Graph coloring

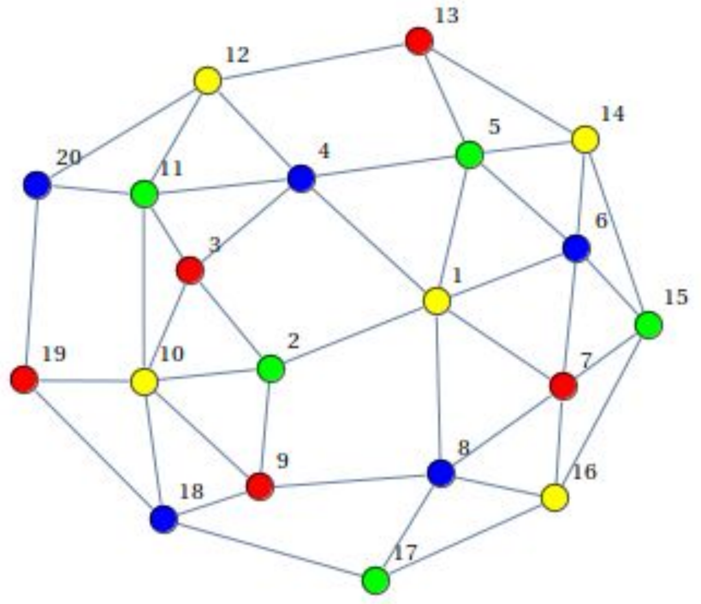
1. Definition
2. Chromatic number
3. Types of graph coloring
4. Method

Graph coloring

- Graph coloring is one of the basic problems of Graph theory, widely used in informatics, to solve practical problems related to zoning, grouping, and maps.

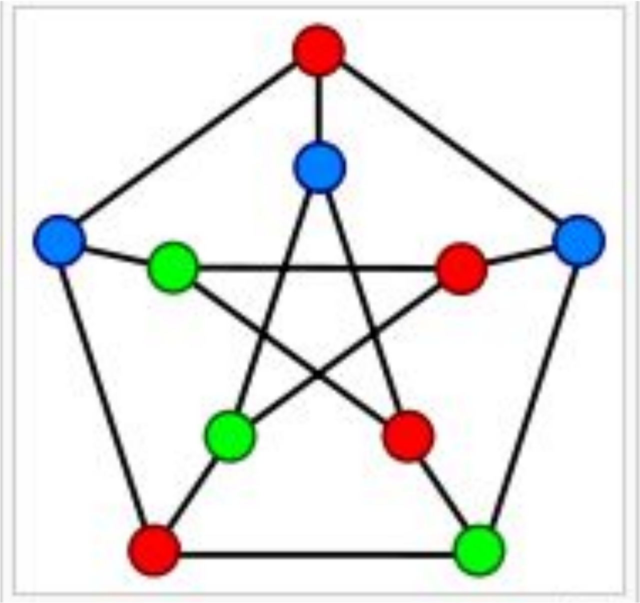
Definition

- In graph theory, graph coloring is a way of coloring the vertices of a graph such that no two adjacent vertices share the same color.



Chromatic number - Definition

- The chromatic number of a graph is the minimum number of colors one can use to color the vertices of the graph so that no two adjacent vertices are the same color.
- Chromatic number of the graph G is denoted by χ(G).
- $1 <= \chi(G) <= n$

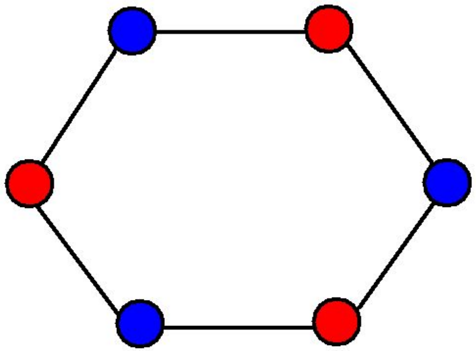


Note: n are vertices of graph

Chromatic number - Types of graphs

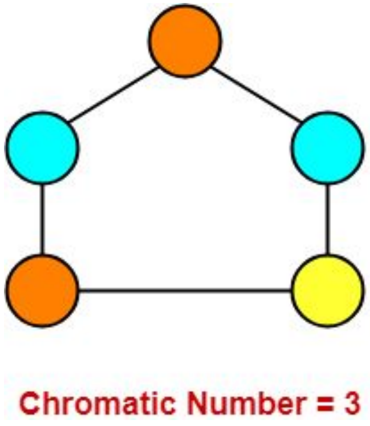
Cycle graph

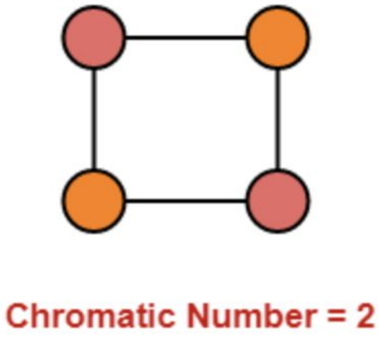
- A simple graph of ‘n’ vertices (n>=3) and ‘n’ edges forming a cycle of length ‘n’ is called as a cycle graph.
- In a cycle graph, all the vertices are of degree 2.



Chromatic number - Types of graphs

Cycle graph





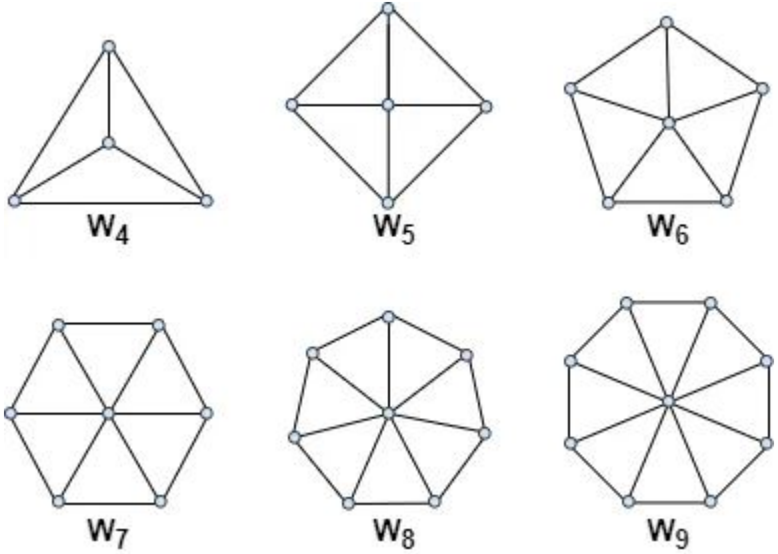
Any odd length cycle will have a chromaticity of 3 : $\chi(C_{2n+1}) = 3$

Any even length cycle will have a chromaticity of 2 : $\chi(C_{2n}) = 2$

Chromatic number - Types of graphs

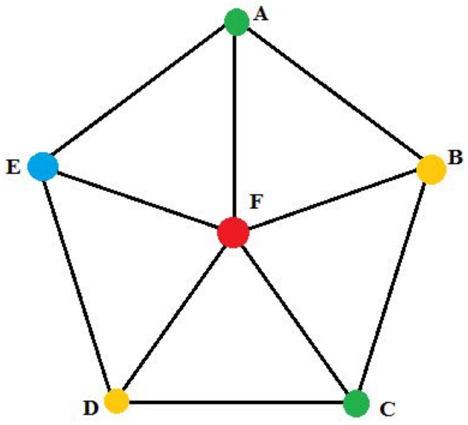
Wheel graph

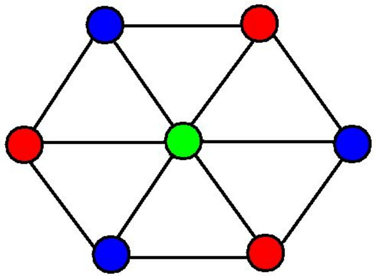
- A wheel graph is obtained by connecting a vertex to all the vertices of a cycle graph.
- It is denoted by Wn, for n > 3 where n is the number of vertices in the graph.



Chromatic number - Types of graphs

Wheel graph





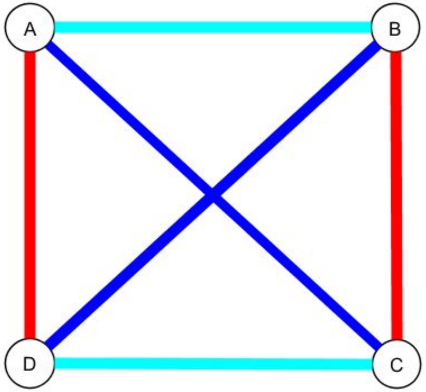
$\chi(W_n) = 4$ if n is even

$\chi(W_n) = 3$ if n is odd

Chromatic number - Types of graphs

Complete Graph

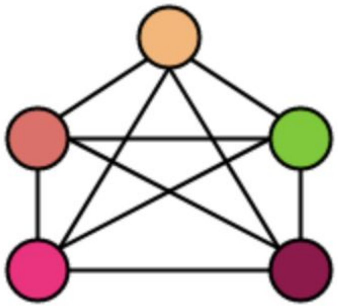
- A complete graph is a graph in which every two distinct vertices are joined by exactly one edge.
- In a complete graph, each vertex is connected with every other vertex.



Chromatic number - Types of graphs

Complete Graph

Chromatic Number = 4



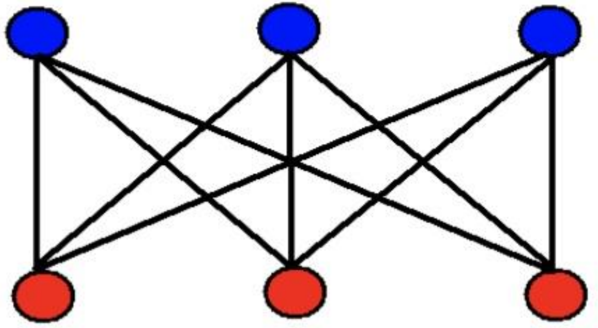
Chromatic Number = 5

- Chromatic Number of any Complete Graph = Number of vertices in that Complete Graph ($\chi(K_n) = n$)

Chromatic number - Types of graphs

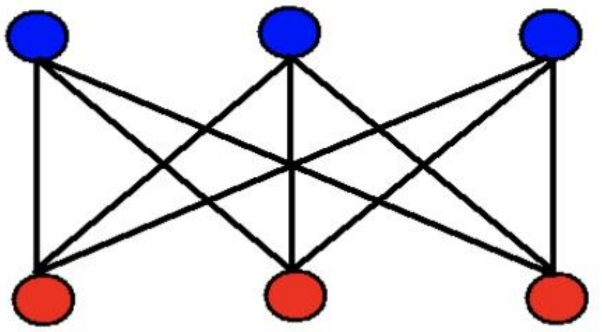
Bipartite Graphs

- A Bipartite Graph consists of two sets of vertices X and Y.
- The edges only join vertices in X to vertices in Y, not vertices within a set.



Chromatic number - Types of graphs

Bipartite Graphs

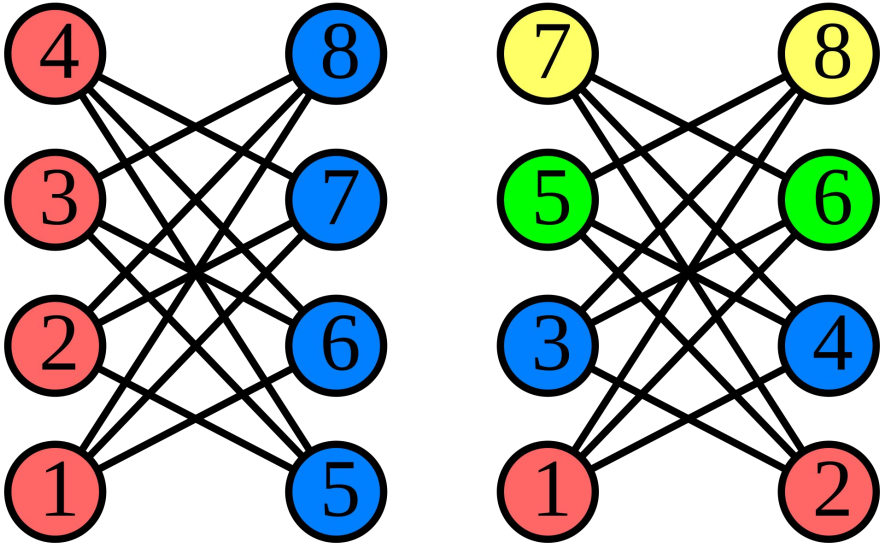


Chromatic Number of any Bipartite Graph = 2

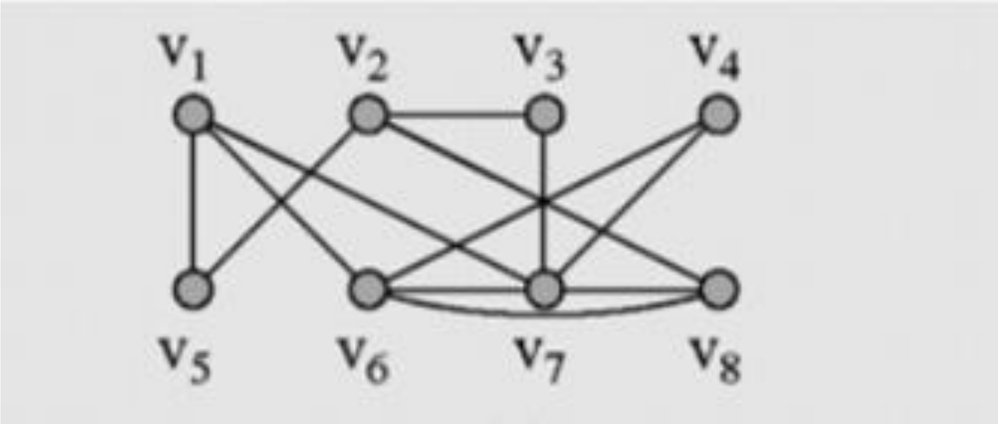
($\chi(G) \leq 2$)

Graph coloring - Sequential coloring

- **Sequential coloring** establishes the sequence of vertices before coloring them, and then color the next vertex with the lowest number possible



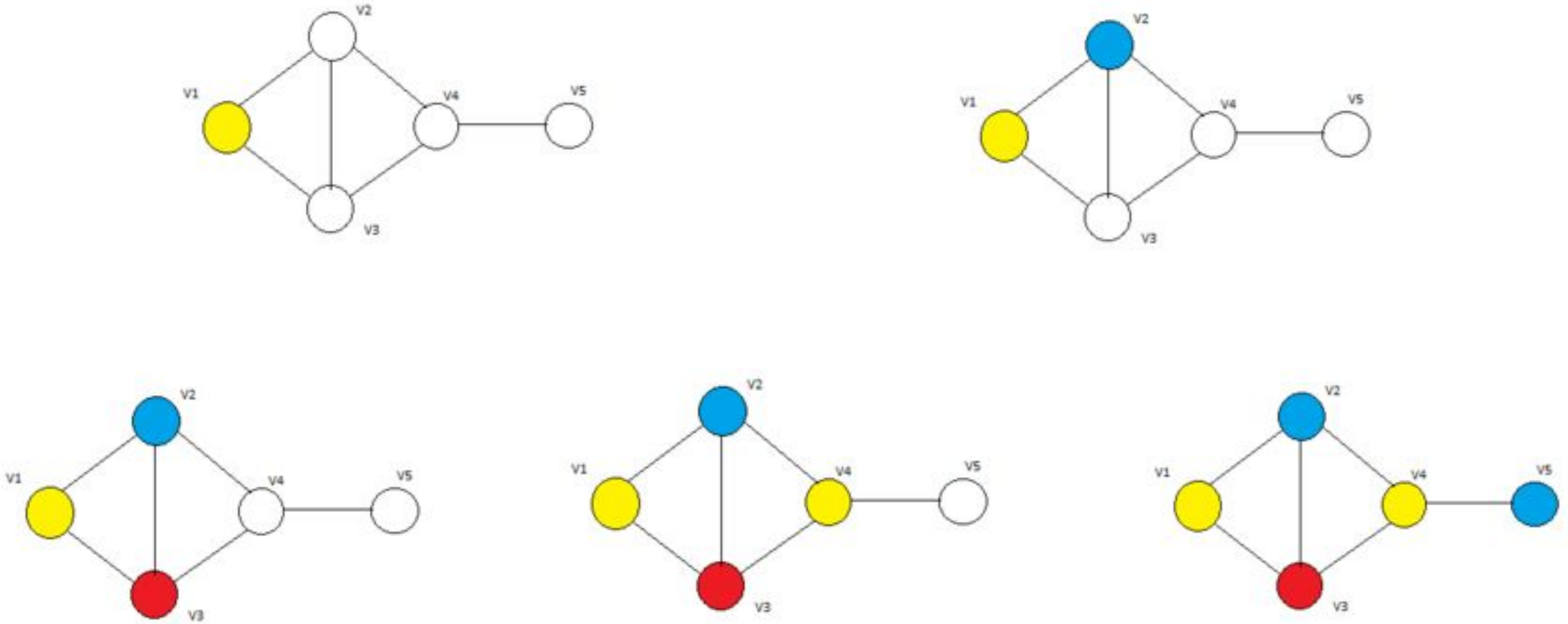
Graph coloring - Sequential coloring



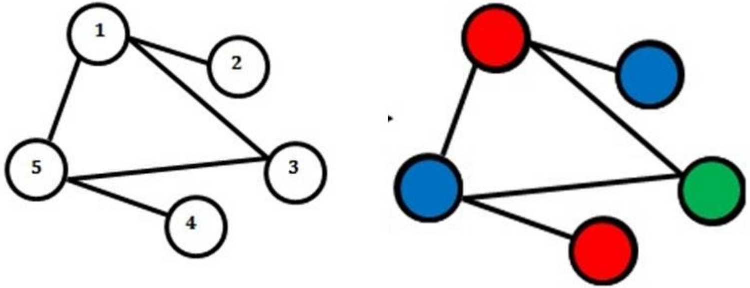
- **Step 1:** Color first vertex with the first color.
- **Step 2:** consider the remaining (V-1) vertices one by one:

+ Color the currently picked vertex with the lowest numbered color if it has not been used to color any of its adjacent vertices.
+ If it has been used, then choose the next least numbered color. If all the previously used colors have been used, then assign a new color to the currently picked vertex.

Sequential coloring - EXAMPLE



Graph coloring - Largest first sequence



- **Step 1:** We have a graph with 5 vertices numbered 1, 2, 3, 4, 5 with the ranks corresponding to each vertex in the order of 3, 1, 2, 1, 3. Hence V 'initially The order is [1, 5, 3, 2, 4]. Assign i = 1 (number of shaded colors).
- **Step 2:** Color 1 (red) for vertex 1. In turn, browse the remaining vertices in V '. We have: Point 5 is adjacent to vertex 1 (vertex 1 is colored 1 - red), so there is no color for vertex 5. Similarly, vertices 3, 2 are adjacent to vertex 1, so vertices 3, 2 have not been colored. Vertex 4 is not adjacent to vertex 1, so color 1 for vertex 4. Vertex 4 has color 1 - red.
- **Step 3:** Check that there are still unpainted vertices in V, so go to step 4.
- **Step 4:** Remove the colored vertices 1, 4 from V ', rearrange V' in descending order, we get V '= [5, 3, 2]. We have i = 2. Repeat step 2: