

Name: Phạm Đức Thể

ID: 19522253

Class: DS303.M21

Thống Kê Bayes

SUMMARY

Task		Status	Page
Bài Tập Lấy Điểm Quá Trình	Câu 1	Hoàn thành	2
	Câu 2	Hoàn thành câu a	5
	Câu 3	Hoàn thành	6
...	...		
	...		

Kiểm Tra Giữa Kỳ

1. Câu 1

dek
TECHNOLOGIES

Date:

D3303.M21 - Thống kê Bayes

Họ và Tên: Phạm Đức Thế

MSGV: 19522253

Câu 1

a) $P(X_1, \dots, X_n | \theta) = \prod_{i=1}^n P(X_i | \theta) = \prod_{i=1}^n \theta^{x_i} (1-\theta)$

$$= \theta^{\sum x_i} (1-\theta)^n$$

$\Rightarrow \operatorname{Argmax}_{\theta} P(X_1, \dots, X_n | \theta) = \operatorname{Argmax}_{\theta} \theta^{\sum x_i} (1-\theta)^n$

Tìm cực trị của $\theta^{\sum x_i} (1-\theta)^n$

$$\operatorname{Argmax}_{\theta} \theta^{\sum x_i} (1-\theta)^n = \operatorname{Argmax}_{\theta} \ln [\theta^{\sum x_i} (1-\theta)^n]$$

$$= \operatorname{Argmax}_{\theta} [\ln \theta^{\sum x_i} + \ln (1-\theta)^n]$$

$$= \operatorname{Argmax}_{\theta} [\underbrace{\sum x_i \ln \theta + n \ln (1-\theta)}_{f(\theta)}]$$

www.dektechnologies.com

www.facebook.com/DEKTechnologiesGlobal

Hình 1: Câu 1

dek
TECHNOLOGIES

Date:

$$f'(\theta) = \sum x_i \frac{1}{\theta} - n \frac{1}{1-\theta} = 0$$

$$\Leftrightarrow \sum x_i (1-\theta) - n \cdot \theta = 0.$$

$$\Leftrightarrow \sum x_i - \theta \sum x_i - n \cdot \theta = 0.$$

$$\Leftrightarrow \sum x_i - \theta (\sum x_i + n) = 0$$

$$\Rightarrow \hat{\theta} = \frac{\sum x_i}{\sum x_i + n}$$

b) Xác suất hậu nghiệm.

Tìm $\theta | x_1, \dots, x_n$ biết $\theta \sim \text{Beta}(\alpha, \beta)$

$$\text{Ta có: } P(\theta | x_1, \dots, x_n) = \frac{P(x_1, \dots, x_n | \theta) \cdot P(\theta)}{P(x_1, \dots, x_n)}$$

$$\sim P(x_1, \dots, x_n | \theta) \cdot P(\theta)$$

$$\sim \theta^{\sum x_i} (1-\theta)^n \theta^{\alpha-1} (1-\theta)^{\beta-1}$$

$$\sim \theta^{\sum x_i + \alpha - 1} (1-\theta)^{n + \beta - 1}$$

$$\text{Ta được } \theta | x_1, \dots, x_n \sim \text{Beta} \left(\sum_{i=1}^n x_i + \alpha, n + \beta \right)$$

Hình 2: Câu 1

Date:

c) Tìm kỳ vọng và phương sai

$$E(\theta | x_1, \dots, x_n) = \frac{\sum x_i + \alpha}{\sum x_i + \alpha + n + \beta} = \frac{\sum x_i + \alpha - 1}{\sum x_i + \alpha + n + \beta}$$

$$\text{Var}(\theta | x_1, \dots, x_n) = \frac{(\sum x_i + \alpha) \cdot (n + \beta)}{(\sum x_i + \alpha + n + \beta)^2 (\sum x_i + \alpha + n + \beta + 1)}$$

d) $n=15$, $x_i = i$

$$\hat{\theta} = \frac{\sum_{i=1}^{15} x_i}{\sum_{i=1}^{15} x_i + 15} = \frac{8}{9} \approx 0,89$$

e) $x_i = i + 1$

$$\hat{\theta} = \frac{\sum_{i=1}^{15} i + \sum_{i=16}^{20} i+1}{\sum_{i=1}^{15} i + \sum_{i=16}^{20} i + n} = \frac{43}{47} \approx 0,915$$

Hình 3: Câu 1

2. Câu 2

dek
TECHNOLOGIES

Date:

Câu 2

a) Ta có: $X|\theta \sim \text{Gamma}\left(\frac{n}{2}, 2\theta\right)$

$$\Rightarrow P(X|\theta) = \frac{1}{\Gamma\left(\frac{n}{2}\right)(2\theta)^{n/2}} \cdot e^{-\frac{x}{2\theta}} \cdot x^{\frac{n}{2}-1}$$

Phân phối xác suất hỗn hợp.

$$p(\theta|x) \propto P(X|\theta) \cdot P(\theta)$$

$$\propto \frac{1}{\Gamma\left(\frac{n}{2}\right)(2\theta)^{n/2}} \cdot e^{-\frac{x}{2\theta}} \cdot x^{\frac{n}{2}-1} \cdot \frac{1}{\Gamma(\alpha)\beta^\alpha} \theta^{\alpha-1} e^{-\frac{1}{\theta\beta}}$$

$$\propto \frac{1}{\theta^{\left(\frac{n}{2}+\alpha\right)+1}} e^{-\frac{1}{\theta}\left(\frac{\alpha}{2} + \frac{1}{\beta}\right)}$$

$$\propto \frac{1}{\theta^{\left(\frac{n}{2}+\alpha\right)+1}} e^{-\frac{1}{\theta}\left(\frac{2\beta}{2+\beta x}\right)}$$

$$\Rightarrow \theta|X \sim \text{Inverse Gamma}\left(\frac{n}{2} + \alpha, \frac{2\beta}{2 + \beta x}\right)$$

Với hàm loss $L(\theta, a) = (\theta - a)^2$

$\hat{\theta} = E(\theta|x)$ (Kỳ vọng của phân phối xác suất hỗn hợp)

$$\hat{\theta} = \frac{1}{\frac{2\beta}{2+\beta x} \left(\frac{n}{2} + \alpha - 1\right)}$$

www.dektechnologies.com

www.facebook.com/DEKTechnologiesGlobal

Hình 4: Câu 2a

3. Câu 3

dek
TECHNOLOGIES

Date:

Câu 3:

a) Ta có $X_i \sim N(\theta; 5)$ $\theta \sim N(6; 4)$
 $\theta | X_1, \dots, X_n \sim N(\theta_1, M_1)$
 $\phi = 5$; $\psi = 4$; $\bar{x} = 4,6$
 $M_1 = \frac{1}{\phi^{-1} + n\psi^{-1}} = \frac{1}{5^{-1} + 5 \cdot 4^{-1}} = \frac{20}{29}$
 $\theta_1 = M_1 \left(\frac{\mu}{\psi} + \frac{\bar{x}}{\frac{\psi}{n}} \right) = \frac{20}{29} \left(\frac{6}{4} + \frac{4,6}{\frac{5}{5}} \right) = 4,21$
 Vậy $\theta | X_1, \dots, X_n \sim N(4,21, \frac{20}{29})$

b) Để kiểm tra $H_0 : \theta \leq 4$, ta có:
 $H_1 : \theta > 4$
 $\alpha_0 = P(\theta \leq 4 | X_1, \dots, X_n)$
 $= P\left(\frac{\theta - 4,21}{\sqrt{\frac{20}{29}}} \leq \frac{4 - 4,21}{\sqrt{\frac{20}{29}}} \right) \approx P(Z \leq -0,253) = 0,4$
 $\alpha_1 = 1 - \alpha_0 = 0,6$
 $\Rightarrow \frac{\alpha_0}{\alpha_1} < 1 \Rightarrow$ Bác bỏ H_0 , ủng hộ H_1

www.dektechnologies.com www.facebook.com/DEKTechnologiesGlobal

Hình 5: Câu 3

dek
TECHNOLOGIES

Date:

$$\text{c) Kiểm tra } \left\{ \begin{array}{l} H_0: \theta = 4 \\ H_1: \theta \neq 4 \end{array} \right.$$

$$\text{Ta sẽ tính: } B = \frac{\frac{p_0}{p_1}}{\frac{\pi_0}{\pi_1}} = \frac{P(X|\theta_0)}{P(X)} = \frac{P(X_1, \dots, 15|\theta=6)}{P(X)}$$

Xác suất phân phối nghiệm $\theta \sim N(6; 4)$

$$B = \frac{\left\{ \frac{2\pi \cdot \sigma}{n} \right\}^{-1/2} \exp\left\{ \frac{-\frac{1}{2}(\bar{x} - \theta_0)^2}{\frac{\sigma}{n}} \right\}}{\left\{ 2\pi \left(\psi + \frac{\sigma}{n} \right) \right\}^{-1/2} \exp\left\{ \frac{-\frac{1}{2}(\bar{x} - \theta_0)^2}{\psi + \frac{\sigma}{n}} \right\}}$$

$$\Rightarrow B = \frac{\left\{ \frac{2\pi \cdot 5}{5} \right\}^{-1/2} \exp\left\{ \frac{-\frac{1}{2}(4,6 - 4)^2}{5/5} \right\}}{\left\{ 2\pi \left(4 + \frac{5}{5} \right) \right\}^{-1/2} \exp\left\{ \frac{-\frac{1}{2}(4,6 - 4)^2}{4 + \frac{5}{5}} \right\}}$$

$$= 1,93 > 1$$

Vậy chấp nhận H_0 .

Hình 6: Câu 3