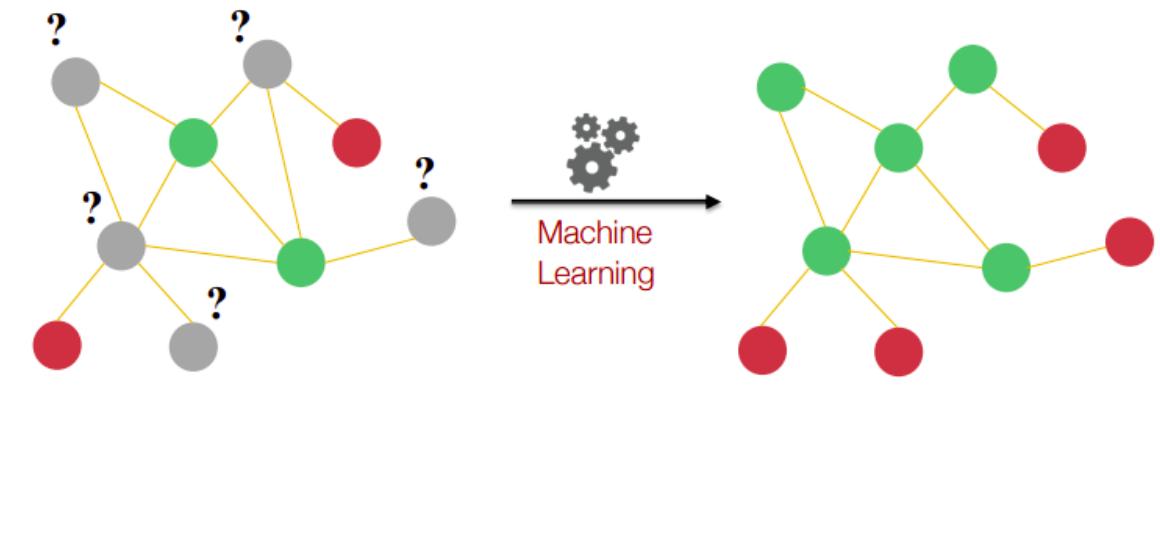
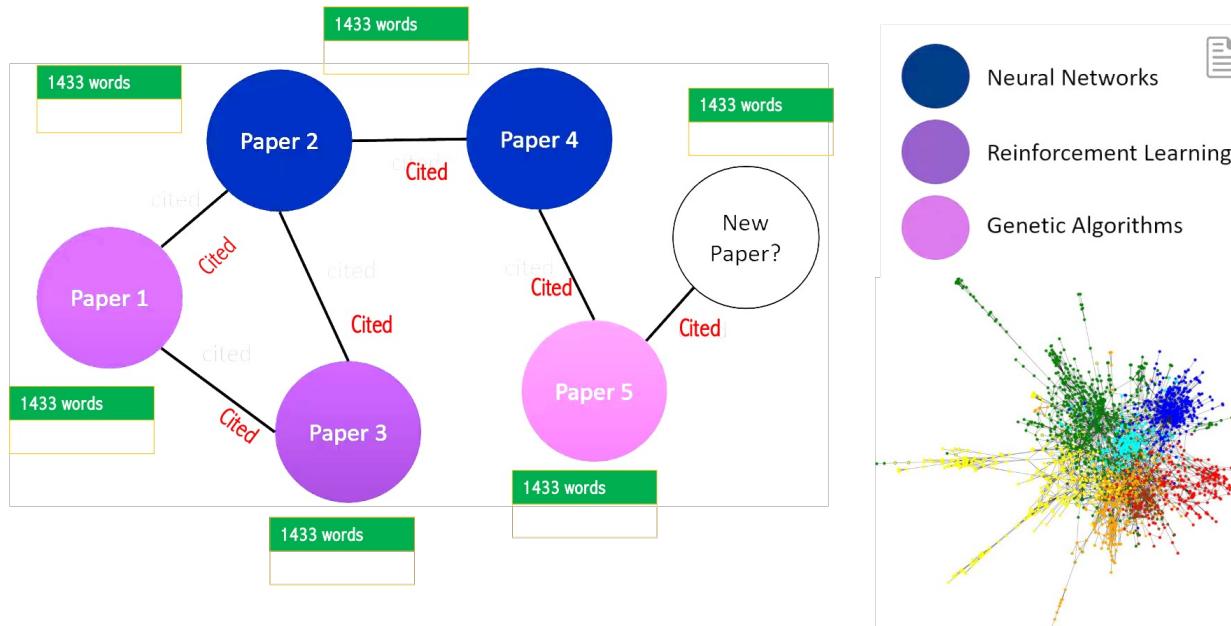


# Introduction to Graph Neural Network

## (Node Classification: Cora Citation Dataset)



Vinh Dinh Nguyen  
PhD in Computer Science

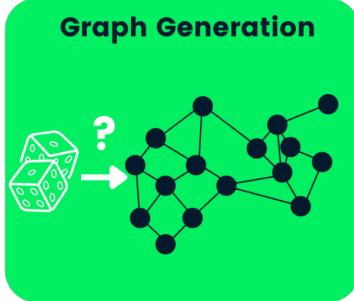
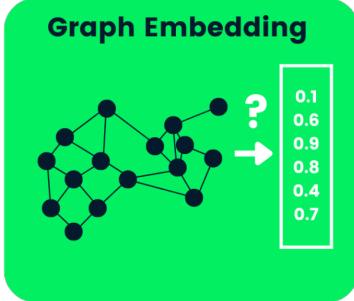
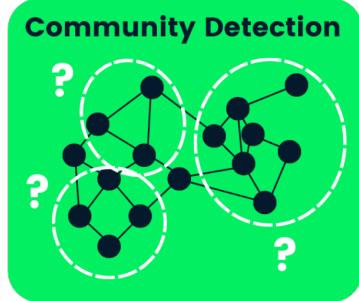
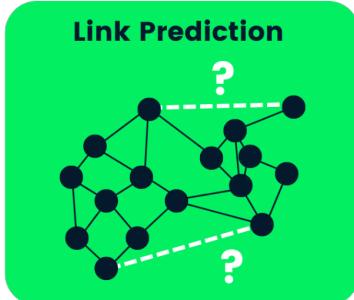
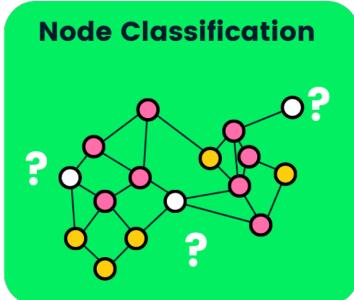
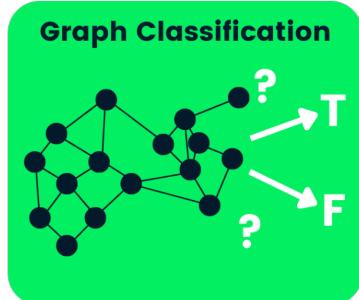
# Outline

- **Objective**
- **Introduction to Graph Data**
- **Graph Data with Neural Network**
- **Node Classification Problem: Cora Citation Dataset**
- **Summary**

# Outline

- **Objective**
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# Objective

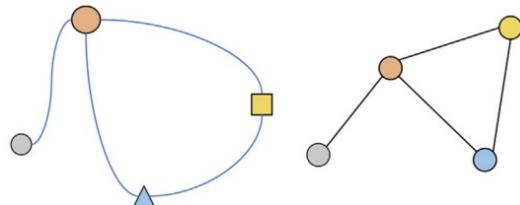


- What is Graph Data Around Us
- Understand Graph Neural Network
- Understand Graph Convolutional Neural Network
- Node Classification with Cora Citation Dataset

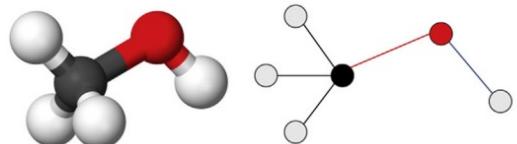
# Outline

- **Objective**
- **Introduction to Graph Data**
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- **Node Classification Problem: Cora Citation Dataset**
- **Summary**

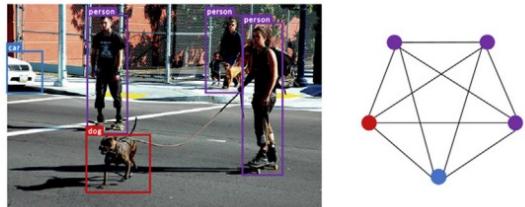
# Applications of GNNs



(a) Physics



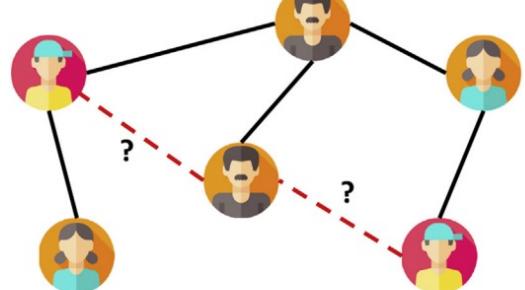
(b) Molecule



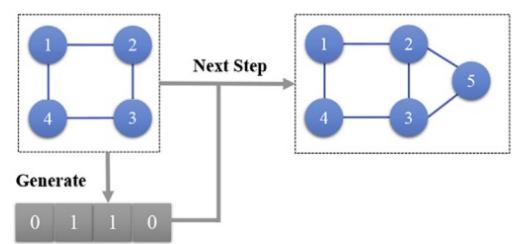
(c) Image

A sentence "The quick brown fox jumped over the lazy dog." is shown with dependency arrows indicating grammatical relations like subject (det), object (amod), and verb (vmod).

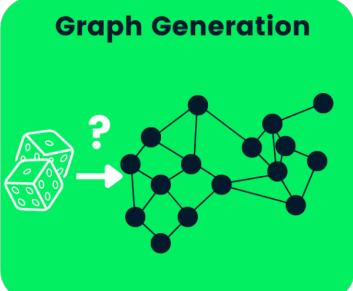
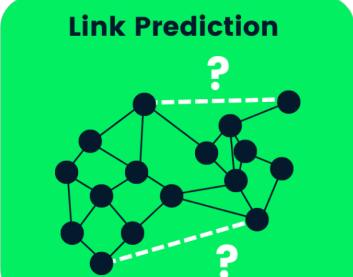
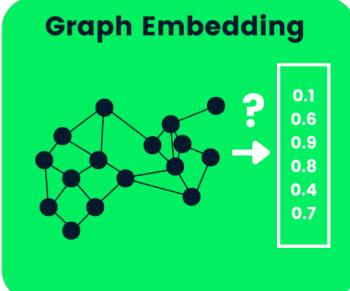
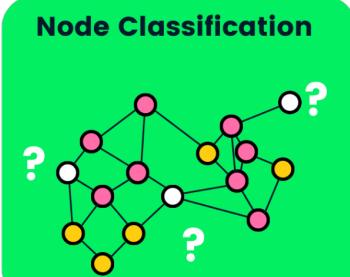
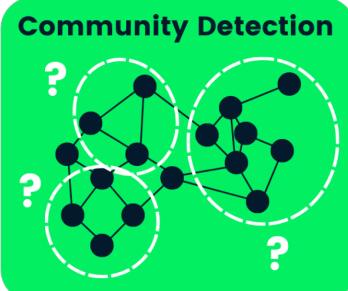
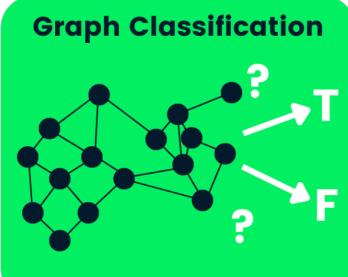
(d) Text



(e) Social Network

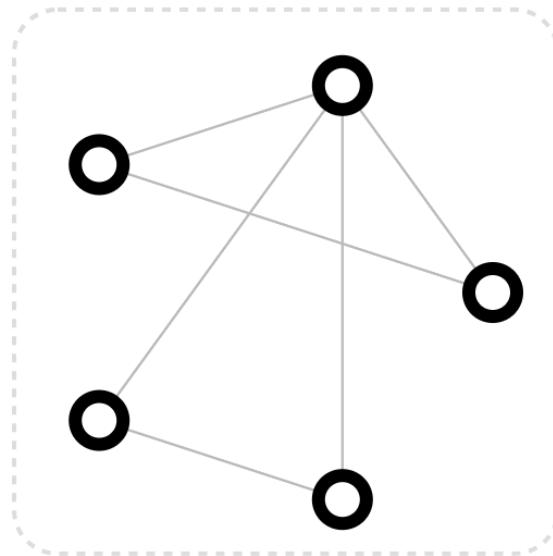


(f) Generation

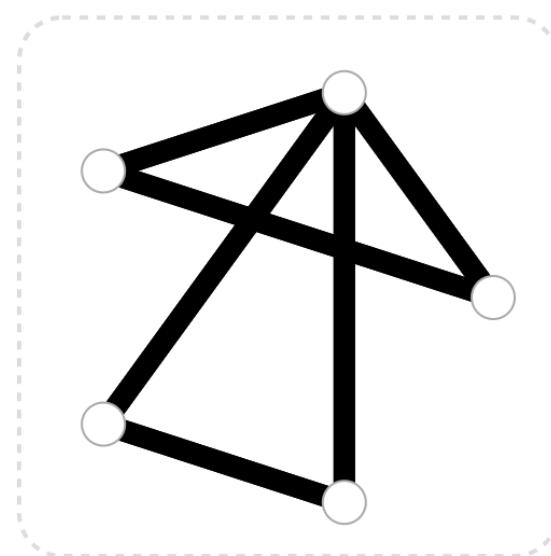


# Graph Definition

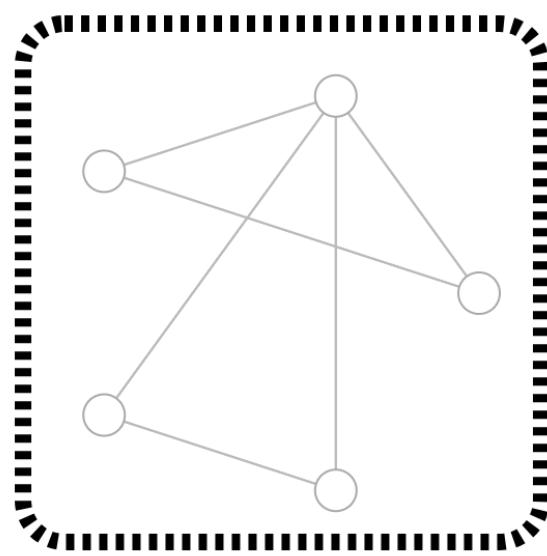
A graph represents the relations (edges) between a collection of entities (nodes).



**V** Vertex (or node) attributes  
e.g., node identity, number of neighbors

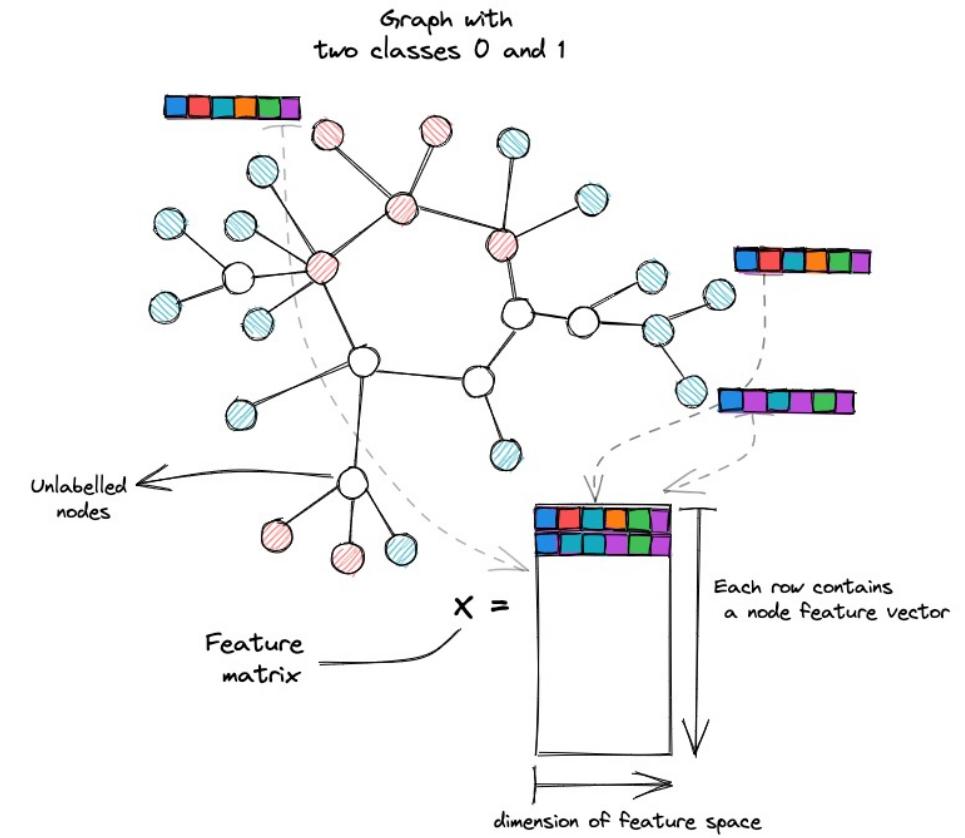
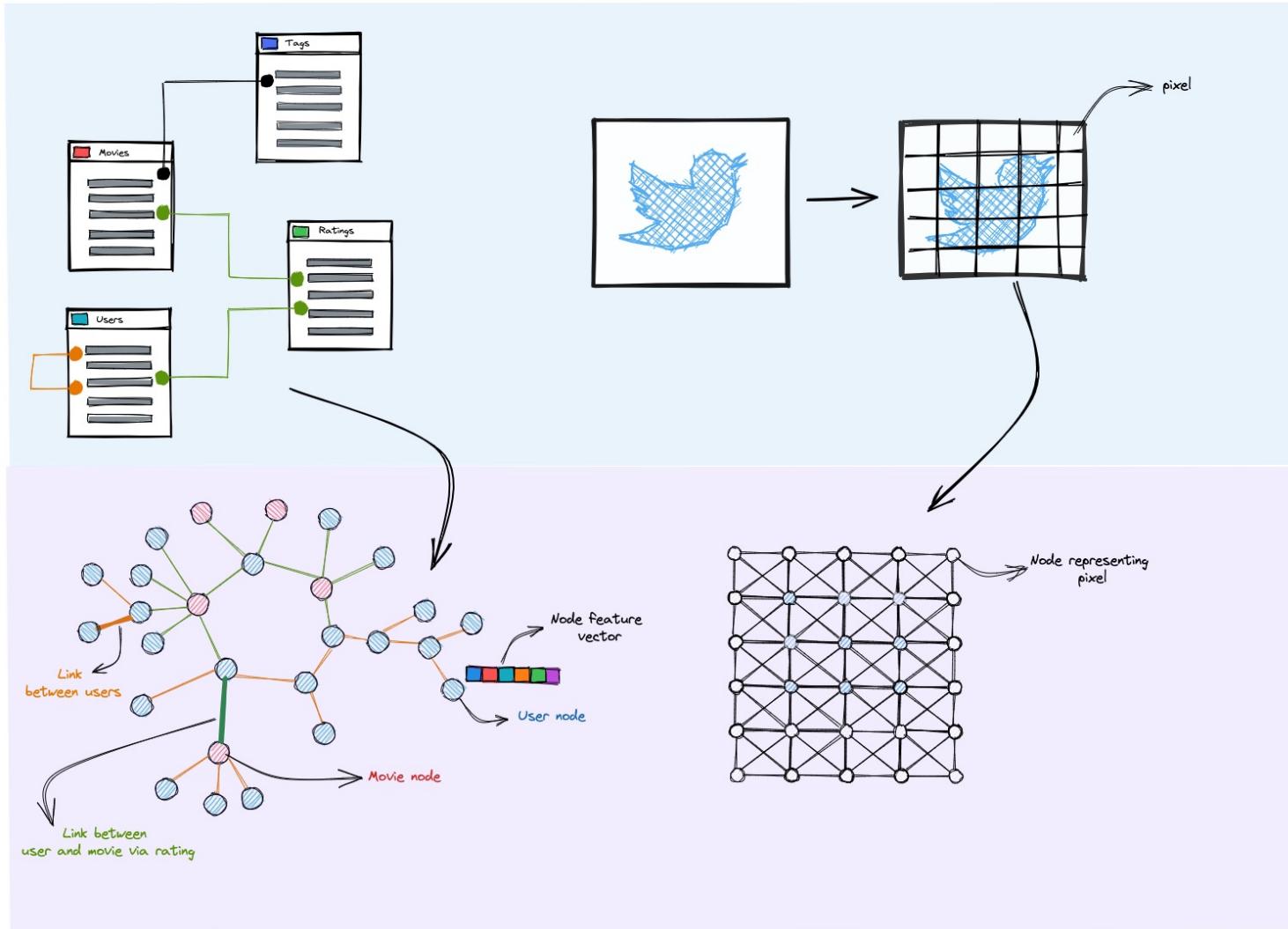


**E** Edge (or link) attributes and directions  
e.g., edge identity, edge weight



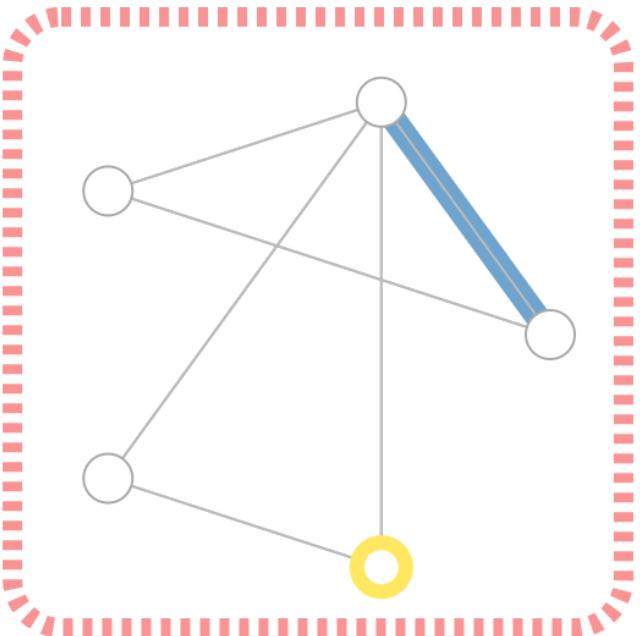
**U** Global (or master node) attributes  
e.g., number of nodes, longest path

# Graphs are everywhere



# Graph Definition

To further describe each node, edge or the entire graph, we can store information in each of these pieces of the graph



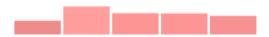
Vertex (or node) embedding



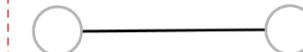
Edge (or link) attributes and embedding



Global (or master node) embedding



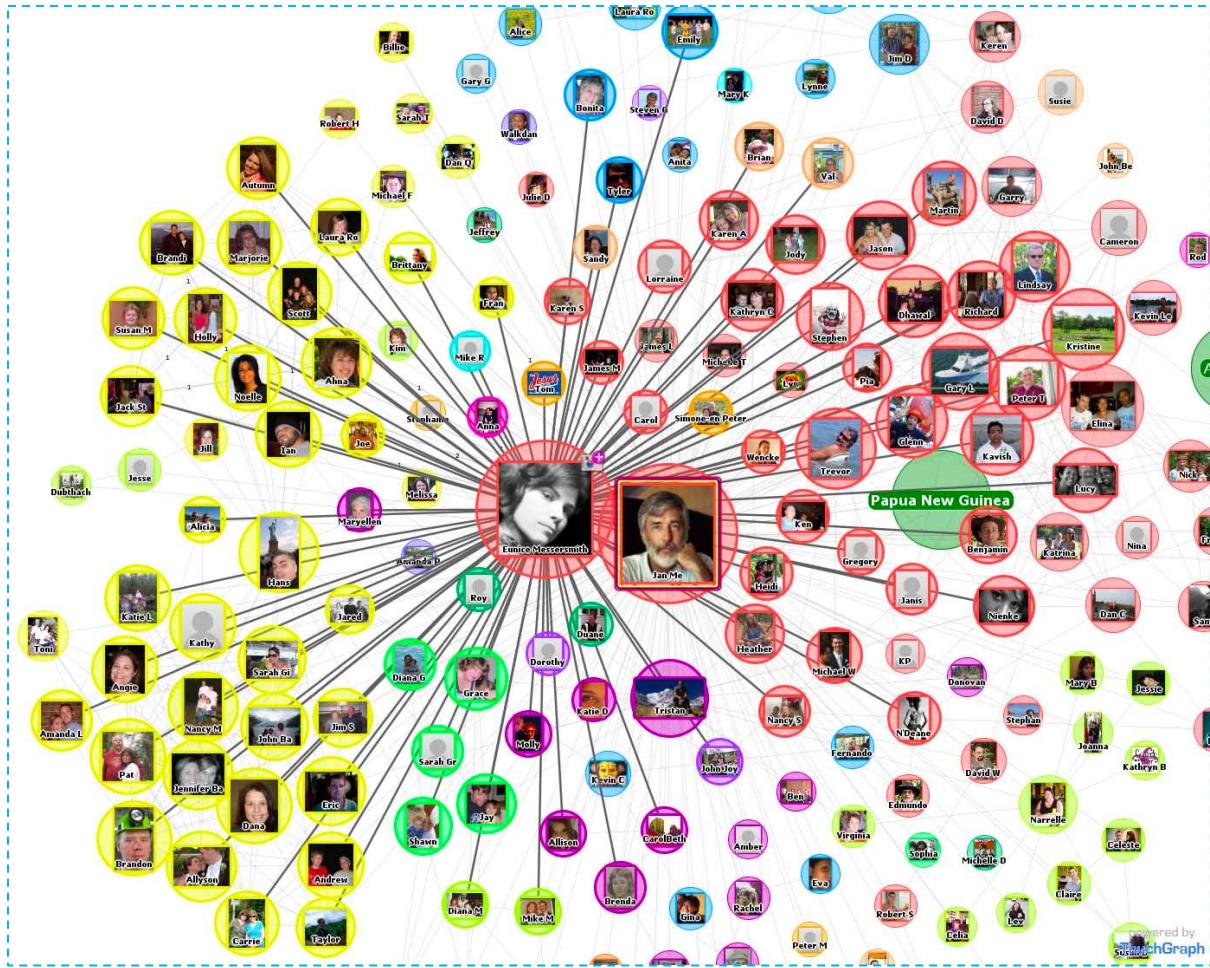
Undirected edge



Directed edge



# Graphs and where to find them



Social networks

Two types of data that you might not think could be modeled as graphs:



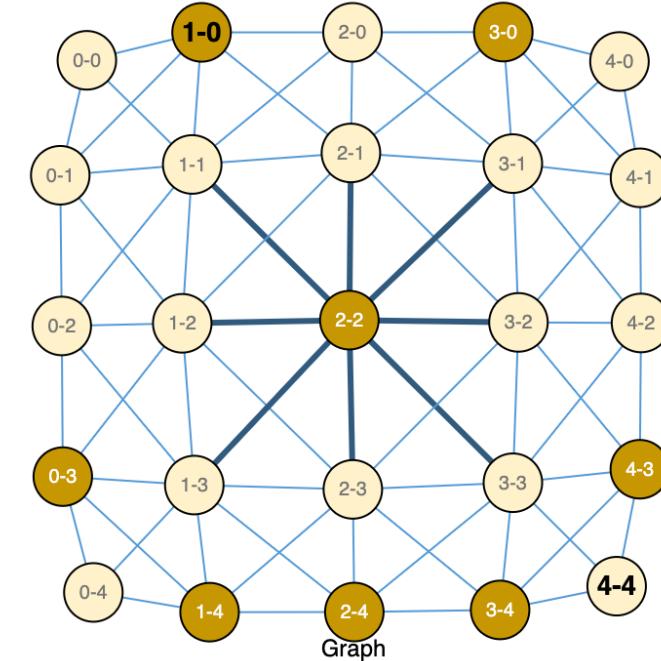
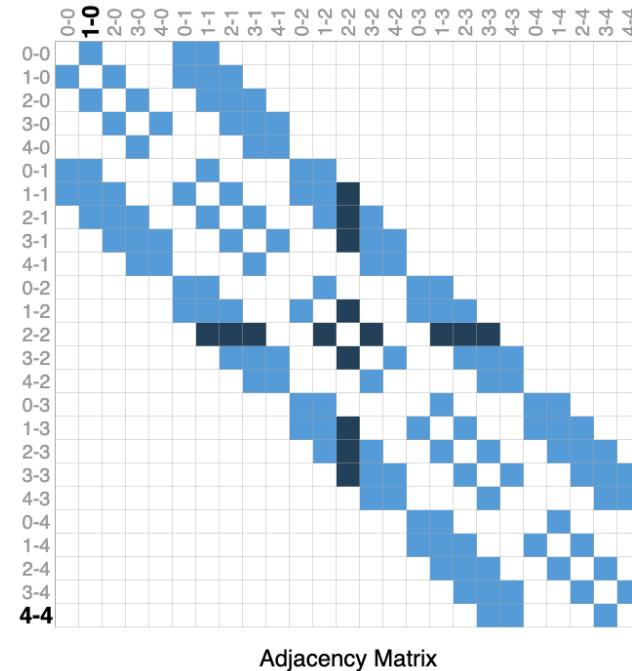
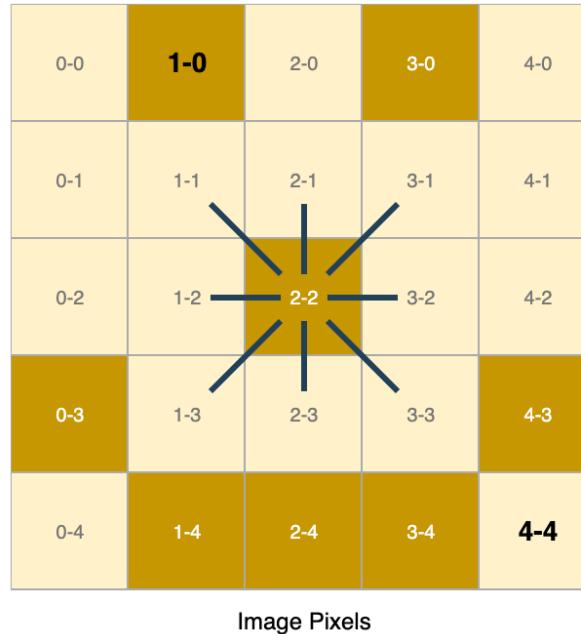
Image



Graphs are all around us

Text

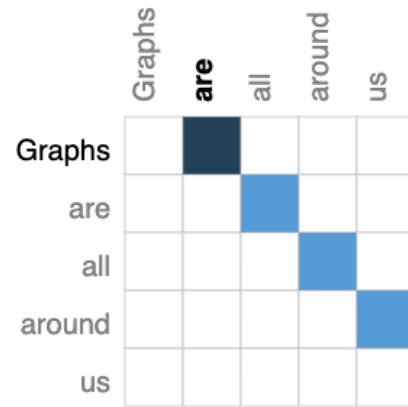
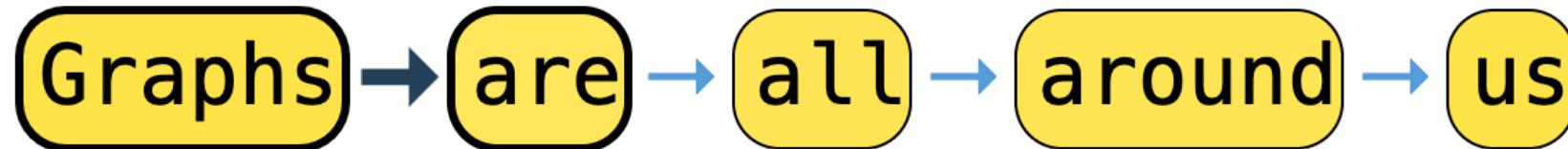
# Image as Graphs



Another way to think of images is as graphs with regular structure, where each pixel represents a node and is connected via an edge to adjacent pixels. Each non-border pixel has exactly 8 neighbors, and the information stored at each node is a 3-dimensional vector representing the RGB value of the pixel.

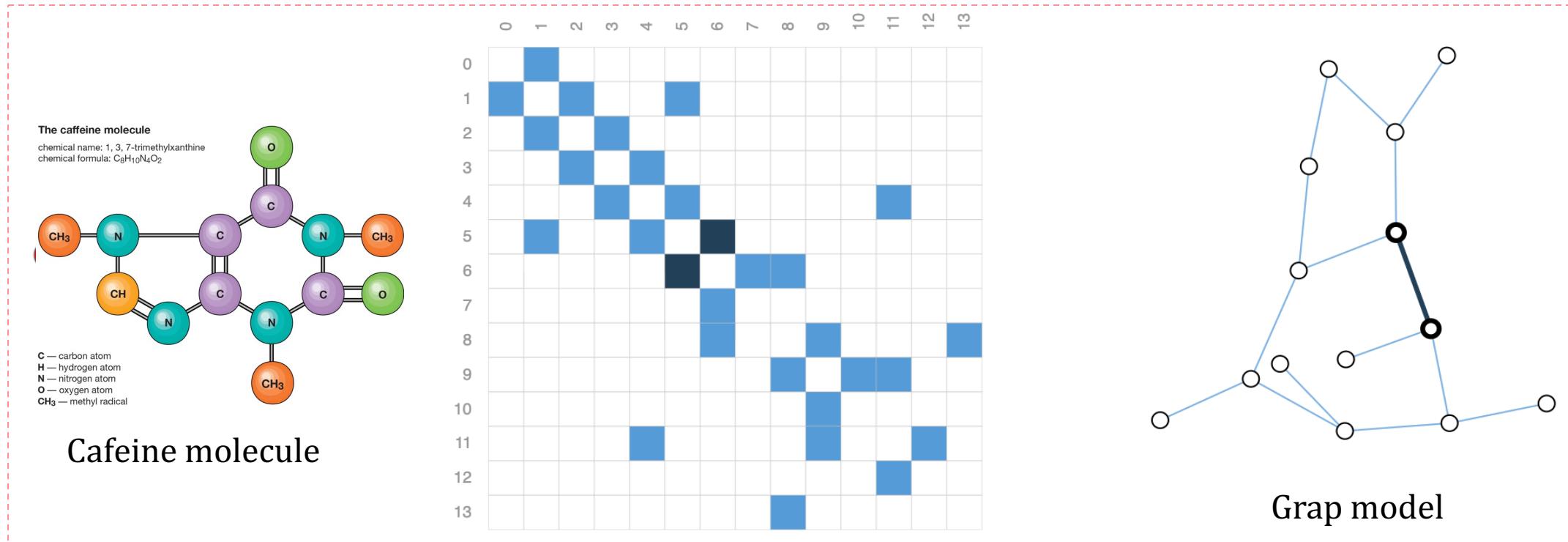
Source: <https://distill.pub/2021/gnn-intro/>

# Text as Graphs



We can digitize text by associating indices to each character, word, or token, and representing text as a sequence of these indices. This creates a simple directed graph, where each character or index is a node and is connected via an edge to the node that follows it.

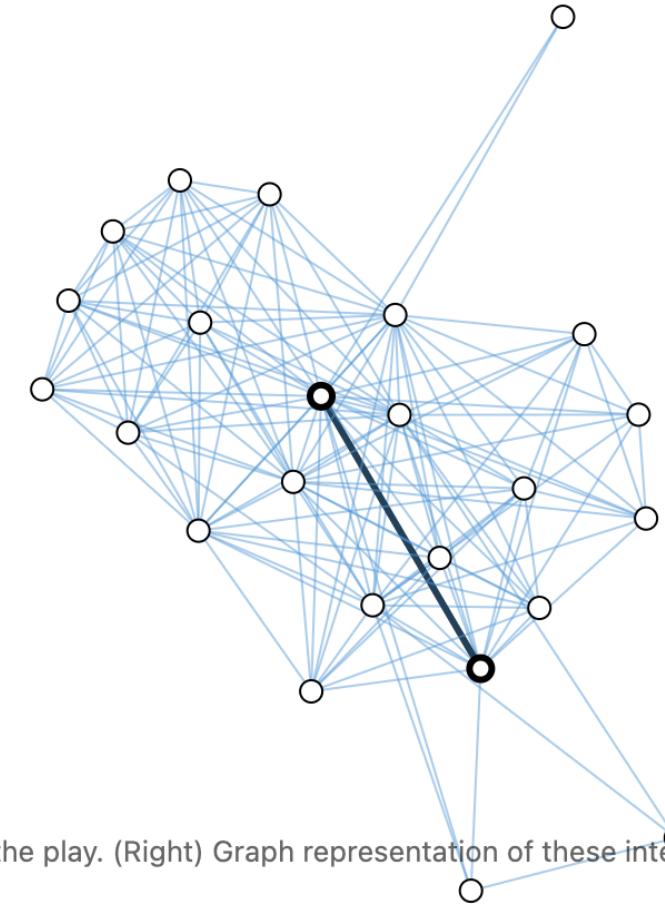
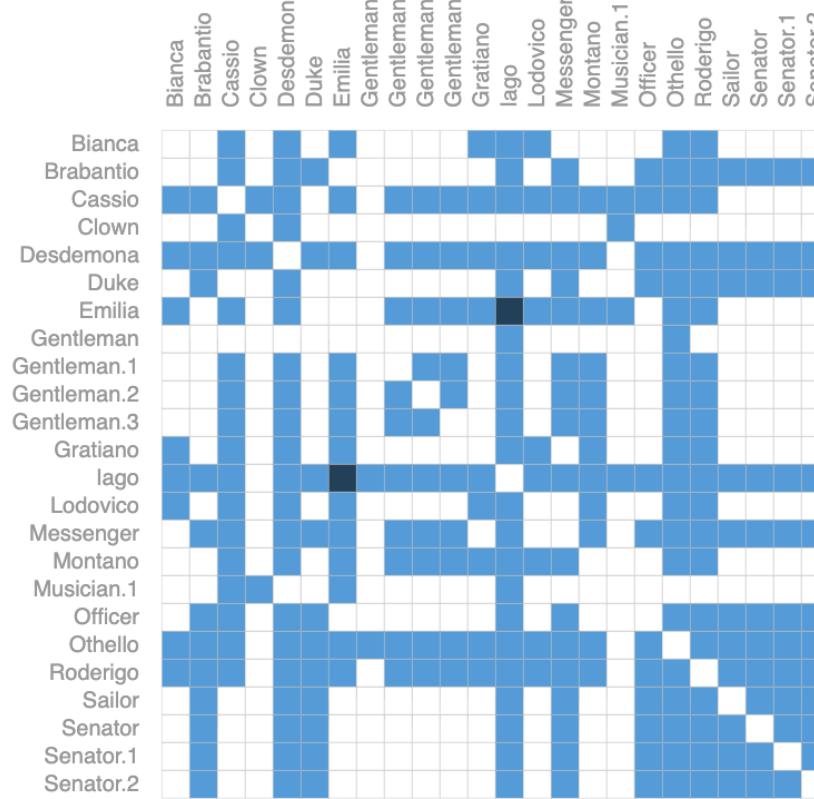
# Other Graph Data



Adjacency matrix

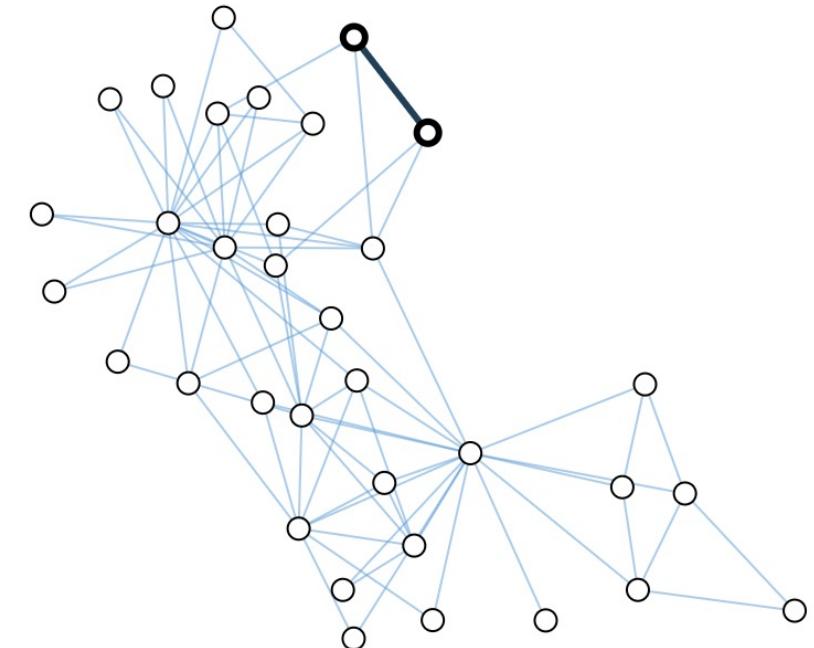
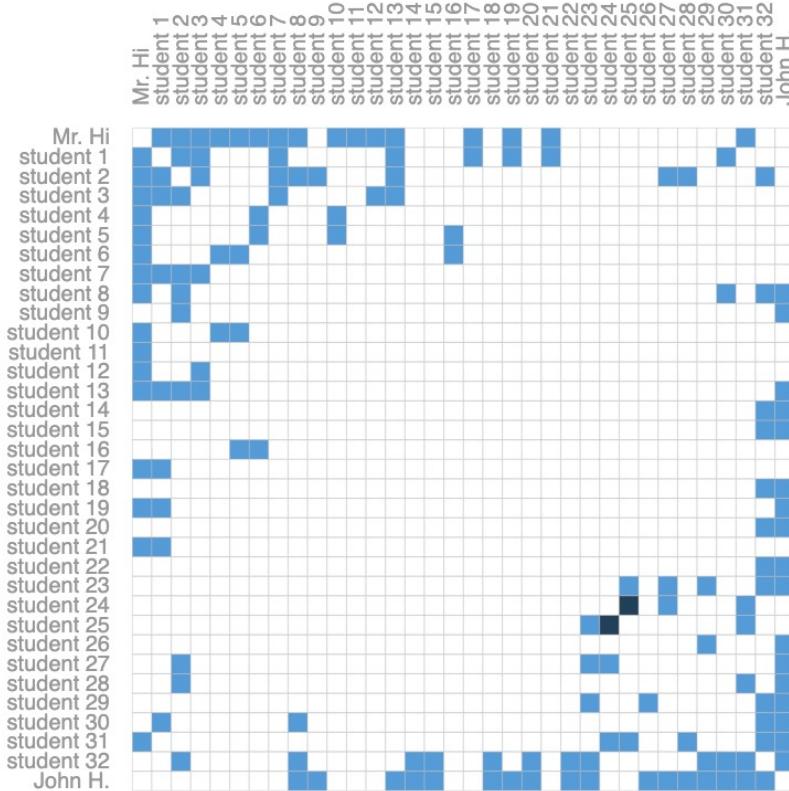
Molecules as graphs

# Other Graph Data



(Left) Image of a scene from the play "Othello". (Center) Adjacency matrix of the interaction between characters in the play. (Right) Graph representation of these interactions.

# Other Graph Data



(Left) Image of karate tournament. (Center) Adjacency matrix of the interaction between people in a karate club. (Right) Graph representation of these interactions.

# Other Graph Data

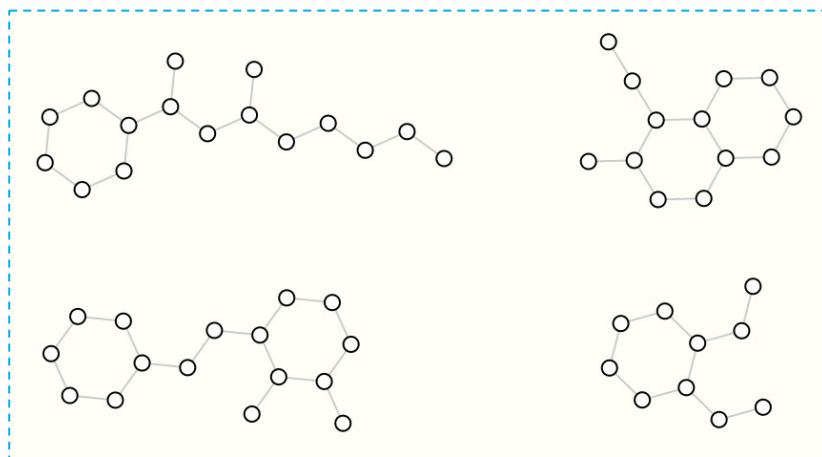
Dataset	Domain	Edges per node (degree)					
		graphs	nodes	edges	min	mean	max
karate club	Social network	1	34	78		4.5	17
qm9	Small molecules	134k	$\leq 9$	$\leq 26$	1	2	5
Cora	Citation network	1	23,166	91,500	1	7.8	379
Wikipedia links, English	Knowledge graph	1	12M	378M		62.24	1M



Summary statistics on graphs found in the real world. Numbers are dependent on featurization decisions.

# Tasks on Graph Data

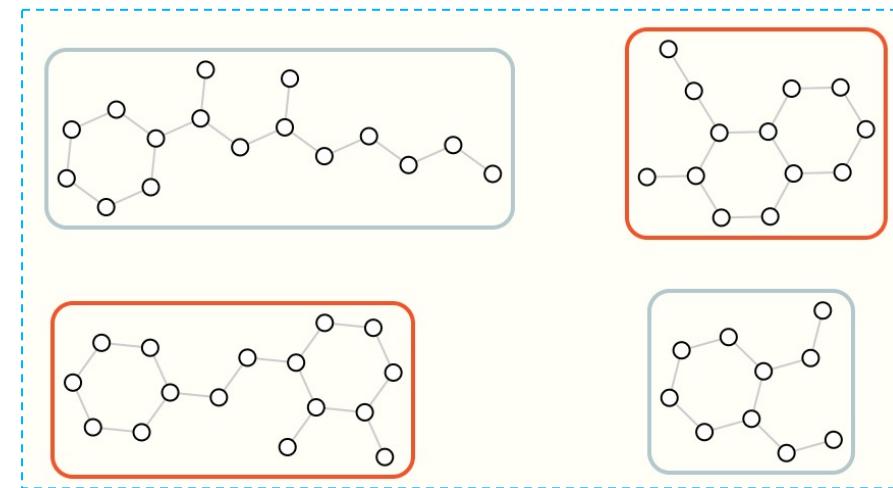
## Graph-level task



Input: graphs



Similar to image classification problems



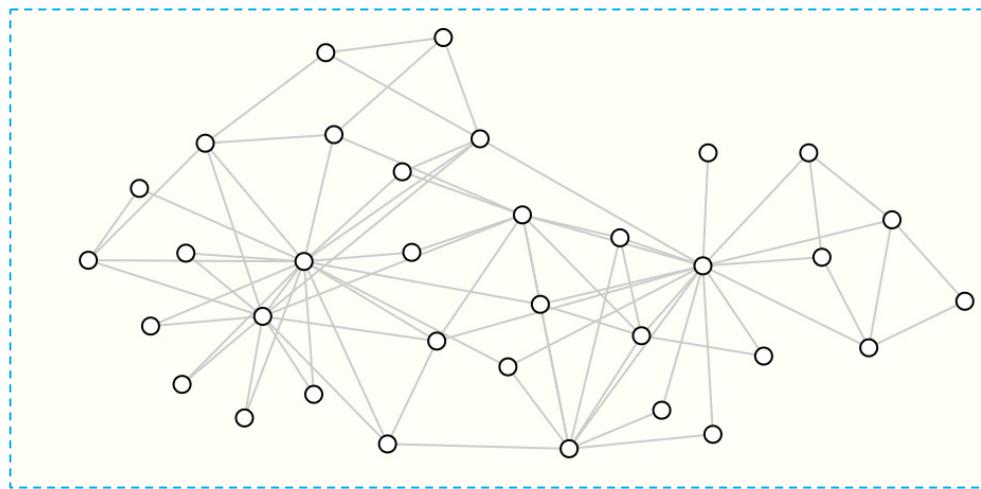
Output: labels for each graph, (e.g., "does the graph contain two rings?")



In a graph-level task, our goal is to predict the property of an entire graph. For example, for a molecule represented as a graph, we might want to predict what the molecule smells like, or whether it will bind to a receptor implicated in a disease.

# Tasks on Graph Data

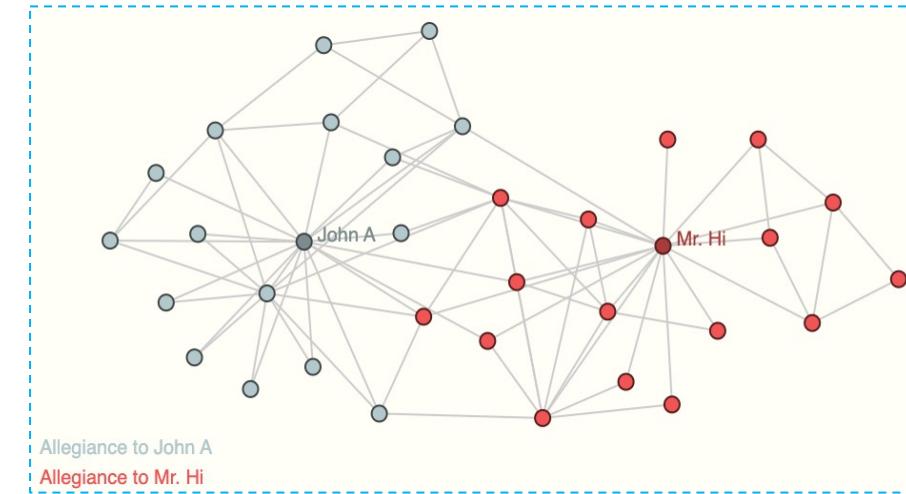
## Node-level task



Input: graph with unlabeled nodes



Similar to image segmentation problems



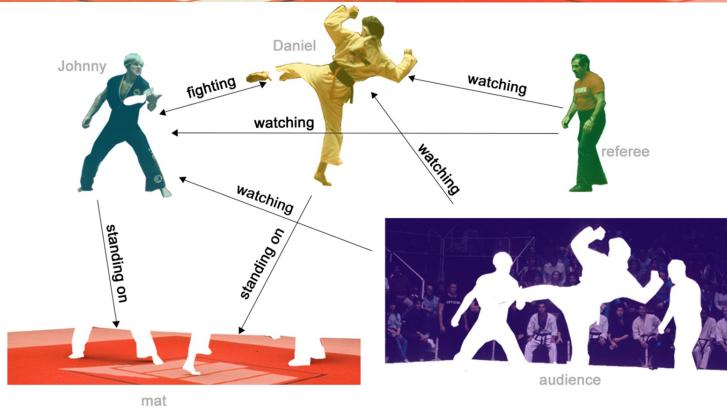
Output: graph node labels



Node-level tasks are concerned with predicting the identity or role of each node within a graph.

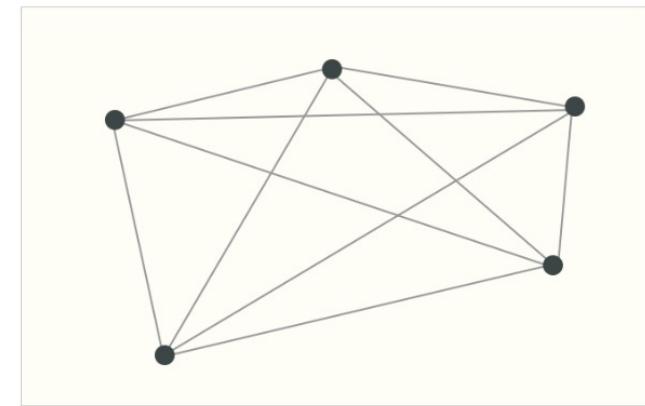
# Tasks on Graph Data

## Edge-level task

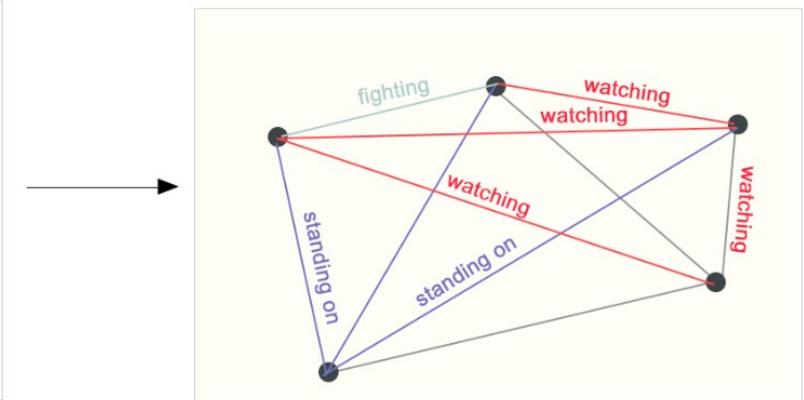


Similar to image scene understanding

Node-level tasks are concerned with predicting the identity or role of each node within a graph.



Input: fully connected graph, unlabeled edges



Output: labels for edges

# A Simple Graph

Node information

x1		x2		...		xn
----	--	----	--	-----	--	----

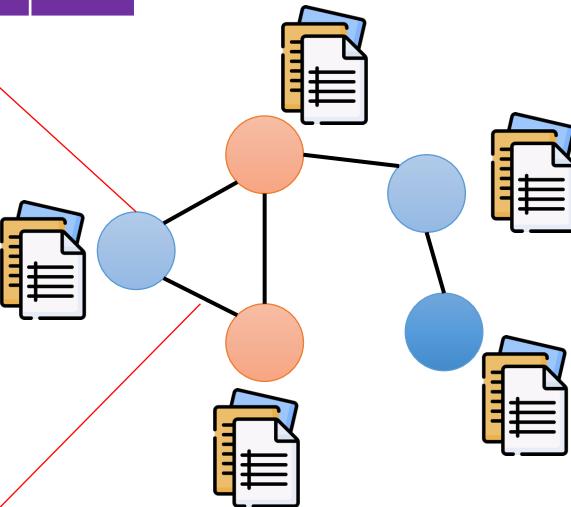
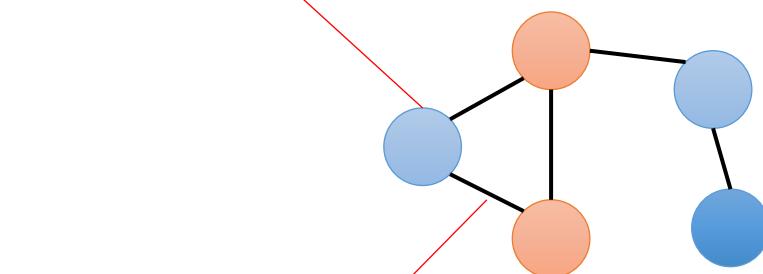
Paper Content

x1		x2		...		xn
----	--	----	--	-----	--	----

Personal Information

x1		x2		...		xn
----	--	----	--	-----	--	----

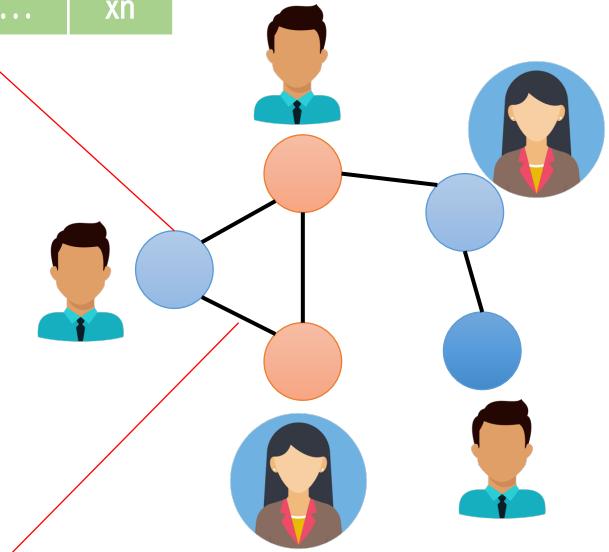
Edge information



Paper citation

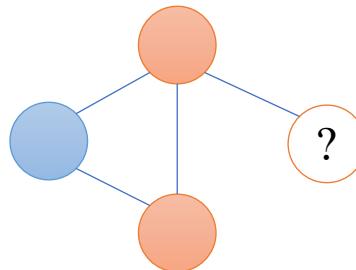
Relationship

x1		x2		...		xn
----	--	----	--	-----	--	----



# Example

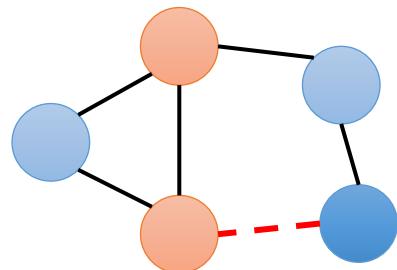
Node level prediction



Does this student smoke?  
(unlabeled node)



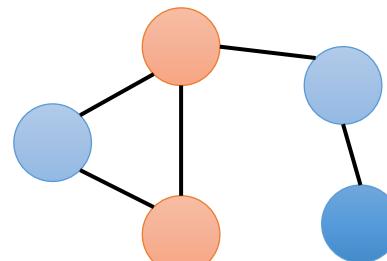
Edge level predictions  
(Link prediction)



Next Youtube  
Video?



Graph Level  
Predictions



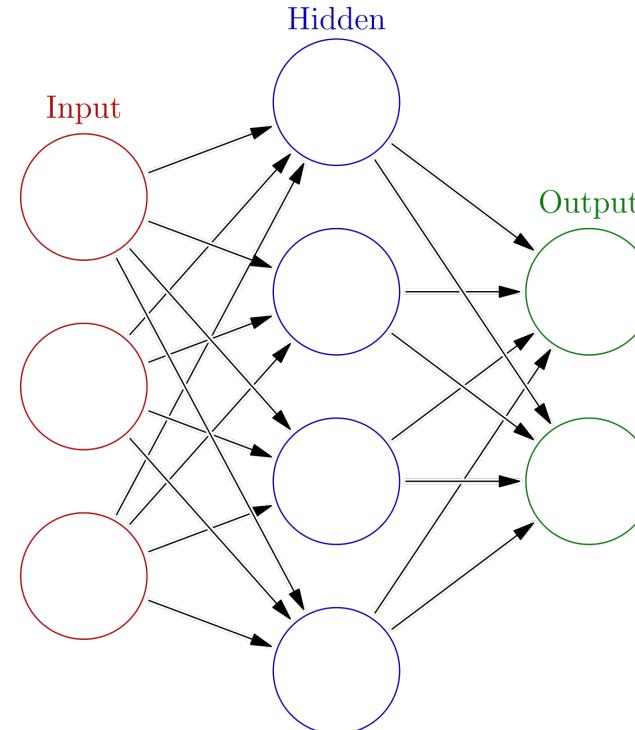
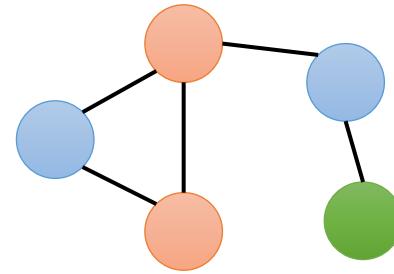
Is this molecule a suitable  
drug?



# Outline

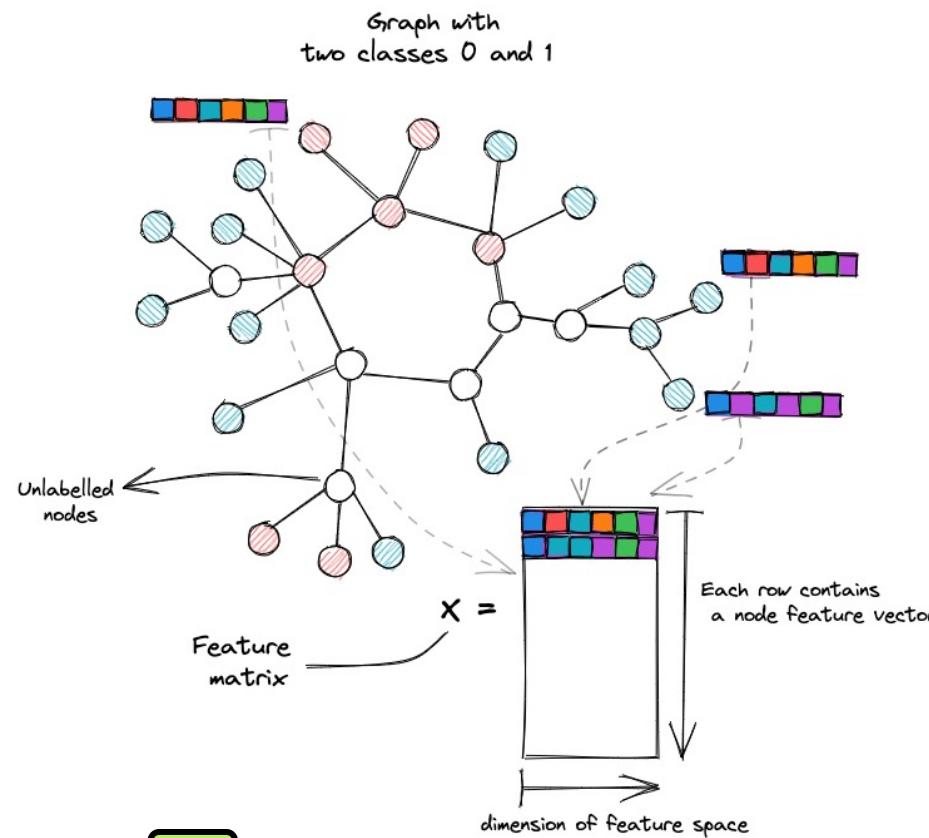
- **Objective**
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- **Summary**

# Graph Data with Neural Network

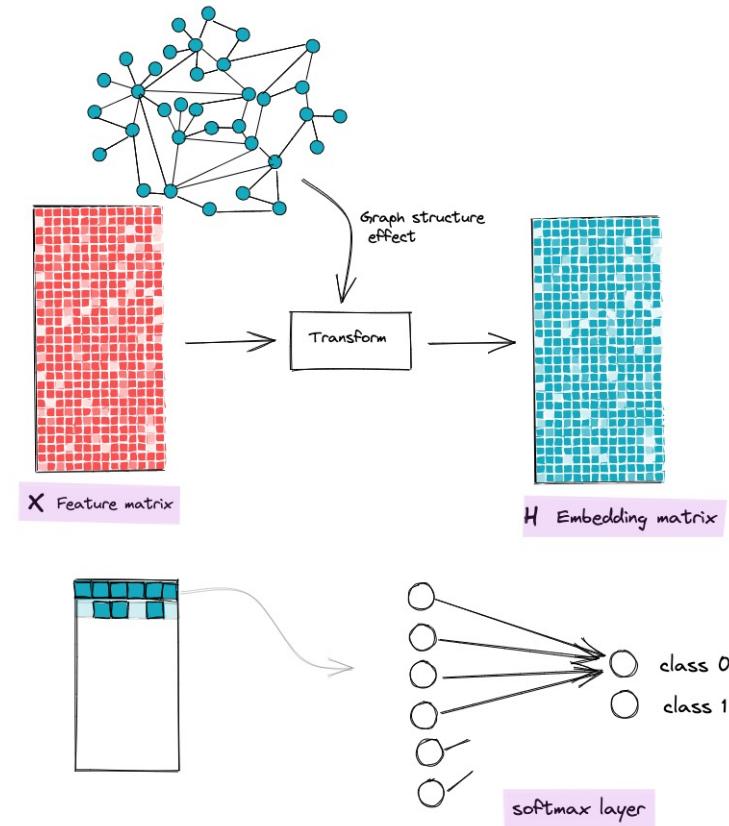


So, how do we go about solving these different graph tasks with neural networks? The first step is to think about how we will represent graphs to be compatible with neural networks.

# Graph Data with Neural Network

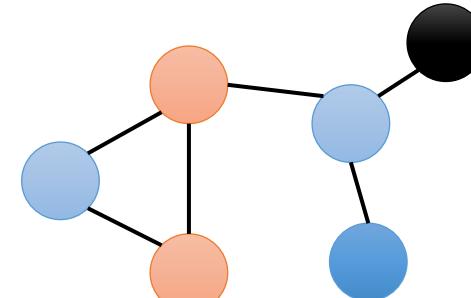
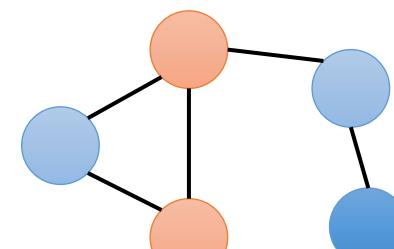


The goal of GNN is to transform node features to features that are aware of the graph structure



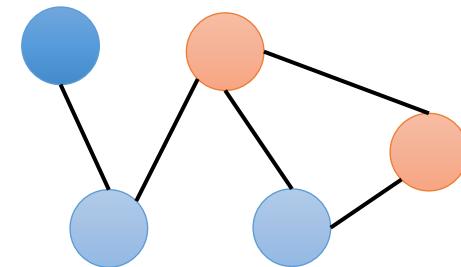
# Problem with Graph Data

## Difference in Size and Shape

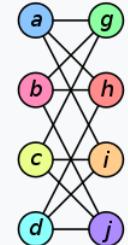
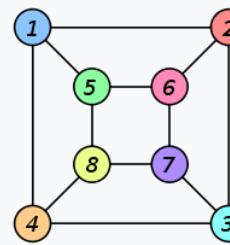


How to handle arbitrary input graph shape?

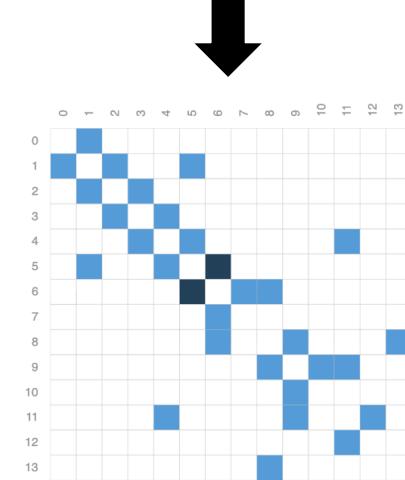
## Isomorphism



Permutation invariant

Graph G	Graph H	An isomorphism between G and H
		$f(a) = 1$ $f(b) = 6$ $f(c) = 8$ $f(d) = 3$ $f(g) = 5$ $f(h) = 2$ $f(i) = 4$ $f(j) = 7$

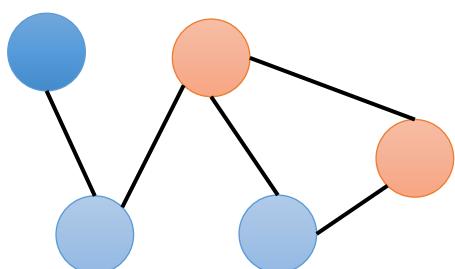
Cannot use adjacency matrix as an input



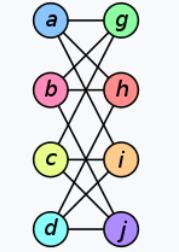
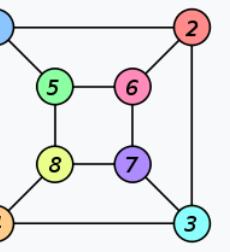
# Problem with Graph Data



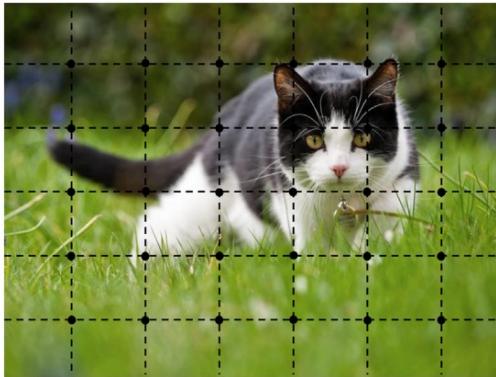
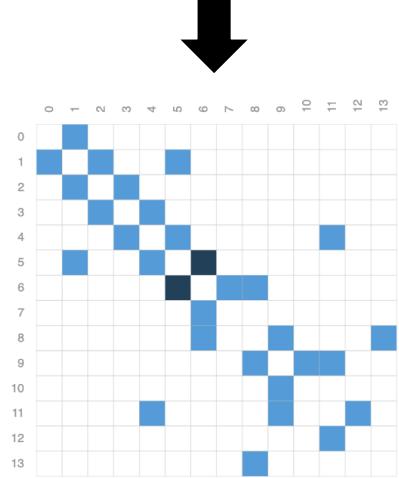
## Isomorphism



Permutation invariant

Graph G	Graph H	An isomorphism between G and H
		$\begin{aligned}f(a) &= 1 \\f(b) &= 6 \\f(c) &= 8 \\f(d) &= 3 \\f(g) &= 5 \\f(h) &= 2 \\f(i) &= 4 \\f(j) &= 7\end{aligned}$

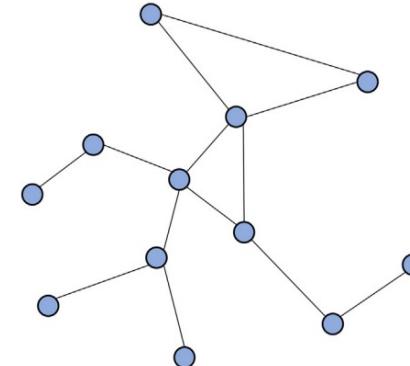
Cannot use adjacency matrix as an input



Euclidean space

## Grid Structure

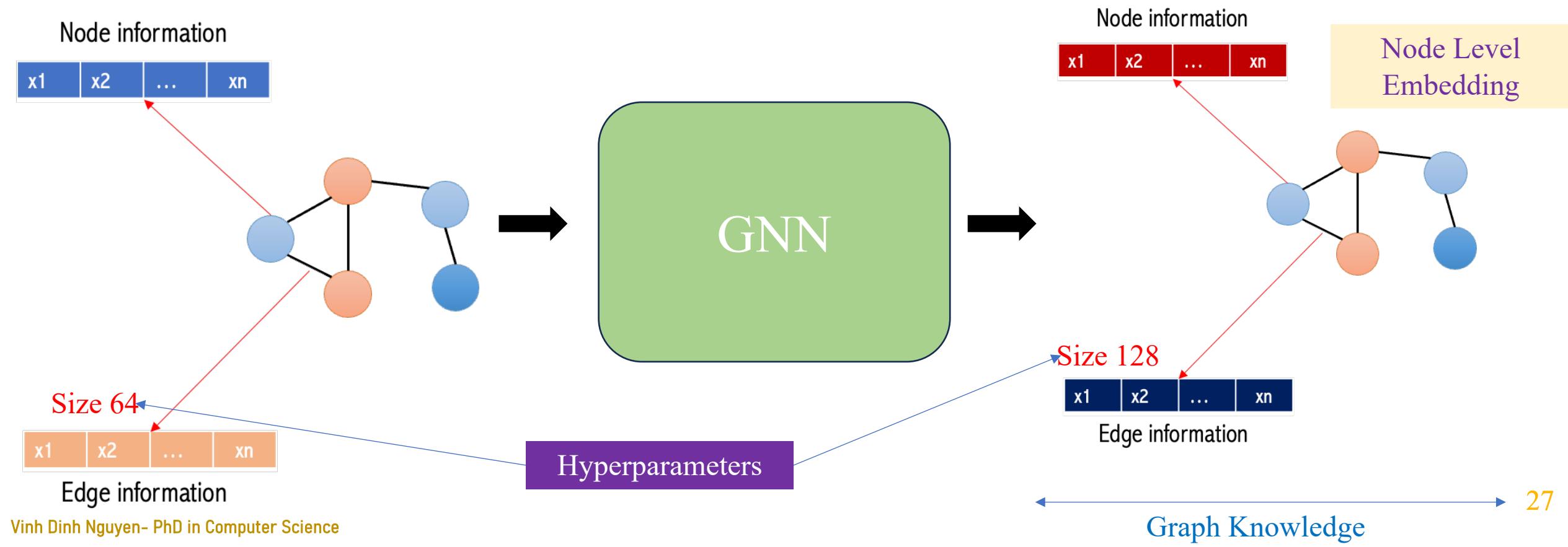
Information between nodes



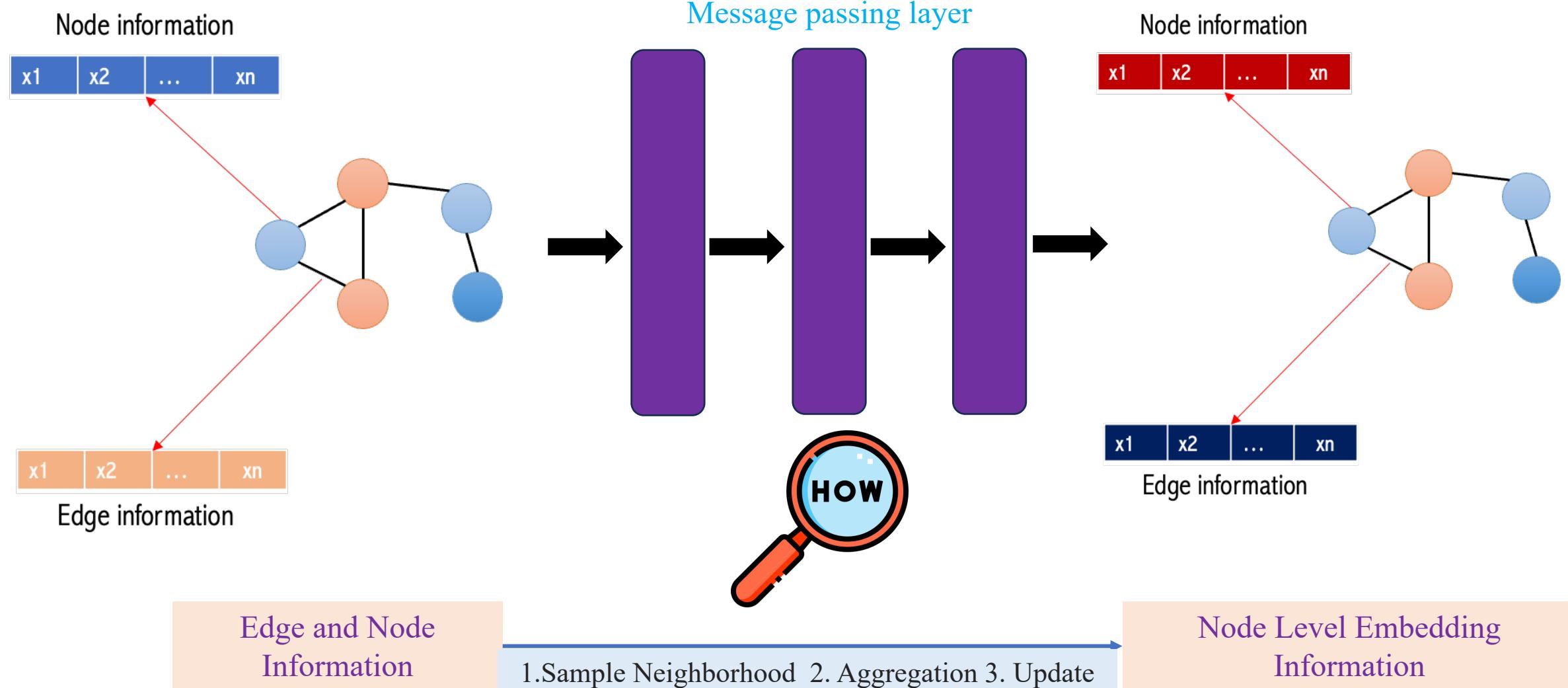
Non-Euclidean space

# Fundamental Idea of GNNs

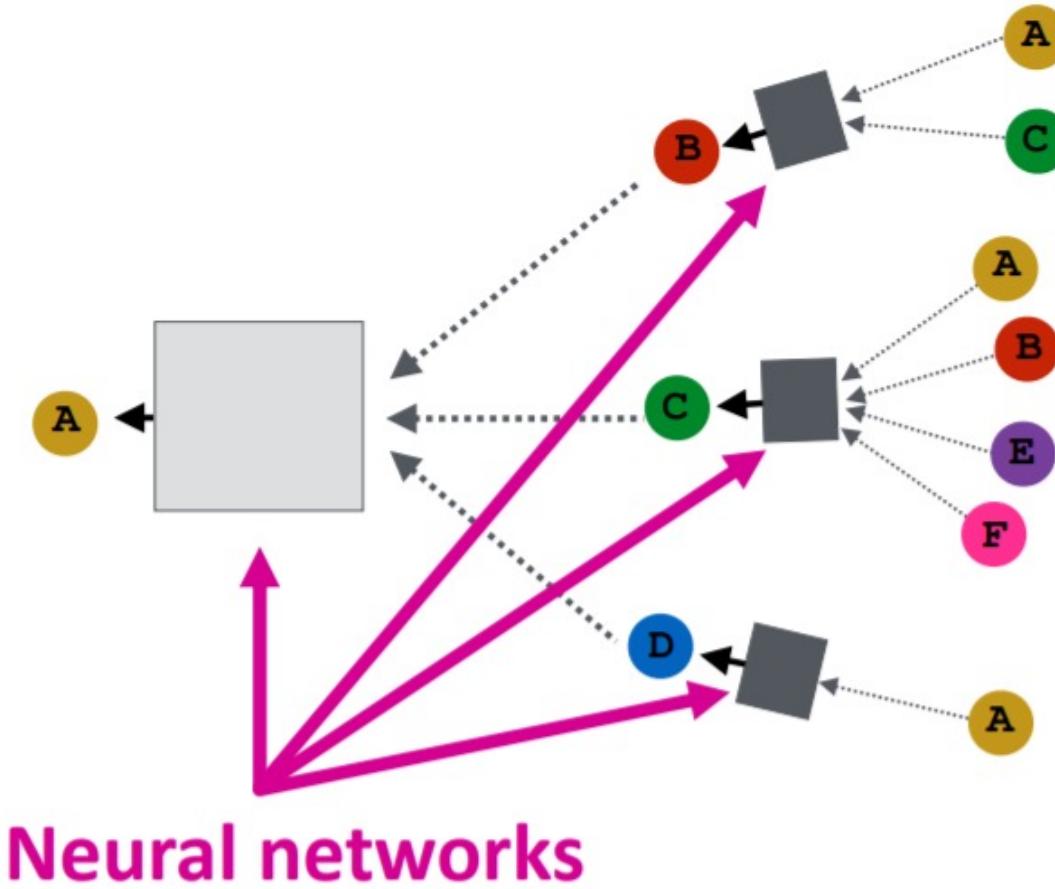
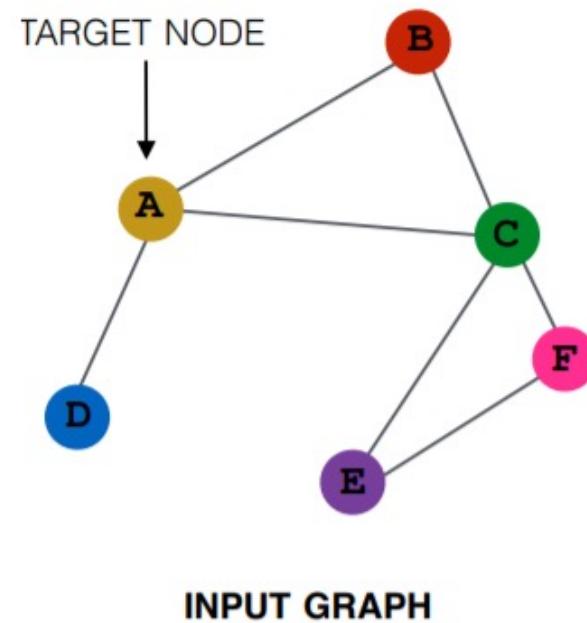
Learning a neural network suitable representation of graph data  
**<Representation Learning>>**



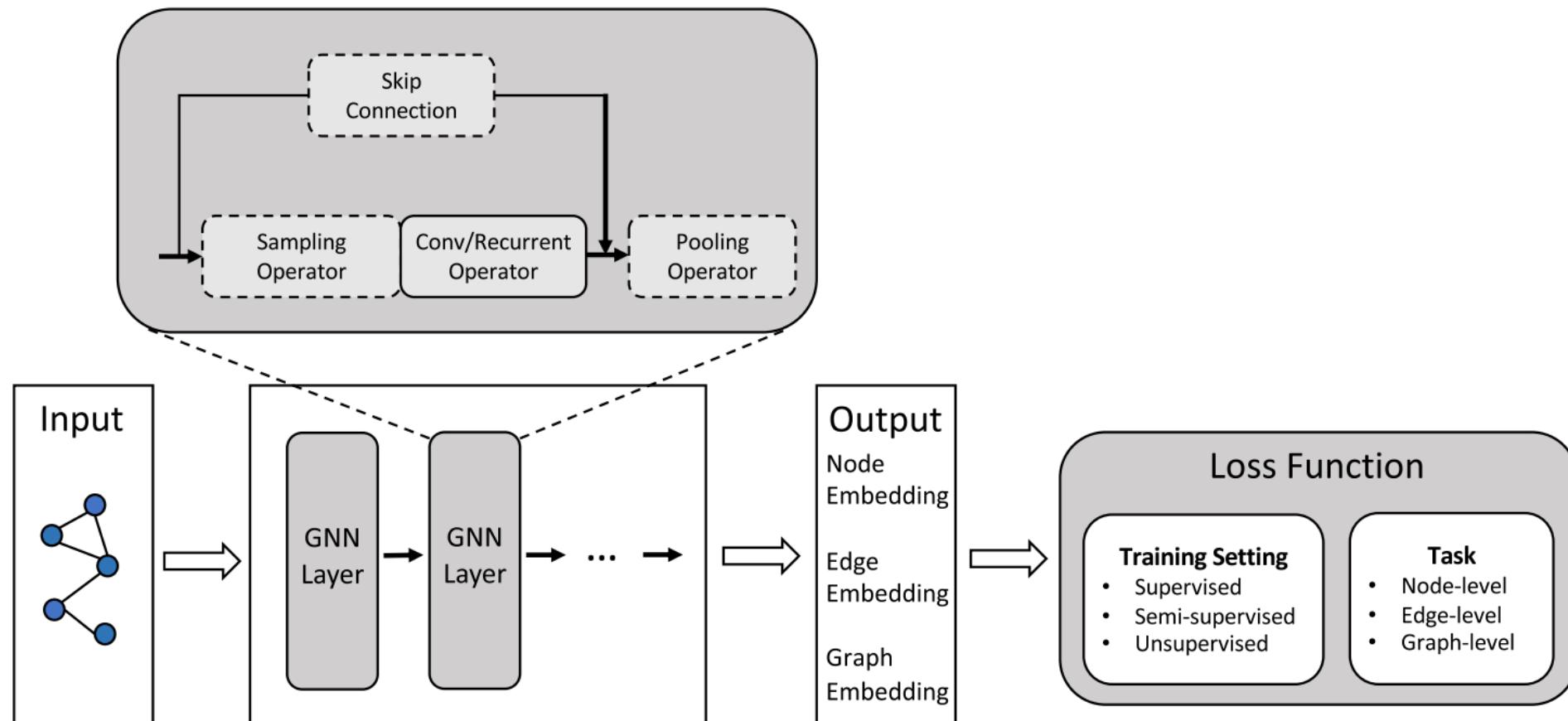
# How do Graph Neural Network Work?



# How do Graph Neural Network Work?



# How do Graph Neural Network Work?



1. Find graph structure.

2. Specify graph type and scale.

4. Build model using computational modules.

3. Design loss function.

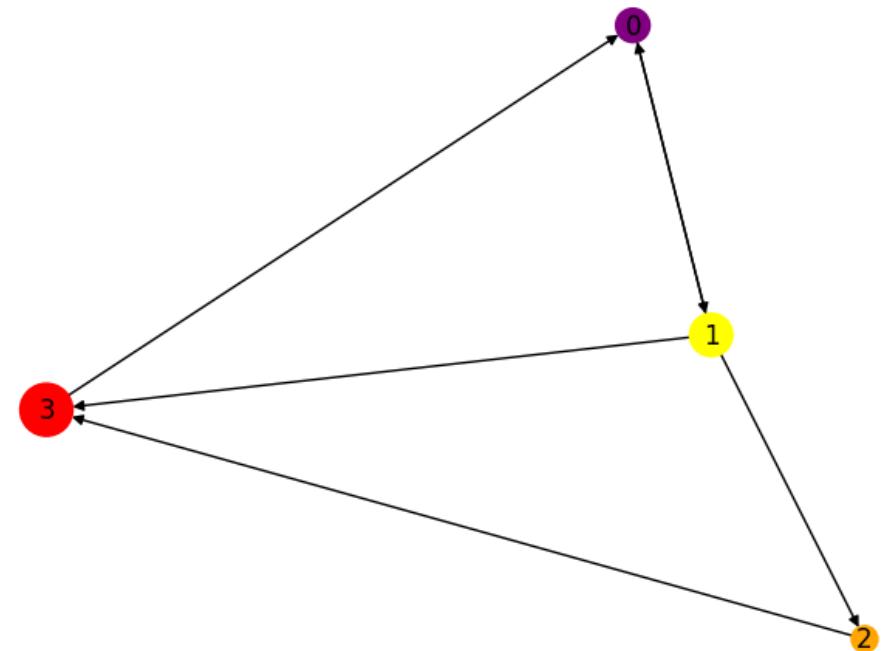
# Create a graph using NetworkX

```
import networkx as nx
H = nx.DiGraph()

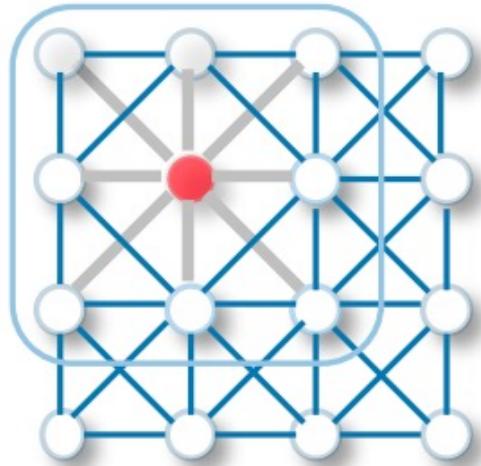
#adding nodes
H.add_nodes_from([
    (0, {"color": "purple", "size": 250}),
    (1, {"color": "yellow", "size": 400}),
    (2, {"color": "orange", "size": 150}),
    (3, {"color": "red", "size": 600})
])

#adding edges
H.add_edges_from([
    (0, 1), (1, 2), (1, 0), (1, 3), (2, 3), (3,0)
])
|
node_colors = nx.get_node_attributes(H, "color").values()
colors = list(node_colors)
node_sizes = nx.get_node_attributes(H, "size").values()
sizes = list(node_sizes)

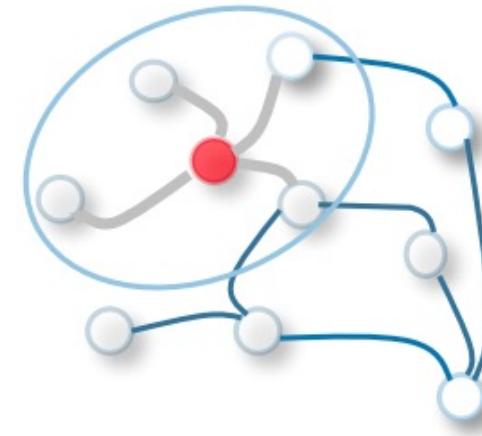
#Plotting Graph
nx.draw(H, with_labels=True, node_color=colors, node_size=sizes)
```



# CNN Vs. GNN: Message Passing

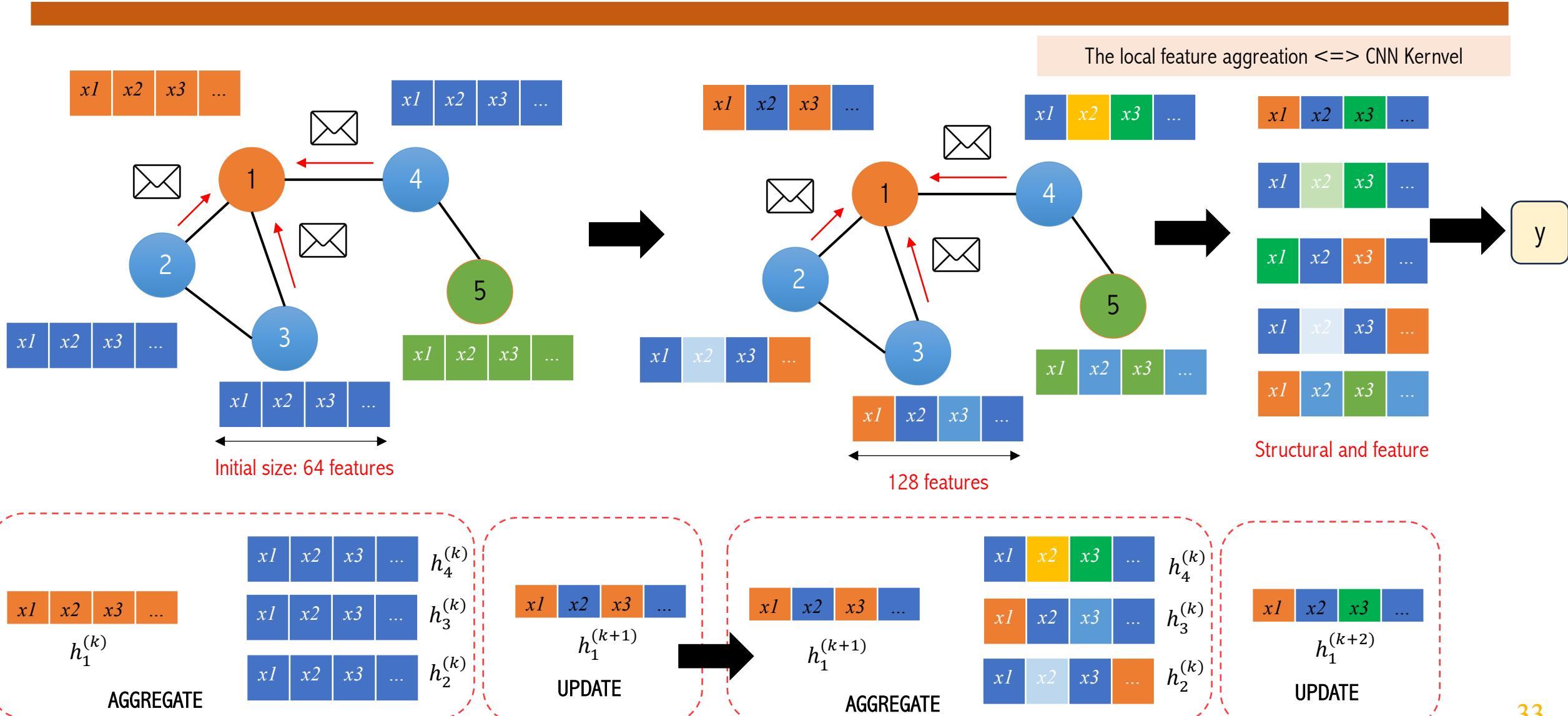


2D Convolution. Analogous to a graph, each pixel in an image is taken as a node where neighbors are determined by the filter size. The 2D convolution takes the weighted average of pixel values of the red node along with its neighbors. The neighbors of a node are ordered and have a fixed size

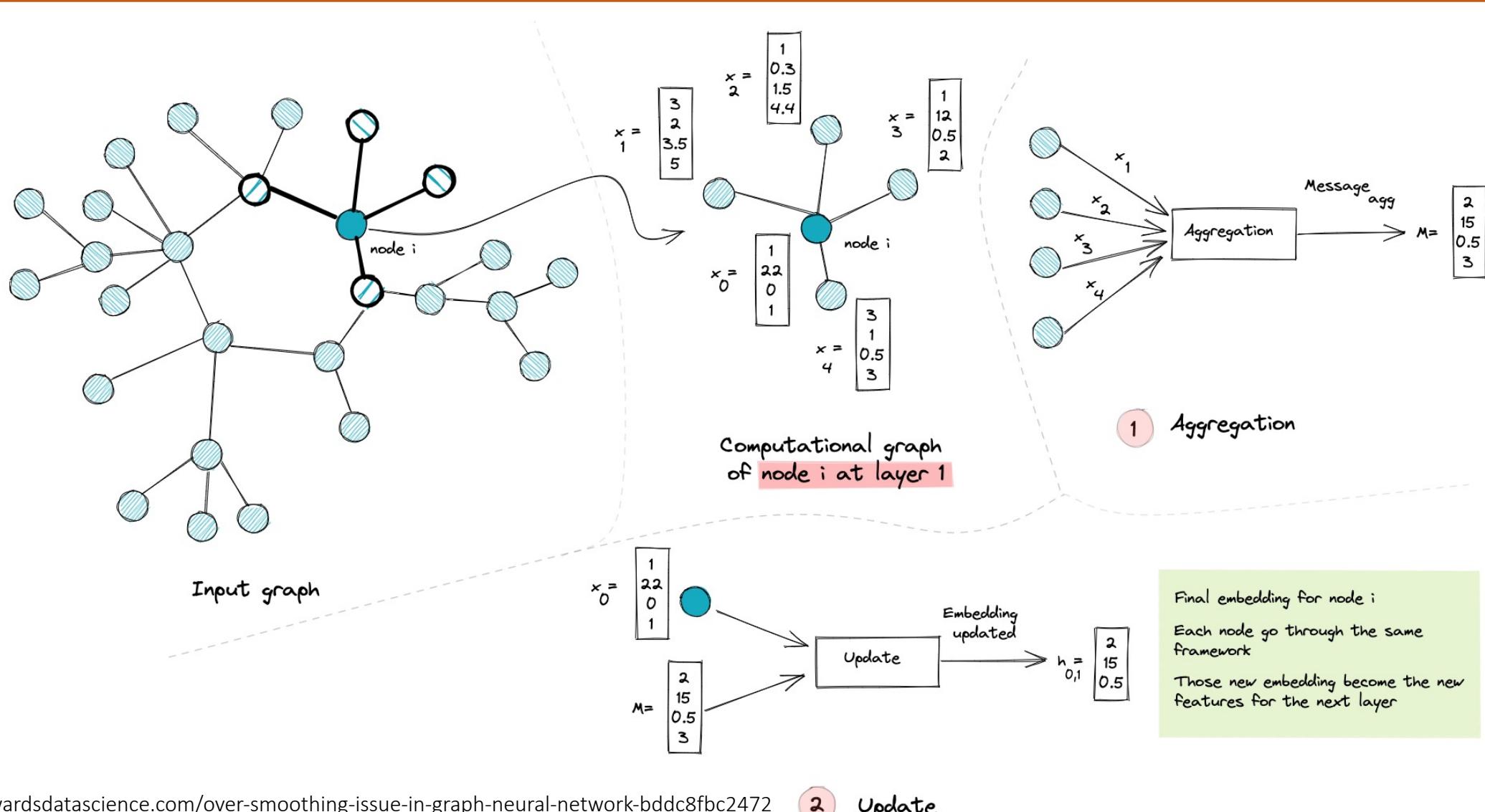


Graph Convolution. To get a hidden representation of the red node, one simple solution of the graph convolutional operation is to take the average value of the node features of the red node along with its neighbors. Different from image data, the neighbors of a node are unordered and variable in size

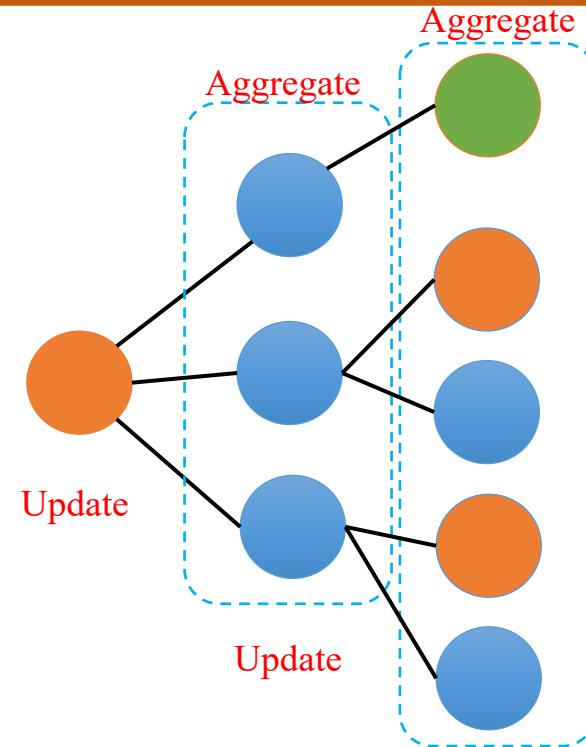
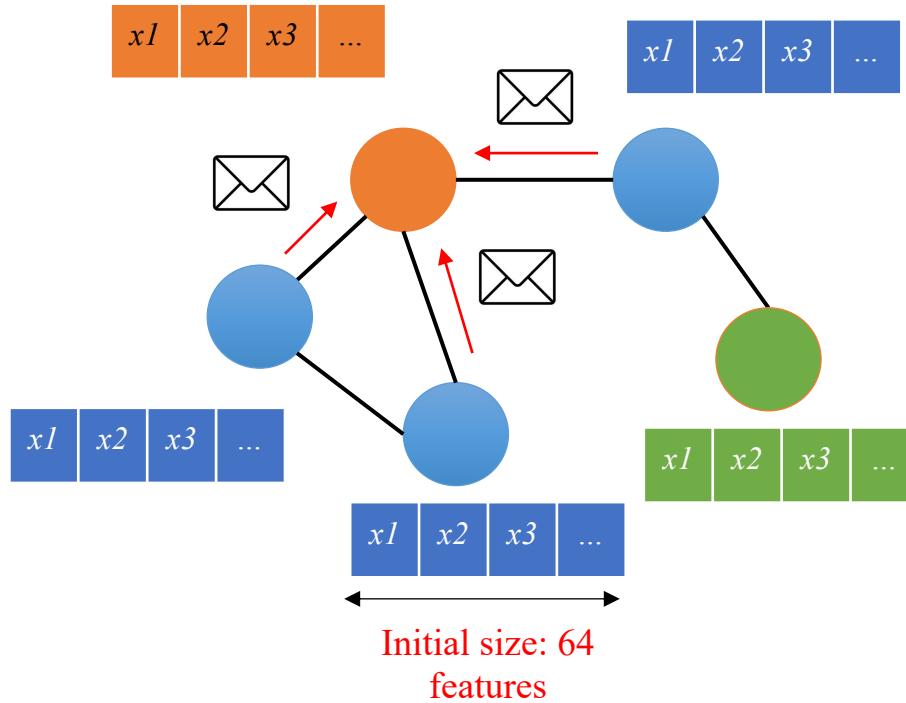
# Message Passing: Behind the Scene



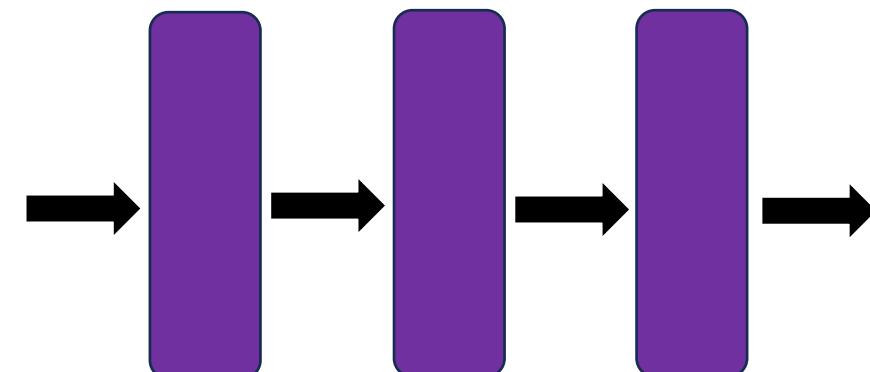
# Graph: Example



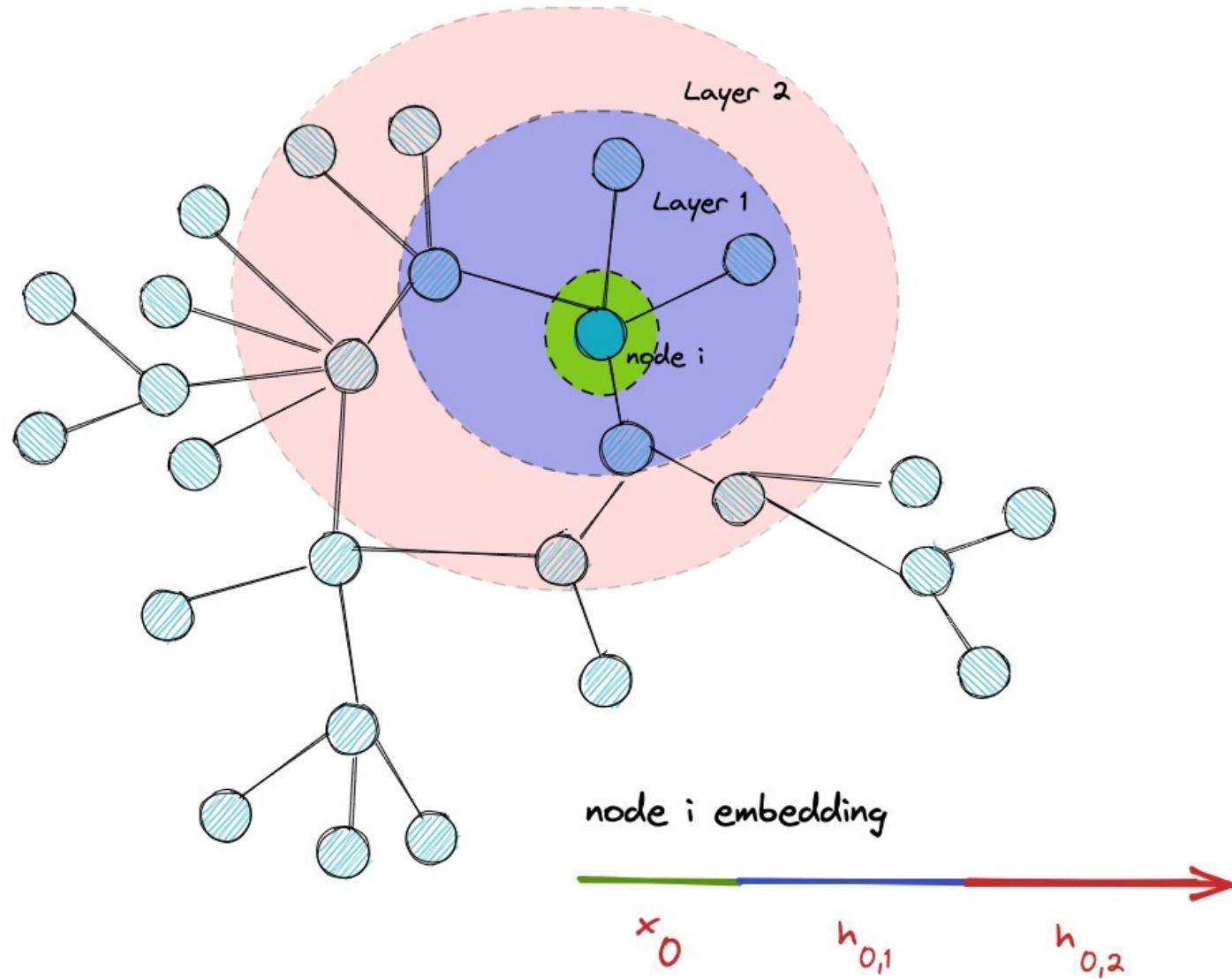
# Computation Graph Representation



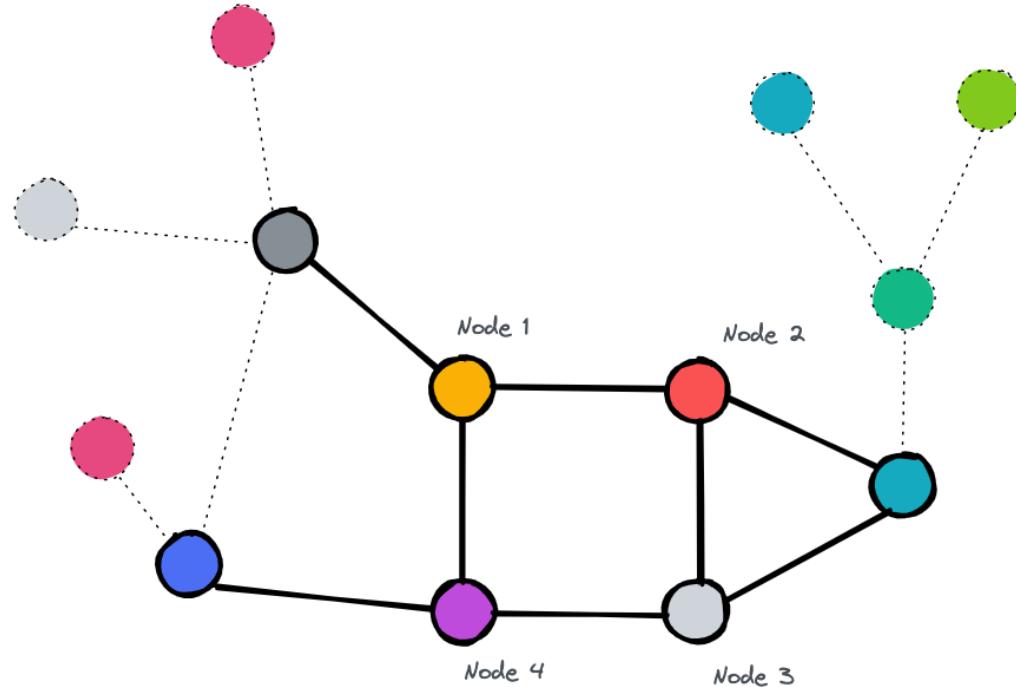
Computational Graph  
for Node  $v$



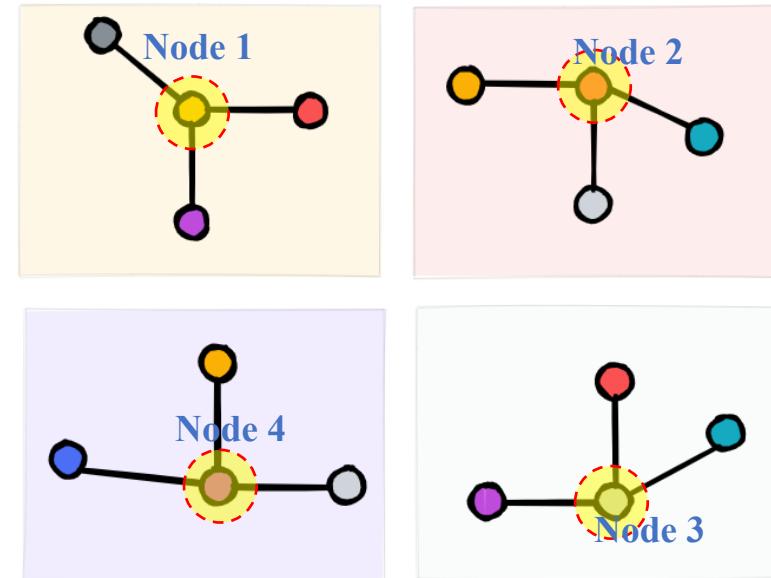
# Graph: Example



# Over-smoothing in GNN



Input graph



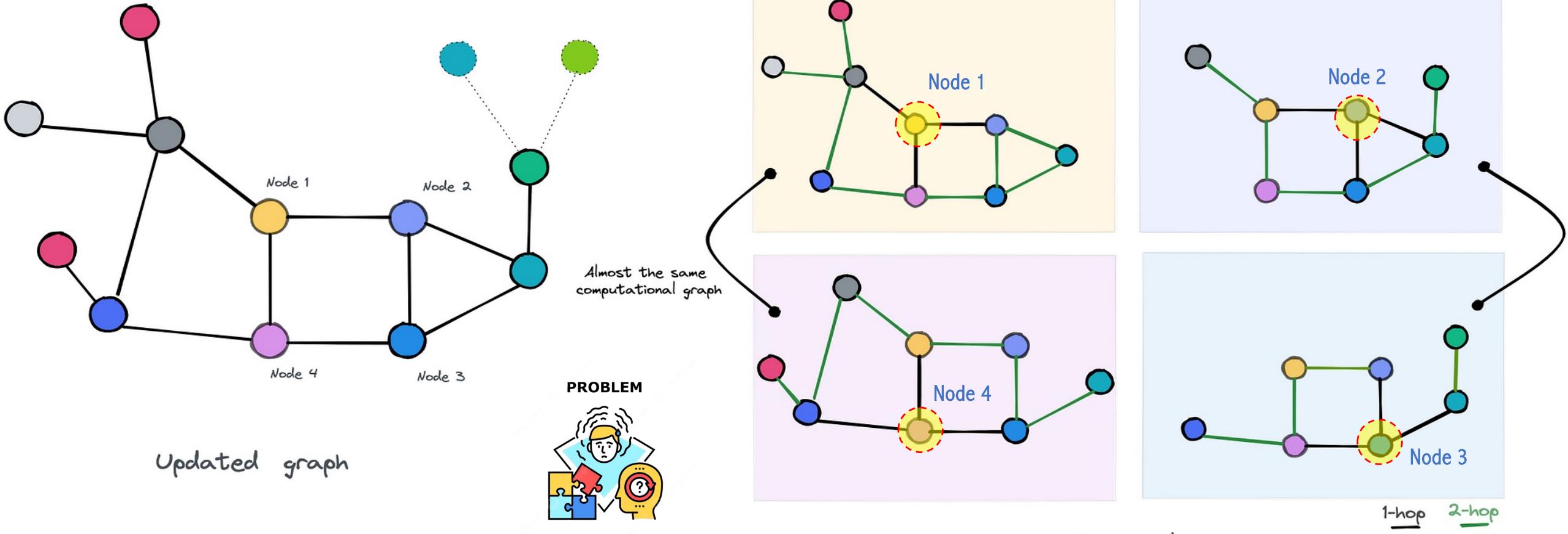
Computational graphs  
at layer 1



Node 2 and node 3 have almost access to the same information -> We can predict that their embeddings will be slightly similar.

Node 1 and Node 4, they interact with each other but have different neighbors -> We may predict that their new embeddings will be different.

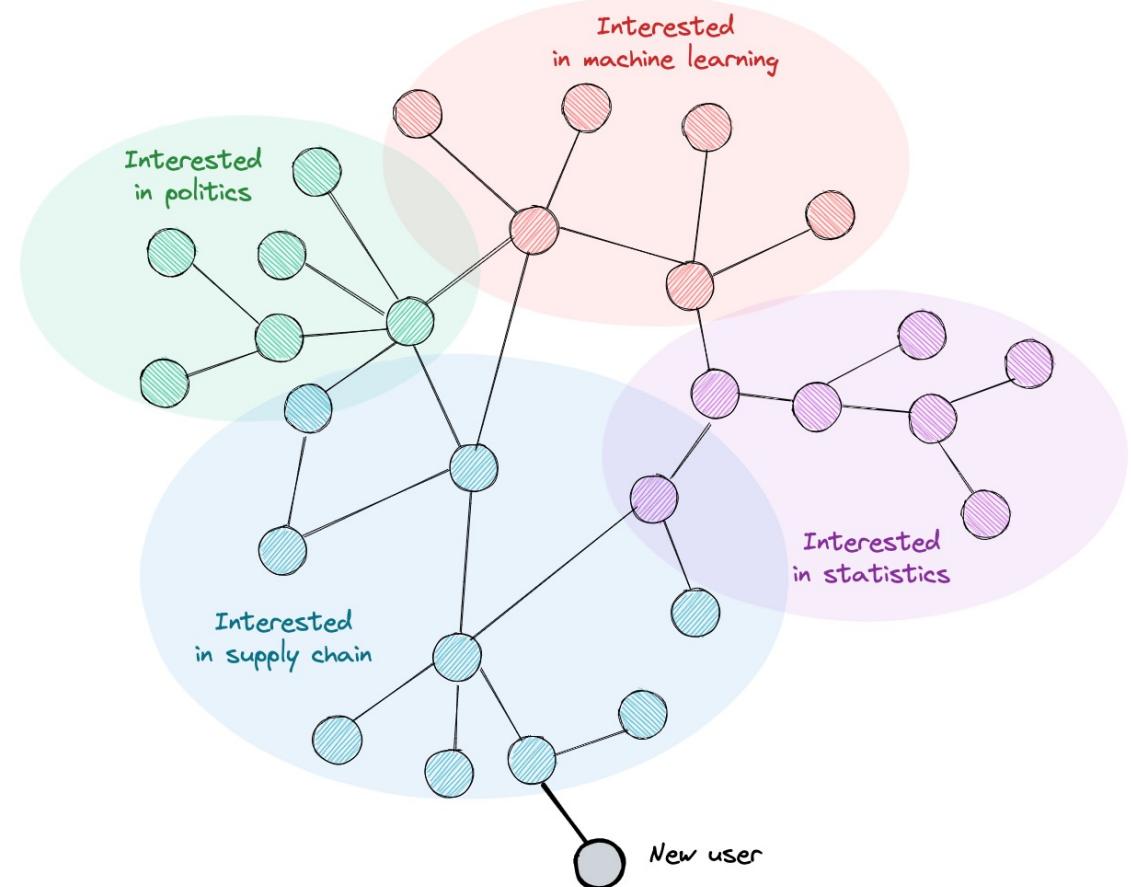
# Over-smoothing in GNN



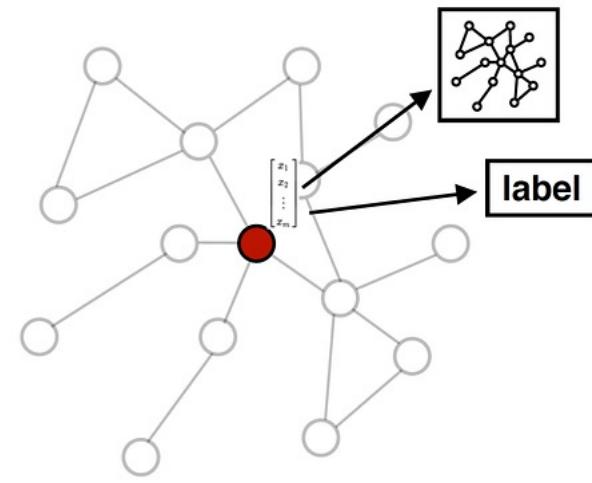
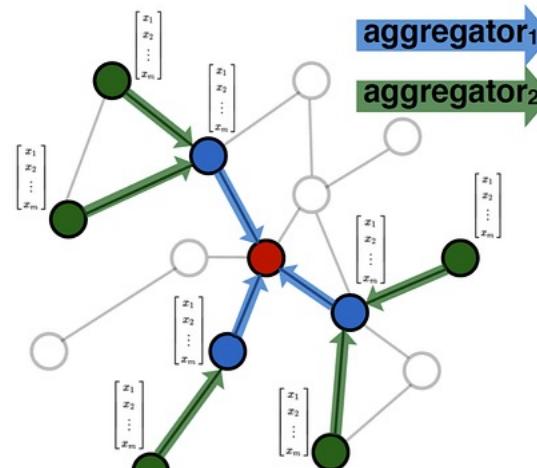
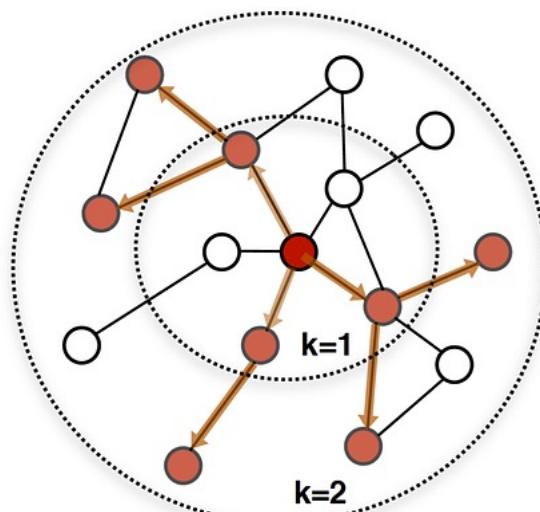
The computational graphs of nodes 1,4, and 2,3 are almost the same respectively

# Over-smoothing in GNN

Graph Neural Networks, or GNNs, are really good at working with data that is organized in a graph structure. But sometimes, when we add more layers to a GNN architecture, it doesn't work as well as we would like. This is called over-smoothing.



# Why over-smoothing happens?

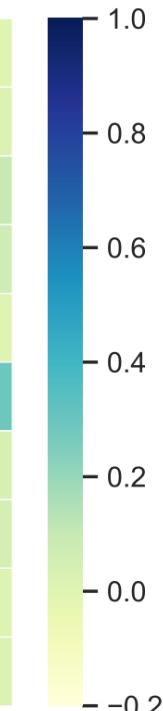


Reason 1 → More number of layers (depth)  
Reason 2 → Default nature of GNNs

# How to detect over-smoothing?

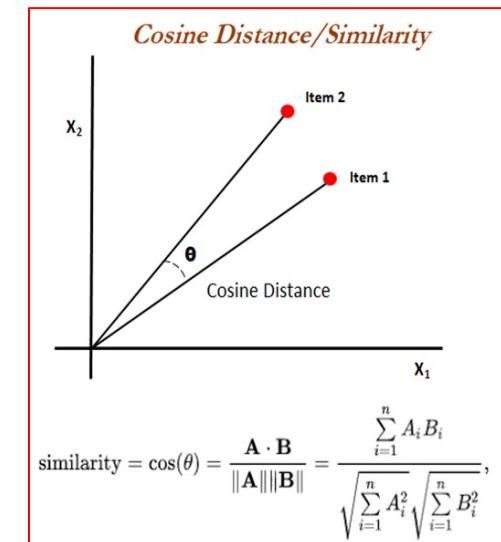
## Metric 1: MAD(Mean Average Distance)

Model	2	3	4	5	6
ARMA	0.629	0.860	0.608	0.305	0.004
ChebGCN	0.557	0.756	0.138	0.024	0.018
DNA	0.665	0.352	0.347	0.172	0.096
FeaSt	0.778	0.770	0.677	0.182	0.072
GAT	0.794	0.704	0.232	0.047	0.005
GCN	0.796	0.765	0.714	0.602	0.289
GGNN	0.661	0.078	0.021	0.033	0.039
GraphSAGE	0.925	0.816	0.632	0.303	0.053
HighOrder	0.629	0.145	0.023	0.004	0.012
HyperGraph	0.828	0.742	0.493	0.046	0.023



MAD computes the Mean Average Distance (MAD) between node representations (embeddings) in the graph

$$D_{ij} = 1 - \frac{\mathbf{H}_{i,:} \cdot \mathbf{H}_{j,:}}{|\mathbf{H}_{i,:}| \cdot |\mathbf{H}_{j,:}|} \quad i, j \in [1, 2, \dots, n],$$



## Measuring and Relieving the Over-smoothing Problem for Graph Neural Networks from the Topological View

Deli Chen,<sup>1</sup> Yankai Lin,<sup>2</sup> Wei Li,<sup>1</sup> Peng Li,<sup>2</sup> Jie Zhou,<sup>2</sup> Xu Sun<sup>1</sup>

<sup>1</sup>MOE Key Lab of Computational Linguistics, School of EECS, Peking University

<sup>2</sup>Pattern Recognition Center, WeChat AI, Tencent Inc., China

{chendeli, liweitj47, xusun}@pku.edu.cn, {yankailin, patrickpli, withtomzhou}@tencent.com,

# How to detect over-smoothing?

## Metric 2 : MADGap

$$\text{MADGap} = \text{MAD}^{\text{rmt}} - \text{MAD}^{\text{neb}}, \quad (5)$$

where  $\text{MAD}^{\text{rmt}}$  is the MAD value of the remote nodes in the graph topology and  $\text{MAD}^{\text{neb}}$  is the MAD value of the neighbouring nodes.

It is based on the main hypothesis that when nodes interact, they have access to either important information from nodes of the same class or noise from nodes of other classes.

### Measuring and Relieving the Over-smoothing Problem for Graph Neural Networks from the Topological View

Deli Chen,<sup>1</sup> Yankai Lin,<sup>2</sup> Wei Li,<sup>1</sup> Peng Li,<sup>2</sup> Jie Zhou,<sup>2</sup> Xu Sun<sup>1</sup>

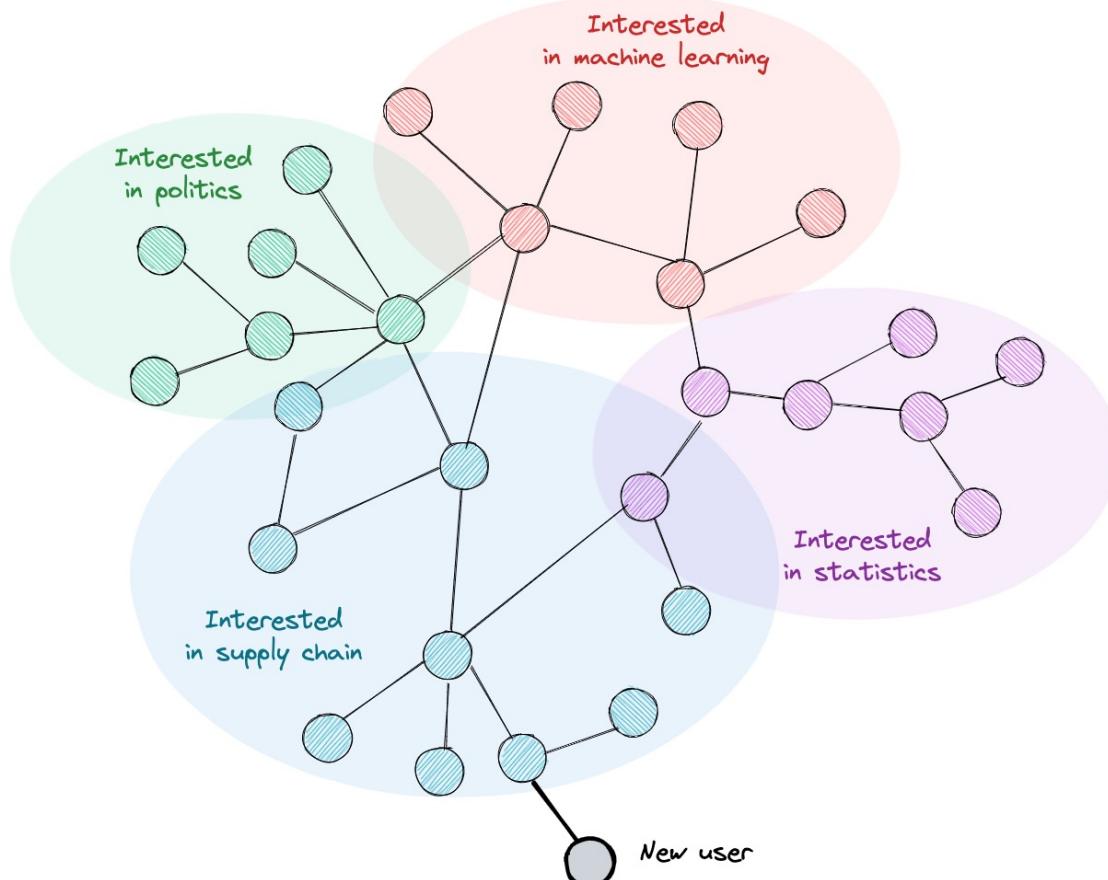
<sup>1</sup>MOE Key Lab of Computational Linguistics, School of EECS, Peking University

<sup>2</sup>Pattern Recognition Center, WeChat AI, Tencent Inc., China

{chendeli,liweitj47,xusun}@pku.edu.cn, {yankailin,patrickpli,withtomzhou}@tencent.com,



# Over-smoothing in GNN



## How to reduce the effect of over-smoothing.

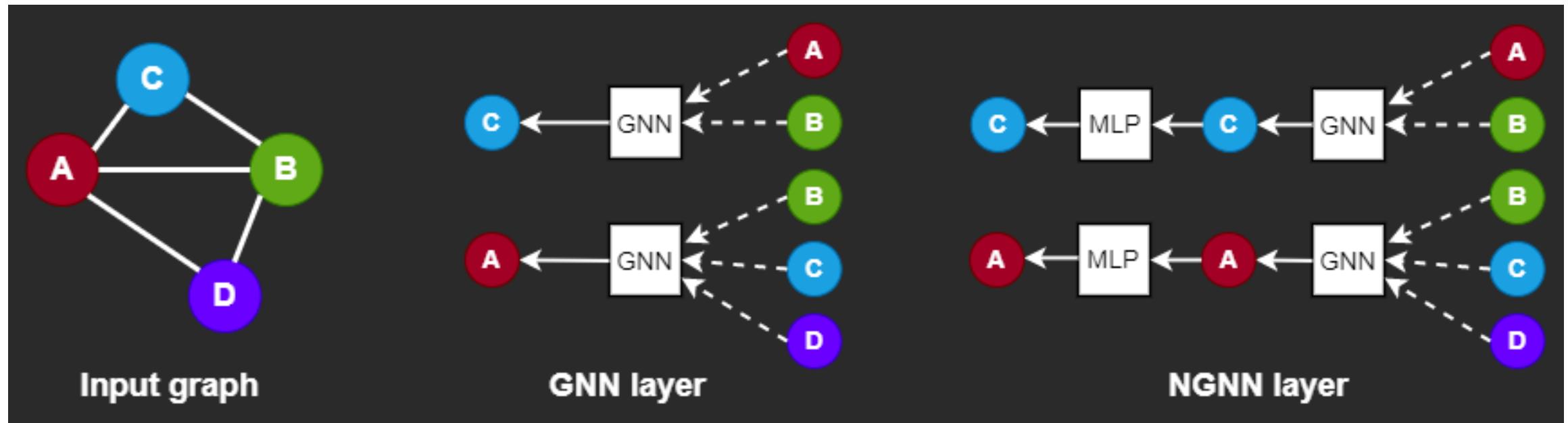
We encounter a trade-off between a low-efficiency model and a model with more depth but less expressivity in terms of node representations



Imagine that we're dealing with a social network graph with thousands of nodes. Some new users just signed in to the platform and subscribed to their friend's profiles. Our goal is to find topic suggestions to fill their feed.

# Over-smoothing in GNN

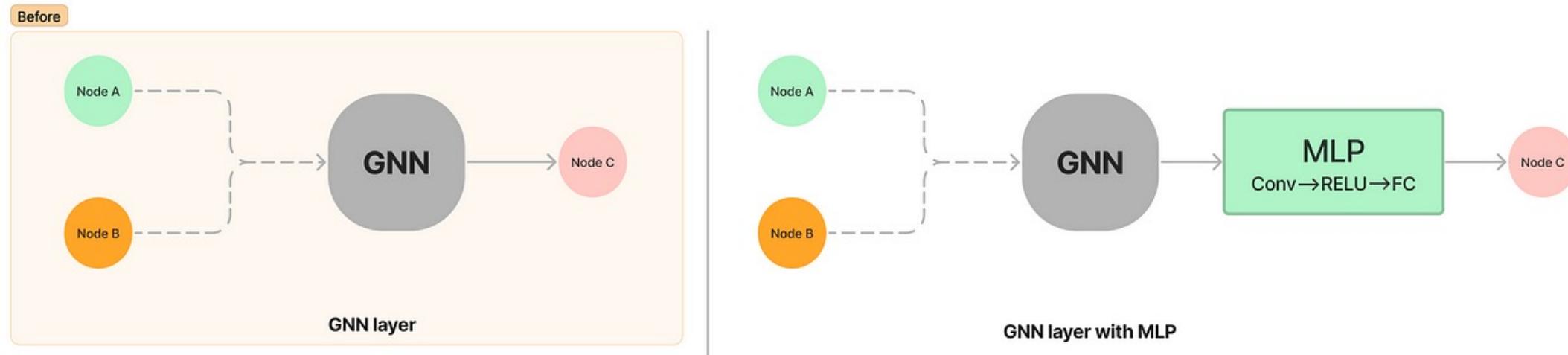
Solution → Inserting nonlinear feedforward neural network layer(s) within each GNN layer.



<https://www.dgl.ai/blog/2022/11/28/ngnn.html>

# Over-smoothing in GNN

Solution → Inserting nonlinear feedforward neural network layer(s) within each GNN layer.



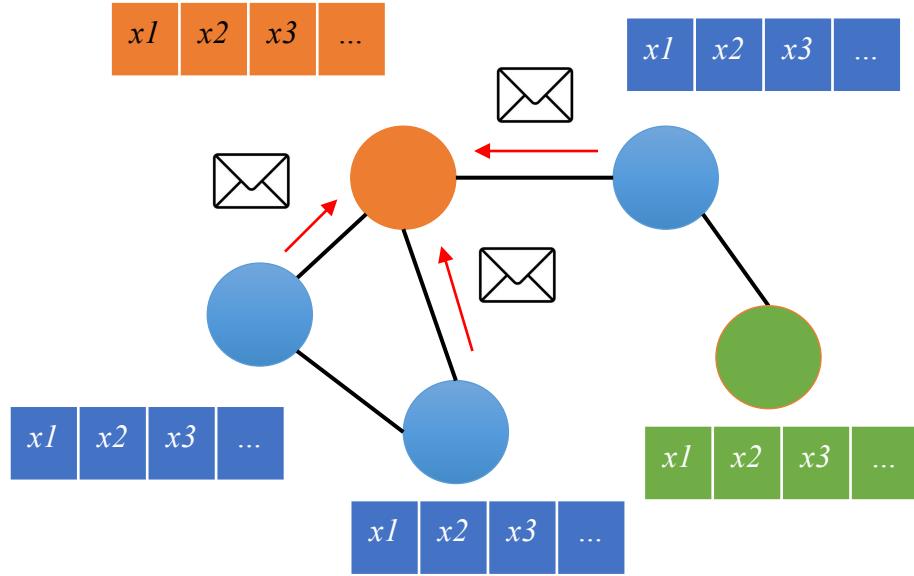
<https://www.dgl.ai/blog/2022/11/28/ngnn.html>

# Over-smoothing in GNN

Solution → Inserting nonlinear feedforward neural network layer(s) within each GNN layer.

Dataset	Metric	Model		Performance
ogbn-proteins	ROC-AUC(%)	GraphSage+Cluster Sampling	Vanilla	$67.45 \pm 1.21$
			+NGNN	<b><math>68.12 \pm 0.96</math></b>
ogbn-products	Accuracy(%)	GraphSage	Vanilla	$78.27 \pm 0.45$
			+NGNN	<b><math>79.88 \pm 0.34</math></b>
		GAT+Neighbor Sampling	Vanilla	$79.23 \pm 0.16$
			+NGNN	<b><math>79.67 \pm 0.09</math></b>
ogbl-collab	hit@50(%)	GCN	Vanilla	$49.52 \pm 0.70$
			+NGNN	<b><math>53.48 \pm 0.40</math></b>
		GraphSage	Vanilla	$51.66 \pm 0.35$
			+NGNN	<b><math>53.59 \pm 0.56</math></b>
ogbl-ppa	hit@100(%)	SEAL-DGCNN	Vanilla	$48.80 \pm 3.16$
			+NGNN	<b><math>59.71 \pm 2.45</math></b>
		GCN	Vanilla	$18.67 \pm 1.32$
			+NGNN	<b><math>36.83 \pm 0.99</math></b>

# Message Passing: Math is Fun



Mean  
Max  
Neural Network  
Recurrent Neural Network

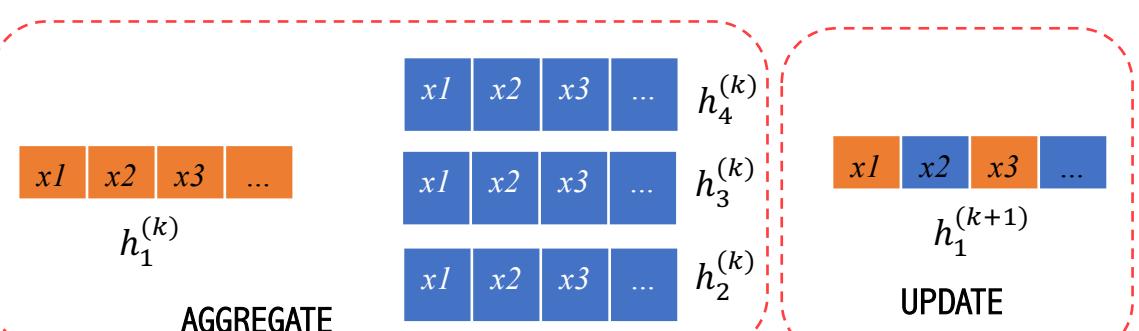
$$h_u^{k+1} = \text{UPDATE}^{(k)} \left( h_u^{k+1}, \text{AGGREGATE}^{(k)} \left( \{h_v^k\}, \forall v \in N(u) \right) \right)$$

$x_1 \ x_2 \ x_3 \ \dots$

$x_1 \ x_2 \ x_3 \ \dots$

$x_1 \ x_2 \ x_3 \ \dots$   
 $x_1 \ x_2 \ x_3 \ \dots$   
 $x_1 \ x_2 \ x_3 \ \dots$

Mean  
Max  
Normalization Sum  
Neural Network



# Message Passing: Variants

*AGGREGATE*  
(permutation invariant)



*UPDATE*



Graph Convolutional Networks,  
Kipf and Welling [2016]

$$\mathbf{h}_v^{(k)} = \sigma \left( \mathbf{W}^{(k)} \sum_{v \in \mathcal{N}(u) \cup \{u\}} \frac{\mathbf{h}_v}{\sqrt{|\mathcal{N}(u)||\mathcal{N}(v)|}} \right)$$

Sum of normalized neighbor embeddings

Multi-Layer-Perceptron as  
Aggregator, Zaheer et al. [2017]

Aggregated message

$$\mathbf{m}_{\mathcal{N}(u)} = \boxed{\text{MLP}_\theta \text{ trainable!}} \left( \sum_{v \in \mathcal{N}(u)} \text{MLP}_\phi(\mathbf{h}_v) \right)$$

Send states through a MLP

Graph Attention Networks,  
Veličković et al. [2017]

$$\mathbf{m}_{\mathcal{N}(u)} = \sum_{v \in \mathcal{N}(u)} \alpha_{u,v} \mathbf{h}_v$$

Attention weights

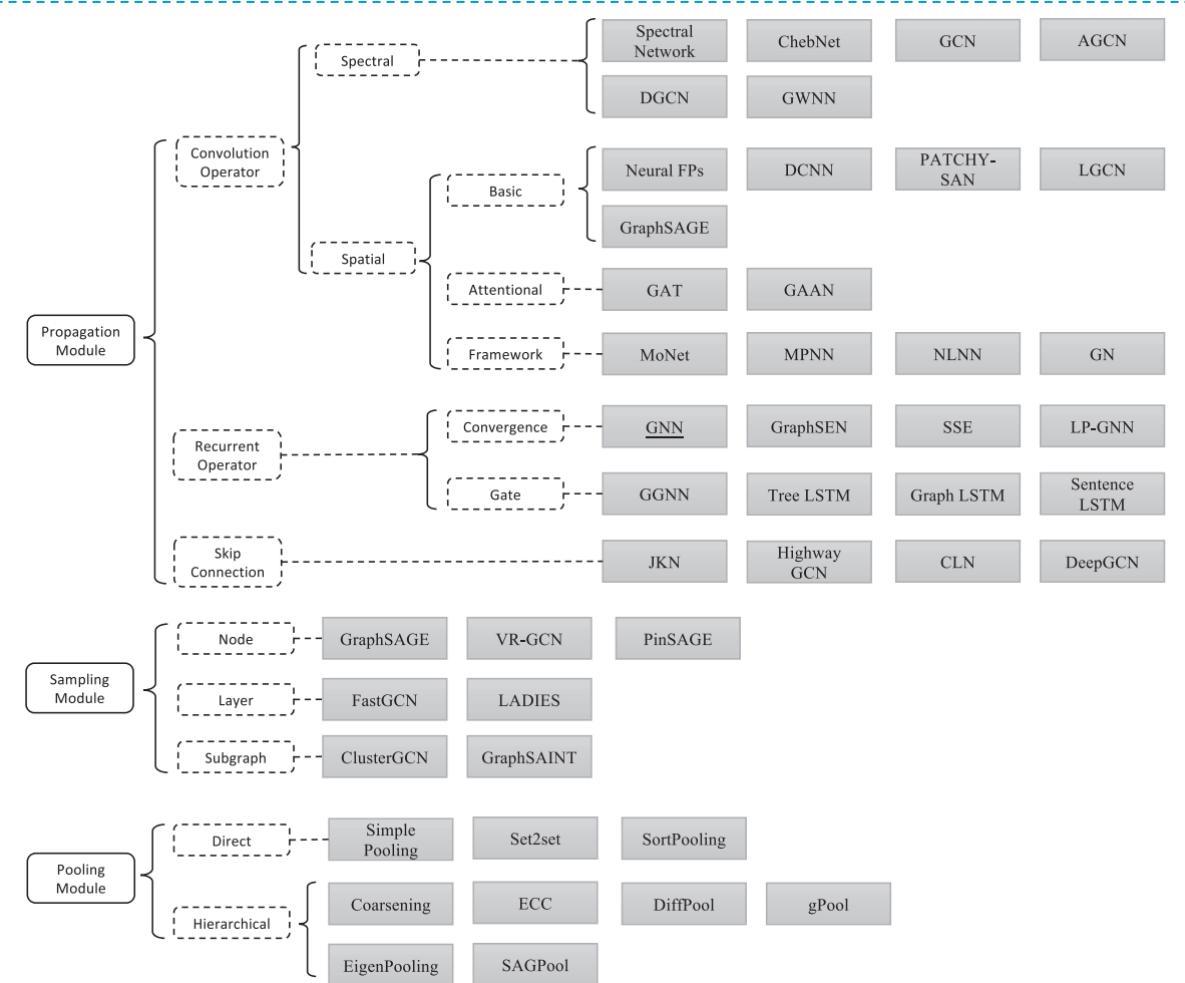
$$\alpha_{u,v} = \frac{\exp(\mathbf{a}^\top [\mathbf{W}\mathbf{h}_u \oplus \mathbf{W}\mathbf{h}_v])}{\sum_{v' \in \mathcal{N}(u)} \exp(\mathbf{a}^\top [\mathbf{W}\mathbf{h}_u \oplus \mathbf{W}\mathbf{h}_{v'}])}$$

Gated Graph Neural Networks,  
Li et al. [2015]

$$\mathbf{h}_u^{(k)} = \text{GRU}(\mathbf{h}_u^{(k-1)}, \mathbf{m}_{\mathcal{N}(u)}^{(k)})$$

Recurrent update of the state

# Message Passing: Variants



Different variants of recurrent operators.

Variant	Aggregator	Updater
GGNN	$\mathbf{h}_{\mathcal{N}_v}^t = \sum_{k \in \mathcal{N}_v} \mathbf{h}_k^{t-1} + \mathbf{b}$	$\mathbf{z}_v^t = \sigma(\mathbf{W}^z \mathbf{h}_{\mathcal{N}_v}^t + \mathbf{U}^z \mathbf{h}_v^{t-1})$ $\mathbf{r}_v^t = \sigma(\mathbf{W}^r \mathbf{h}_{\mathcal{N}_v}^t + \mathbf{U}^r \mathbf{h}_v^{t-1})$ $\tilde{\mathbf{h}}_v^t = \tanh(\mathbf{W}^h \mathbf{h}_{\mathcal{N}_v}^t + \mathbf{U}^h (\mathbf{r}_v^t \odot \mathbf{h}_v^{t-1}))$ $\mathbf{h}_v^t = (1 - \mathbf{z}_v^t) \odot \mathbf{h}_v^{t-1} + \mathbf{z}_v^t \odot \tilde{\mathbf{h}}_v^t$
Tree LSTM (Child sum)	$\mathbf{h}_{\mathcal{N}_v}^t = \sum_{k \in \mathcal{N}_v} \mathbf{U}^i \mathbf{h}_k^{t-1}$ $\mathbf{h}_{\mathcal{N}_v, k}^f = \mathbf{U}^f \mathbf{h}_k^{t-1}$ $\mathbf{h}_{\mathcal{N}_v}^{to} = \sum_{k \in \mathcal{N}_v} \mathbf{U}^o \mathbf{h}_k^{t-1}$ $\mathbf{h}_{\mathcal{N}_v}^u = \sum_{k \in \mathcal{N}_v} \mathbf{U}^u \mathbf{h}_k^{t-1}$	$\mathbf{i}_v^t = \sigma(\mathbf{W}^i \mathbf{x}_v^t + \mathbf{h}_{\mathcal{N}_v}^t + \mathbf{b}^i)$ $\mathbf{f}_k^t = \sigma(\mathbf{W}^f \mathbf{x}_v^t + \mathbf{h}_{\mathcal{N}_v, k}^f + \mathbf{b}^f)$ $\mathbf{o}_v^t = \sigma(\mathbf{W}^o \mathbf{x}_v^t + \mathbf{h}_{\mathcal{N}_v}^{to} + \mathbf{b}^o)$ $\mathbf{u}_v^t = \tanh(\mathbf{W}^u \mathbf{x}_v^t + \mathbf{h}_{\mathcal{N}_v}^u + \mathbf{b}^u)$ $\mathbf{c}_v^t = \mathbf{i}_v^t \odot \mathbf{u}_v^t + \sum_{k \in \mathcal{N}_v} \mathbf{f}_k^t \odot \mathbf{c}_k^{t-1}$ $\mathbf{h}_v^t = \mathbf{o}_v^t \odot \tanh(\mathbf{c}_v^t)$
Tree LSTM (N-ary)	$\mathbf{h}_{\mathcal{N}_v}^t = \sum_{l=1}^K \mathbf{U}_l^i \mathbf{h}_{vl}^{t-1}$ $\mathbf{h}_{\mathcal{N}_v, k}^f = \sum_{l=1}^K \mathbf{U}_{kl}^f \mathbf{h}_{vl}^{t-1}$ $\mathbf{h}_{\mathcal{N}_v}^{to} = \sum_{l=1}^K \mathbf{U}_l^o \mathbf{h}_{vl}^{t-1}$ $\mathbf{h}_{\mathcal{N}_v}^u = \sum_{l=1}^K \mathbf{U}_l^u \mathbf{h}_{vl}^{t-1}$	
Graph LSTM in (Peng et al., 2017)	$\mathbf{h}_{\mathcal{N}_v}^t = \sum_{k \in \mathcal{N}_v} \mathbf{U}_{m(v,k)}^i \mathbf{h}_k^{t-1}$ $\mathbf{h}_{\mathcal{N}_v, k}^f = \mathbf{U}_{m(v,k)}^f \mathbf{h}_k^{t-1}$ $\mathbf{h}_{\mathcal{N}_v}^{to} = \sum_{k \in \mathcal{N}_v} \mathbf{U}_{m(v,k)}^o \mathbf{h}_k^{t-1}$ $\mathbf{h}_{\mathcal{N}_v}^u = \sum_{k \in \mathcal{N}_v} \mathbf{U}_{m(v,k)}^u \mathbf{h}_k^{t-1}$	

## Graph neural networks: A review of methods and applications

Jie Zhou <sup>a,1</sup>, Ganqu Cui <sup>a,1</sup>, Shengding Hu <sup>a</sup>, Zhengyan Zhang <sup>a</sup>, Cheng Yang <sup>b</sup>, Zhiyuan Liu <sup>a,\*</sup>, Lifeng Wang <sup>c</sup>, Changcheng Li <sup>c</sup>, Maosong Sun <sup>a</sup>

<sup>a</sup> Department of Computer Science and Technology, Tsinghua University, Beijing, China

<sup>b</sup> School of Computer Science, Beijing University of Posts and Telecommunications, China

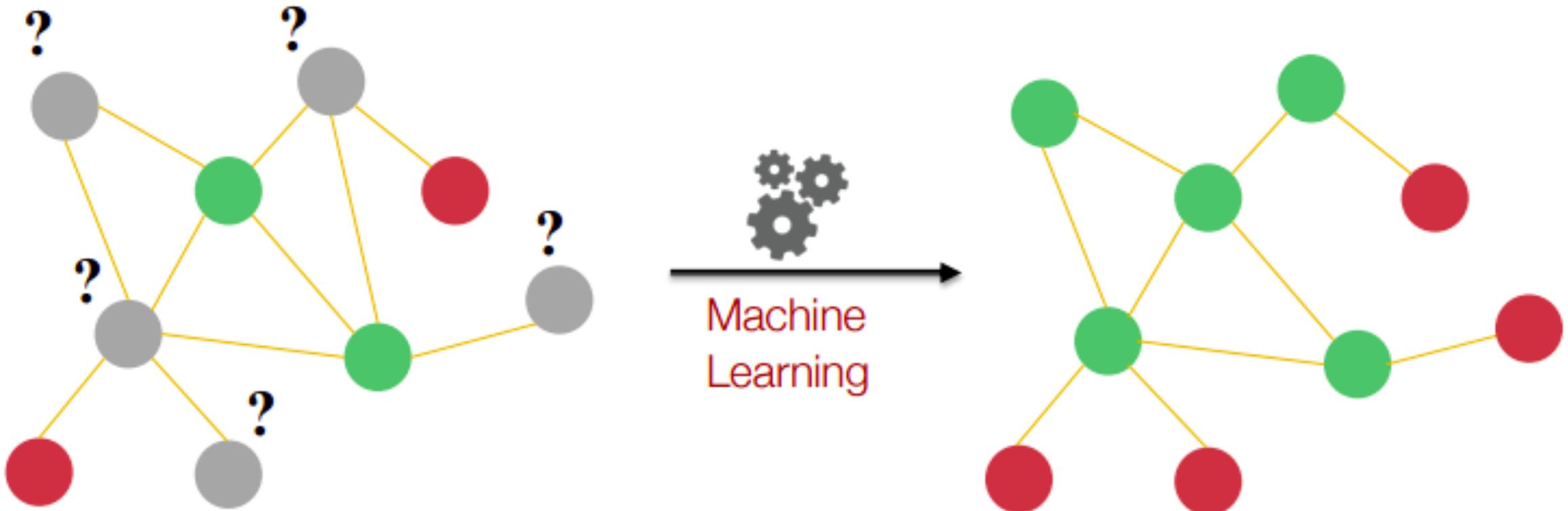
<sup>c</sup> Tencent Incorporation, Shenzhen, China

QUIZ TIME

# Outline

- **Objective**
- **Introduction to Graph Data**
- **Graph Data with Neural Network**
- **Node Classification Problem: Cora Citation Dataset**
- **Summary**

# Node Classification Problem





## Knowledge Graphs and Node Classification

We have one large graph and not many individual graphs (like molecules)

We infer on unlabeled nodes in this large graph and hence perform node-level predictions

--> We have to use different nodes of the graph depending on what we want to do



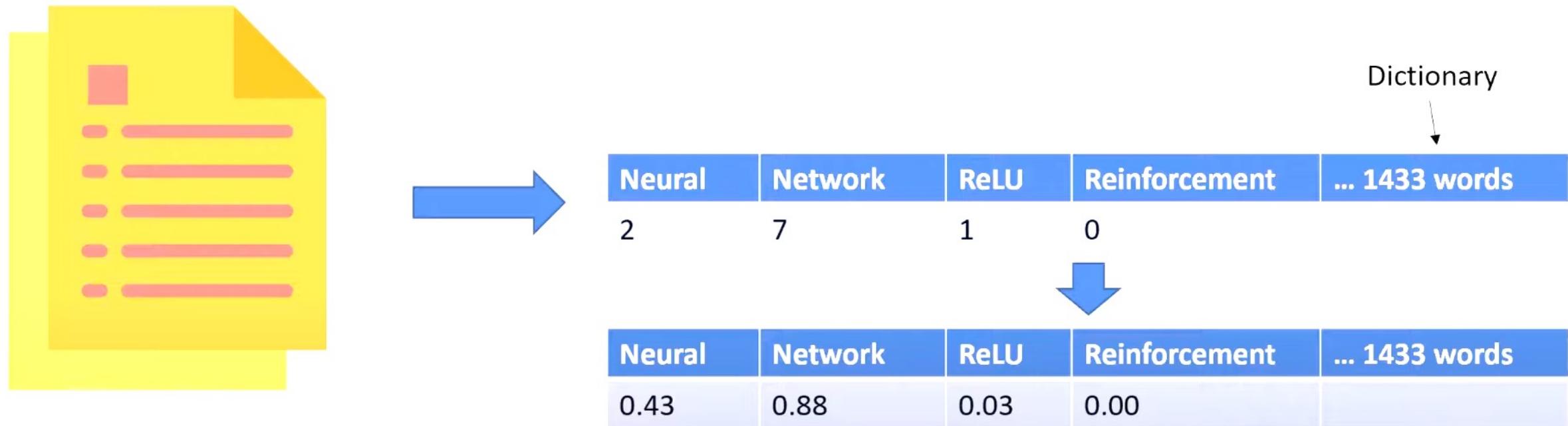
## Dataset Introduction: Cora Citation Dataset in PyTorch Geometric

The Cora dataset consists of 2708 scientific publications classified into one of seven classes.

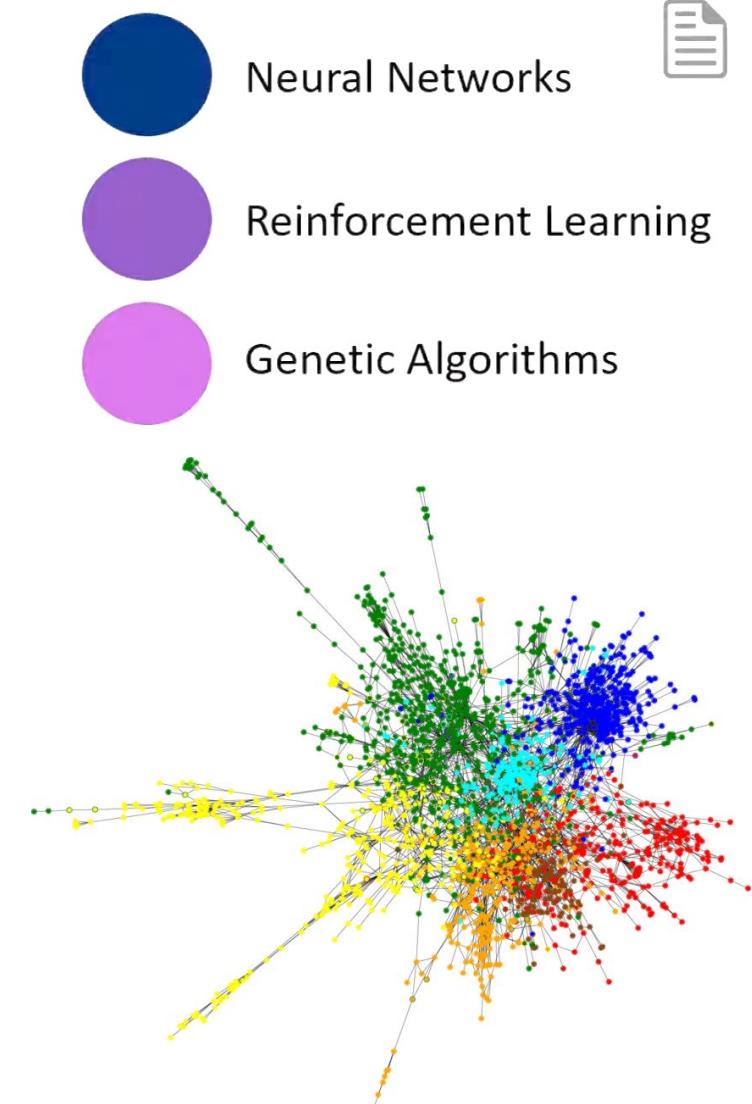
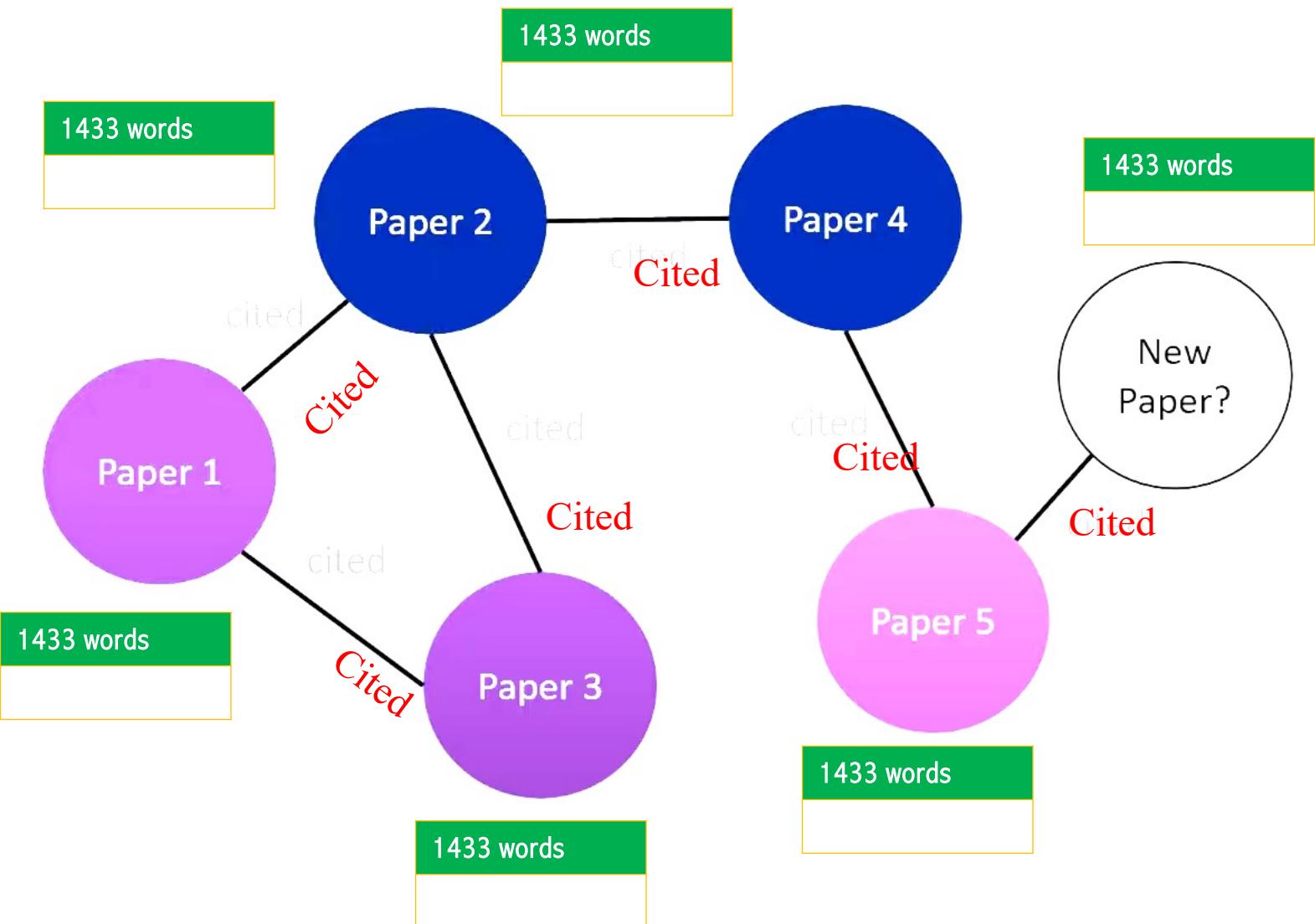
Each publication in the dataset is described by a 0/1-valued word vector indicating the absence/presence of the corresponding word from the dictionary.

- The dictionary consists of 1433 unique words.
- Nodes = Publications (Papers, Books ...)
- Edges = Citations Node Features = word vectors
- 7 Labels = Publication
- type e.g. Neural\_Networks, Rule\_Learning, Reinforcement\_Learning, Probabilistic\_Methods...

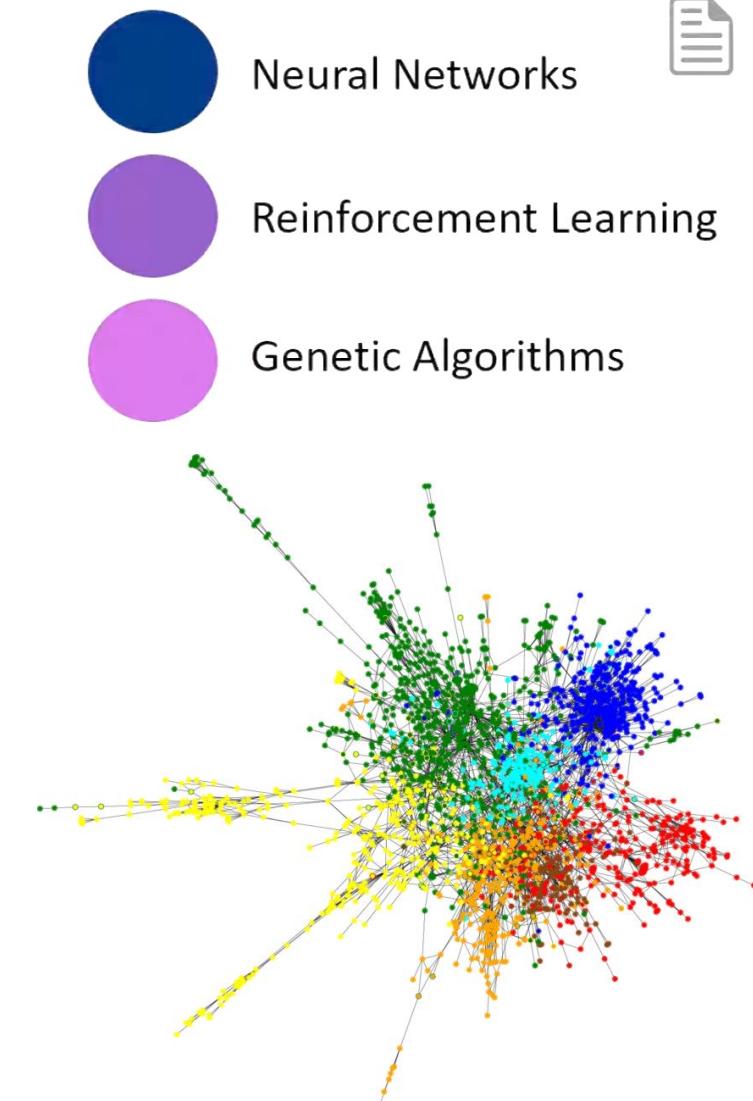
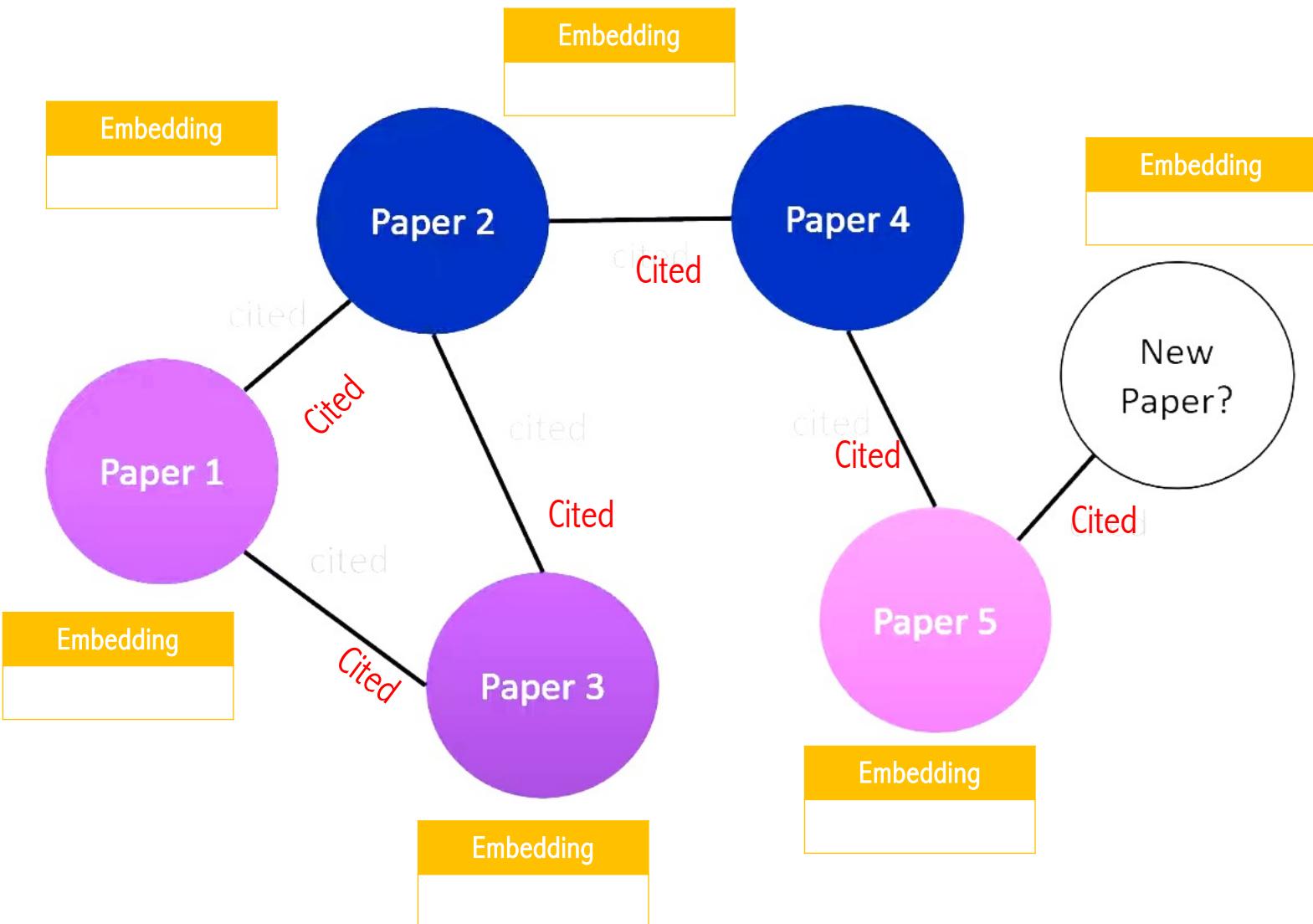
# BoW Representation



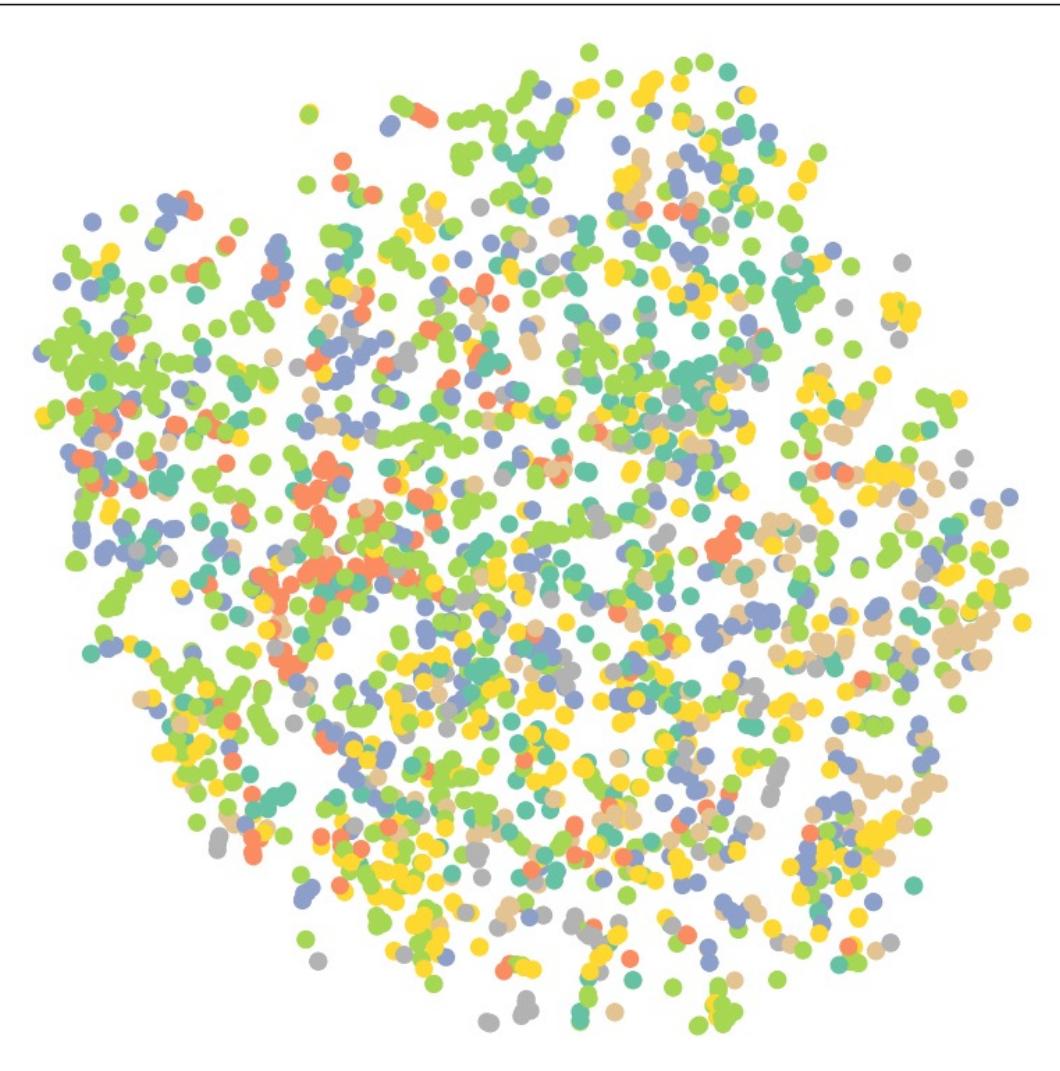
# Cora Citation Dataset



# Cora Citation Dataset



# Visualize the node embeddings of the untrained GCN network



```
model = GCN(hidden_channels=16)  
model.eval()  
  
out = model(data.x, data.edge_index)  
visualize(out, color=data.y)
```



## Install Pytorch Geometric

```
# Check CUDA Version
!python -c "import torch; print(torch.version.cuda)"

# Add this in a Google Colab cell to install the correct version of Pytorch Geometric.
import torch

def format_pytorch_version(version):
    return version.split('+')[0]

TORCH_version = torch.__version__
TORCH = format_pytorch_version(TORCH_version)

def format_cuda_version(version):
    return 'cu' + version.replace('.', '')

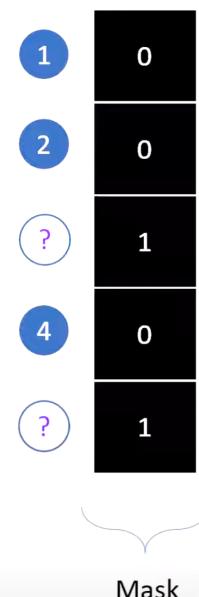
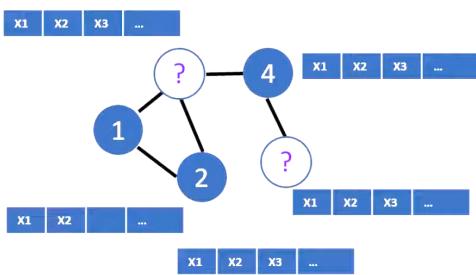
CUDA_version = torch.version.cuda
CUDA = format_cuda_version(CUDA_version)

!pip install torch-scatter      -f https://pytorch-geometric.com/whl/torch-{TORCH}+{CUDA}.html
!pip install torch-sparse       -f https://pytorch-geometric.com/whl/torch-{TORCH}+{CUDA}.html
!pip install torch-cluster      -f https://pytorch-geometric.com/whl/torch-{TORCH}+{CUDA}.html
!pip install torch-spline-conv  -f https://pytorch-geometric.com/whl/torch-{TORCH}+{CUDA}.html
!pip install torch-geometric
```

# GNN: Node Classification



## Load Cora Dataset



```
from torch_geometric.datasets import Planetoid
from torch_geometric.transforms import NormalizeFeatures

dataset = Planetoid(root='data/Planetoid', name='Cora', transform=NormalizeFeatures())

# Get some basic info about the dataset
print(f'Number of graphs: {len(dataset)}')
print(f'Number of features: {dataset.num_features}')
print(f'Number of classes: {dataset.num_classes}')
print(50*'=')

# There is only one graph in the dataset, use it as new data object
data = dataset[0]

# Gather some statistics about the graph.
print(data)
print(f'Number of nodes: {data.num_nodes}')
print(f'Number of edges: {data.num_edges}')
print(f'Number of training nodes: {data.train_mask.sum()}')
print(f'Training node label rate: {int(data.train_mask.sum()) / data.num_nodes:.2f}')
print(f'Is undirected: {data.is_undirected()}'
```

```
Number of graphs: 1
Number of features: 1433
Number of classes: 7
=====
Data(x=[2708, 1433], edge_index=[2, 10556], y=[2708], train_mask=[2708], val_mask=[2708], test_mask=[2708])
Number of nodes: 2708
Number of edges: 10556
Number of training nodes: 140
Training node label rate: 0.05
Is undirected: True
```

train\_mask=[2708], val\_mask=[2708], test\_mask=[2708])

# GNN: Node Classification MLP



## Define MLP

Here, we first reduce the 1433-dimensional feature vector to a low-dimensional embedding (hidden\_channels=16), while the second linear layer acts as a classifier that should map each low-dimensional node embedding to one of the 7 classes.

```
import torch
from torch.nn import Linear
import torch.nn.functional as F

class MLP(torch.nn.Module):
    def __init__(self, hidden_channels):
        super().__init__()
        torch.manual_seed(12345)
        self.lin1 = Linear(dataset.num_features, hidden_channels)
        self.lin2 = Linear(hidden_channels, dataset.num_classes)

    def forward(self, x):
        x = self.lin1(x)
        x = x.relu()
        x = F.dropout(x, p=0.5, training=self.training)
        x = self.lin2(x)
        return x

model = MLP(hidden_channels=16)
print(model)
```

```
test_acc = test()
print(f'Test Accuracy: {test_acc:.4f}')
```

Test Accuracy: 0.5900

# GNN: Node Classification MLP



## How to Train

Here, we first reduce the 1433-dimensional feature vector to a low-dimensional embedding (hidden\_channels=16), while the second linear layer acts as a classifier that should map each low-dimensional node embedding to one of the 7 classes.

```
test_acc = test()  
print(f'Test Accuracy: {test_acc:.4f}')
```

Test Accuracy: 0.5900

```
from IPython.display import Javascript # Restrict height of output cell.  
display(Javascript("google.colab.output.setIframeHeight(0, true, {maxHeight: 300})"))  
  
model = MLP(hidden_channels=16)  
criterion = torch.nn.CrossEntropyLoss() # Define loss criterion.  
optimizer = torch.optim.Adam(model.parameters(), lr=0.01, weight_decay=5e-4) # Define optimizer.  
  
def train():  
    model.train()  
    optimizer.zero_grad() # Clear gradients.  
    out = model(data.x) # Perform a single forward pass.  
    loss = criterion(out[data.train_mask], data.y[data.train_mask]) # Compute the loss solely based on the  
    loss.backward() # Derive gradients.  
    optimizer.step() # Update parameters based on gradients.  
    return loss  
  
def test():  
    model.eval()  
    out = model(data.x)  
    pred = out.argmax(dim=1) # Use the class with highest probability.  
    test_correct = pred[data.test_mask] == data.y[data.test_mask] # Check against ground-truth labels.  
    test_acc = int(test_correct.sum()) / int(data.test_mask.sum()) # Derive ratio of correct predictions.  
    return test_acc
```

# GNN: Node Classification GCN



Define GCN

```
class GCN(torch.nn.Module):
    def __init__(self, hidden_channels):
        super().__init__()
        torch.manual_seed(1234567)
        self.conv1 = GCNConv(dataset.num_features, hidden_channels)
        self.conv2 = GCNConv(hidden_channels, dataset.num_classes)

    def forward(self, x, edge_index):
        x = self.conv1(x, edge_index)
        x = x.relu()
        x = F.dropout(x, p=0.5, training=self.training)
        x = self.conv2(x, edge_index)
        return x

model = GCN(hidden_channels=16)
print(model)
```

GCN(
 (conv1): GCNConv(1433, 16)
 (conv2): GCNConv(16, 7)
)

we will use one of the most simple GNN operators, the **GCN layer** ([Kipf et al. \(2017\)](#)), which is defined as

$$\mathbf{x}_v^{(\ell+1)} = \mathbf{W}^{(\ell+1)} \sum_{w \in \mathcal{N}(v) \cup \{v\}} \frac{1}{c_{w,v}} \cdot \mathbf{x}_w^{(\ell)}$$

# GNN: Node Classification GCN



## Define GCN

Dropout is only applied in the training step, but not for predictions

We have 2 Message Passing Layers and one Linear output layer

We use the softmax function for the classification problem

The output of the model are 7 probabilities, one for each class

```
class GCN(torch.nn.Module):
    def __init__(self, hidden_channels):
        super(GCN, self).__init__()
        torch.manual_seed(42)

        # Initialize the layers
        self.conv1 = GCNConv(dataset.num_features, hidden_channels)
        self.conv2 = GCNConv(hidden_channels, hidden_channels)
        self.out = Linear(hidden_channels, dataset.num_classes)

    def forward(self, x, edge_index):
        # First Message Passing Layer (Transformation)
        x = self.conv1(x, edge_index)
        x = x.relu()
        x = F.dropout(x, p=0.5, training=self.training)

        # Second Message Passing Layer
        x = self.conv2(x, edge_index)
        x = x.relu()
        x = F.dropout(x, p=0.5, training=self.training)

        # Output layer
        x = F.softmax(self.out(x), dim=1)
        return x
```

we will use one of the most simple GNN operators, the **GCN layer** ([Kipf et al. \(2017\)](#)), which is defined as

$$\mathbf{x}_v^{(\ell+1)} = \mathbf{W}^{(\ell+1)} \sum_{w \in \mathcal{N}(v) \cup \{v\}} \frac{1}{c_{w,v}} \cdot \mathbf{x}_w^{(\ell)}$$

# GNN: Node Classification GCN

```
from IPython.display import Javascript # Restrict height of output cell.
display(Javascript('''google.colab.output.setIframeHeight(0, true, {maxHeight: 300})'''))

model = GCN(hidden_channels=16)
optimizer = torch.optim.Adam(model.parameters(), lr=0.01, weight_decay=5e-4)
criterion = torch.nn.CrossEntropyLoss()

def train():
    model.train()
    optimizer.zero_grad() # Clear gradients.
    out = model(data.x, data.edge_index) # Perform a single forward pass.
    loss = criterion(out[data.train_mask], data.y[data.train_mask]) # Compute the loss solely based on the training set.
    loss.backward() # Derive gradients.
    optimizer.step() # Update parameters based on gradients.
    return loss

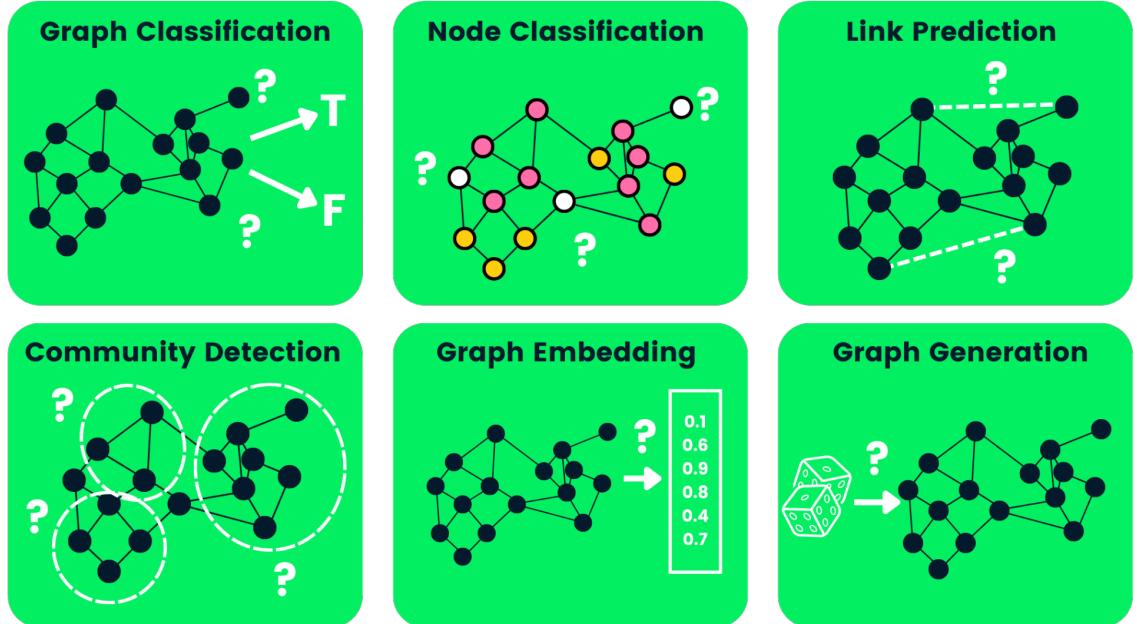
def test():
    model.eval()
    out = model(data.x, data.edge_index)
    pred = out.argmax(dim=1) # Use the class with highest probability.
    test_correct = pred[data.test_mask] == data.y[data.test_mask] # Check against ground-truth labels.
    test_acc = int(test_correct.sum()) / int(data.test_mask.sum()) # Derive ratio of correct predictions.
    return test_acc

for epoch in range(1, 101):
    loss = train()
    print(f'Epoch: {epoch:03d}, Loss: {loss:.4f}')
```

```
▶ test_acc = test()
print(f'Test Accuracy: {test_acc:.4f}')

Test Accuracy: 0.8150
```

# Summary



- 
- A vertical stack of four colored triangles, each containing a number from 1 to 4. To the right of the stack is a list of four bullet points corresponding to the numbers.
- What is Graph Data Around Us
  - Understand Graph Neural Network
  - Understand Graph Convolutional Neural Network
  - Node Classification with Cora Citation Dataset

