$\neg\forall x (P(x) \to Q(x))$$

so that the negation precedes the predicates.

Translating Sentences into Logical Expressions

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Example 1. "Every students of class SE0000 passed Calculus"

Translating Sentences into Logical Expressions

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Example 2. "Each student of SE0000 has visited Canada or Mexico"

Example 3. "Some student of SE0000 has visited Canada or Mexico"

1.4 Nested Quantifiers

$\forall x \forall y P(x, y)$

$\forall x \forall y P(x, y)$ = For all x and for all y, $P(x, y)$ is true

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$\forall x \forall y (x + y = 1)$

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Translate Logical Expressions into Sentences

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Example 1. $\forall x \forall y[(x > 0) \land (y > 0) \rightarrow (xy > 0)]$

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Example 1. $\forall x \forall y[(x > 0) \wedge (y > 0) \rightarrow (xy > 0)]$

where x, y are real numbers.

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Example 2. Let x, y represent students in a university, and

$C(x) =$ " x has a laptop"

Translate Logical Expressions into Sentences

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Translate the logical expression $\forall x[C(x) \vee \exists y(C(y) \wedge F(x, y))]$

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Translate the logical expression

$$\exists x \forall y \forall z[(F(x, y) \land F(x, z) \land (y \neq z)) \to \neg F(y, z)]$$

Translate Sentences into Logical Expression using Nested Quantifiers

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Example 1. "Each student has sent emails to each other, but not to him/herself."

Translate Sentences into Logical Expression using Nested Quantifiers

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Use:

Translate Sentences into Logical Expression using Nested Quantifiers

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Use: $E(x, y)$="x has sent emails to y"

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Example 1. "Each student has sent emails to each other, but not to him/herself."

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Example 2. "Each student either has a car or has a room-mate in the same class who has a car"

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Example 3. (a) There is exactly one student in the class that was born in Hanoi.

Translate Sentences into Logical Expression using Nested Quantifiers

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Use: $C(x)$ = "x has a car"
$R(x, y)$="x and y are room-mates "

Example 3. (a) There is exactly one student in the class that was born in Hanoi.

(b) There are exactly two students in the class that was born in Hanoi.

Negating Nested Quantifiers

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$$\neg(\forall x \forall y P(x, y)) = \exists x \exists y \neg P(x, y)$$

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$$\neg(\forall x \forall y P(x,y)) = \exists x \exists y \neg P(x,y) \quad \neg(\forall x \exists y P(x,y)) = \exists x \forall y \neg P(x,y)$$
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Example.

Negating Nested Quantifiers

$$\neg(\forall x \forall y P(x,y)) \equiv \exists x \exists y \neg P(x,y) \quad \neg(\forall x \exists y P(x,y)) \equiv \exists x \forall y \neg P(x,y)$$
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Example. Translate the following statements into logical expressions, then find the negation statement.

Negating Nested Quantifiers

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Example. Translate the following statements into logical expressions, then find the negation statement.

(a) " For all real numbers x there is a real number y such that $x = y^3$"

Negating Nested Quantifiers

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Example. Translate the following statements into logical expressions, then find the negation statement.

(a) " For all real numbers x there is a real number y such that $x = y^3$"

(b) " For all $\epsilon > 0$, for all real numbers x there exists a rational number p such that $|p - x| < \epsilon$"

1.5 Rules of Inference

- An argument is a sequence of statements that end with a conclusion.

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- An argument is a sequence of statements that end with a conclusion. An argument is valid if the conclusion follows from the truth of the preceding statements (premises or hypotheses).
- In propositional logic, an argument is valid if it is based on a tautology.
- Arguments that are not based on tautology are called fallacies.

Name	Rule of Inference	Tautology
Addition	$\begin{array}{l} \quad p \\ \hline \therefore p \vee q \end{array}$	$p \rightarrow (p \vee q)$
Simplification	$\begin{array}{l} \quad p \wedge q \\ \hline \therefore p \end{array}$	$(p \wedge q) \rightarrow p$
Modus ponens	$\begin{array}{l} \quad p \\ \quad p \rightarrow q \\ \hline \therefore q \end{array}$	$p \wedge (p \rightarrow q) \rightarrow q$
Modus tollens	$\begin{array}{l} \quad \neg q \\ \quad p \rightarrow q \\ \hline \therefore \neg p \end{array}$	$(\neg q) \wedge (p \rightarrow q) \rightarrow \neg p$
Hypothetical syllogism	$\begin{array}{l} \quad p \rightarrow q \\ \quad q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$	$(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$
Disjunctive syllogism	$\begin{array}{l} \quad \neg p \\ \quad p \vee q \\ \hline \therefore q \end{array}$	$(p \vee q) \wedge (\neg p) \rightarrow q$

Example 1.

Example 1. Given the hypotheses:

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- "It is not sunny and is cold"

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- "It is not sunny and is cold"
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- "If we play soccer then we will go home by sunset"

Example 1. Given the hypotheses:

- "It is not sunny and is cold"
- "We go swimming only if it is sunny"
- "If we do not go swimming then we will play soccer"
- "If we play soccer then we will go home by sunset"

Show that these hypotheses lead to the conclusion: "We will go home by sunset".

Example 2.

Example 2. Given the hypotheses:

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- "If you send me an email, I will finish writing the program"

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Show that these hypotheses lead to the conclusion: "If I do not finish writing the program then I will go jogging tomorrow morning".

Some fallacies

Some fallacies

- Fallacy of affirming the conclusion: $[(p \to q) \land q] \to p$

Some fallacies

- Fallacy of affirming the conclusion: $[(p \rightarrow q) \wedge q] \rightarrow p$
- Fallacy of denying the hypothesis: $[(p \rightarrow q) \wedge \neg p] \rightarrow \neg q$

Some fallacies

- Fallacy of affirming the conclusion: $[(p \to q) \land q] \to p$
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Rules of Inference for Quantified Statements

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Name	Rule of Inference
Universal instantiation	$\dfrac{\forall x P(x)}{\therefore P(c),\ c \text{ is arbitrary}}$
Universal generalization	$\dfrac{P(c),\ \text{c is arbitrary}}{\therefore \forall x P(x)}$
Existential instantiation	$\dfrac{\exists x P(x)}{\therefore P(c),\ \text{for some } c}$
Existential generalization	$\dfrac{P(c),\ \text{for some } c}{\therefore \exists x P(x)}$

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Example 1. Given the hypotheses:

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- "Each student of SE0000 must take Discrete Math",
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Show that these hypotheses lead to the conclusion "Jenifer must take Discrete Math".

Example 1. Given the hypotheses:

- "Each student of SE0000 must take Discrete Math",
- "Jenifer is a student of SE0000".

Show that these hypotheses lead to the conclusion "Jenifer must take Discrete Math".

Example 2. Given the hypotheses:

Example 1. Given the hypotheses:

- "Each student of SE0000 must take Discrete Math",
- "Jenifer is a student of SE0000".

Show that these hypotheses lead to the conclusion "Jenifer must take Discrete Math".

Example 2. Given the hypotheses:

- "Some student of SE0000 has not read this book",

Example 1. Given the hypotheses:

- "Each student of SE0000 must take Discrete Math",
- "Jenifer is a student of SE0000".

Show that these hypotheses lead to the conclusion "Jenifer must take Discrete Math".

Example 2. Given the hypotheses:

- "Some student of SE0000 has not read this book",
- "Every student of SE0000 passed the exam".

Example 1. Given the hypotheses:

- "Each student of SE0000 must take Discrete Math",
- "Jenifer is a student of SE0000".

Show that these hypotheses lead to the conclusion "Jenifer must take Discrete Math".

Example 2. Given the hypotheses:

- "Some student of SE0000 has not read this book",
- "Every student of SE0000 passed the exam".

Show that these hypotheses lead to the conclusion "Some student of SE0000 who passed the exam has not read this book".