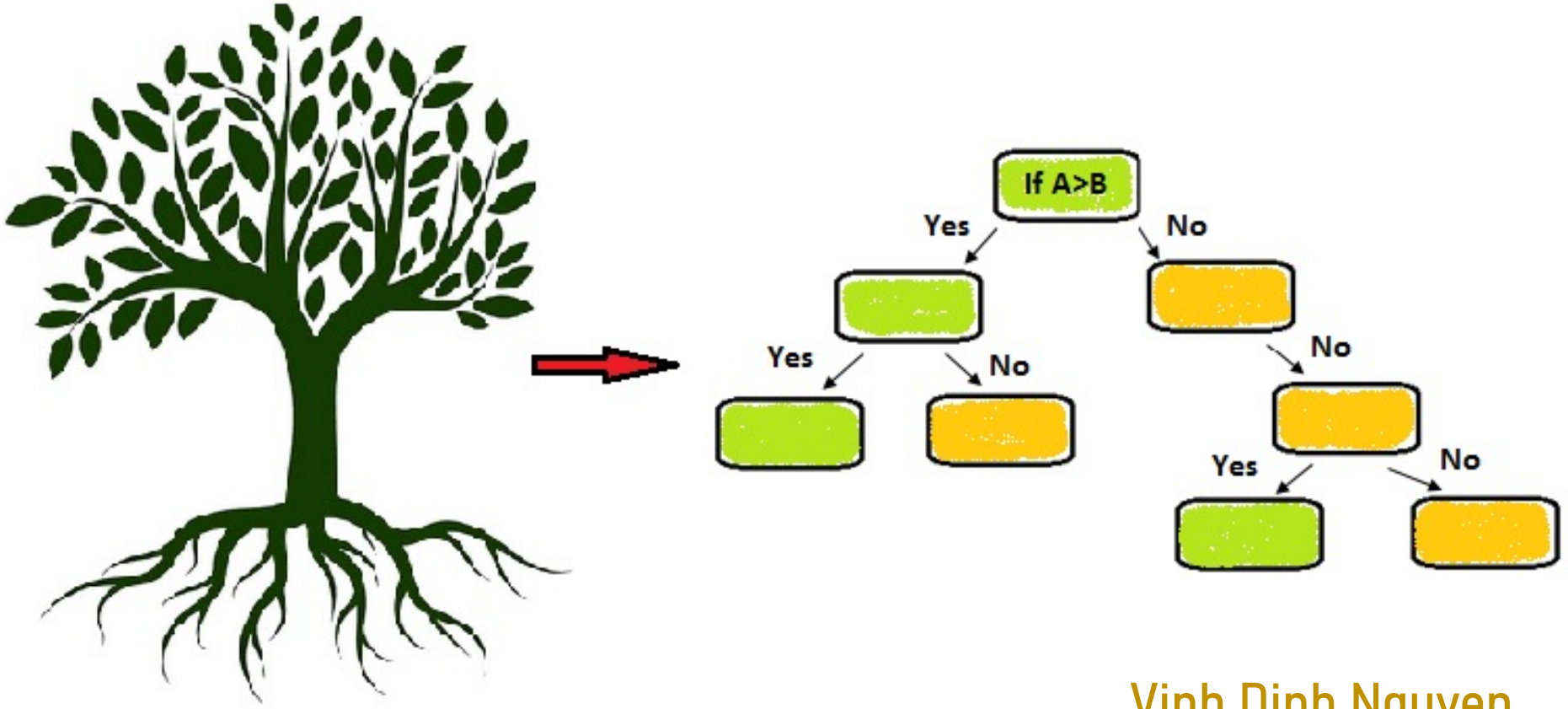
# Decision Tree for Classification




Vinh Dinh Nguyen
PhD in Computer Science

# Outline

- Introduction to Tree
- Decision Tree
- Decision Tree with Gini
- Decision Tree with Entropy
- Several Examples

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- Decision Tree with Entropy
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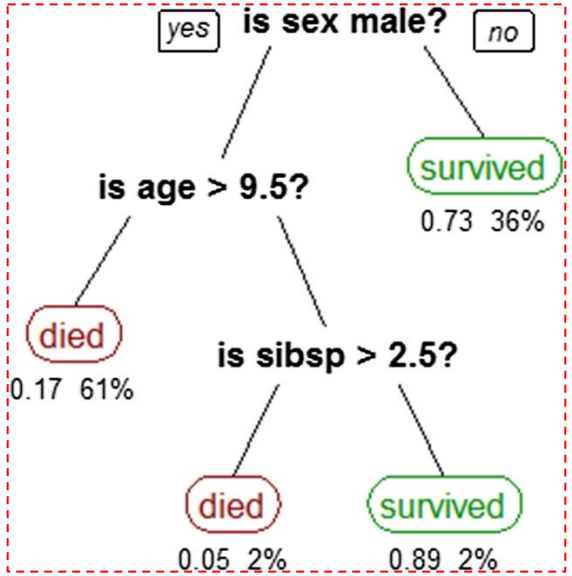
# Sample Decision Tree

A decision tree predicting the survival of a passenger on the Titanic using the sex, age, and siblings or spouses onboard attributes

| | PassengerId | Survived | Pclass | Name | Sex | Age | SibSp | Parch | Ticket | Fare | Cabin | Embarked |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 0 | 3 | Braund, Mr. Owen Harris | male | 22.0 | 1 | 0 | A/5 21171 | 7.2500 | NaN | S |
| 1 | 2 | 1 | 1 | Cumings, Mrs. John Bradley (Florence Briggs Th... | female | 38.0 | 1 | 0 | PC 17599 | 71.2833 | C85 | C |
| 2 | 3 | 1 | 3 | Heikkinen, Miss. Laina | female | 26.0 | 0 | 0 | STON/O2. 3101282 | 7.9250 | NaN | S |
| 3 | 4 | 1 | 1 | Futrelle, Mrs. Jacques Heath (Lily May Peel) | female | 35.0 | 1 | 0 | 113803 | 53.1000 | C123 | S |
| 4 | 5 | 0 | 3 | Allen, Mr. William Henry | male | 35.0 | 0 | 0 | 373450 | 8.0500 | NaN | S |

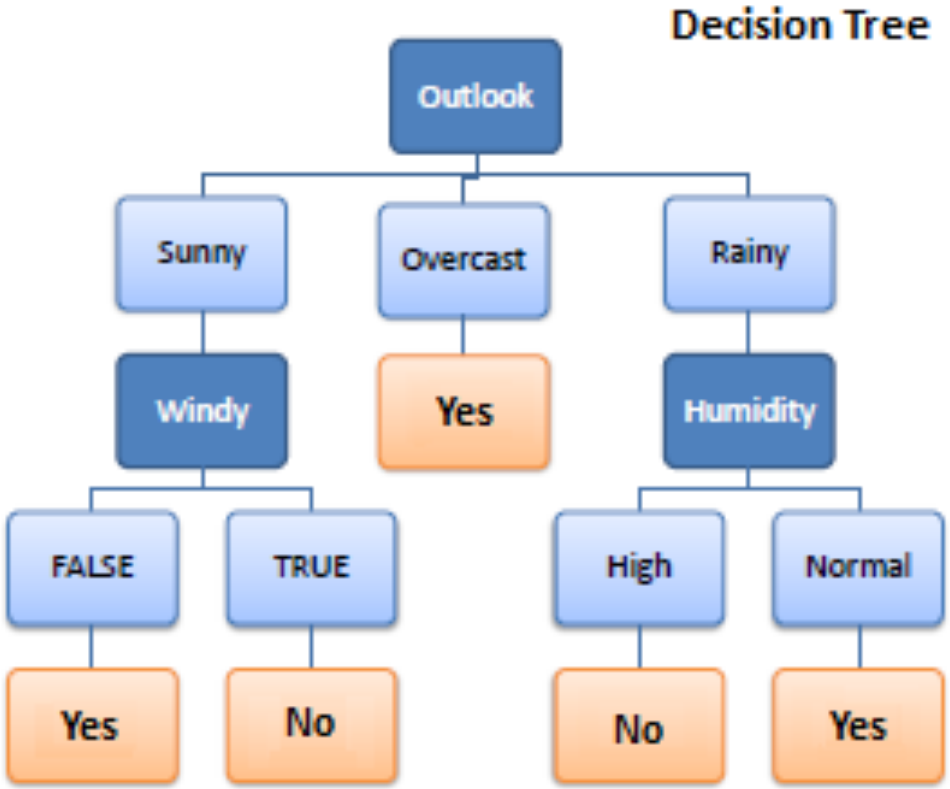



*Milborrow S (2011) Titanic decision tree. https://commons.wikimedia. org/wiki/File:CART_tree_titanic_survivors.png. Accessed Mar 2019*

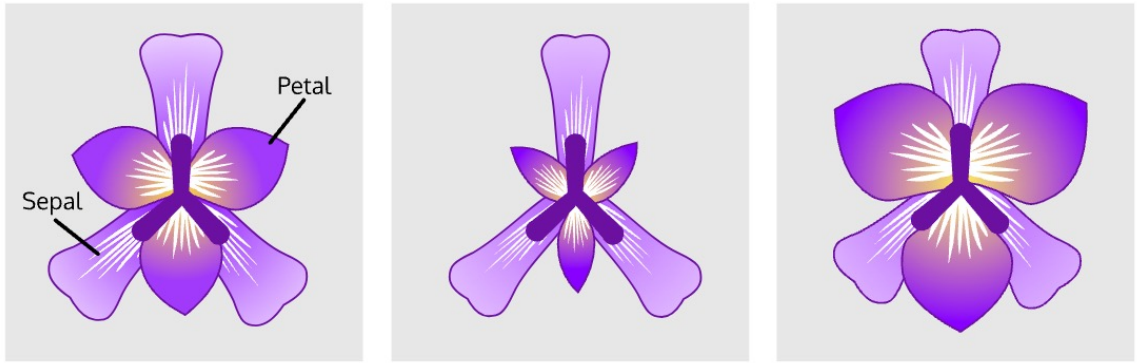
# Sample Decision Tree

Predictors | Target

| Outlook | Temp. | Humidity | Windy | Play Golf |
|---|---|---|---|---|
| Rainy | Hot | High | False | No |
| Rainy | Hot | High | True | No |
| Overcast | Hot | High | False | Yes |
| Sunny | Mild | High | False | Yes |
| Sunny | Cool | Normal | False | Yes |
| Sunny | Cool | Normal | True | No |
| Overcast | Cool | Normal | True | Yes |
| Rainy | Mild | High | False | No |
| Rainy | Cool | Normal | False | Yes |
| Sunny | Mild | Normal | False | Yes |
| Rainy | Mild | Normal | True | Yes |
| Overcast | Mild | High | True | Yes |
| Overcast | Hot | Normal | False | Yes |
| Sunny | Mild | High | True | No |

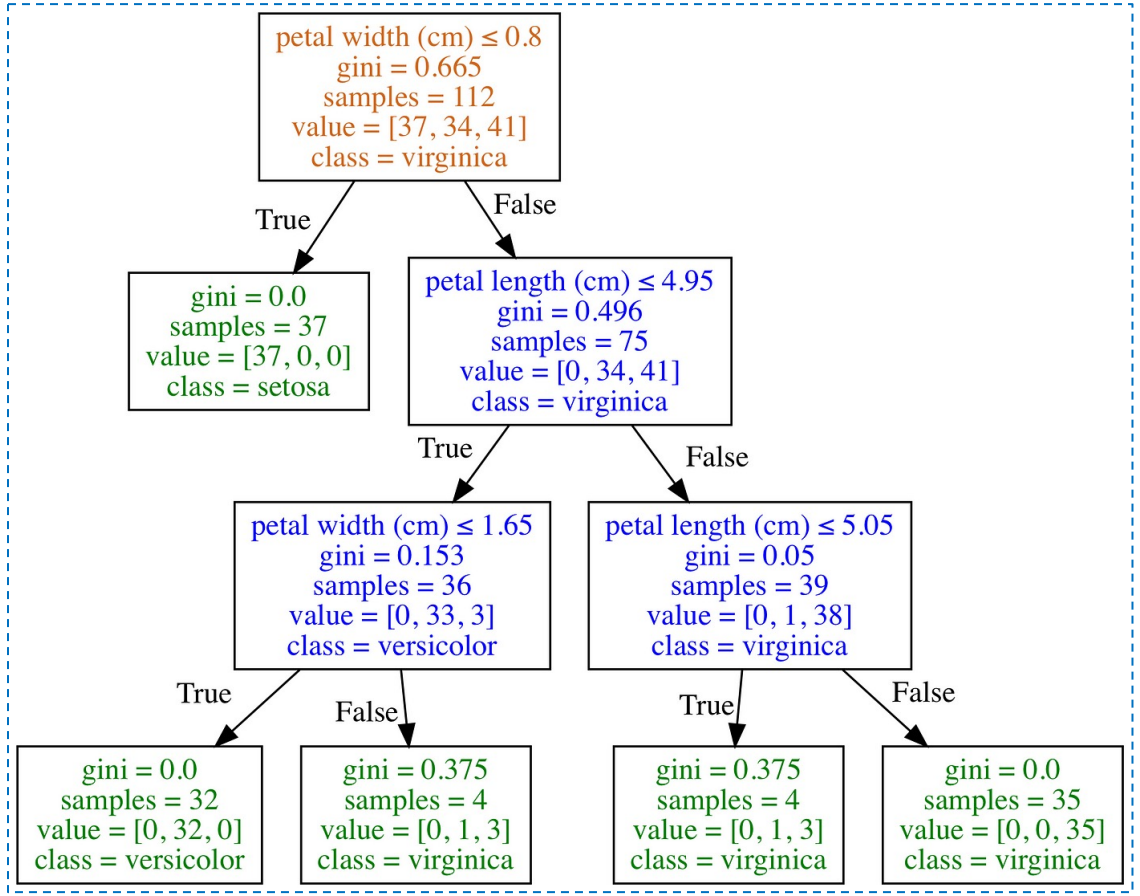


# Sample Decision Tree

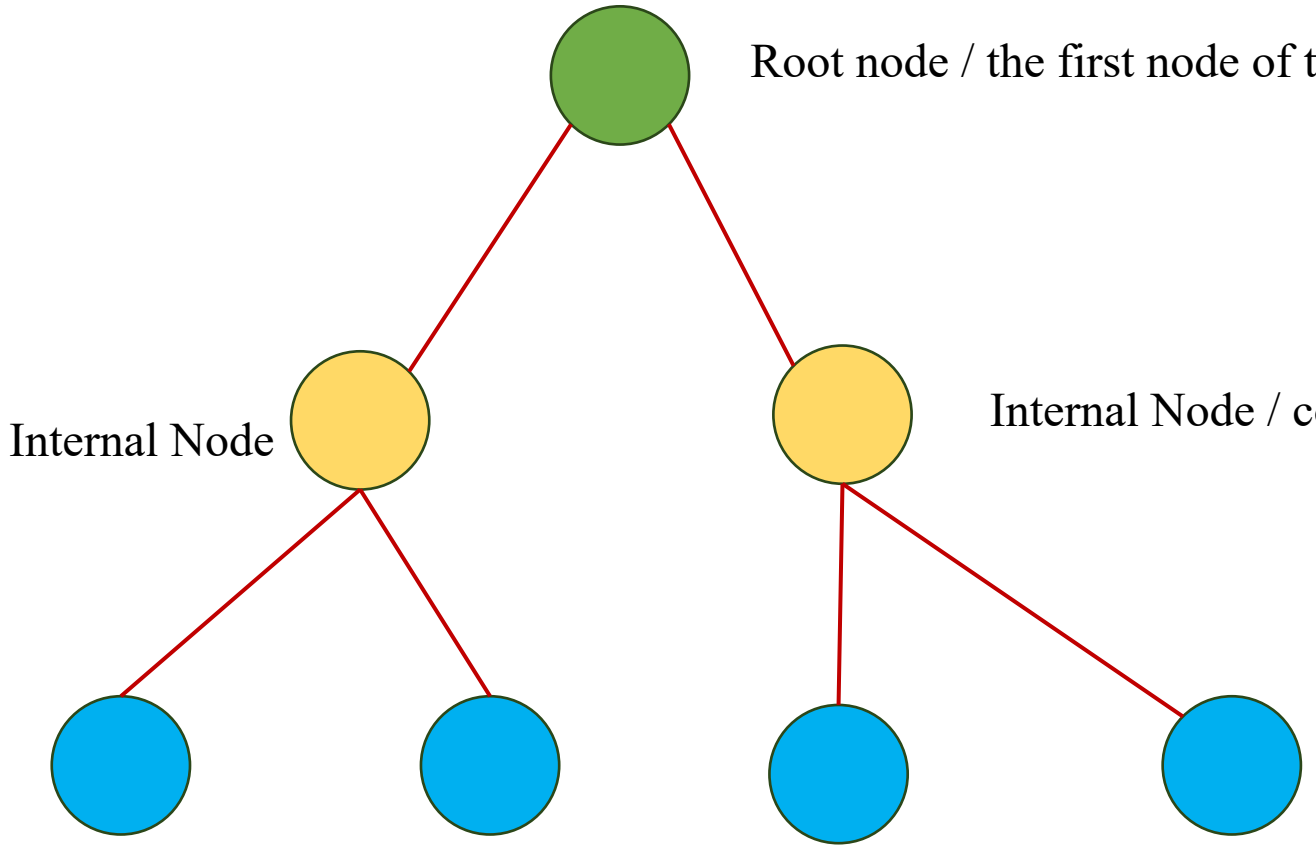



Iris Versicolor | Iris Setosa | Iris Virginica

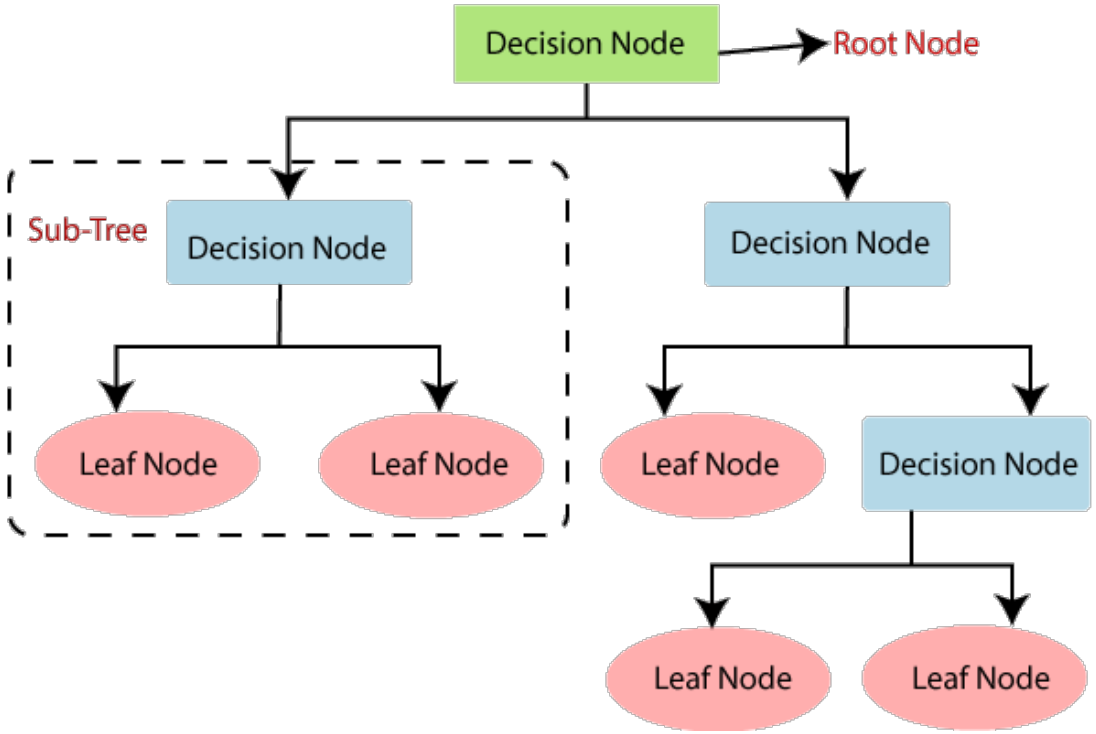
| | sepal_length | sepal_width | petal_length | petal_width | species |
|---|---|---|---|---|---|
| 0 | 5.1 | 3.5 | 1.4 | 0.2 | Iris-setosa |
| 1 | 4.9 | 3.0 | 1.4 | 0.2 | Iris-setosa |
| 2 | 4.7 | 3.2 | 1.3 | 0.2 | Iris-setosa |
| 3 | 4.6 | 3.1 | 1.5 | 0.2 | Iris-setosa |
| 4 | 5.0 | 3.6 | 1.4 | 0.2 | Iris-setosa |

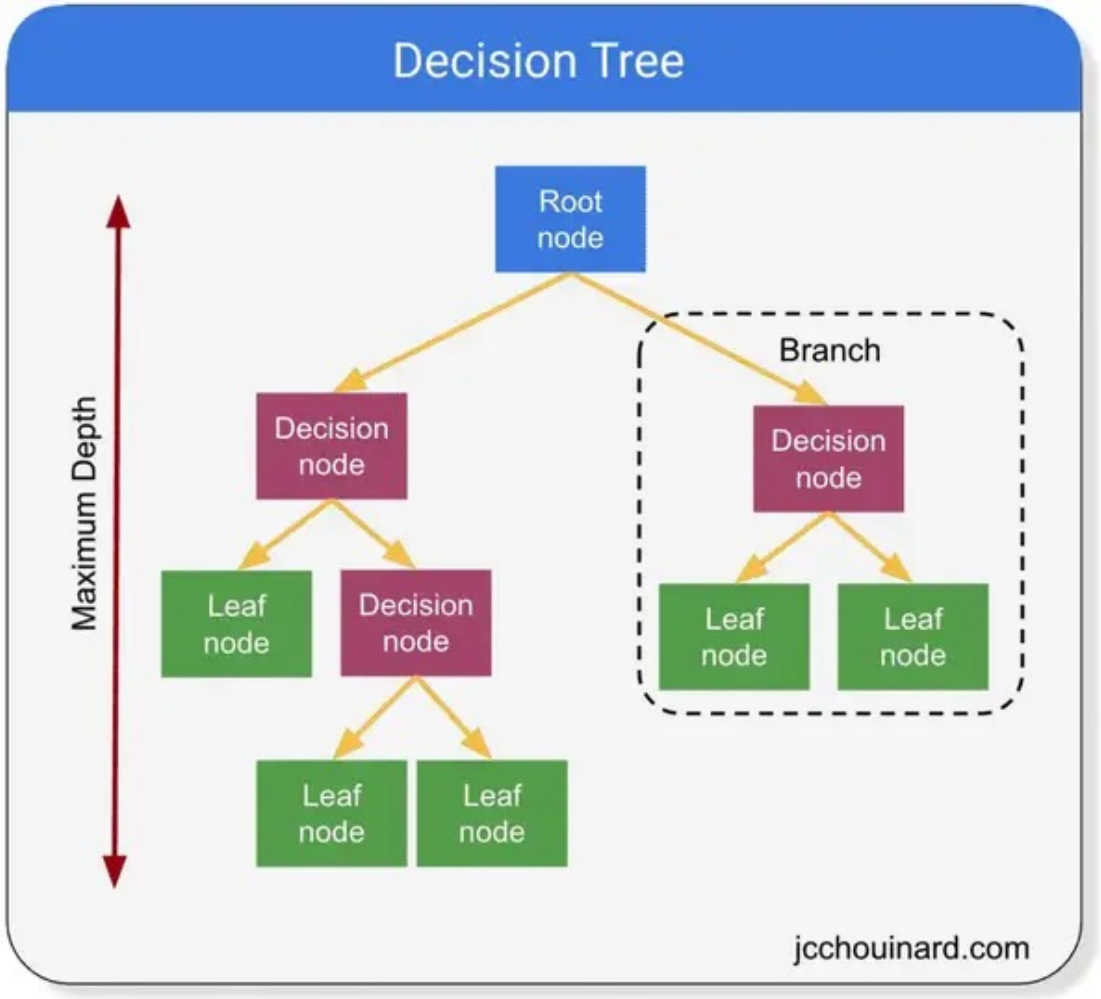


# Basic Tree Terminology



# Basic Tree Terminology



# Outline

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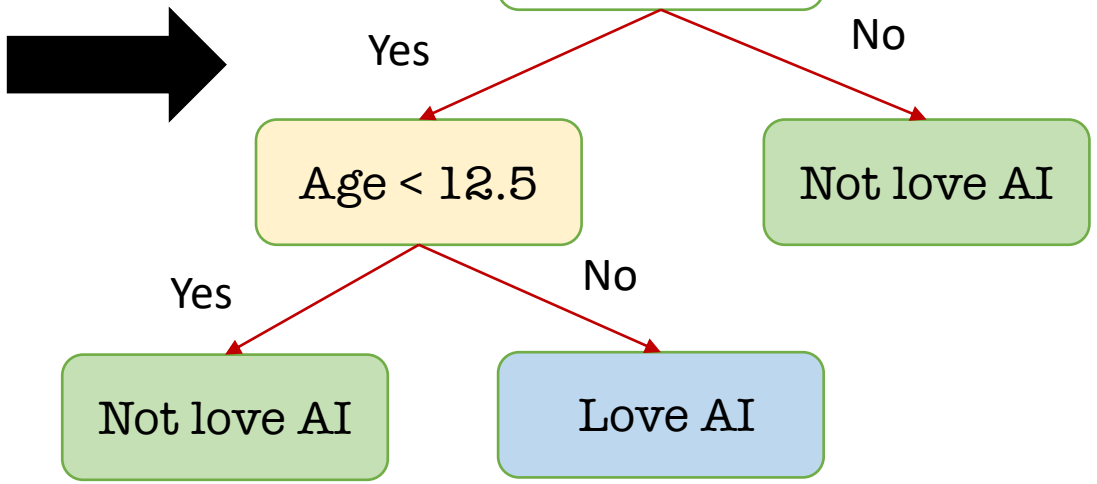
# Decision Tree From Dataset

| Love Math | Love Art | Age | Love AI |
|---|---|---|---|
| Yes | Yes | 7 | No |
| Yes | No | 12 | No |
| No | Yes | 18 | Yes |
| No | Yes | 35 | Yes |
| Yes | Yes | 38 | Yes |
| Yes | No | 50 | No |
| No | No | 83 | No |




*The StatQuest Illustrated Guide To Machine Learning,* Josh Starmer PhD

# How to Build a Decision Tree

•**Step-1:** Begin the tree with the root node, says S, which contains the complete dataset.

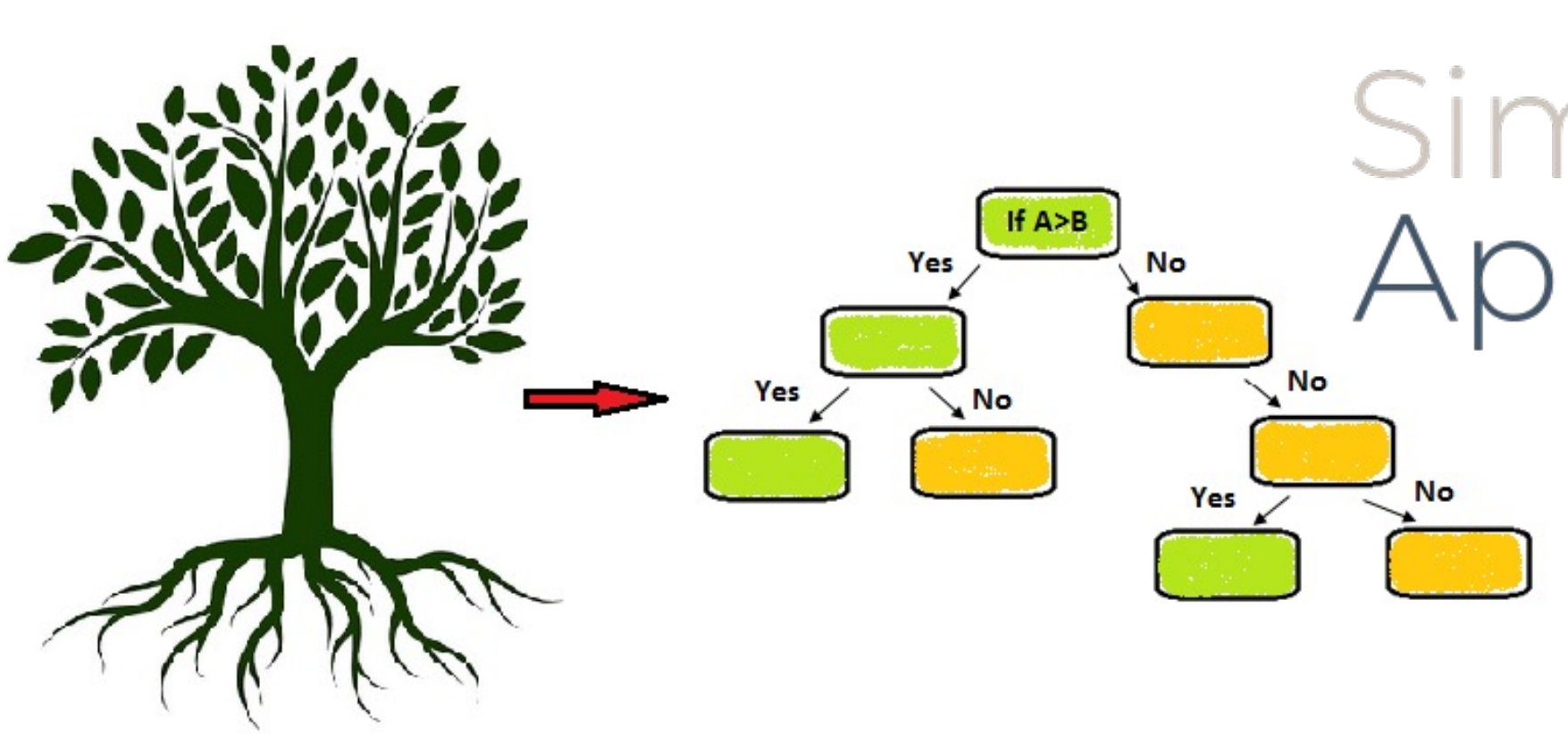
•**Step-2:** Find the best attribute in the dataset using **Attribute Selection Measure (ASM).**

•**Step-3:** Divide the S into subsets that contains possible v attributes.

•**Step-4:** Generate the decision tree node, which contains the b

•**Step-5:** Recursively make new decision trees using the sub created in step -3. Continue this process until a stage is r cannot further classify the nodes and called the final node as a

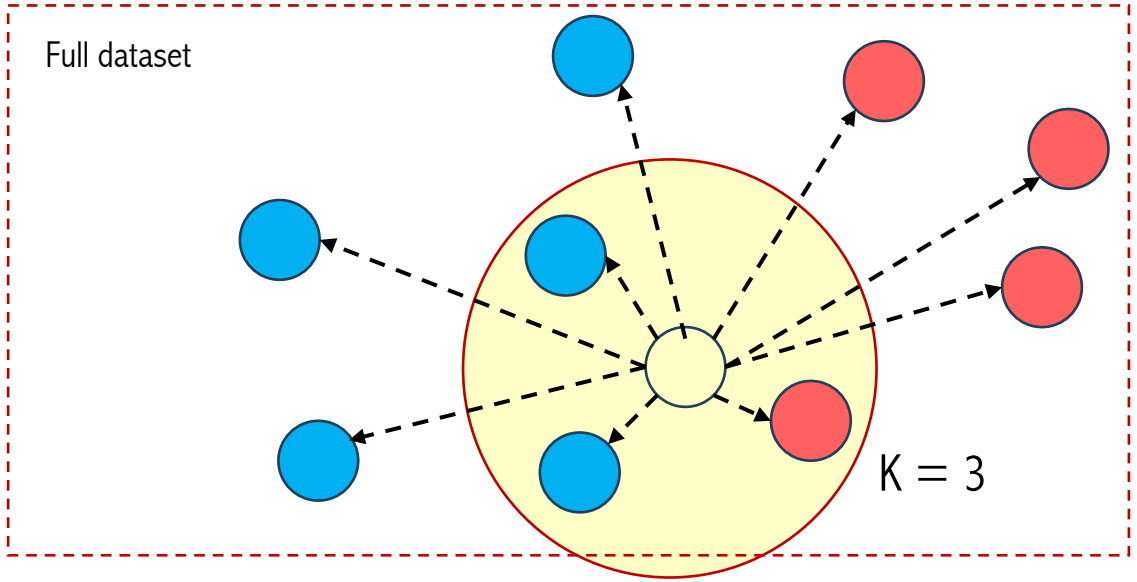
# Decision Tree in Simple Concepts




AI VIETNAM
All-in-One Course

# Decision Tree Motivation

## KNN Limitations

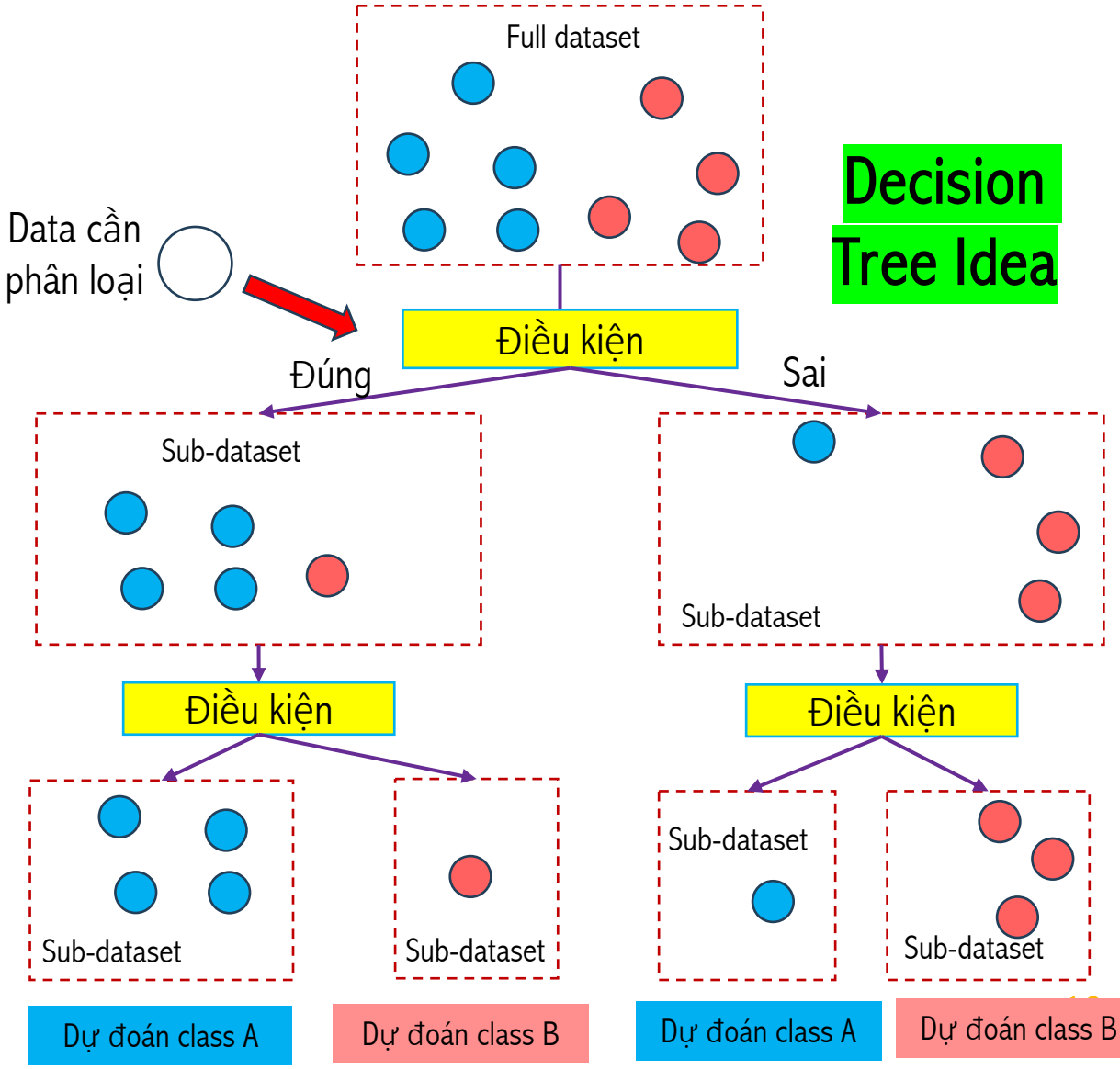



Data cần phân loại

Class A

Class B

KNN has some drawbacks and challenges, such as computational expense, slow speed, for large datasets




## Decision Tree Idea

# How to Build Decision Tree

| No. | Love Math | Love Art | Age | Love AI |
|---|---|---|---|---|
| 1 | Yes | Yes | 7 | No |
| 2 | Yes | No | 12 | No |
| 3 | No | Yes | 18 | Yes |
| 4 | No | Yes | 35 | Yes |
| 5 | Yes | Yes | 38 | Yes |
| 6 | Yes | No | 50 | No |
| 7 | No | No | 83 | No |

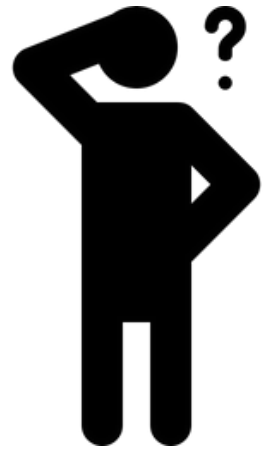
**How to select the first node in the tree**

Love Math

Love Art

Age

**Which one is a Root Node** ??



# How to Build Decision Tree

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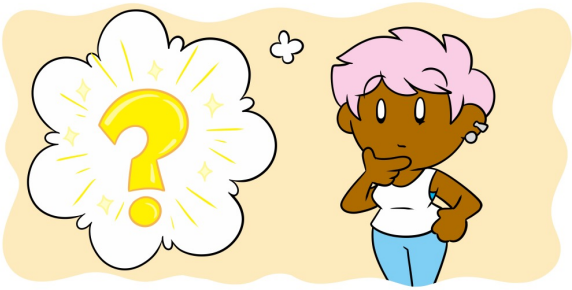
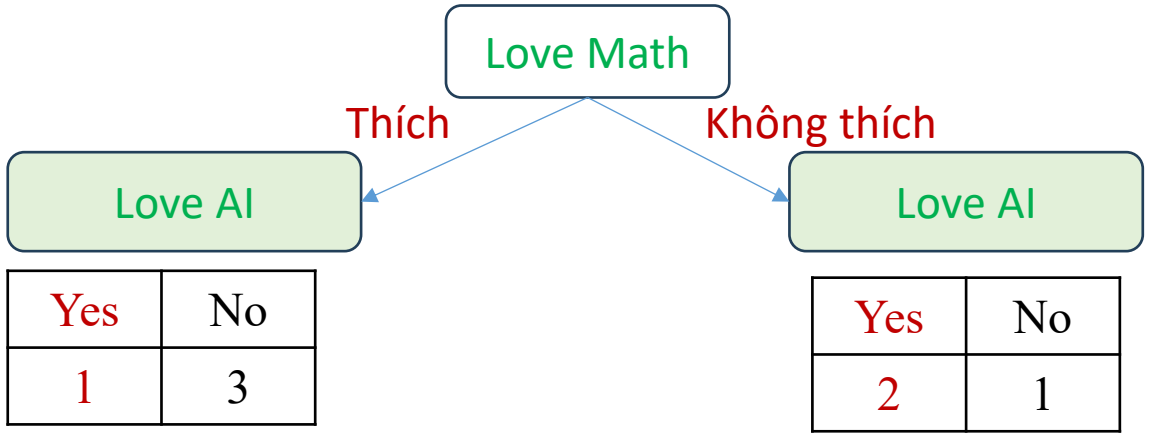
| 7 | No | No | 83 | No |

**How to select the first node in the tree**

Vì hiện tại chúng ta chưa biết chọn thông tin nào làm root node. Nên cách đơn giản là giả sử lấy thông tin "love math" làm root node.






| Yes | No |
|---|---|
| 1 | 3 |

| Yes | No |
|---|---|
| 2 | 1 |

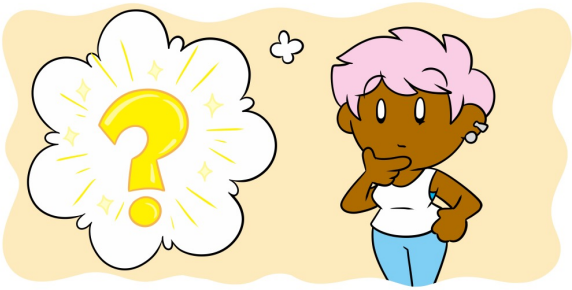
Để cho công bằng, chúng ta cũng nên thử chọn thêm Love Art hoặc Age làm root node nhé.

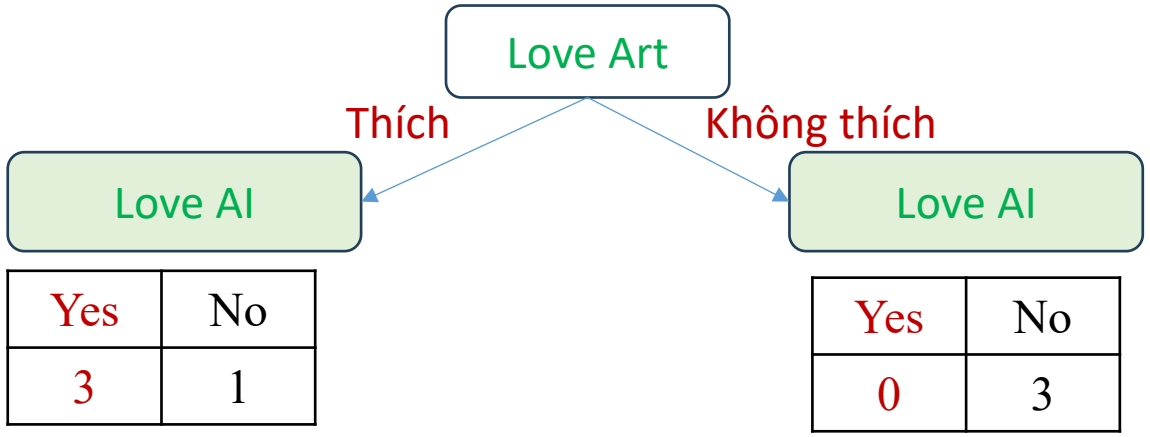
# How to Build Decision Tree

| No. | Love Math | Love Art | Age | Love AI |
|---|---|---|---|---|
| 1 | Yes | Yes | 7 | No |
| 2 | Yes | No | 12 | No |
| 3 | No | Yes | 18 | Yes |
| 4 | No | Yes | 35 | Yes |
| 5 | Yes | Yes | 38 | Yes |
| 6 | Yes | No | 50 | No |
| 7 | No | No | 83 | No |

**How to select the first node in the tree**

Chọn Love Art như là root node.

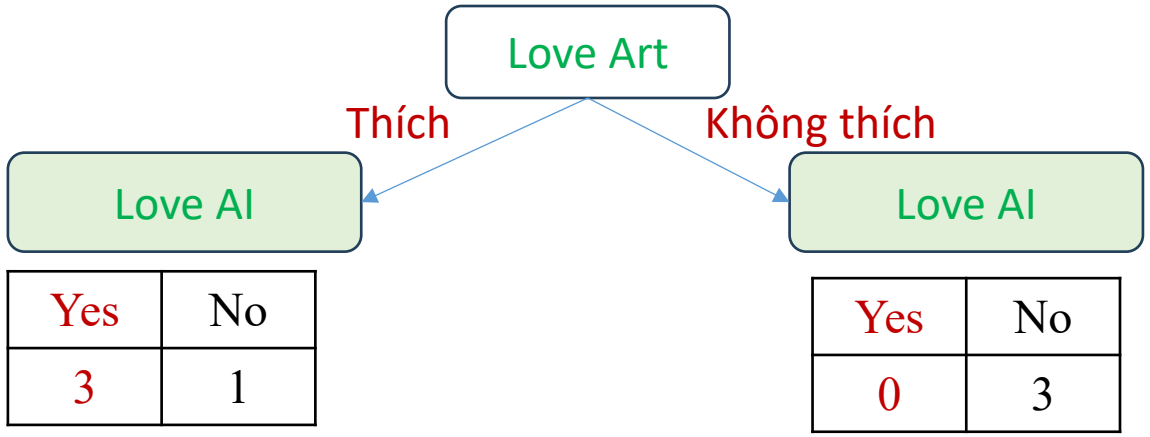





| Yes | No |
|---|---|
| 3 | 1 |

| Yes | No |
|---|---|
| 0 | 3 |

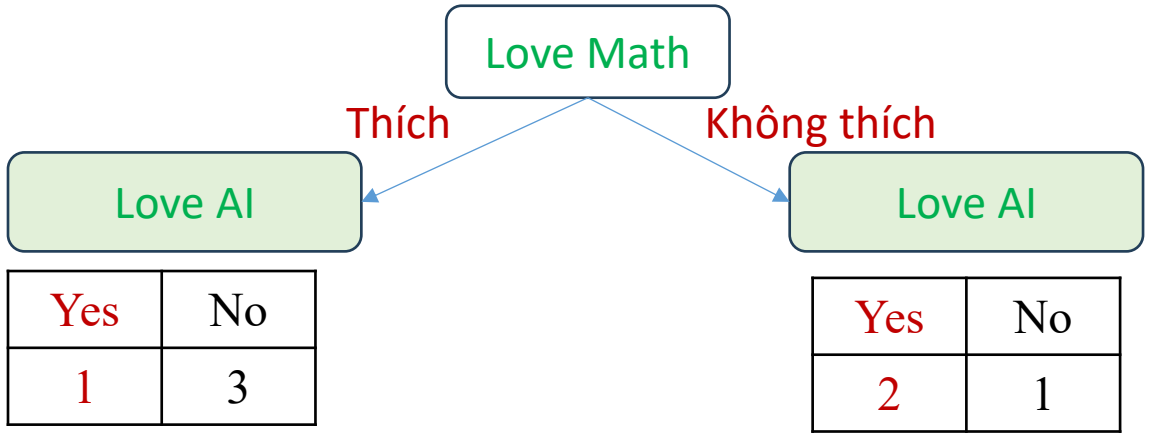
# Review Two Approaches




| Yes | No |
|---|---|
| 3 | 1 |

| Yes | No |
|---|---|
| 0 | 3 |

Do you have any ***comments*** on these two simple trees.

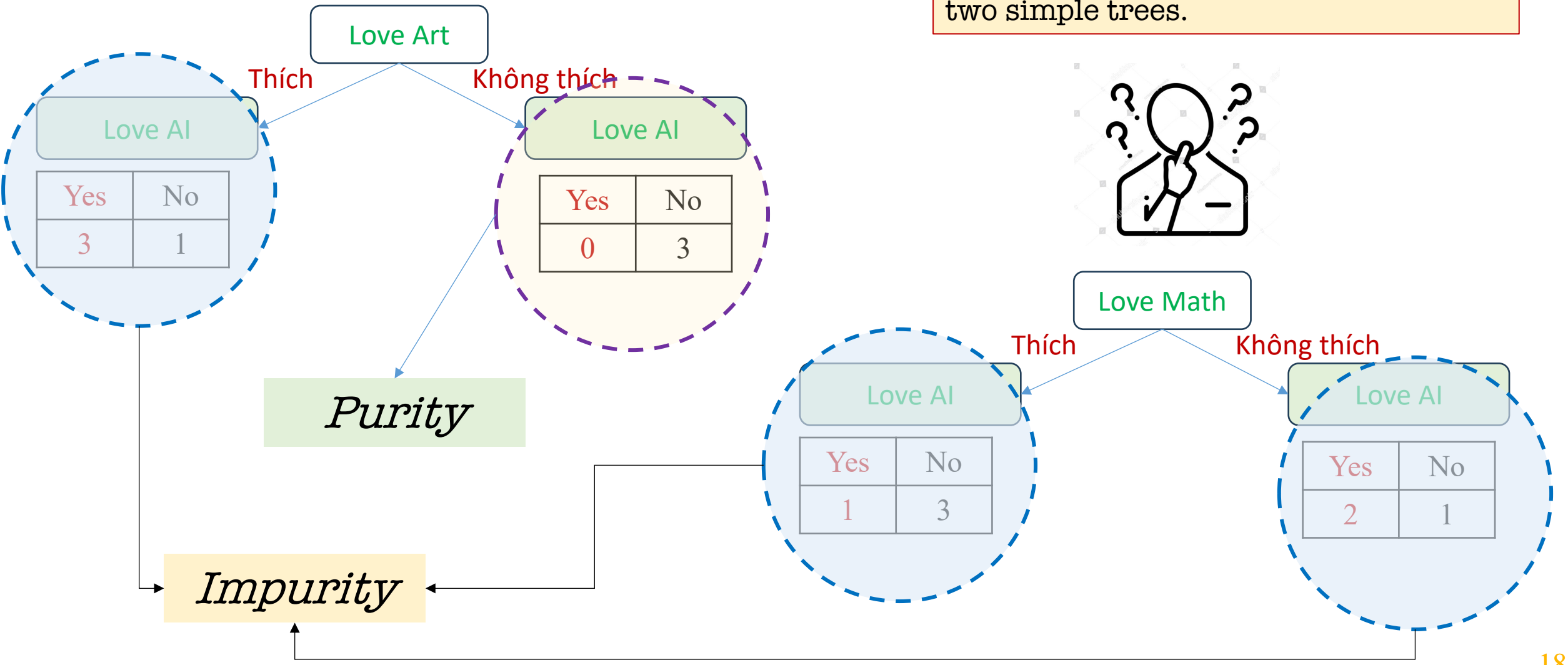



| Yes | No |
|---|---|
| 1 | 3 |

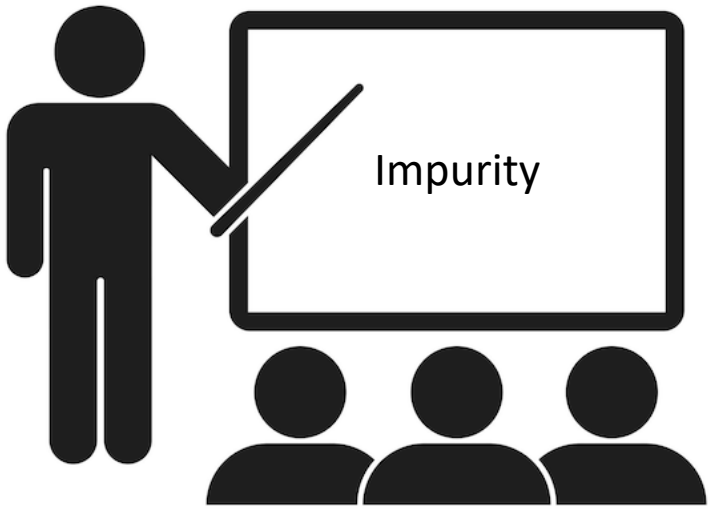
| Yes | No |
|---|---|
| 2 | 1 |

# Review Two Approaches

Do you have any ***comments*** on these two simple trees.

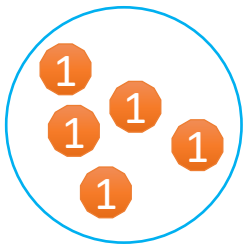


# What is an Impurity

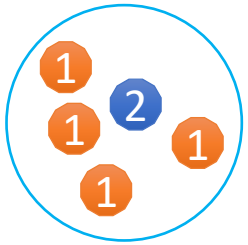



| | | | | | |
|---|---|---|---|---|---|
| Set 1 | 1 | 1 | 1 | 1 | 1 |
| Set 2 | 1 | 1 | 2 | 1 | 1 |
| Set 3 | 1 | 2 | 4 | 6 | 7 |

| | Impurity Score |
|---|---|
| Set 1 | Thấp |
| Set 2 | Trung Bình |
| Set 3 | Cao |



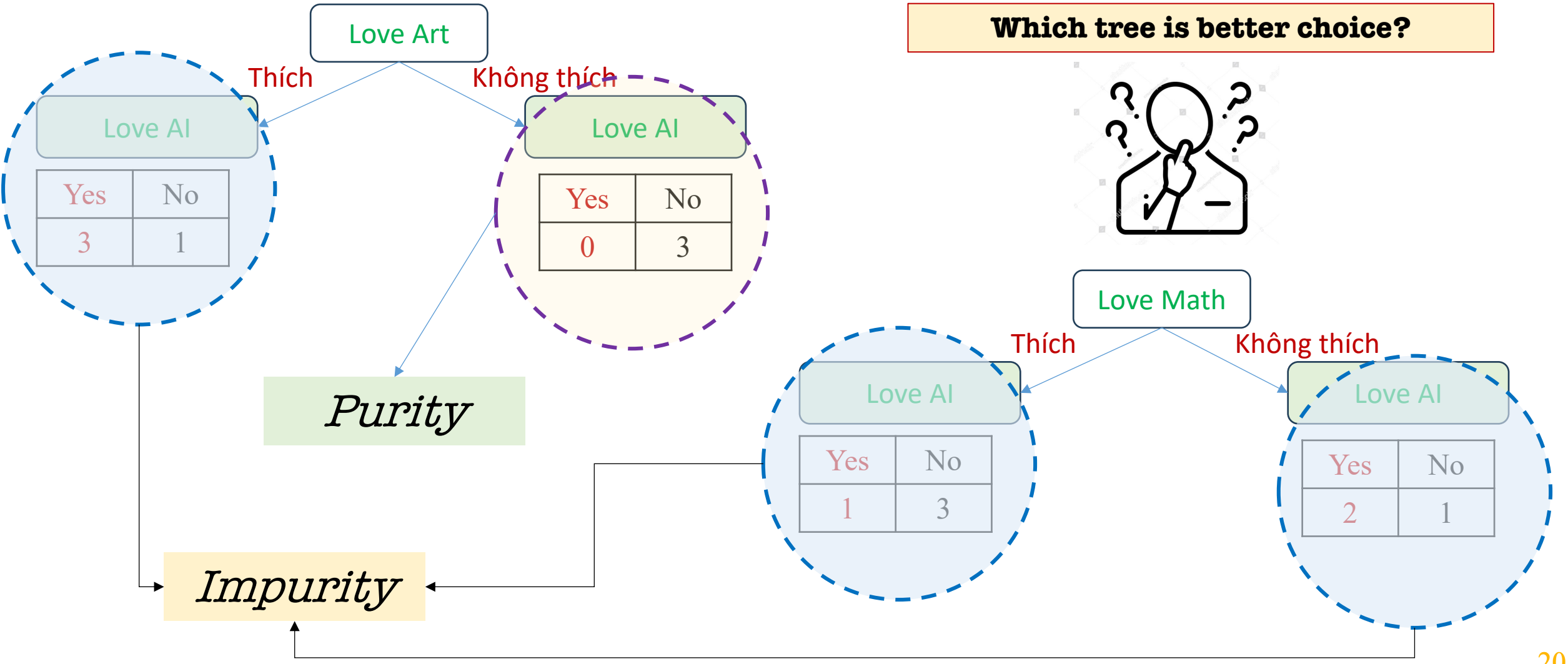

Set 1



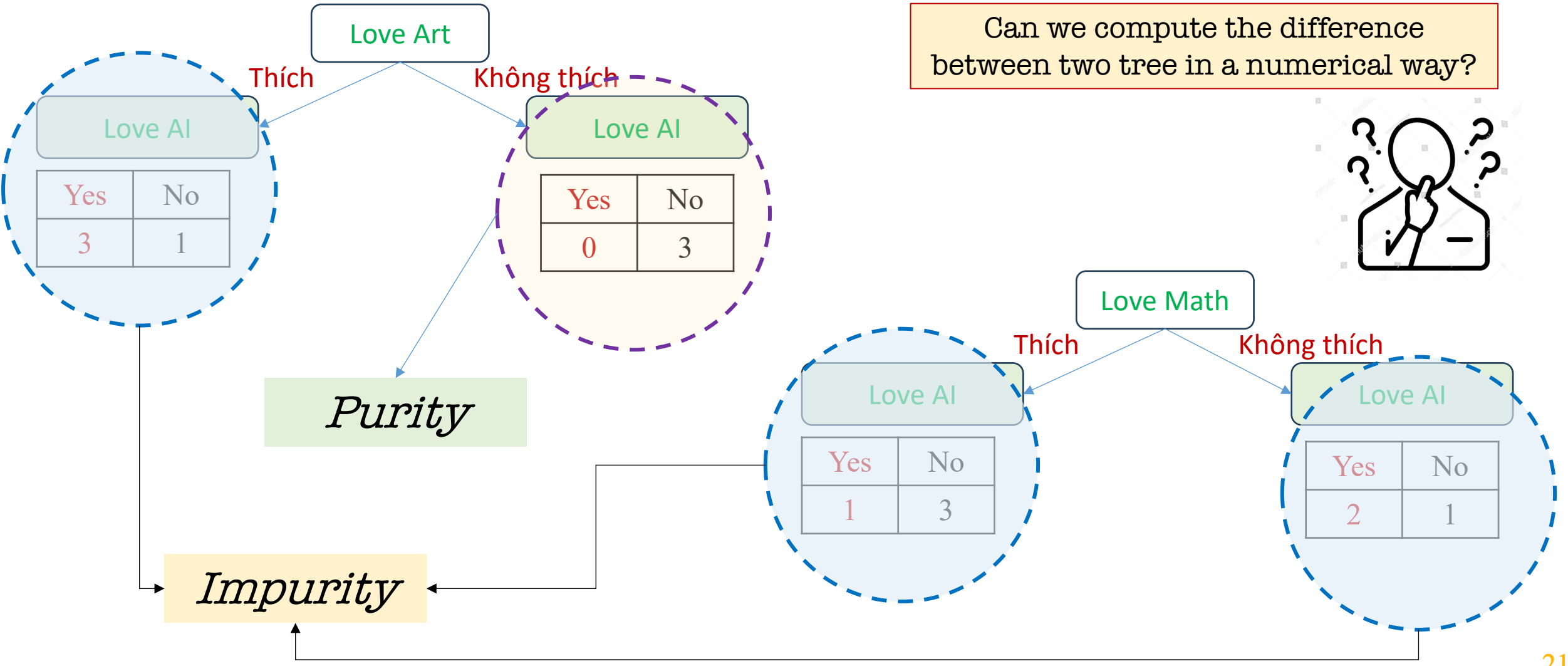

Set 2


Set 3

# Review Two Approaches



# Review Two Approaches



# Evaluation Metrics

**Entropy – Information Gain**

**GNI IMPURITY**

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- Introduction to Tree
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# Evaluation Metrics

**Entropy – Information Gain**

**GNI IMPURITY**

# GINI Impurity

*It was developed by statistician and sociologist Corrado Gini.*

Consider a dataset D that contains samples from k classes. The probability of samples belonging to class i at a given node can be denoted as $p_i$. Then the Gini Impurity of D is defined as:

$$Gini(D) = 1 - \sum_{i=1}^{k} p_i^2$$

| | Count | | Probability | | Gini Impurity |
|---|---|---|---|---|---|
| | $n_1$ | $n_2$ | $p_1$ | $p_2$ | $1 - p_1^2 - p_2^2$ |
| Node A | 0 | 10 | 0 | 1 | $1 - 0^2 - 1^2 = 0$ |
| Node B | 3 | 7 | 0.3 | 0.7 | $1 - 0.3^2 - 0.7^2 = 0.42$ |
| Node C | 5 | 5 | 0.5 | 0.5 | $1 - 0.5^2 - 0.5^2 = 0.5$ |

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| Node B | 3 | 7 | 0.3 | 0.7 | $1 - 0.3^2 - 0.7^2 = 0.42$ |
| Node C | 5 | 5 | 0.5 | 0.5 | $1 - 0.5^2 - 0.5^2 = 0.5$ |

# GINI Impurity

$$Gini(D) = 1 - \sum_{i=1}^{k} p_i^2$$

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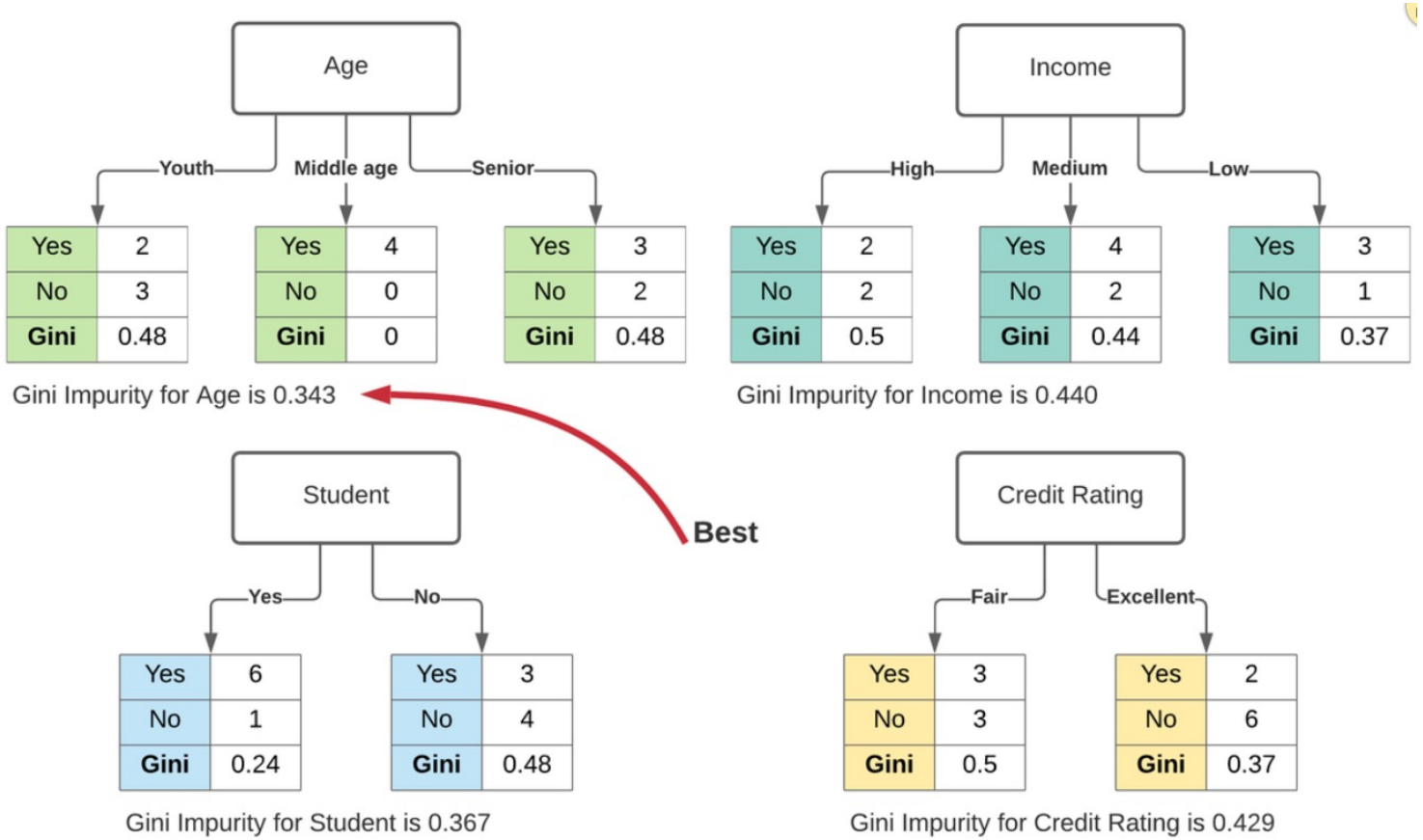

# GINI Impurity

$$Gini(D) = 1 - \sum_{i=1}^{k} p_i^2$$

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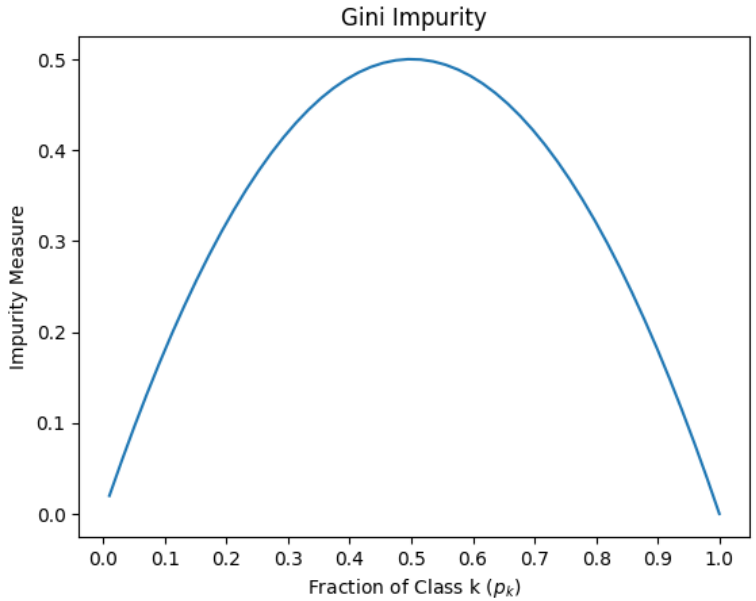



```
#A figure is created to show Gini ımpurity measures
plt.figure()
x = np.linspace(0.01,1)
y = 1 - (x*x) - (1-x)*(1-x)
plt.plot(x,y)
plt.title('Gini Impurity')
plt.xlabel("Fraction of Class k ($p_k$)")
plt.ylabel("Impurity Measure")
plt.xticks(np.arange(0,1.1,0.1))

plt.show()
```

This figure shows that Gini impurity is maximum for the 50-50 sample (p1=0.5 ) and minimum for the homogeneous sample (p1=0 or p1=1)

# GINI Impurity

| | | | | | |
|---|---|---|---|---|---|
| Set 1 | 1 | 1 | 1 | 1 | 1 |
| Set 2 | 1 | 1 | 2 | 1 | 1 |
| Set 3 | 1 | 2 | 4 | 6 | 7 |

| | No of Unique Element | Count of unique elements | Probability |
|---|---|---|---|
| Set 1 | 1 | 5 | 5/5 |
| Set 2 | 1,2 | 4, 1 | 4/5, 1/5 |
| Set 3 | 1,2,4,6,7 | 1,1,1,1,1 | 1/5,1/5,1/5,1/5,1/5,1/5 |

# GINI Impurity

| | | | | | |
|---|---|---|---|---|---|
| Set 1 | 1 | 1 | 1 | 1 | 1 |
| Set 2 | 1 | 1 | 2 | 1 | 1 |
| Set 3 | 1 | 2 | 4 | 6 | 7 |

| | No of Unique Element | Count of unique elements | Probability |
|---|---|---|---|
| Set 1 | 1 | 5 | 5/5 |

$$n = 1, p_1 = 1$$

$$GINI = 1 - \sum_{i=1}^{n} (p_i)^2 \qquad Gini = 1 - [1^2] = 0$$

# GINI Impurity

| | | | | | |
|---|---|---|---|---|---|
| Set 1 | 1 | 1 | 1 | 1 | 1 |
| Set 2 | 1 | 1 | 2 | 1 | 1 |
| Set 3 | 1 | 2 | 4 | 6 | 7 |

| | No of Unique Element | Count of unique elements | Probability |
|---|---|---|---|
| Set 3 | 2 | 4,1 | 4/5, 1/5 |

$$GINI = 1 - \sum_{i=1}^{n}(p_i)^2$$

$$n = 2,\ p1 = 4/5,\ p2 = 1/5$$

$$Gini = 1 - [4/5^2 + 1/5^2] = 0.32$$

# GINI Impurity

| | | | | | |
|---|---|---|---|---|---|
| Set 1 | 1 | 1 | 1 | 1 | 1 |
| Set 2 | 1 | 1 | 2 | 1 | 1 |
| Set 3 | 1 | 2 | 4 | 6 | 7 |

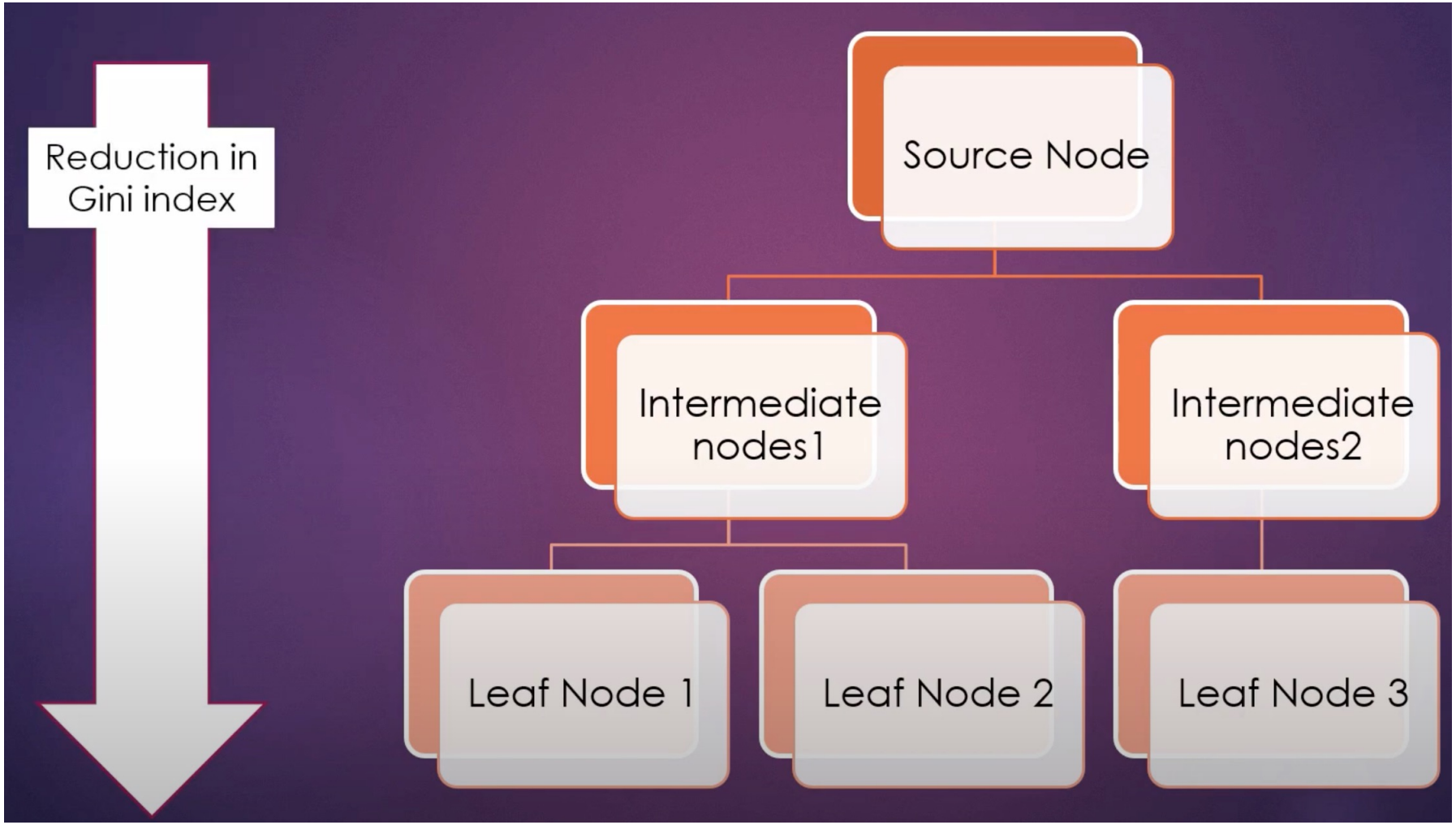
| | No of Unique Element | Count of unique elements | Probability |
|---|---|---|---|
| Set 3 | 1 | 5 | 5/5 |

$$GINI = 1 - \sum_{i=1}^{n}(p_i)^2$$

$n = 5,\ p1 = p2 = p3 = p4 = p5 = 1/5$

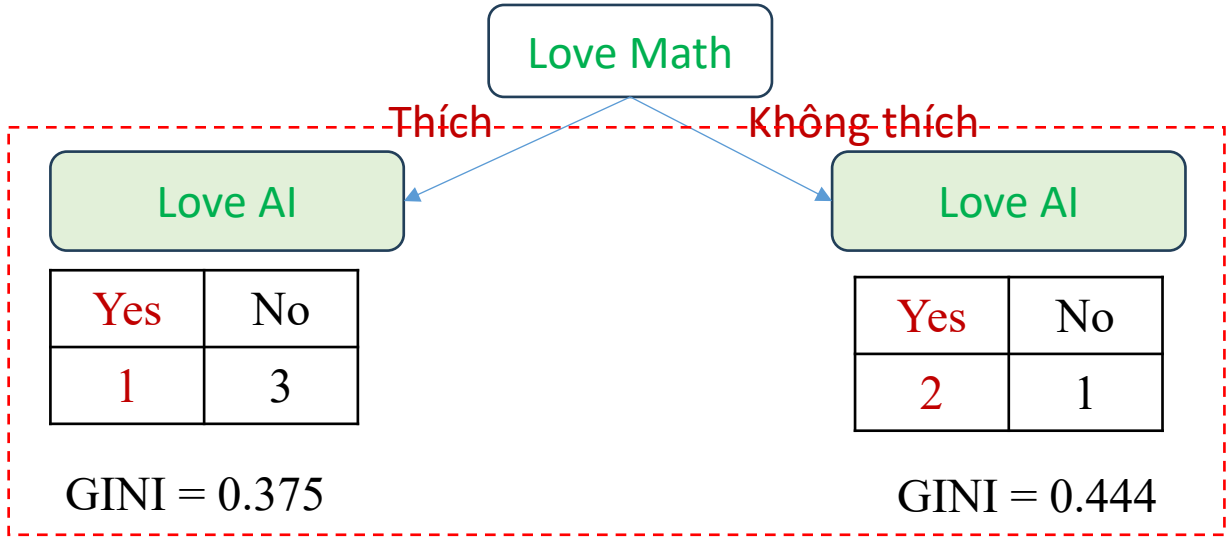
$$Gini = 1 - [1/5^2 + 1/5^2 + 1/5^2 + 1/5^2 + 1/5^2] = 0.8$$

# Idea of GNI Impurity



# Gini Impurity for Leaf Nodes

$$\text{Gini} = 1 - \sum_{i=1}^{n} P_i^2$$

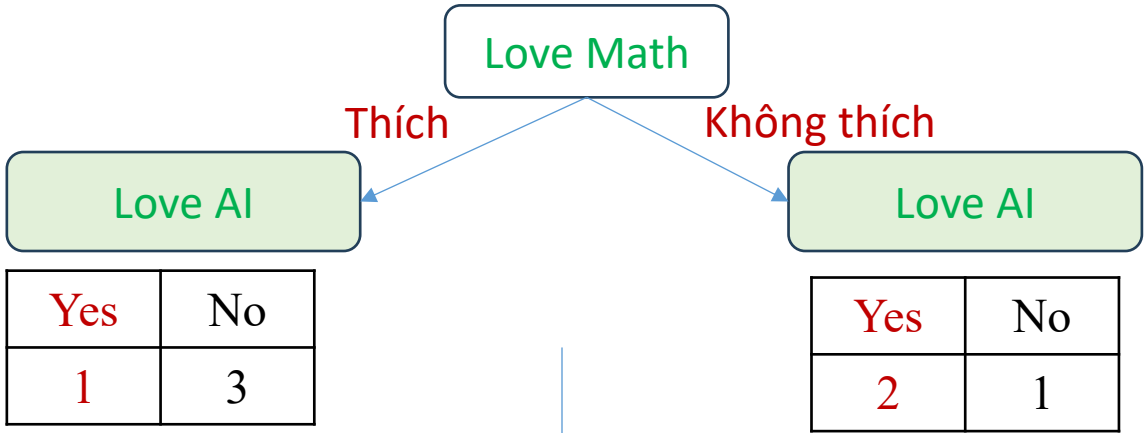



Cách tính Gini Impurity cho Leaf Nodes

1. Calculate the probabilties of all classess.
2. Square the calculated probabilities
3. Sum all the squared probabilities into a single integer
4. Substract the single integer from 1

# Gini Impurity for Leaf Nodes

$$\text{Gini} = 1 - \sum_{i=1}^{n} P_i^2$$




| Yes | No |
|---|---|
| 1 | 3 |

GINI = 0.375

| Yes | No |
|---|---|
| 2 | 1 |

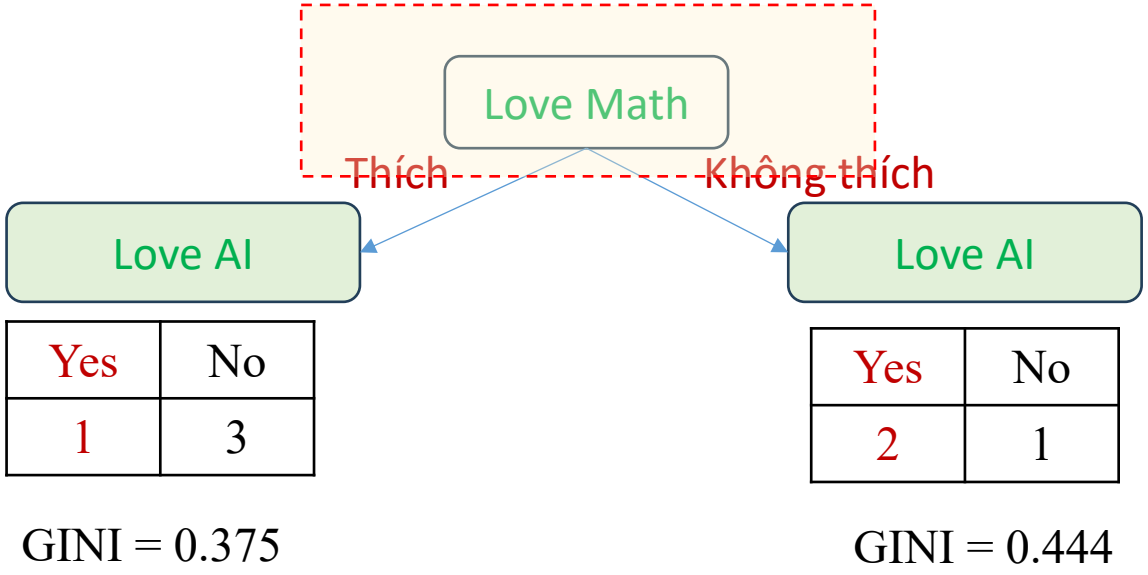
GINI = 0.444

Gini = 1 – (the probability of Yes)$^2$ – (the probability of No)$^2$

GINI = $1 - [(1/4)^2 + (3/4)^2] = 0.375$

GINI = $1 - [(2/3)^2 + (1/3)^2] = 0.444$

# Gini Impurity for Root/Internal Nodes




| Yes | No |
|---|---|
| 1 | 3 |

GINI = 0.375

| Yes | No |
|---|---|
| 2 | 1 |

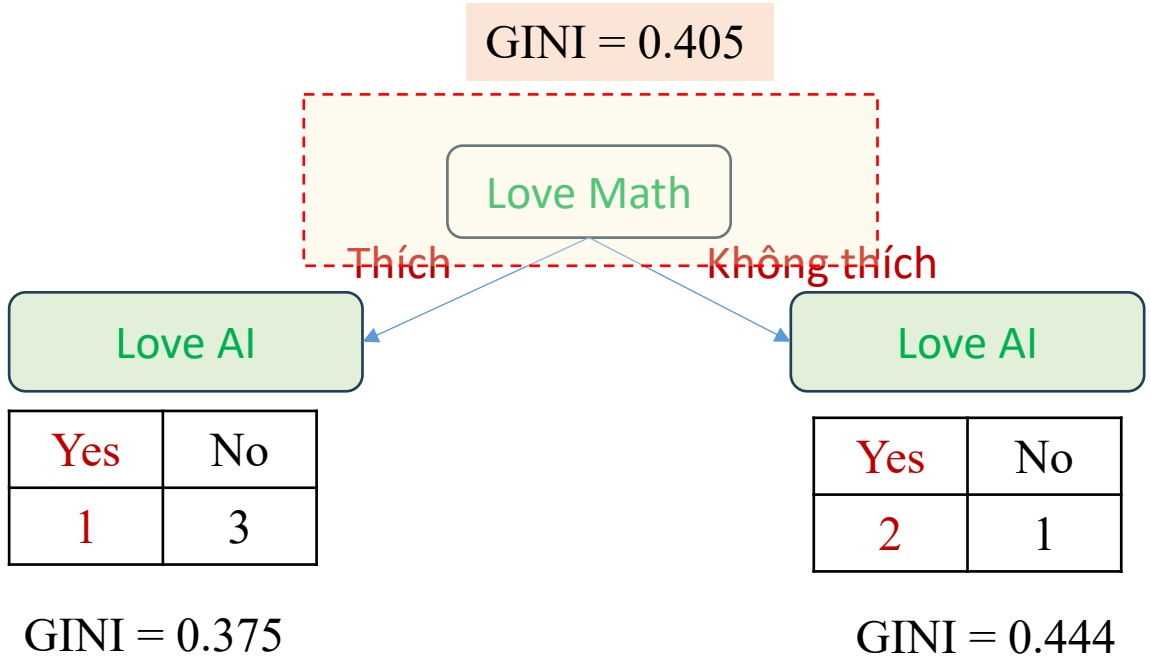
GINI = 0.444

Cách tính Gini Impurity cho Root/Internal Nodes

1. Loop over the leaf node, from k = 1 all the way to the K leaf nodes
2. Find the gini impurity of the urrent k'th leaf
3. Count the number of observations in the k'th leaf
4. Divide by the total number of observation in the all leaf nodes
5. The result of each leaf is summed for a final gini impurity

$$Gini(Internal_j) = \sum_{k'th\ leaf=1}^{K\ leaf\ nodes} \left( \frac{count(L_k)}{count(L_1, \ldots, L_k)} \right) Gini(L_k)$$

# Gini Impurity for Root/Internal Nodes




***Total Impurity = Weight average of Gini Impurities for The leaves***

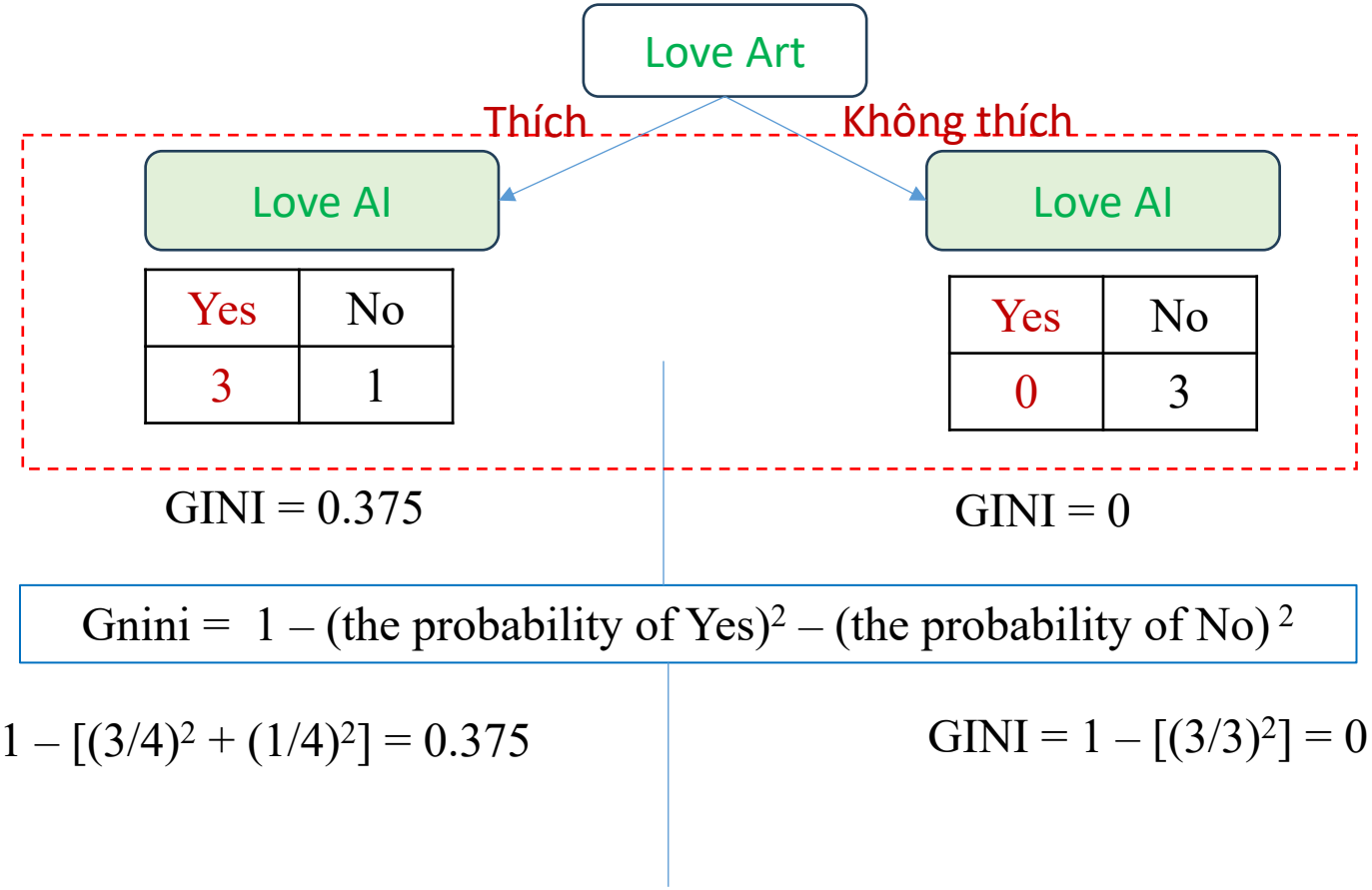
**Total Impurity = $\frac{4}{7}$×0.375 + $\frac{3}{7}$×0.444 = 0.405**

If a data set $D$ is split on an attribute $A$ into two subsets $D_1$ and $D_2$ with sizes $n_1$ and $n_2$, respectively, the Gini Impurity can be defined as:

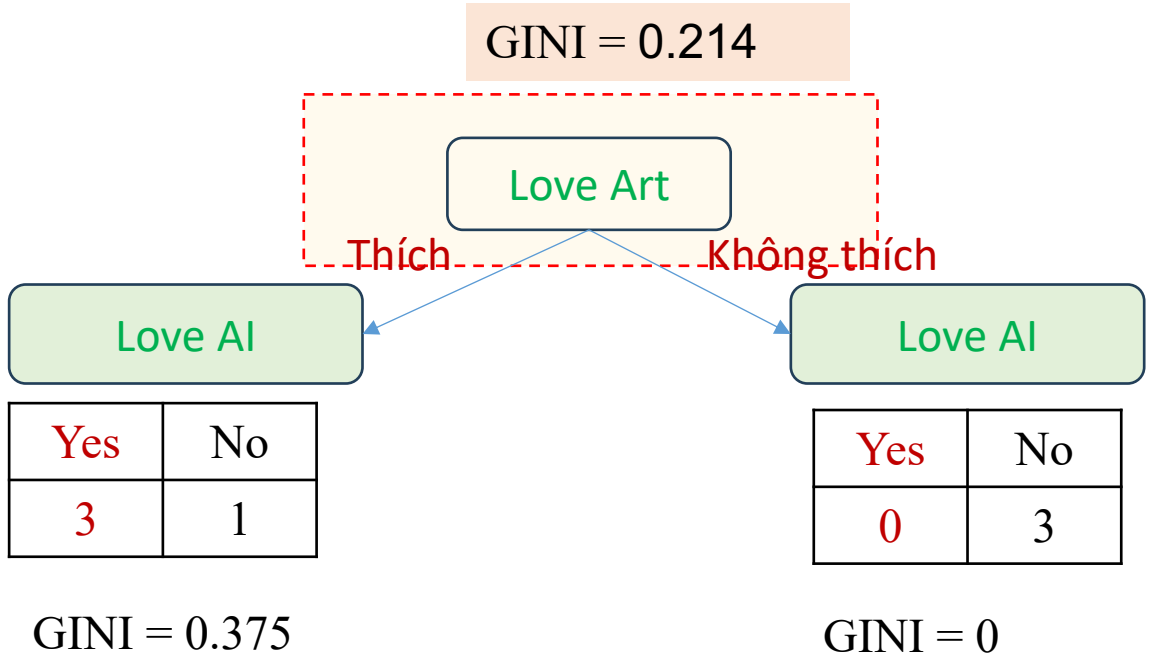
$$Gini_A(D) = \frac{n_1}{n} Gini(D_1) + \frac{n_2}{n} Gini(D_2)$$

# Gini Impurity for Leaf Nodes

$$GINI = 1 - \sum_{i=1}^{n} (p_i)^2$$



# Gini Impurity for Root/Internal Nodes

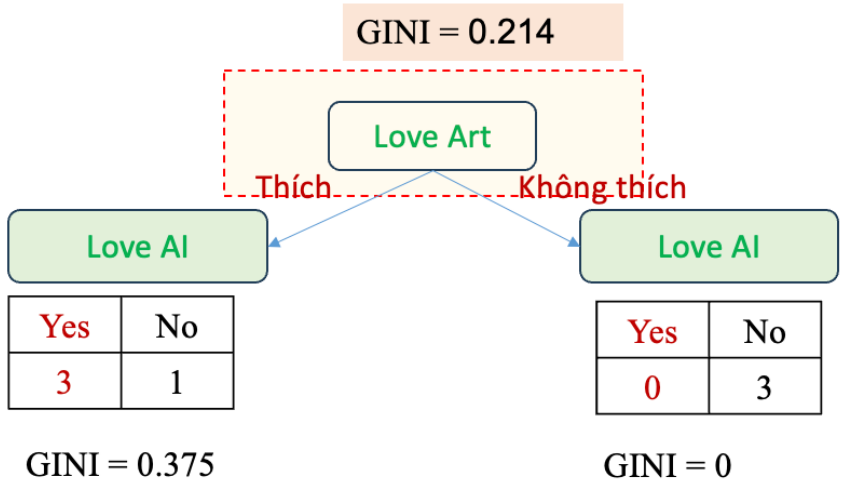


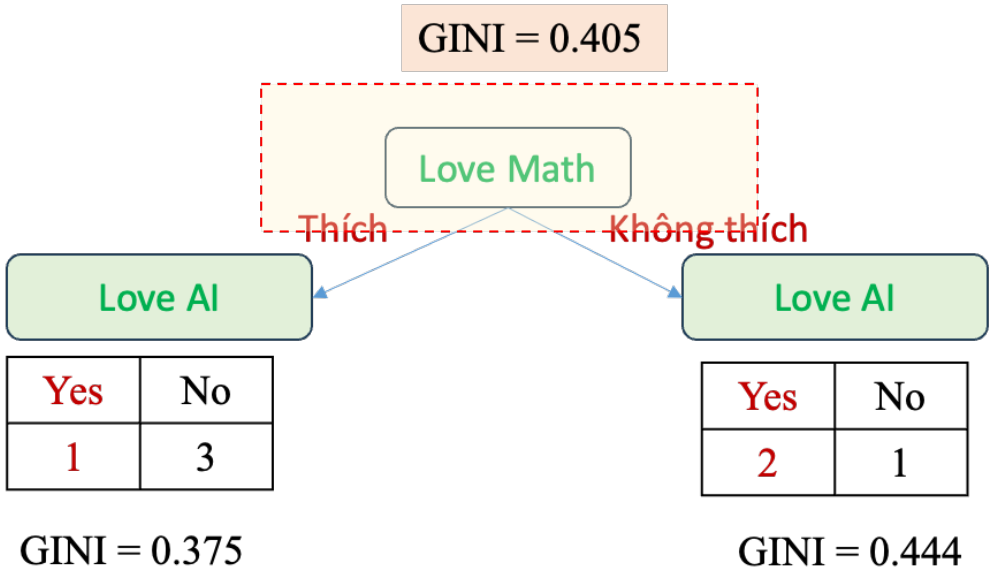

***Total Impurity = Weight average of Gini Impurities for The leaves***

**Total Impurity = $\frac{4}{7} \times 0.375 + \frac{3}{7} \times 0 = 0.214$**

# What is a GINI of "AGE"?

| No. | Love Math | Love Art | Age | Love AI |
|---|---|---|---|---|
| 1 | Yes | Yes | 7 | No |
| 2 | Yes | No | 12 | No |
| 3 | No | Yes | 18 | Yes |
| 4 | No | Yes | 35 | Yes |
| | | Yes | 38 | Yes |
| | | No | 50 | No |
| | | No | 83 | No |



# Sort By Age

**Before**

| Age | Love AI |
|---|---|
| 7 | No |
| 12 | No |
| 18 | Yes |
| 35 | Yes |
| 38 | Yes |
| 50 | No |
| 83 | No |

**Sort by Age** →

**After**

| Age | Love AI |
|---|---|
| 7 | No |
| 12 | No |
| 18 | Yes |
| 35 | Yes |
| 38 | Yes |
| 50 | No |
| 83 | No |

→ **Average of two adjacent cells**

(7 + 12)/2 = 9.5

(18 + 12)/2 = 15

(18 + 35)/2 = 26.5

(38 + 35)/2 = 36.5

(38 + 50)/2 = 44

(50 + 83)/2 = 66.5

| Age | Love AI |
|---|---|
| 7 | No |
| 12 | No |
| 18 | Yes |
| 35 | Yes |
| 38 | Yes |
| 50 | No |
| 83 | No |

# GINI IMPURITY FOR EACH OF AGE

| Age | Love AI |
|---|---|
| 7 | No |
| 12 | No |
| 18 | Yes |
| 35 | Yes |
| 38 | Yes |
| 50 | No |
| 83 | No |

(7 + 12)/2 = 9.5

(18 + 12)/2 = 15

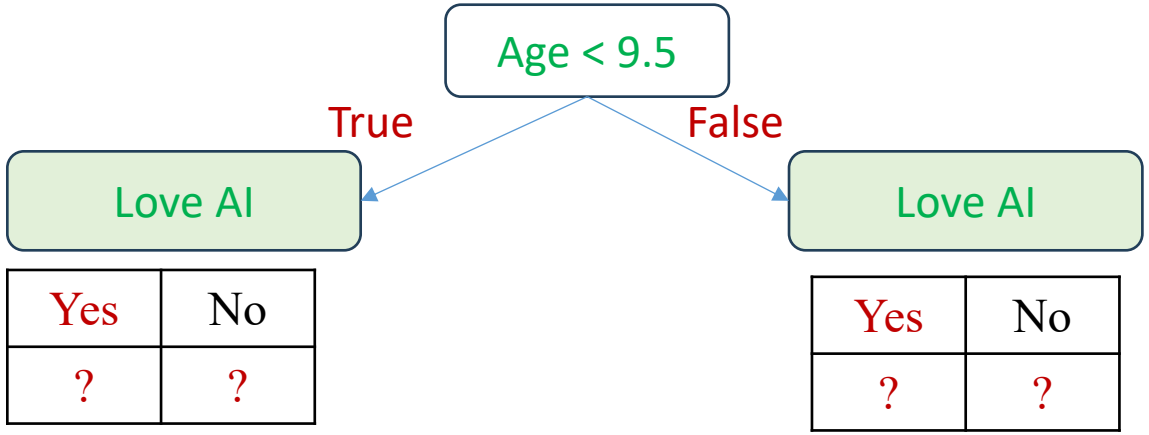
(18 + 35)/2 = 26.5

(38 + 35)/2 = 36.5

(38 + 50)/2 = 44

(50 + 83)/2 = 66.5

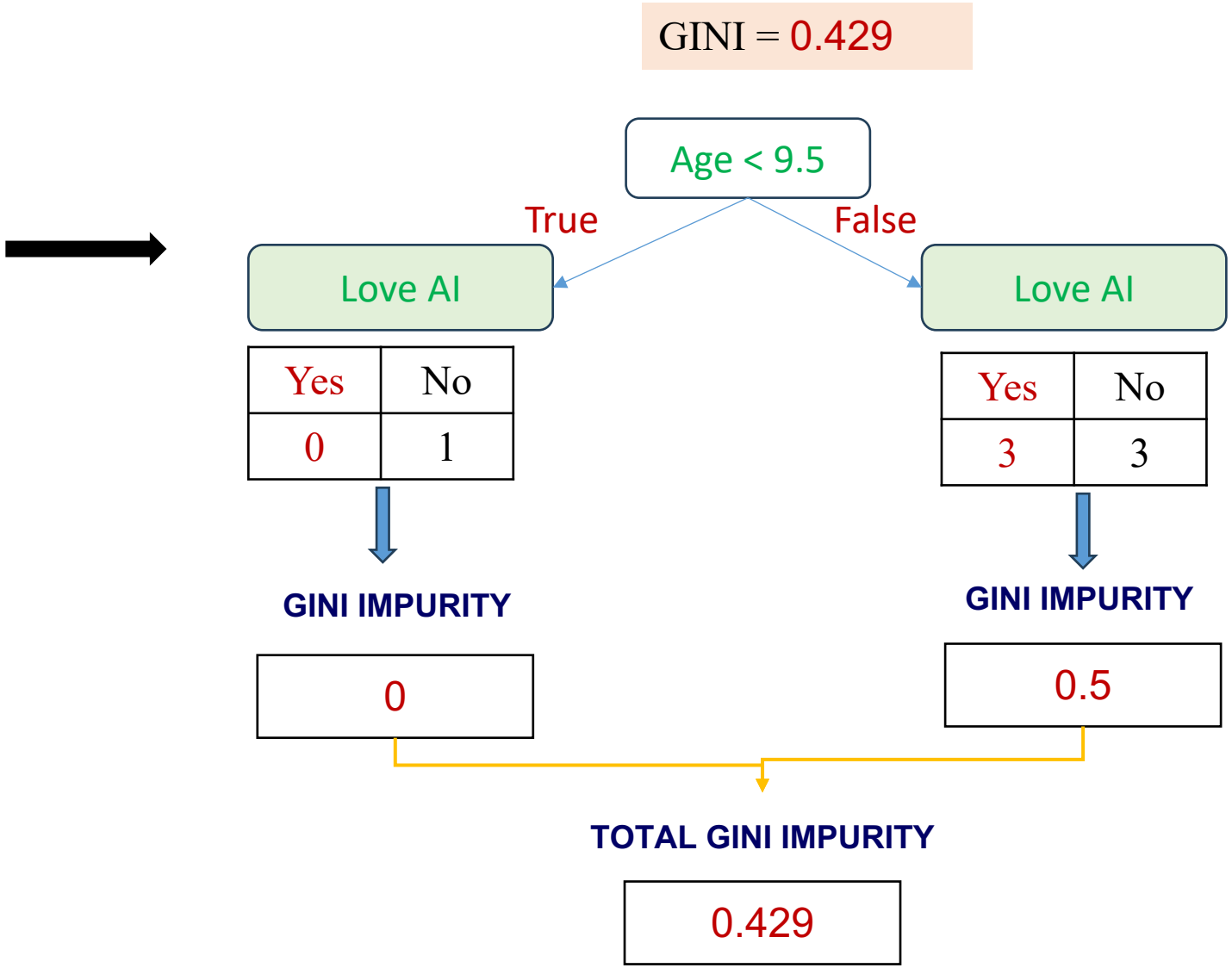
GINI = ???



# GINI IMPURITY FOR EACH OF AGE

| Age | Love AI |
|---|---|
| 7 | No |
| 12 | No |
| 18 | Yes |
| 35 | Yes |
| 38 | Yes |
| 50 | No |
| 83 | No |

- (7 + 12)/2 = 9.5
- (18 + 12)/2 = 15
- (18 + 35)/2 = 26.5
- (38 + 35)/2 = 36.5
- (38 + 50)/2 = 44
- (50 + 83)/2 = 66.5



# GINI IMPURITY FOR EACH OF AGE

| Age | Love AI |
|---|---|
| 7 | No |
| 12 | No |
| 18 | Yes |
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| 50 | No |
| 83 | No |

(7 + 12)/2 = 9.5

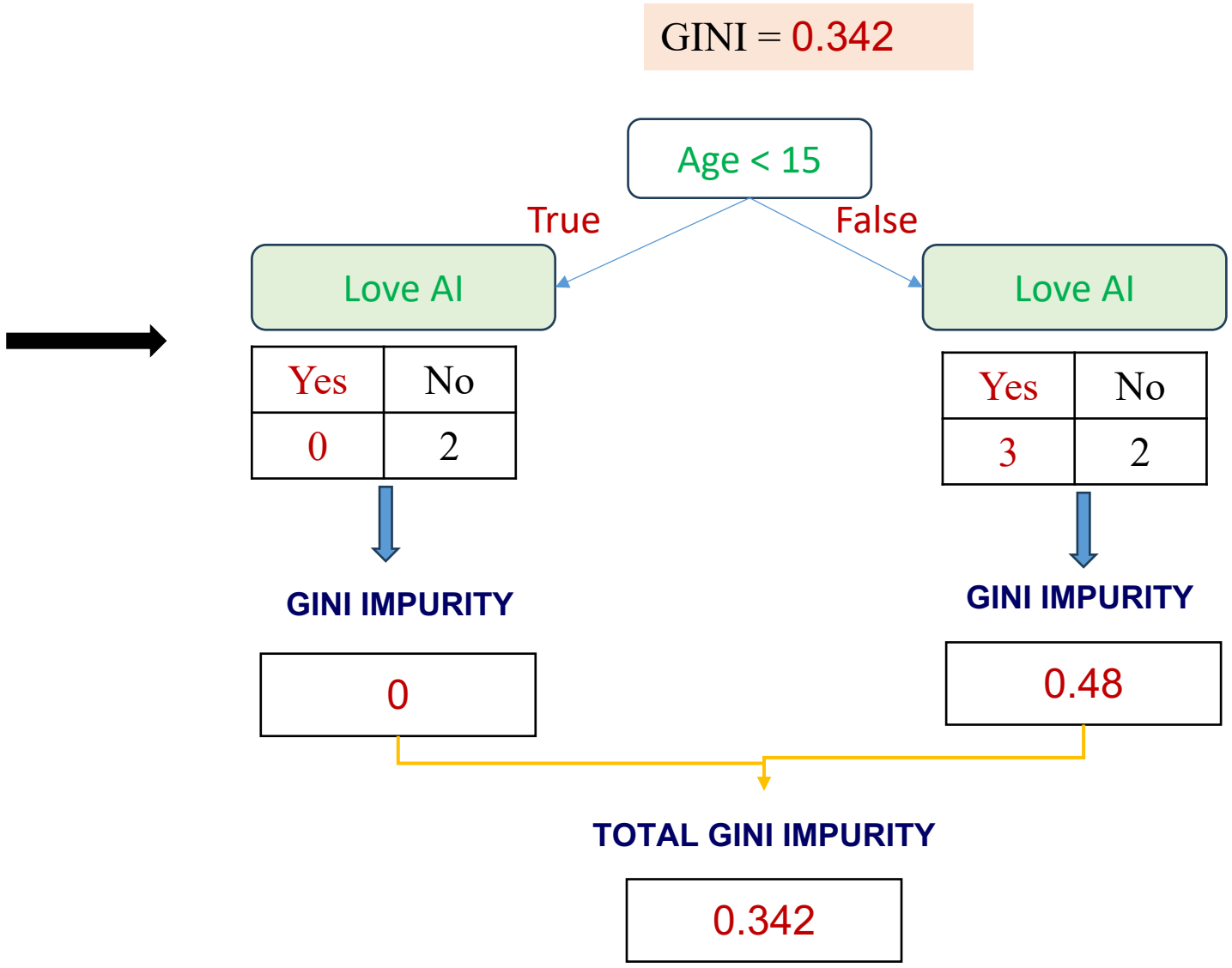
(18 + 12)/2 = 15

(18 + 35)/2 = 26.5

(38 + 35)/2 = 36.5

(38 + 50)/2 = 44

(50 + 83)/2 = 66.5



# GINI IMPURITY FOR EACH OF AGE

| Age | Love AI |
|---|---|
| 7 | No |
| 12 | No |
| 18 | Yes |
| 35 | Yes |
| 38 | Yes |
| 50 | No |
| 83 | No |

(7 + 12)/2 = 9.5

(18 + 12)/2 = 15

(18 + 35)/2 = 26.5

(38 + 35)/2 = 36.5

(38 + 50)/2 = 44

(50 + 83)/2 = 66.5

GINI for each threshold

GINI

0.429

0.343

0.476

0.476

0.343

0.429

Best GINI

GINI

0.429

0.343

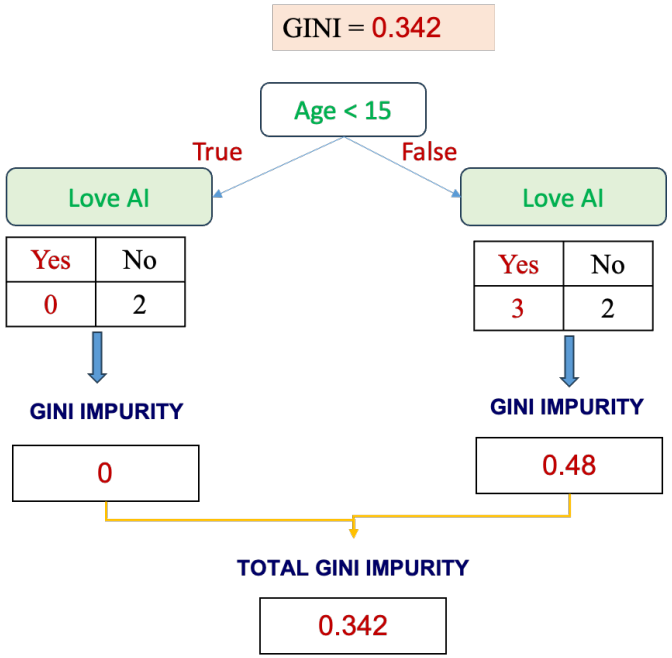
0.476

0.476

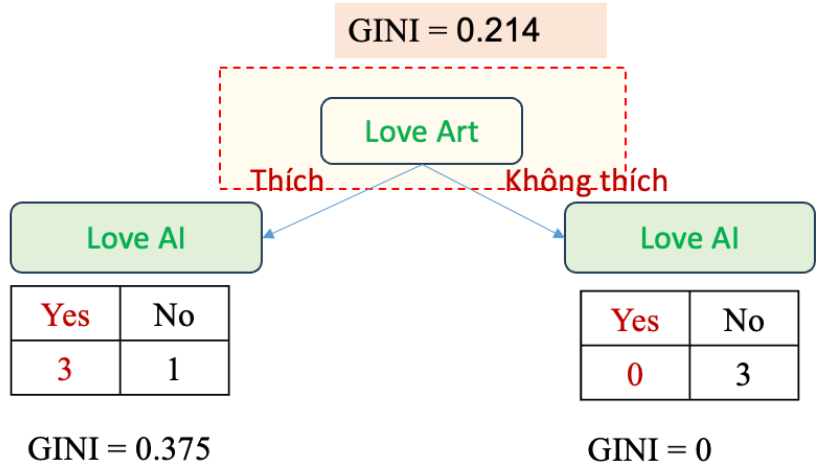
0.343

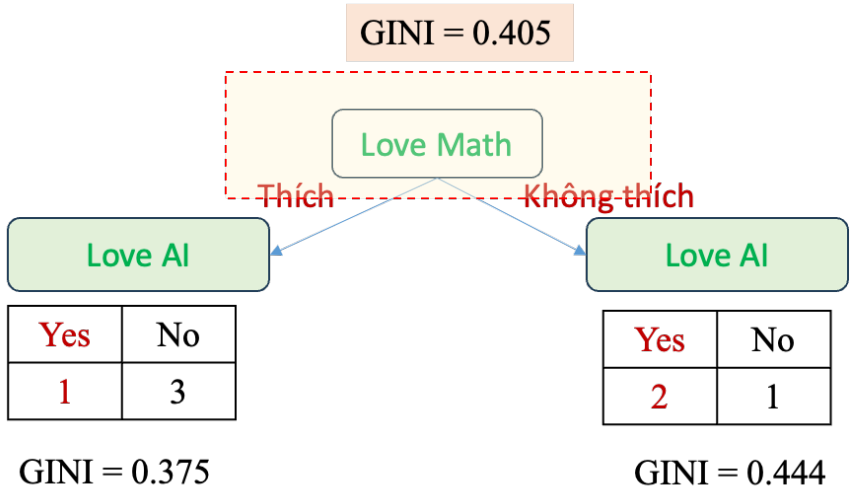
0.429

# GINI for Three Attributes




**Which attribute is in the first node?**

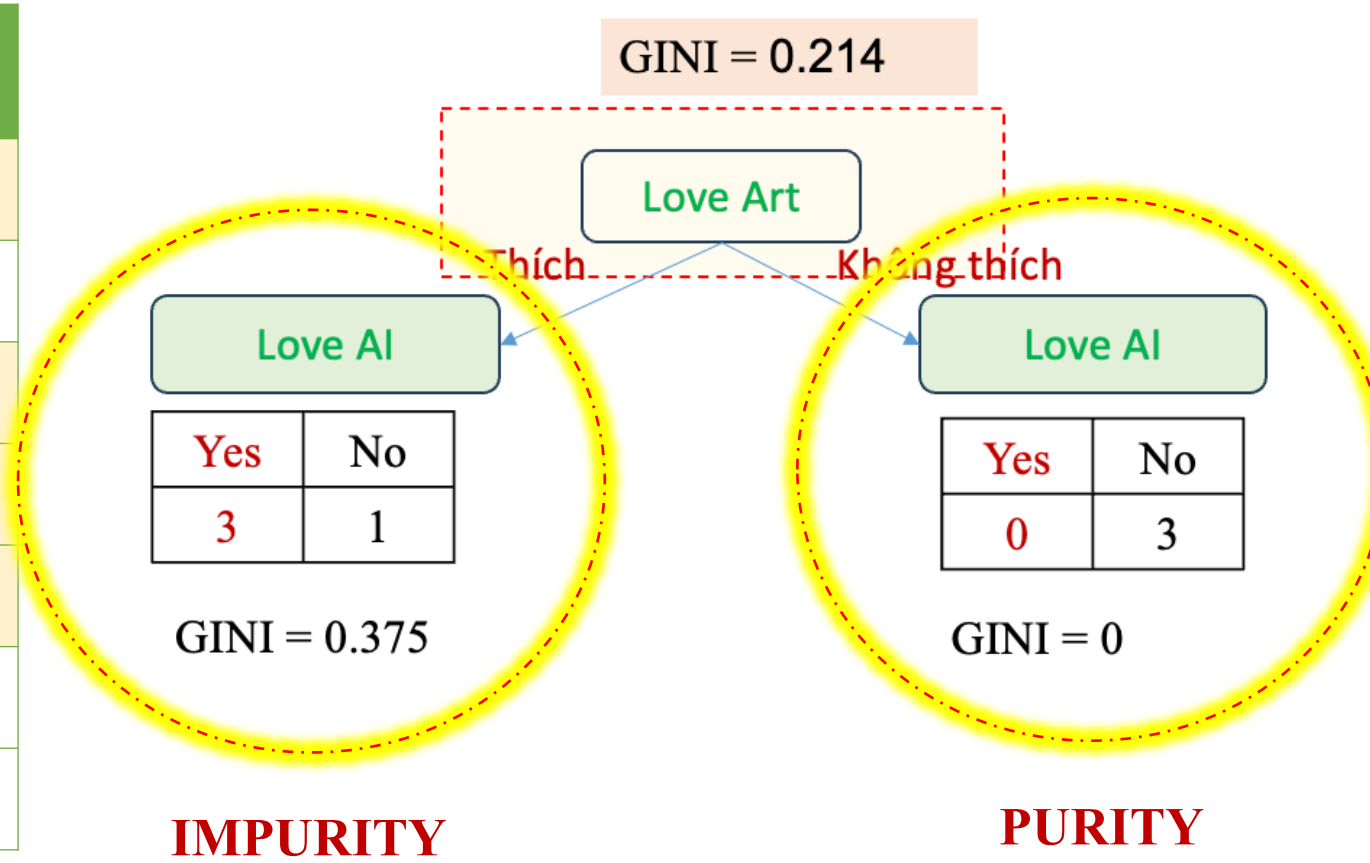



**Love Art is the best one**

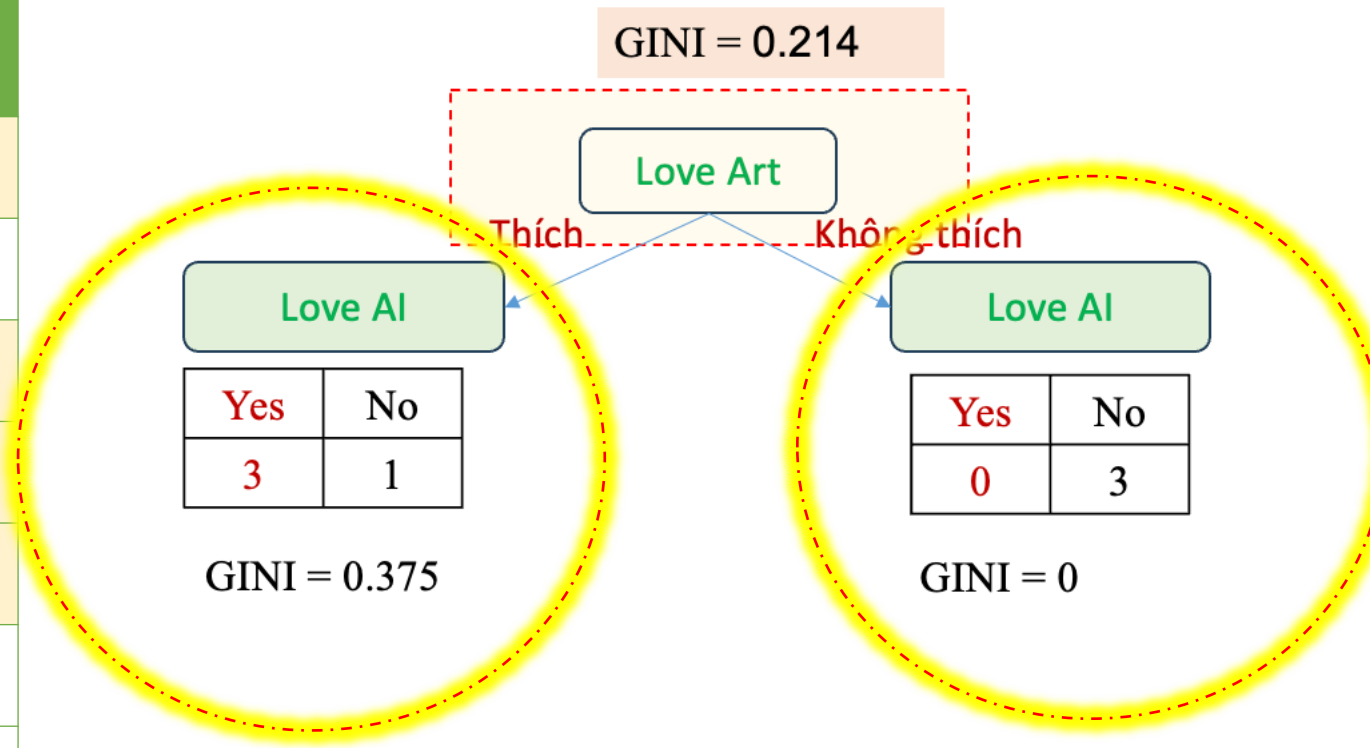
# First Node in The Tree: Love Art

| No. | Love Math | Love Art | Age | Love AI |
|---|---|---|---|---|
| 1 | Yes | Yes | 7 | No |
| 2 | Yes | No | 12 | No |
| 3 | No | Yes | 18 | Yes |
| 4 | No | Yes | 35 | Yes |
| 5 | Yes | Yes | 38 | Yes |
| 6 | Yes | No | 50 | No |
| 7 | No | No | 83 | No |



# First Node in The Tree: Love Art

| No. | Love Math | Love Art | Age | Love AI |
|---|---|---|---|---|
| 1 | Yes | Yes | 7 | No |
| 2 | Yes | No | 12 | No |
| 3 | No | Yes | 18 | Yes |
| 4 | No | Yes | 35 | Yes |
| 5 | Yes | Yes | 38 | Yes |
| 6 | Yes | No | 50 | No |
| 7 | No | No | 83 | No |

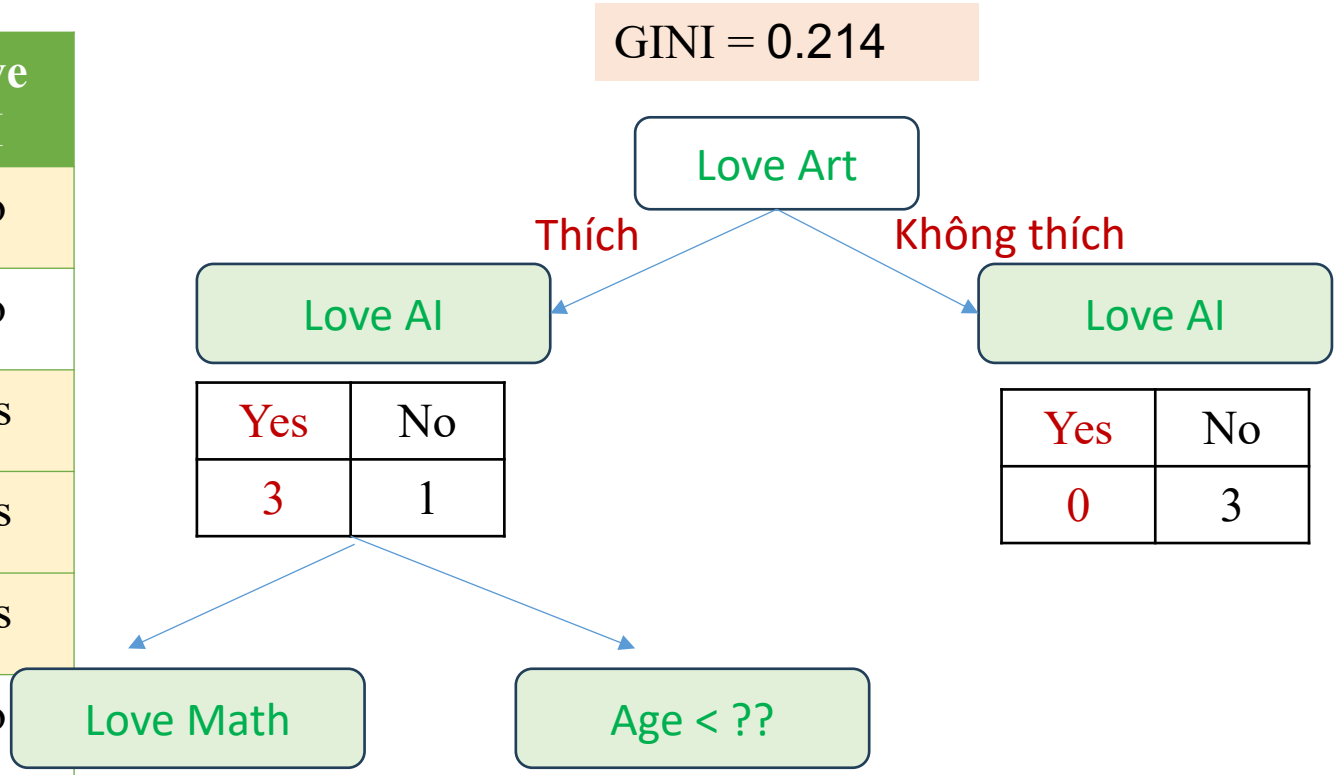



**Not easy to make decision**

**Easy to make decision**

# How to build a Tree

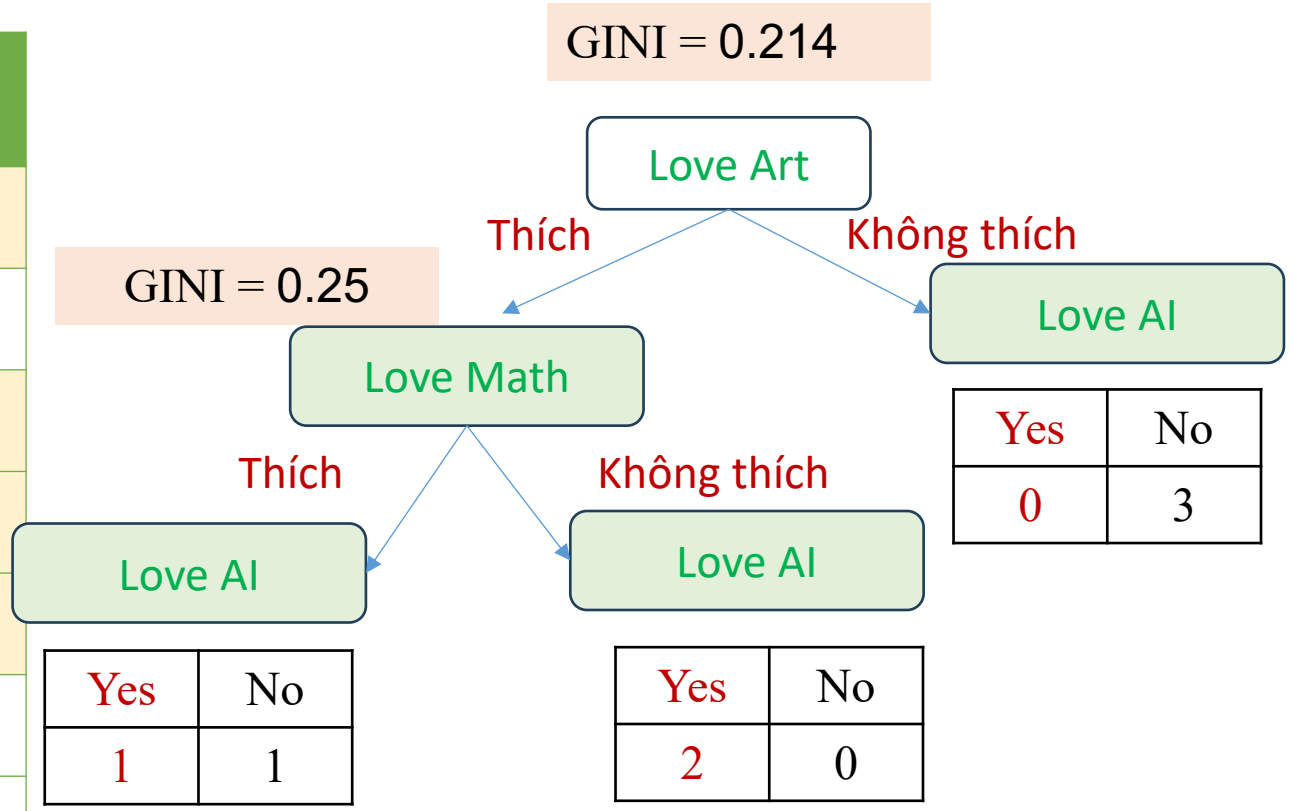
| No. | Love Math | Love Art | Age | Love AI |
|---|---|---|---|---|
| 1 | Yes | Yes | 7 | No |
| 2 | Yes | No | 12 | No |
| 3 | No | Yes | 18 | Yes |
| 4 | No | Yes | 35 | Yes |
| 5 | Yes | Yes | 38 | Yes |
| 6 | Yes | No | 50 | No |
| 7 | No | No | 83 | No |




WHICH ONE IS THE NEXT INTERNODE? Love Math or Age

# How to build a Tree

| No. | Love Math | Love Art | Age | Love AI |
|---|---|---|---|---|
| 1 | Yes | Yes | 7 | No |
| 2 | Yes | No | 12 | No |
| 3 | No | Yes | 18 | Yes |
| 4 | No | Yes | 35 | Yes |
| 5 | Yes | Yes | 38 | Yes |
| 6 | Yes | No | 50 | No |
| 7 | No | No | 83 | No |




Giả sử chúng ta chọn "Love Math" là node kế tiếp, cần tính GINI trong trường hợp này

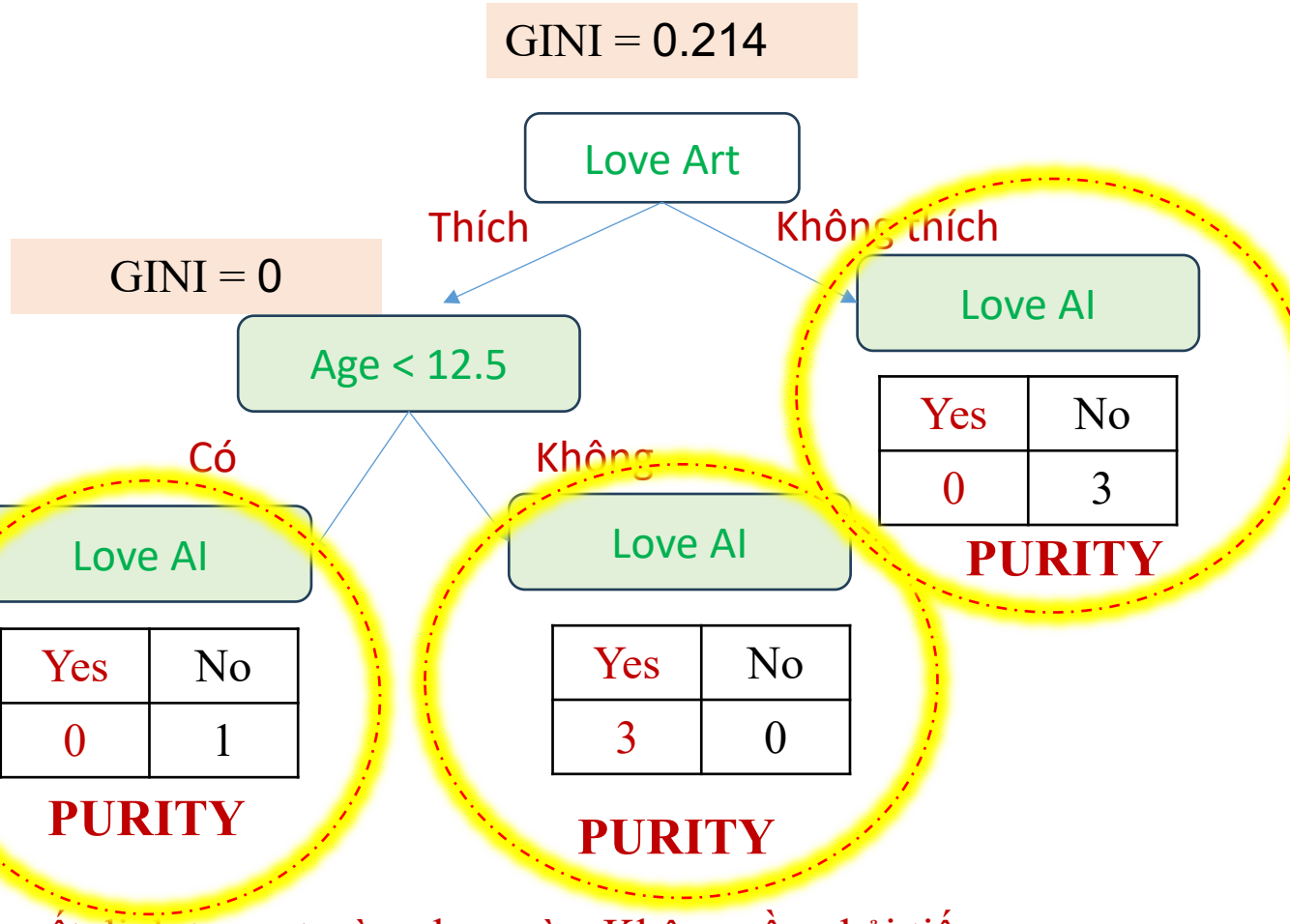
# How to build a Tree

| Love Math | Love Art | Age | Love AI |
|---|---|---|---|
| Yes | Yes | 7 | No |
| No | Yes | 18 | Yes |
| No | Yes | 35 | Yes |
| Yes | Yes | 38 | Yes |

12.5

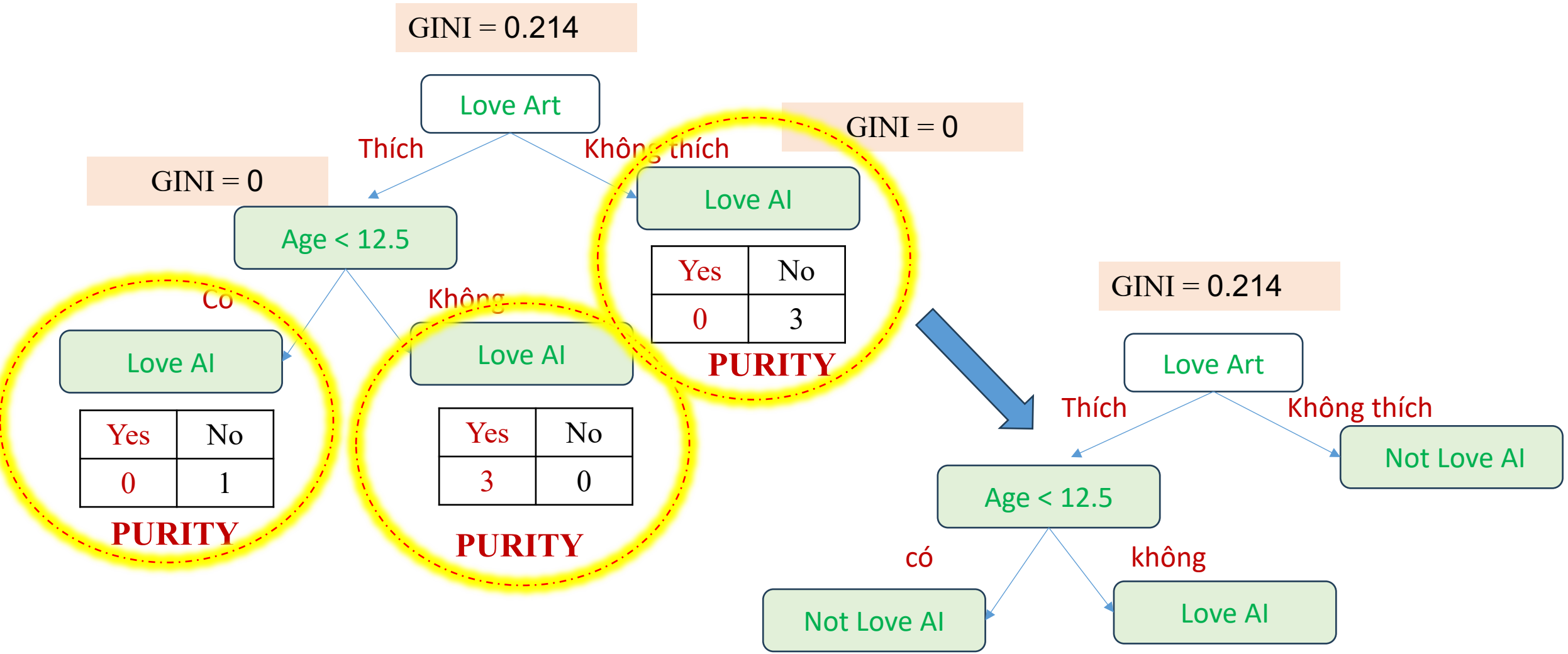
26.5

36.5




Dễ dàng đưa ra quyết định trong trường hợp này. Không cần phải tiếp tục xây dựng Tree.

# Final Tree

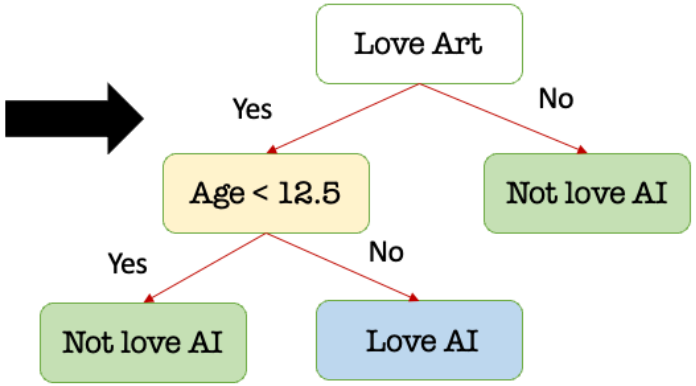

GINI = 0.214
Love Art
Thích
Không thích
GINI = 0
Age < 12.5
Love AI
Yes
No
0
3
PURITY
Có
Không
Love AI
Yes
No
0
1
PURITY
Love AI
Yes
No
3
0
PURITY
GINI = 0.214
Love Art
Thích
Không thích
Not Love AI
Age < 12.5
có
không
Not Love AI
Love AI

# Slides at The First Look

AI VIETNAM
All-in-One Course

## Decision Tree From Dataset

| Love Math | Love Art | Age | Love AI |
|---|---|---|---|
| Yes | Yes | 7 | No |
| Yes | No | 12 | No |
| No | Yes | 18 | Yes |
| No | Yes | 35 | Yes |
| Yes | Yes | 38 | Yes |
| Yes | No | 50 | No |
| No | No | 83 | No |




*The StatQuest Illustrated Guide To Machine Learning,* Josh Starmer PhD

Vinh Dinh Nguyen- PhD in Computer Science

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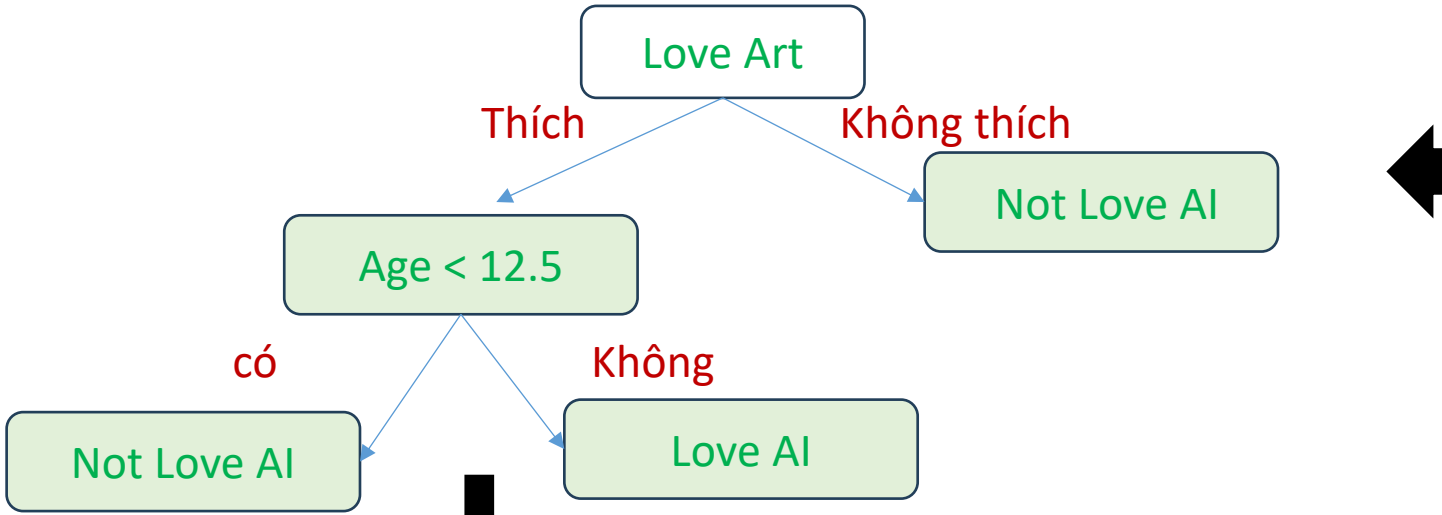
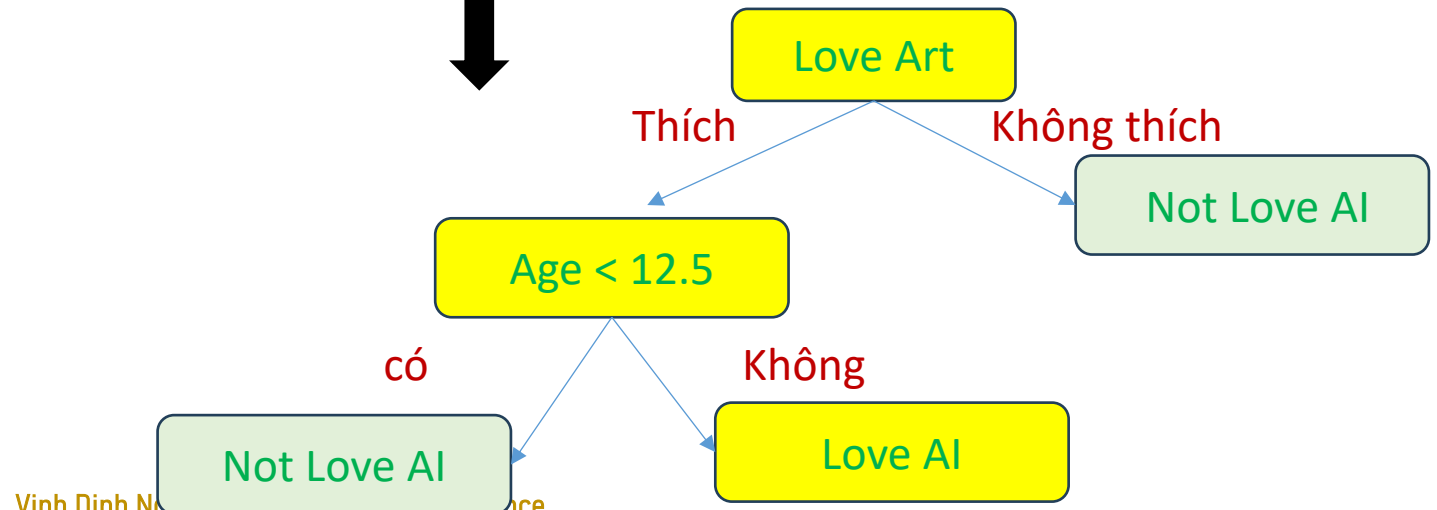
# Decision Tree: Prediction

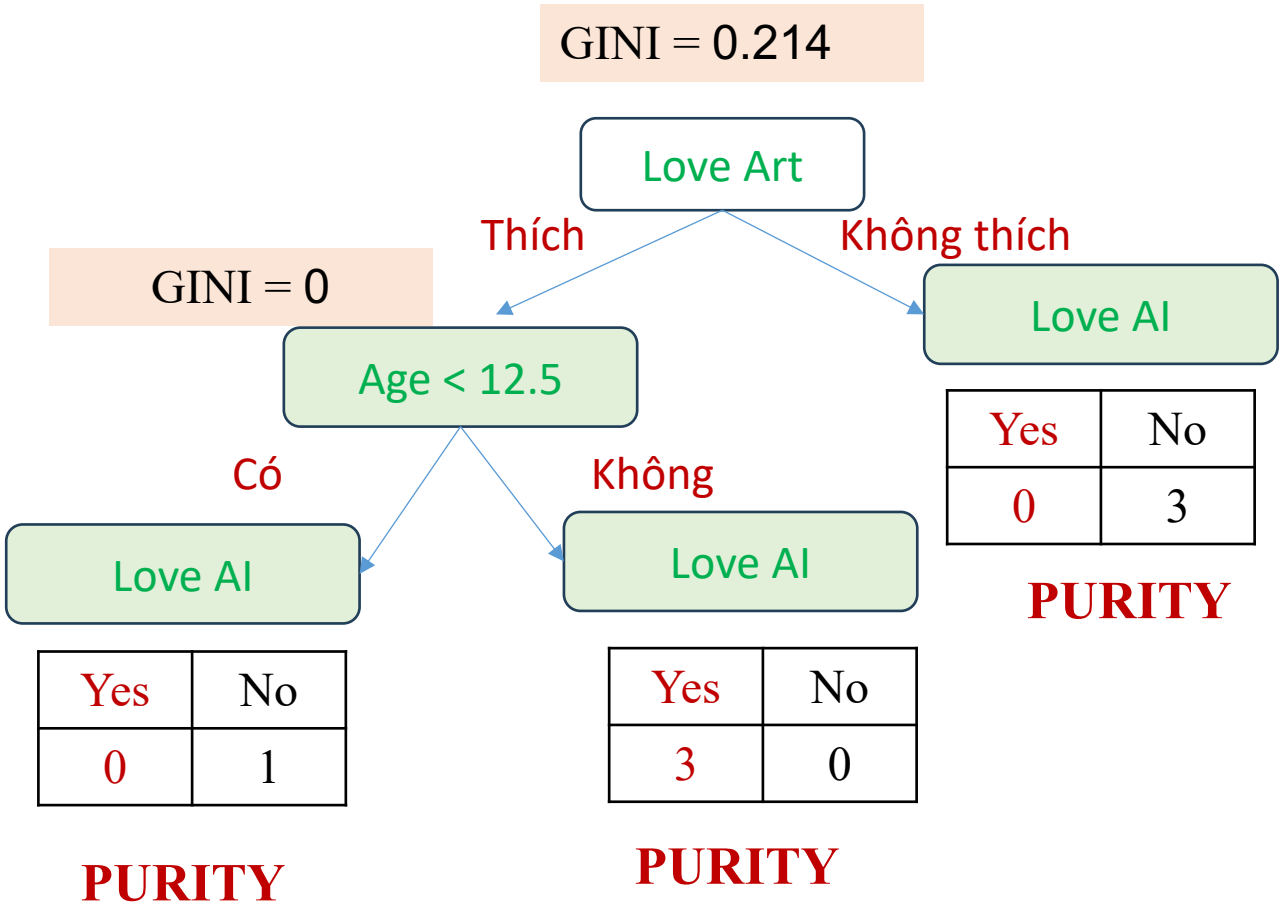



| Love Math | Love Art | Age | Love AI |
|---|---|---|---|
| Yes | Yes | 15 | ??? |




| Love Math | Love Art | Age | Love AI |
|---|---|---|---|
| Yes | Yes | 15 | Love AI |

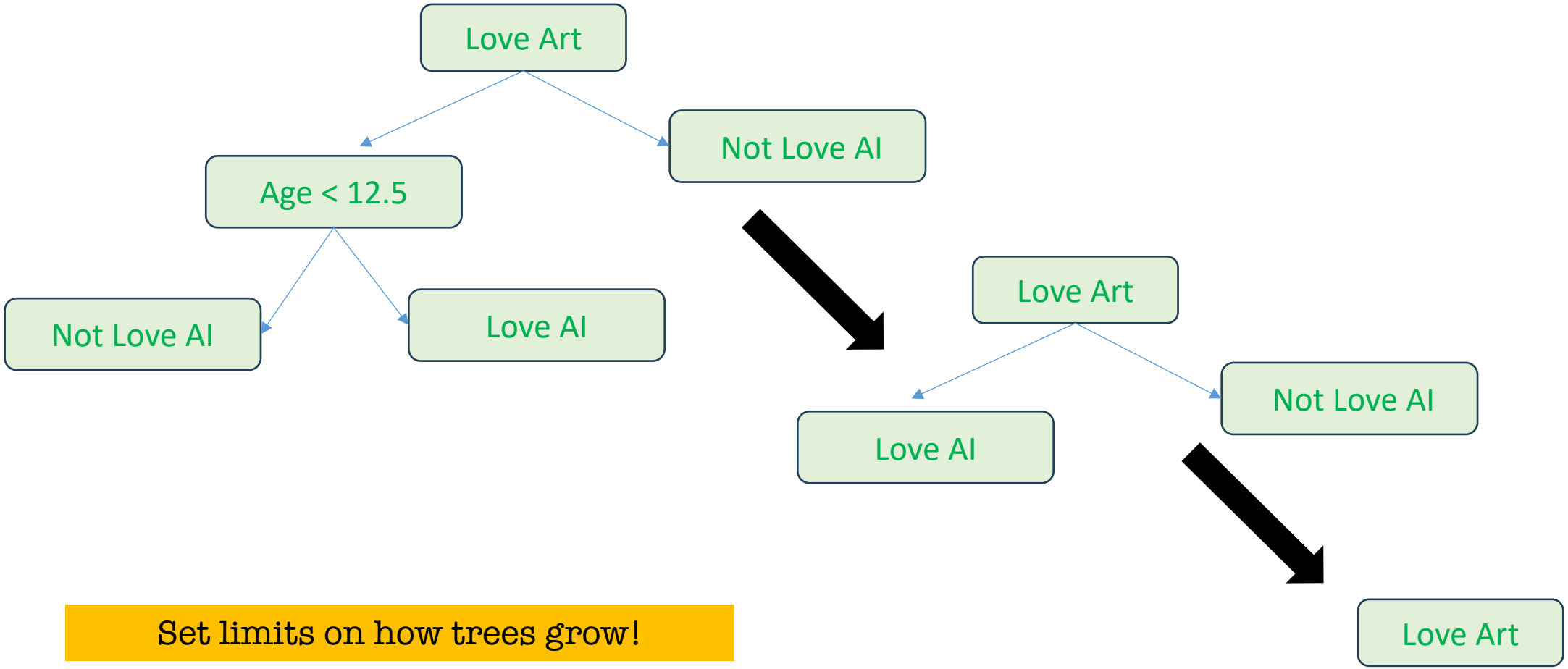
# Overfitting in Tree




## How to Reduce overfitting in Decision Tree

Set limits on how trees grow!

# Pruning Algorithm




Set limits on how trees grow!

# Outline

- Introduction to Tree
- Decision Tree
- Decision Tree with Gini
- Decision Tree with Entropy
- Several Examples

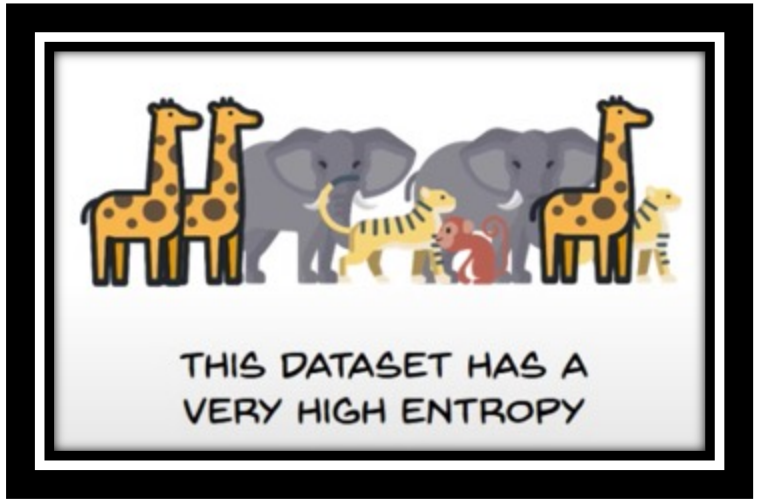
# Evaluation Metrics

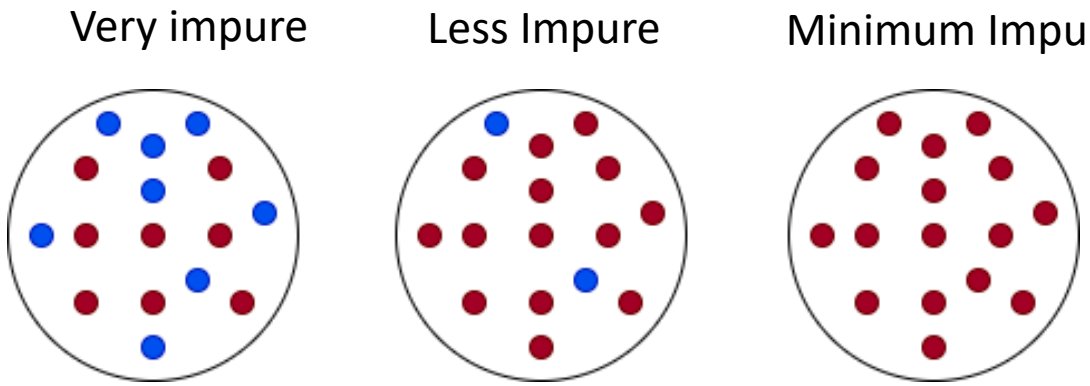
**Entropy – Information Gain**

**GNI IMPURITY**

# What is Entropy

Entropy is an information theory metric that measures the impurity or uncertainty in a group of observations (dataset)




$$E = -\sum_{i=1}^{N} p_i log_2 p_i$$

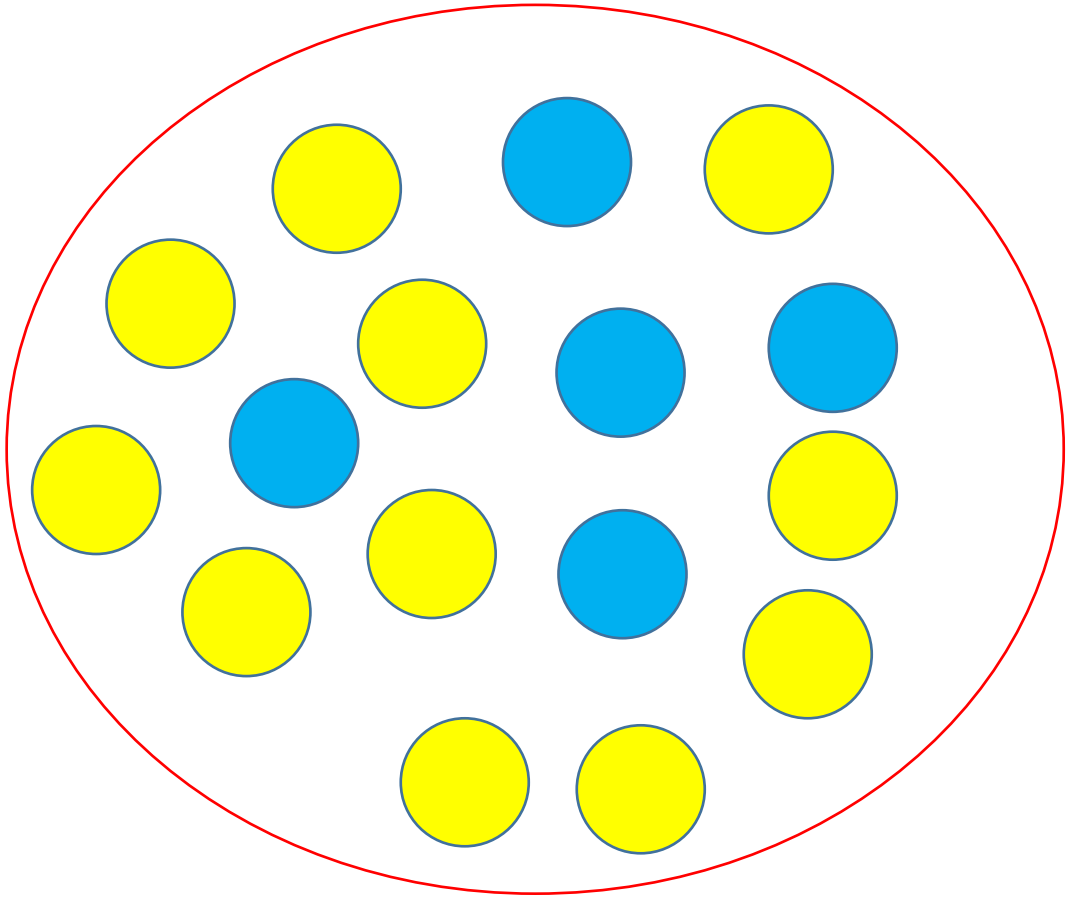
Này là gì vậy?
Nhìn rối quá

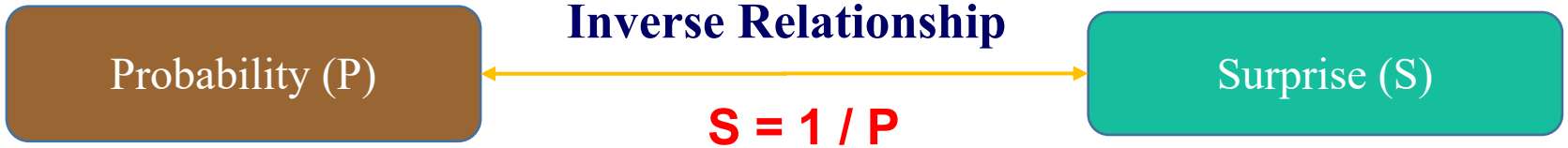
# What is Entropy

$$E = -\sum_{i=1}^{N} p_i log_2 p_i$$

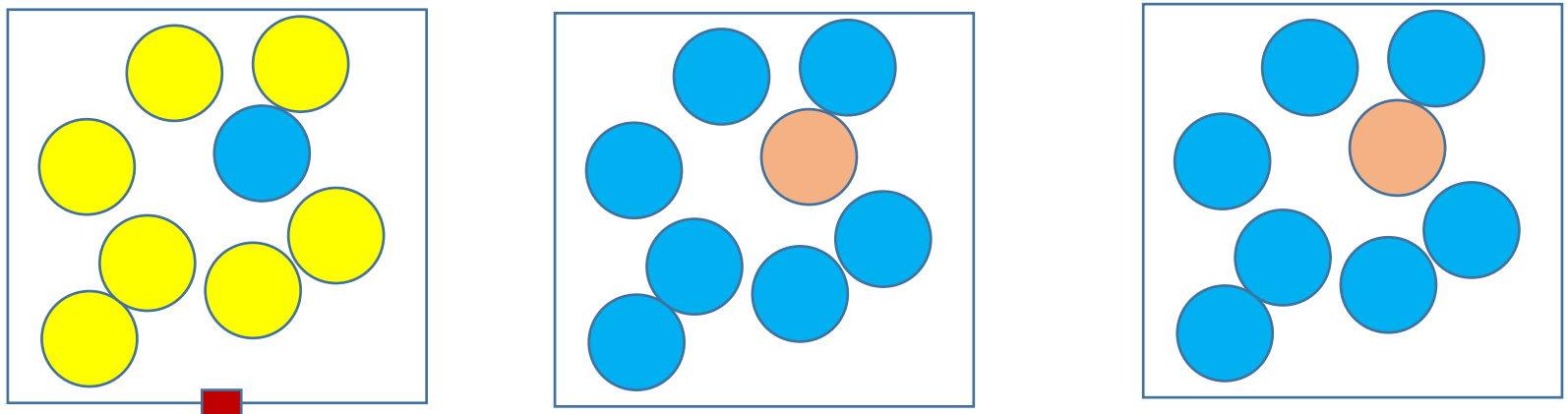
# What is Entropy

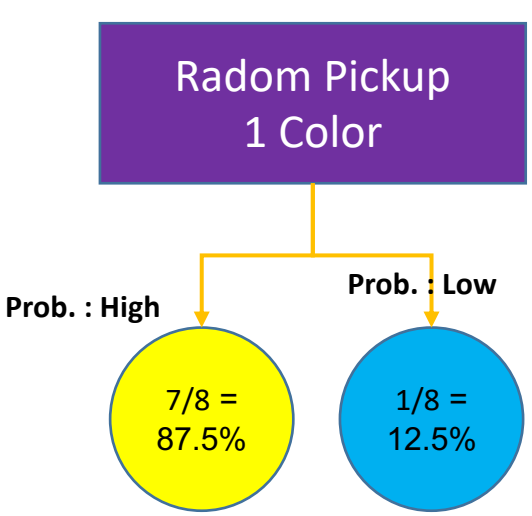
- Sample dataset





# What is Entropy






**Inverse Relationship: S = 1/P**


**For Yellow: P = 7/8 => S = 1/(7/8) = 8/7**
**For Blue: P = 1/8 => S = 1/(1/8) = 8**

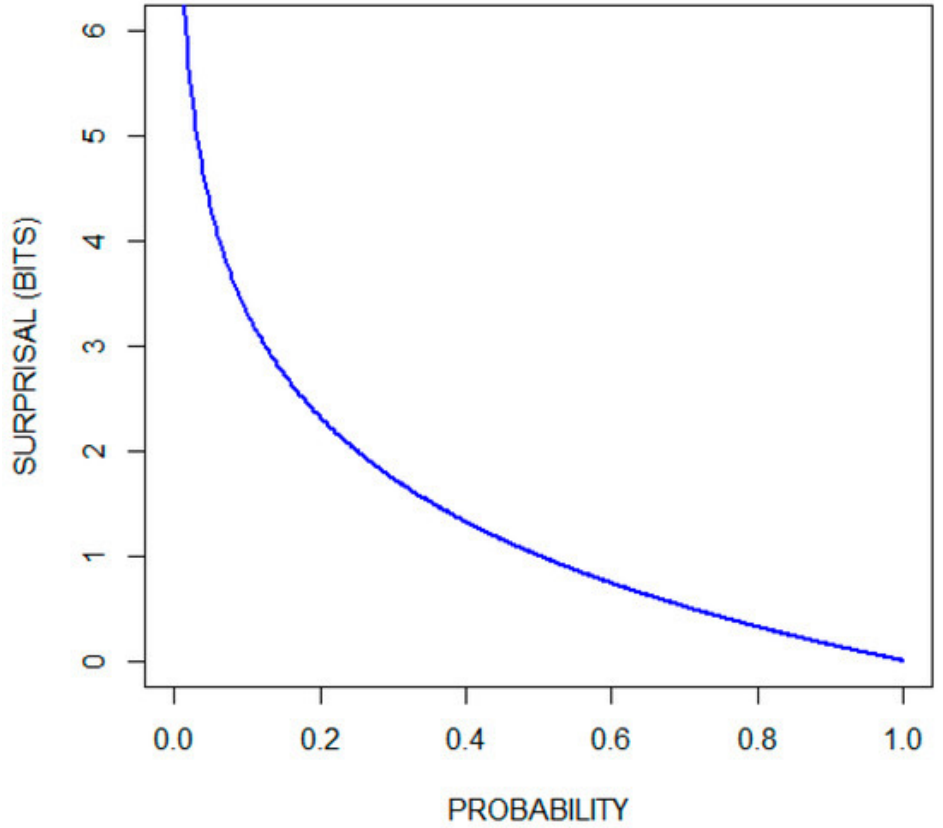
**Surprise: low** **Surprise: High**

# Probability Vs Surprise

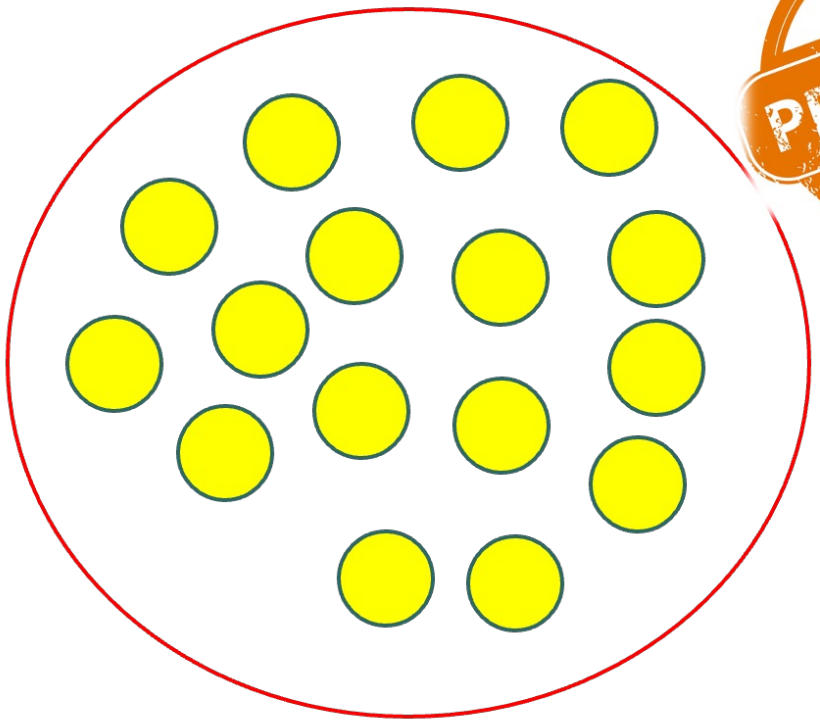
Probability (P) ⟷ Surprise (S)

**Inverse Relationship**

**S = 1 / P**

# Probability Vs Surprise




P = to get  = 1

S = to get  = 1/P = 1

S should be 0

# Logs (Logarithms)

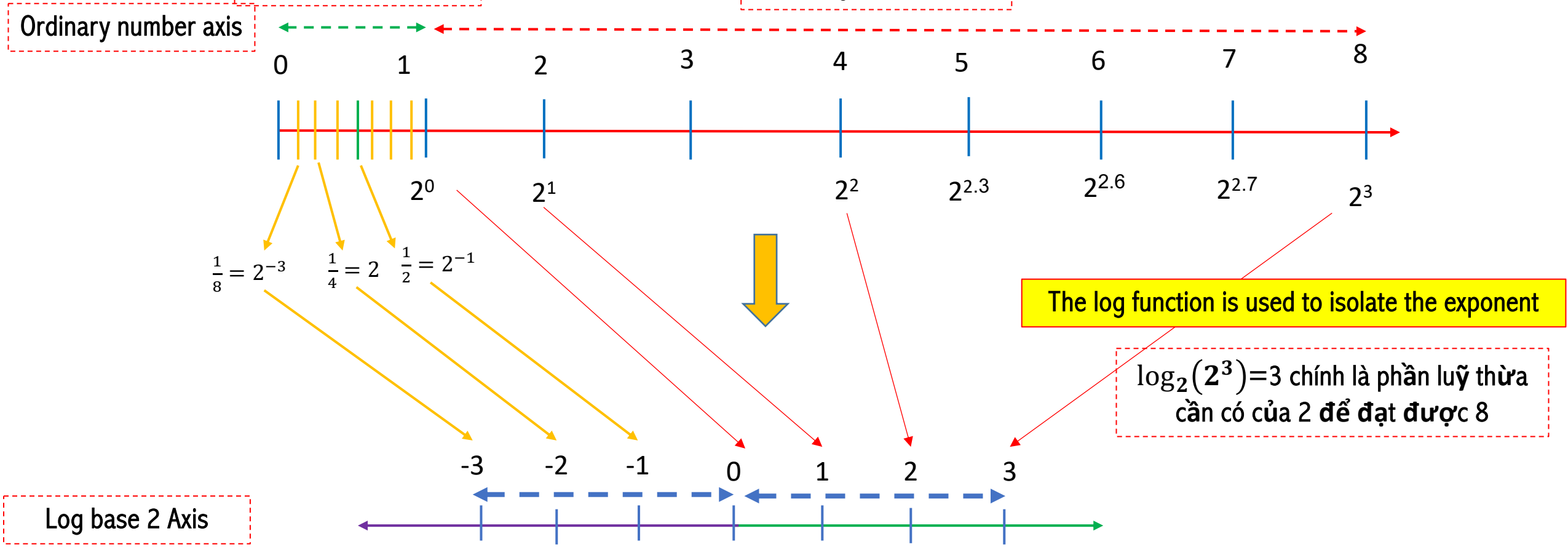
AI VIETNAM
All-in-One Course

# Logs (Logarithms)

Nhận xét: cả 2 measurement đều thể hiện độ lớn so với 1. Nhưng distance không đối xứng tại 1

8 time less than 1

8 time greater than 1

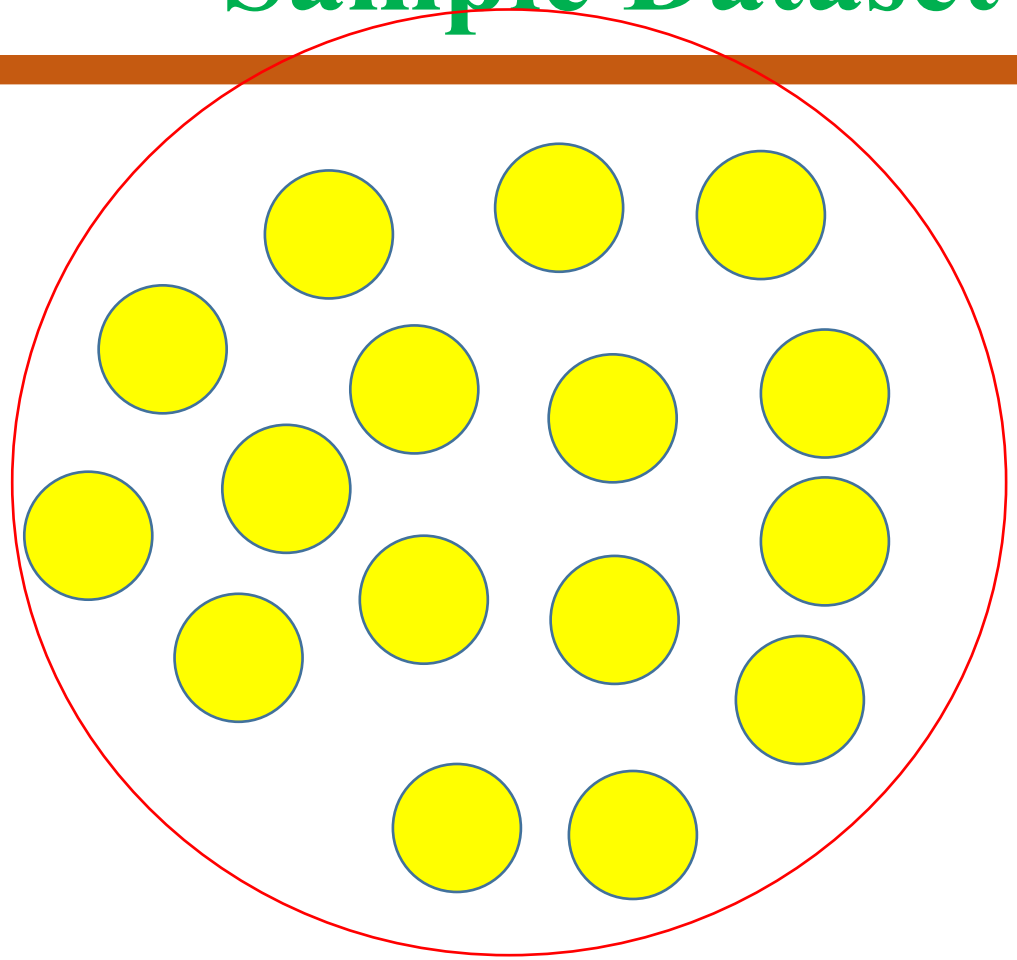


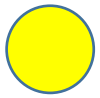

The log function is used to isolate the exponent

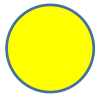
$\log_2(2^3)=3$ chính là phần luỹ thừa cần có của 2 để đạt được 8

Nhận xét: cả 2 measurement đều thể hiện độ lớn so với 1. Nhưng distance đối xứng tại 1

# Sample Dataset

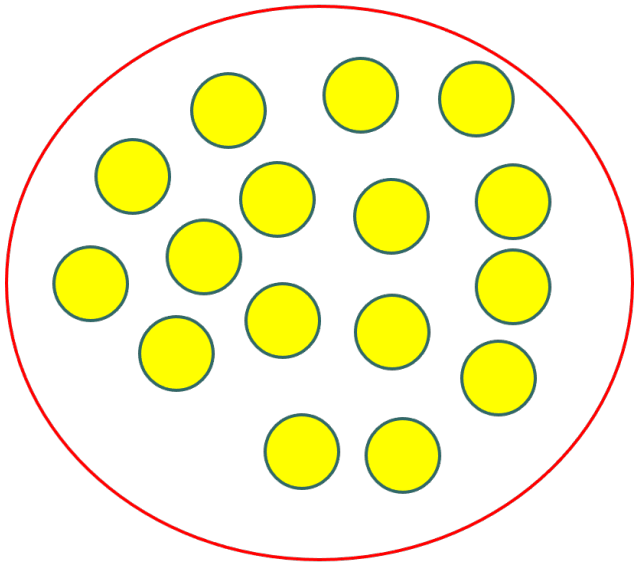


P = to get  = 1

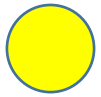
S = to get  = 1/P = 1 

**We use the log of 1/P to calculate S**

# Sample Dataset



P = to get  = 1

S = to get  = log2(1/1) = 0

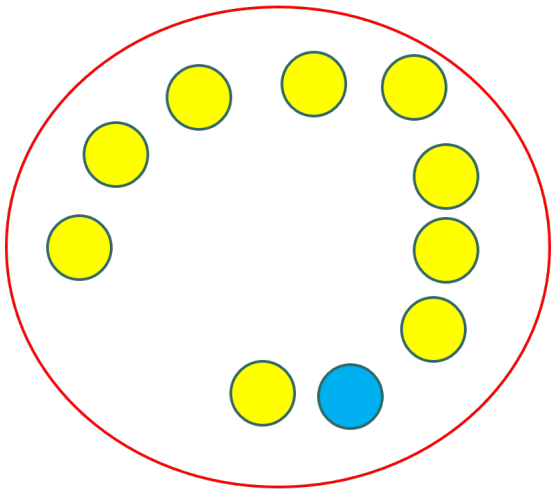
No surprise

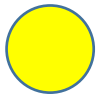
P = to get  = 0

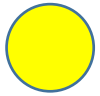
S = to get  = log2(1/0) = log2(1) – log2(0) = Undefined

Big surprise

# Sample Dataset



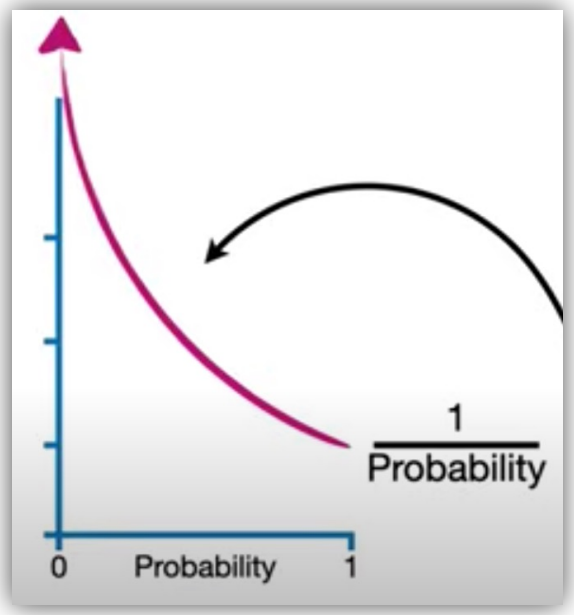
P = to get  = 0.9

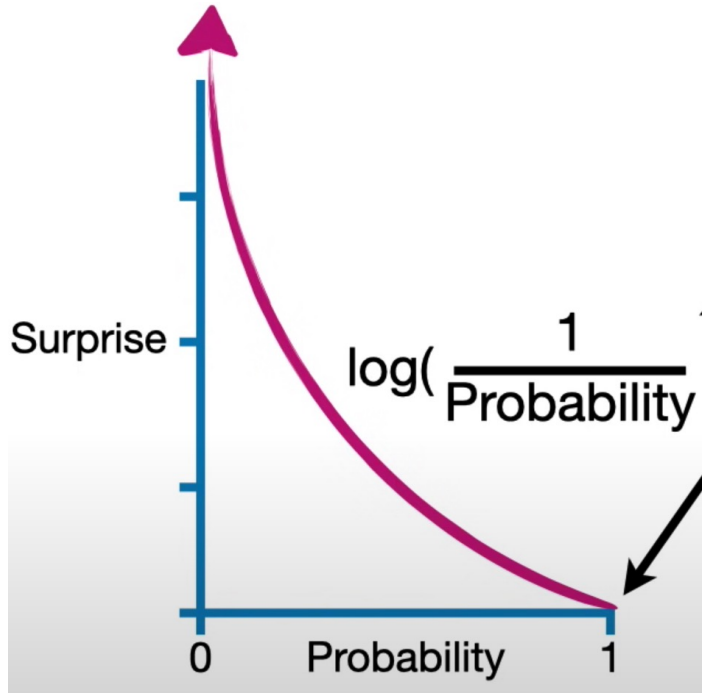
S = to get  = log2(1/0.9) = 0.15

P = to get  = 0.1

S = to get  = log2(1/0.1) = 3.32

# Probability vs Surprise



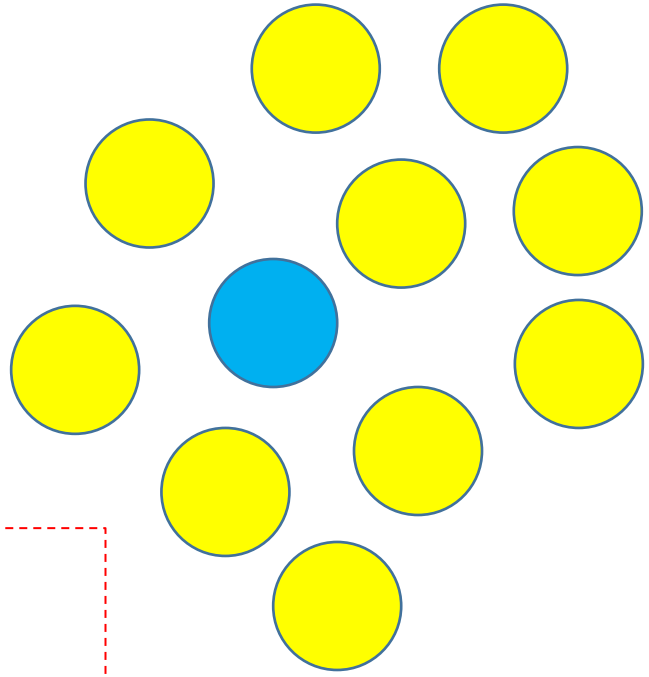




$$S = \log_2( 1 / P)$$

# Sample Dataset

P to get = 9/10

P to get = 1/10



S to get = $\log_2( 1 / 0.9) = 0.15$

S to get = $\log_2( 1 / 0.1) = 3.32$

$$\log_2 \frac{1}{0.9\times0.9\times0.1} = \log_2 1 - \log_2(0.9\times0.9\times0.1)$$
$$= 0 - \log_2(0.9\times0.9\times0.1)$$
$$= 0 - \log_2(0.9) - \log_2(0.9) - \log_2(0.1)$$
$$= 0.15 + 0.15 + 3.32$$

Gợi nhớ!

If two events A and B are independent , then the probability of happening of both A and B is:

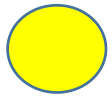
$P ( A \cap B ) = P(A). P(B)$

Flip the coin 3 times

**P to get** = **0.9 * 0.9 * 0.1**

**S to get** = **S** + **S** + **S**

# Sample Dataset

| |  |  |
|---|---|---|
| Probability | 9/10 | 1/10 |
| Surprise | 0.15 | 3.32 |

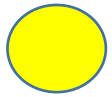
**HOW MUCH SURPRISE**
**FROM GETTING YELLOW COLORS IN 100 TIMES**

(0.9*100) * 0.15

**Expected number of yellows**

**Total surprise from getting Yellow**

# Sample Dataset

| |  |  |
|---|---|---|
| Probability | 9/10 | 1/10 |
| Surprise | 0.15 | 3.32 |

**HOW MUCH SURPRISE**
**FROM GETTING BLUE COLORS IN 100 TIMES**

(0.1*100) * 3.32

**Expected number of blues**

**Total surprise from getting Blue**

# AVERAGE SURPRISE FOR 100 TIMES

S_B = HOW MUCH SURPRISE FROM GETTING BLUE COLORS IN 100 TIMES

(0.1*100) * 3.32

+

S_Y = HOW MUCH SURPRISE FROM GETTING YELLOW COLORS IN 100 TIMES

(0.9*100) * 0.15

$$E = -\sum_{i=1}^{N} p_i log_2 p_i$$

How ?

Total Surprise= S_B + S_Y = (0.1 x 100) x 3.32 + (0.9 x 100) x 0.15 = 46.7

Average = Total Surprise / 100 = (S_B + S_Y = (0.1 x 100) x 3.32 + (0.9 x 100) x 0.15 ) / 100 = 0.467

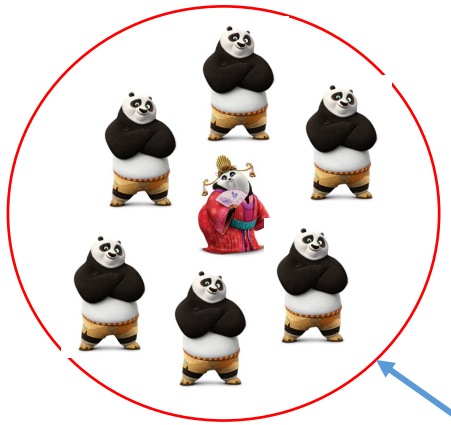
Entropy = Total Surprise / 100 = (0.1 x 3.32) + (0.9 x 0.15 ) = 0.467

$p(x)$ $\log(\frac{1}{p(x)})$

$$\text{Entropy} = \sum \log(\frac{1}{p(x)}) p(x)$$

Surprise — The probability of the Surprise.

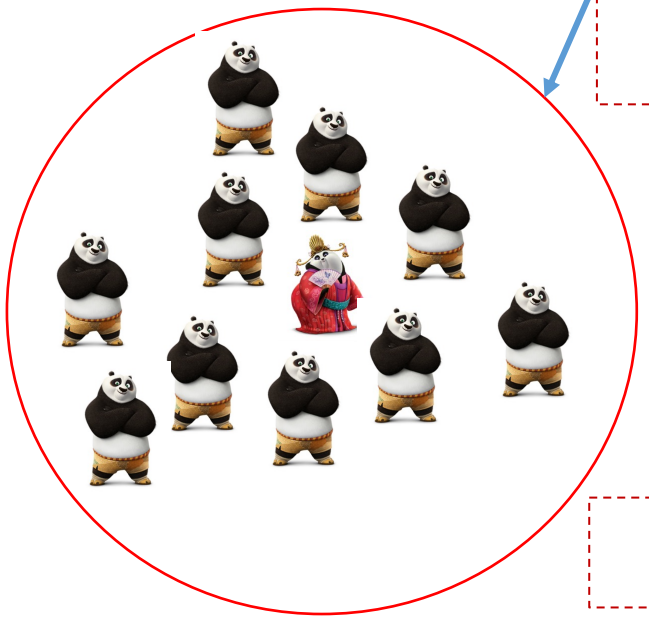
# Sample Dataset



$$\text{Entropy} = \sum \; p(x)\log\left(\frac{1}{p(x)}\right) = \frac{6}{7}\log_2\frac{7}{6} + \frac{1}{7}\log_2 7$$

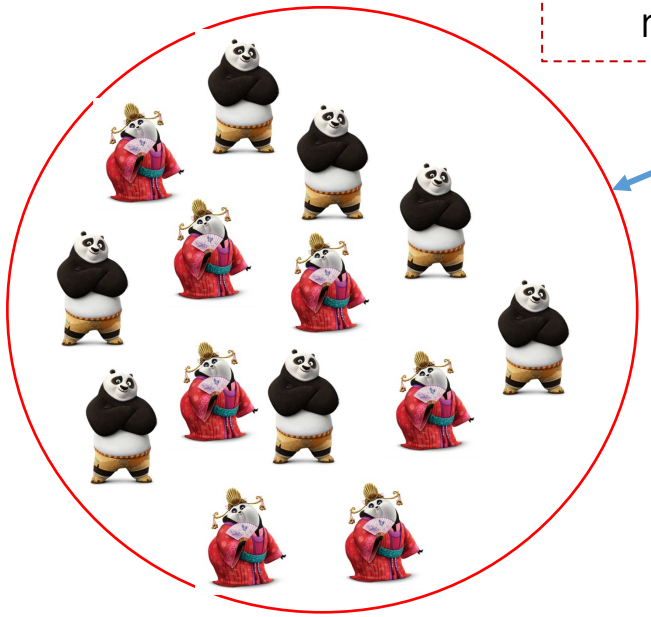
**Entropy = 0.59**

When we increase the difference in the number of male and femal panda. Entropy is lower



**Entropy = 0.44**

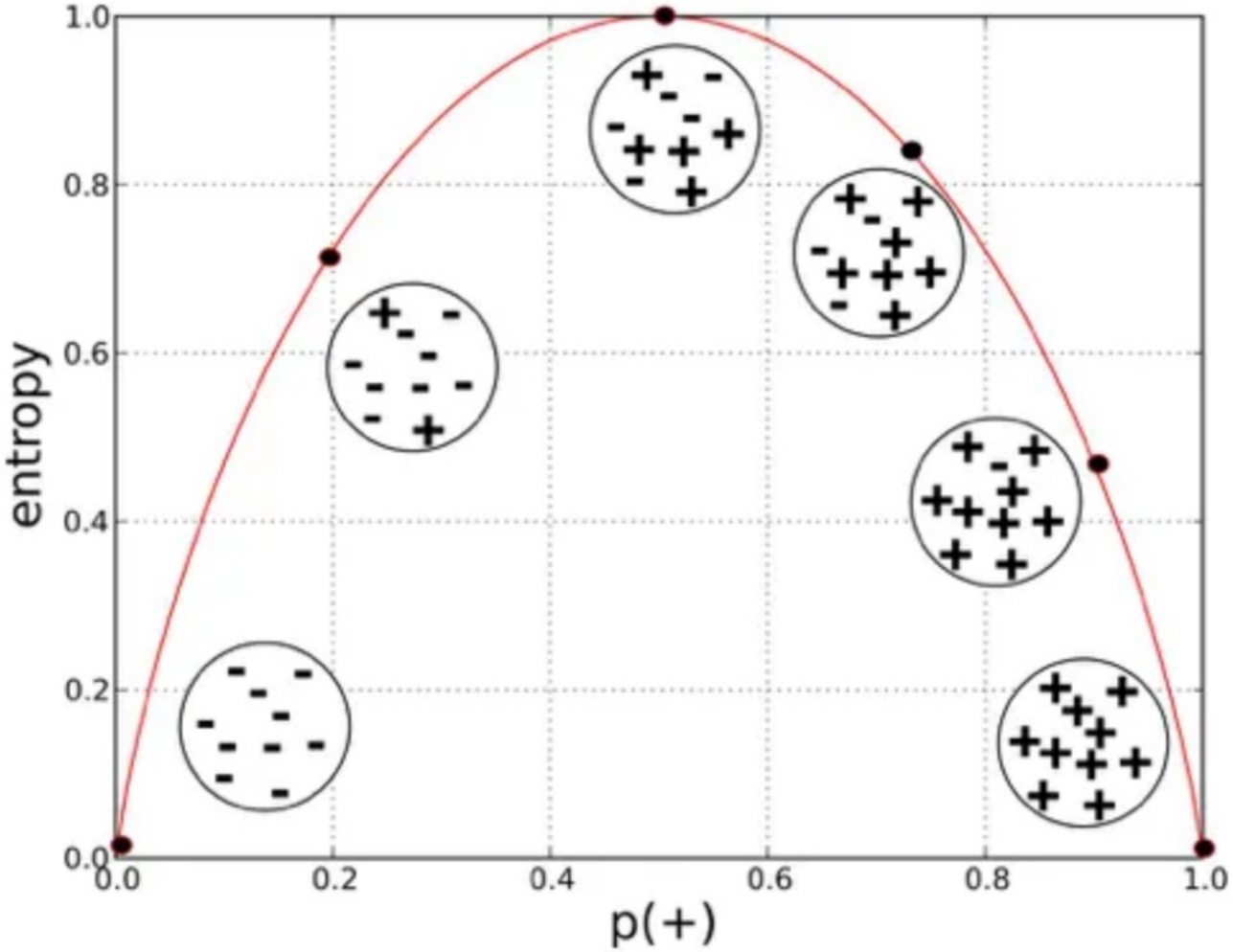
Entropy is highest when we have same number of female and male panda



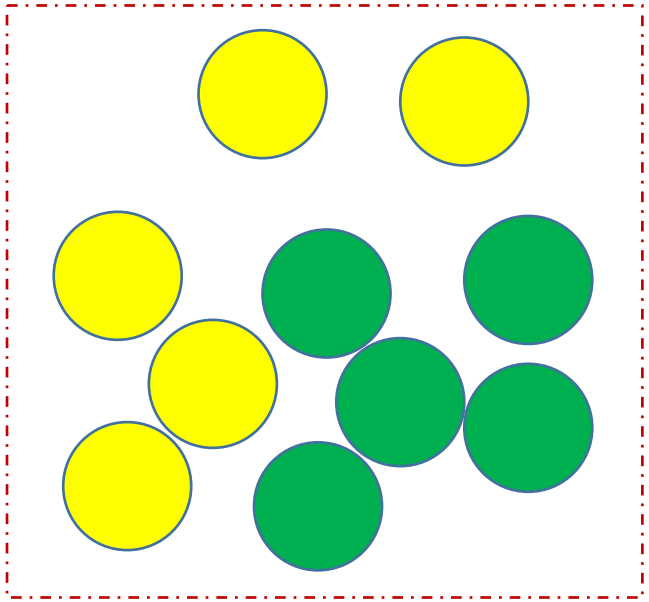
**Entropy = 1.0**

Entropy is used to quantify the similarity or difference in the dataset

# Entropy and Probability

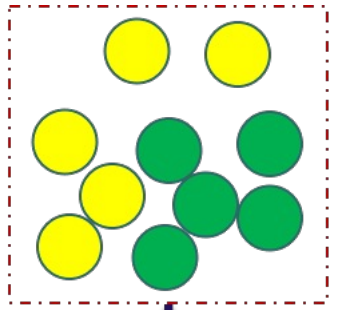


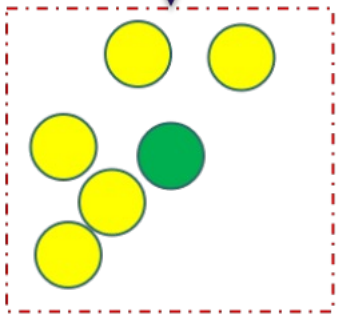
# Sample Dataset



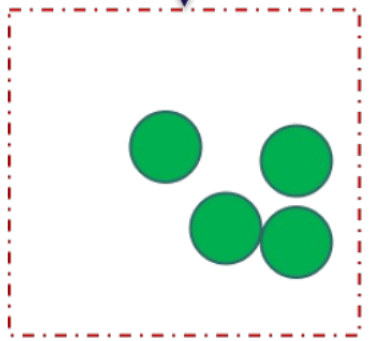
$$E_{initial} = -((0.5log_20.5) + (0.5log_20.5))$$

$$= 1$$



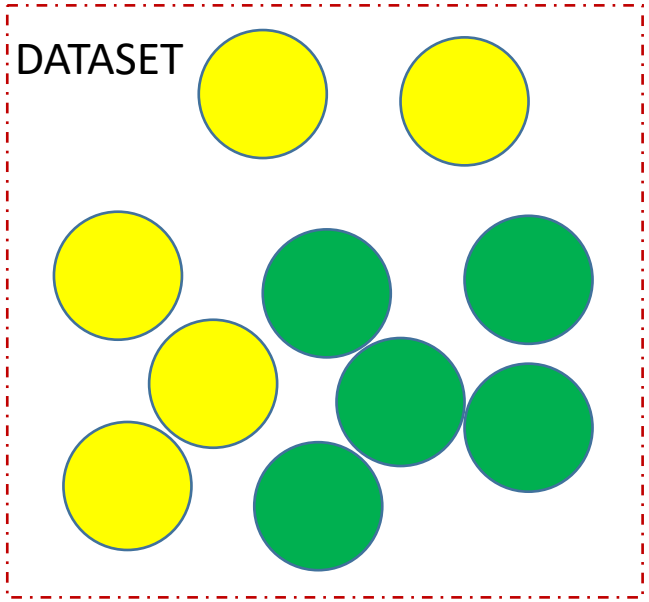


E= 0.65



E= 0

# Information Gain (IG)



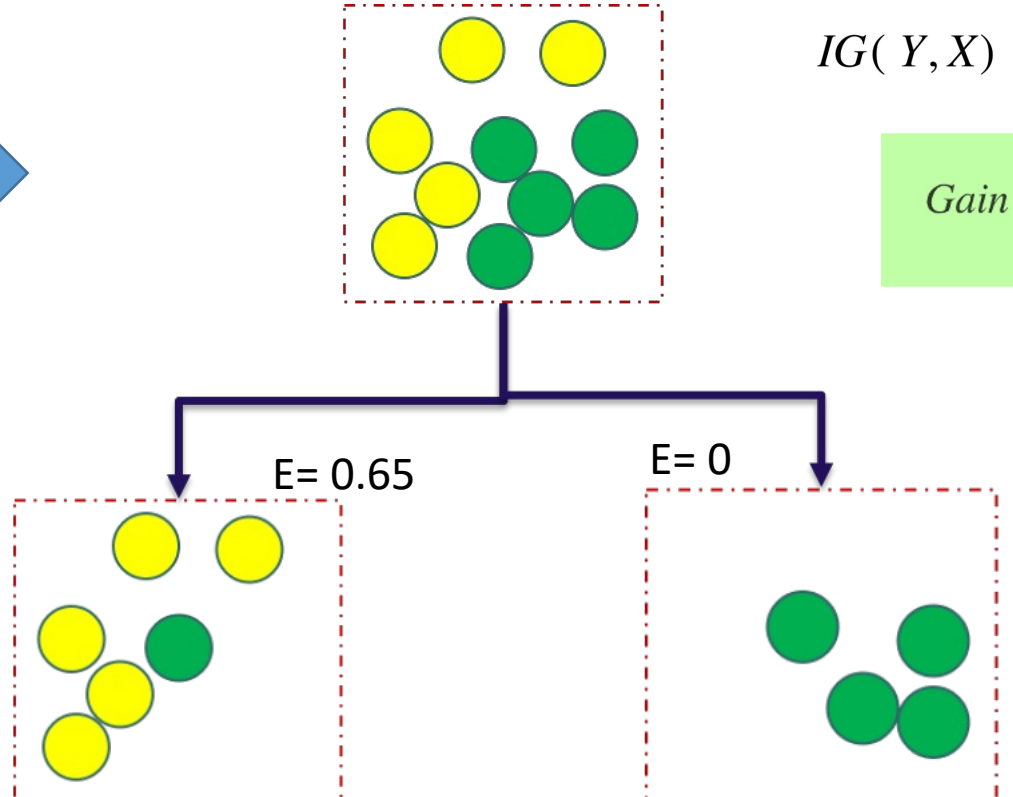

Maximum the information gain

$$E_{initial} = -((0.5log_20.5) + (0.5log_20.5))$$

$$= 1$$

$$IG(Y, X) = E(Y) - E(Y|X)$$

$$Gain = E_{parent} - E_{children}$$




$E_{\_after\_slit} = 0.65 * 0.6 + 0.4 * 0 = 0.39$

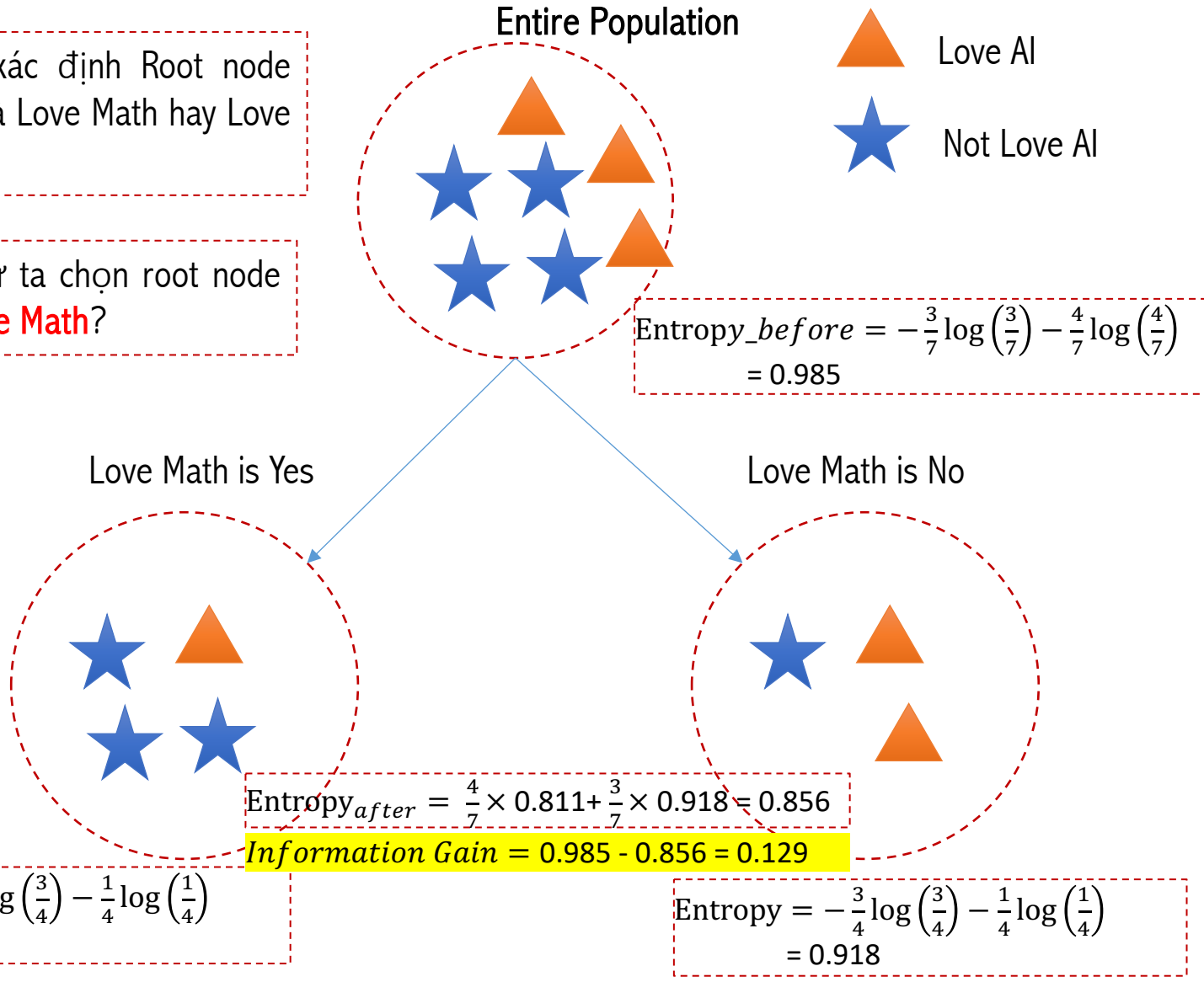
$Gain = 0.61 = E_{initial} - E_{\_after\_slit}$

# Example

| No. | Love Math | Love Art | Love AI |
|---|---|---|---|
| 1 | Yes | Yes | No |
| 2 | Yes | No | No |
| 3 | No | Yes | Yes |
| 4 | No | Yes | Yes |
| 5 | Yes | Yes | Yes |
| 6 | Yes | No | No |
| 7 | No | No | No |

Cần xác định Root node nên là Love Math hay Love Art?

Giả sử ta chọn root node là Love Math?

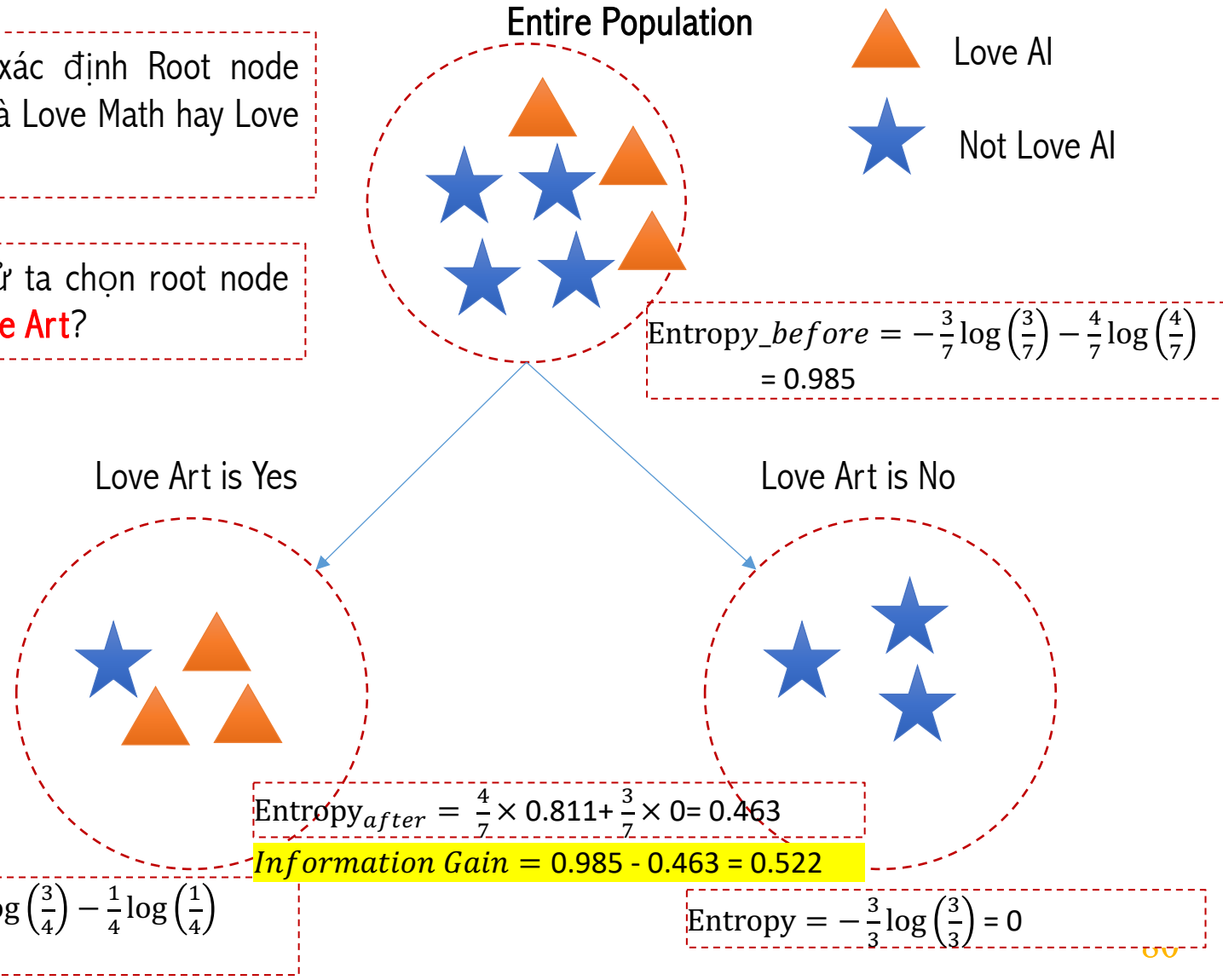


# Example

| No. | Love Math | Love Art | Love AI |
|---|---|---|---|
| 1 | Yes | Yes | No |
| 2 | Yes | No | No |
| 3 | No | Yes | Yes |
| 4 | No | Yes | Yes |
| 5 | Yes | Yes | Yes |
| 6 | Yes | No | No |
| 7 | No | No | No |

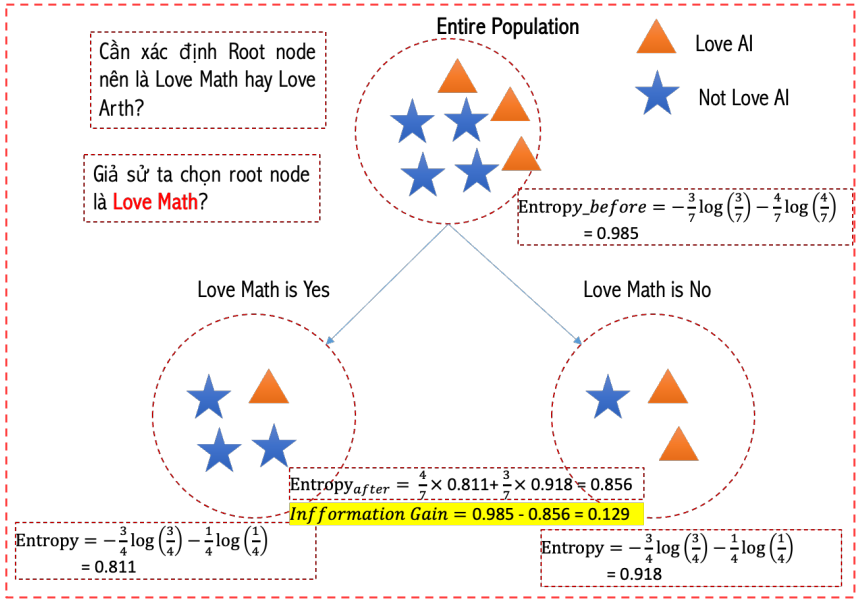
Cần xác định Root node nên là Love Math hay Love Art?

Giả sử ta chọn root node là Love Art?



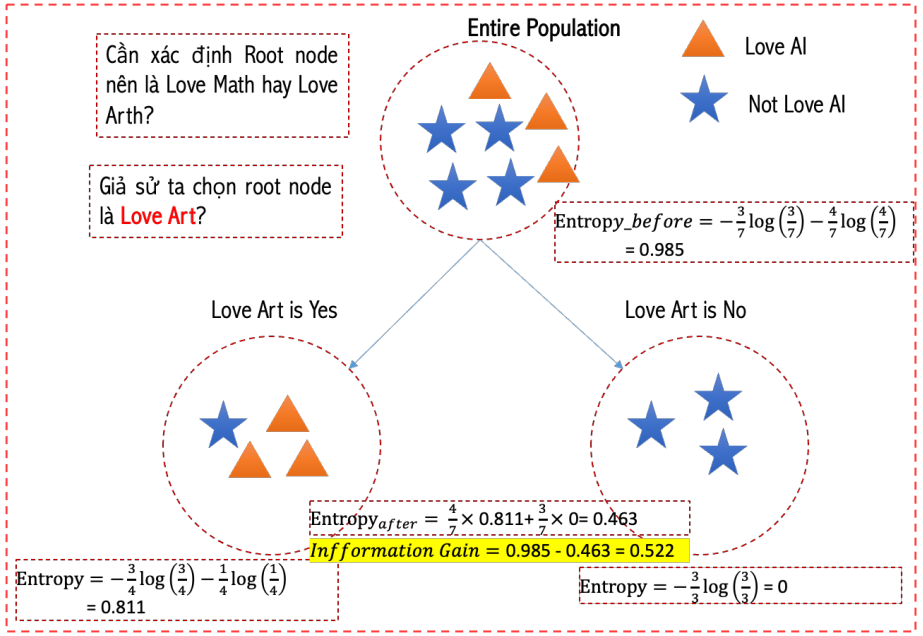
# Example

## Love Math is a Root Node




$Information\ Gain$ = 0.129

## Love Art is a Root Node



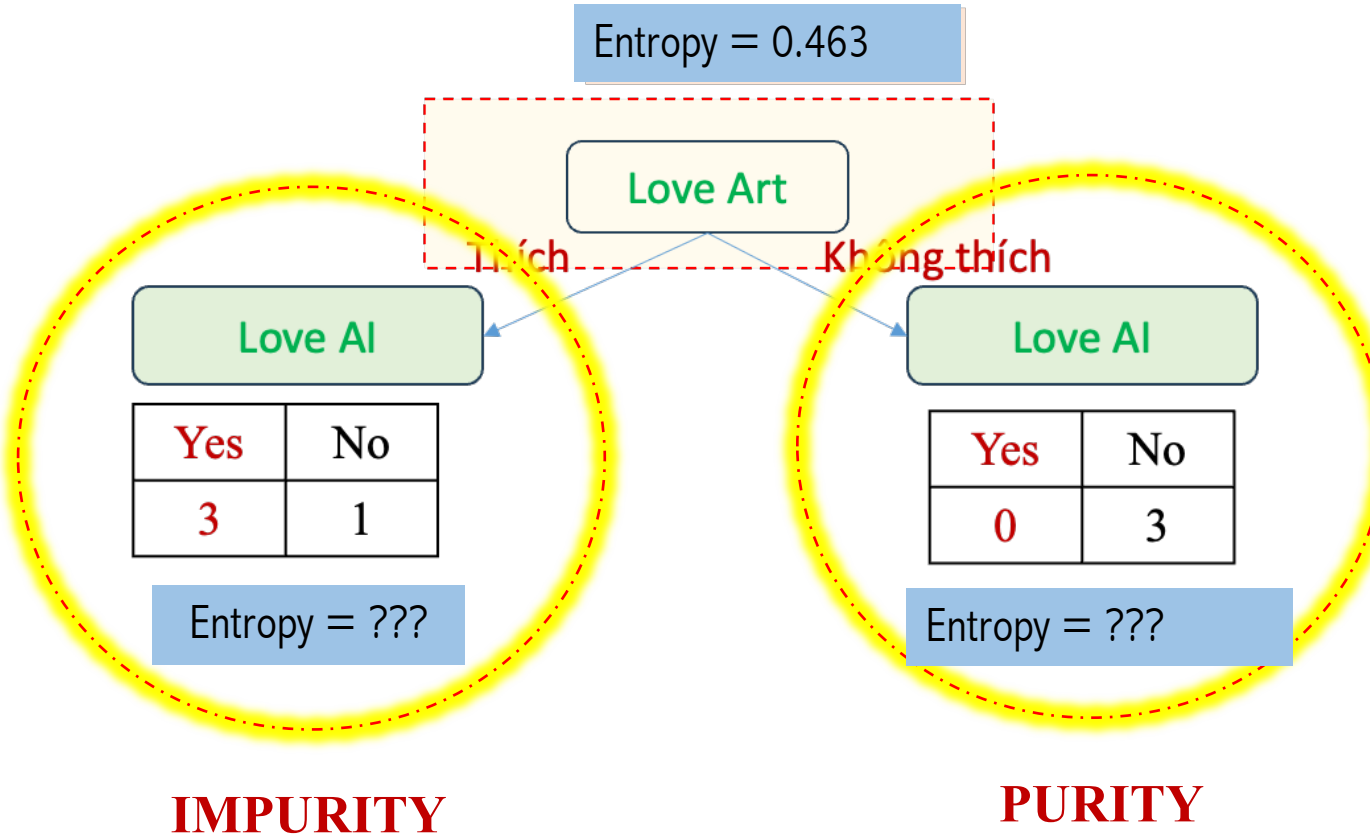

Infformation Gain = 0.522

Which attribute is in the first node?

Maximum the information gain

# Example

| No. | Love Math | Love Art | Love AI |
|---|---|---|---|
| 1 | Yes | Yes | No |
| 2 | Yes | No | No |
| 3 | No | Yes | Yes |
| 4 | No | Yes | Yes |
| 5 | Yes | Yes | Yes |
| 6 | Yes | No | No |
| 7 | No | No | No |




Continue to expand this tree using Entropy metric

# Outline

- Introduction to Tree
- Decision Tree
- Decision Tree with Gini
- Decision Tree with Entropy
- Several Examples

# Example

| | age | income | student | credit_rate | Default |
|---|---|---|---|---|---|
| 0 | youth | high | no | fair | no |
| 1 | youth | high | no | excellent | no |
| 2 | middle_age | high | no | fair | yes |
| 3 | senior | medium | no | fair | yes |
| 4 | senior | low | yes | fair | yes |
| 5 | senior | low | yes | excellent | no |
| 6 | middle_age | low | yes | excellent | yes |
| 7 | youth | medium | no | fair | no |
| 8 | youth | low | yes | fair | yes |
| 9 | senior | medium | yes | | |
| 10 | youth | medium | yes | ex | |
| 11 | middle_age | medium | no | ex | |
| 12 | middle_age | high | yes | | |
| 13 | senior | medium | no | ex | |

This data set is used to predict whether a person will default on their credit card. There are two classes ( default = 'yes', no_default = 'no' ):

```
# Defining a simple dataset
attribute_names =  ['age', 'income','student', 'credit_rate']
class_name = 'default'
data1 ={
    'age' : ['youth', 'youth', 'middle_age', 'senior', 'senior', 'senior','middle_age', 'youth', 'youth', 'senior',
    'income' : ['high', 'high', 'high', 'medium', 'low', 'low', 'low', 'medium','low', 'medium', 'medium', 'medium',
    'student' : ['no','no','no','no','yes','yes','yes','no','yes','yes','yes','no','yes','no'],
    'credit_rate' : ['fair', 'excellent', 'fair', 'fair', 'fair', 'excellent', 'excellent', 'fair', 'fair', 'fair','
    'default' : ['no', 'no', 'yes', 'yes', 'yes', 'no', 'yes', 'no', 'yes', 'yes','yes', 'yes', 'yes', 'no']
}
df1 = pd.DataFrame (data1, columns=data1.keys())
```

# Example

```
# STEP 1: Calculate gini(D)
def gini_impurity (value_counts):
    n = value_counts.sum()
    p_sum = 0
    for key in value_counts.keys():
        p_sum = p_sum  +  (value_counts[key] / n ) * (value_counts[key] / n )
    gini = 1 - p_sum
    return gini

class_value_counts = df1[class_name].value_counts()
print(f'Number of samples in each class is:\n{class_value_counts}')

gini_class = gini_impurity(class_value_counts)
print(f'\nGini Impurity of the class is {gini_class:.3f}')
```

```
Number of samples in each class is:
yes    9
no     5
Name: default, dtype: int64

Gini Impurity of the class is 0.459
```

# Example

```
# STEP 2:
# Calculating  gini impurity for the attiributes
def gini_split_a(attribute_name):
    attribute_values = df1[attribute_name].value_counts()
    gini_A = 0
    for key in attribute_values.keys():
        df_k = df1[class_name][df1[attribute_name] == key].value_counts()
        n_k = attribute_values[key]
        n = df1.shape[0]
        gini_A = gini_A + (( n_k / n) * gini_impurity(df_k))
    return gini_A

gini_attiribute ={}
for key in attribute_names:
    gini_attiribute[key] = gini_split_a(key)
    print(f'Gini for {key} is {gini_attiribute[key]:.3f}')
```

```
Gini for age is 0.343
Gini for income is 0.440
Gini for student is 0.367
Gini for credit_rate is 0.429
```

# Example

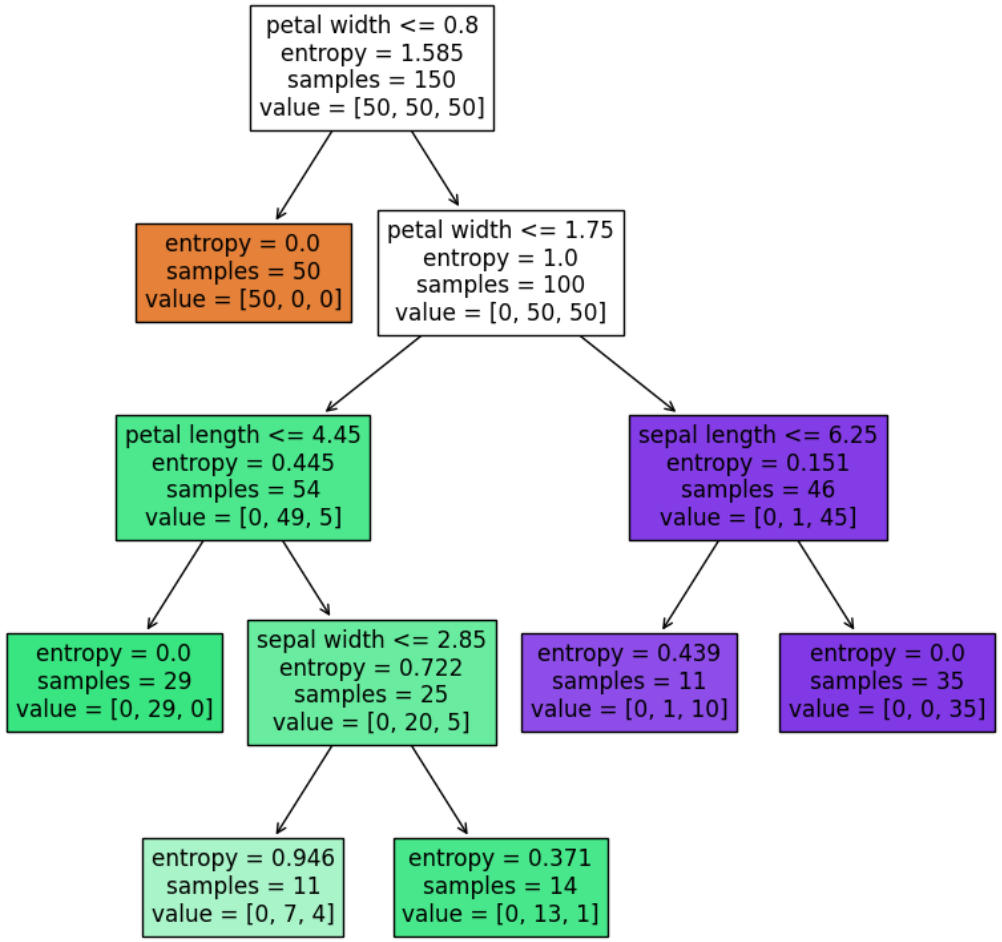
```
# STEP 3:
# Compute Gini gain values to find the best split
# An attribute has maximum Gini gain is selected for splitting.

min_value = min(gini_attiribute.values())
print('The minimum value of Gini Impurity : {0:.3} '.format(min_value))
print('The maximum value of Gini Gain     : {0:.3} '.format(1-min_value))

selected_attribute = min(gini_attiribute.keys())
print('The selected attiribute is: ', selected_attribute)
```

```
The minimum value of Gini Impurity : 0.343
The maximum value of Gini Gain     : 0.657
The selected attiribute is   :  age
```

# Iris Flower Classification

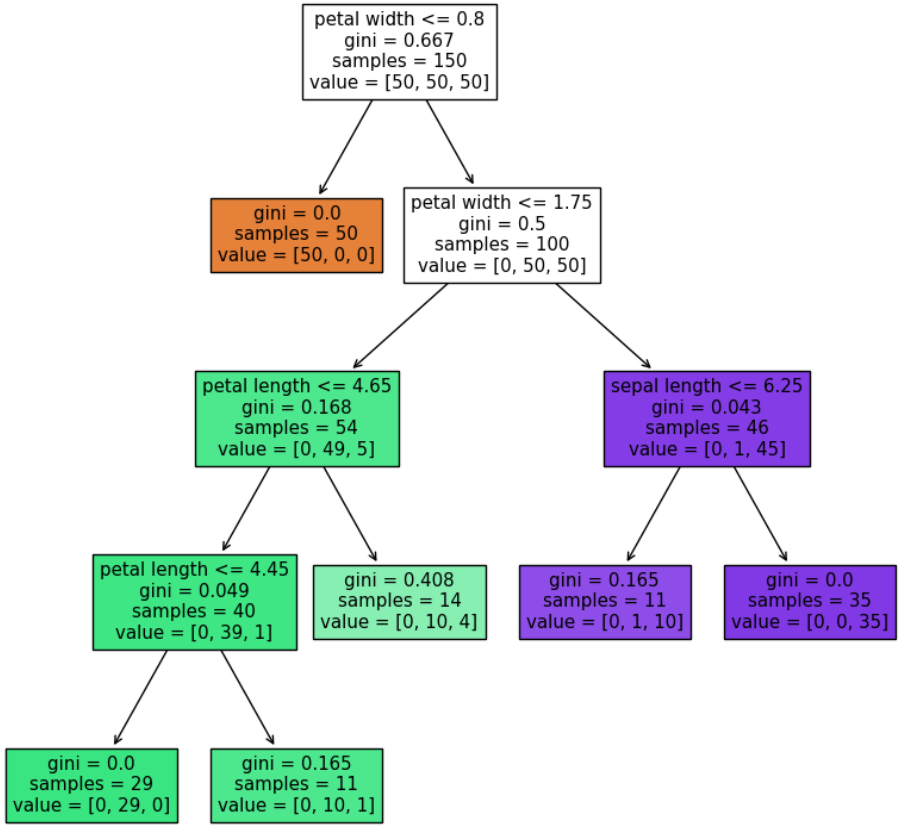

| | Sepal length | Sepal width | Petal length | Petal width | Class |
|---|---|---|---|---|---|
| 1 | 5.1 | 3.5 | 1.4 | 0.2 | setosa |
| 2 | 4.9 | 3.0 | 1.4 | 0.2 | setosa |
| 3 | 4.7 | 3.2 | 1.3 | 0.2 | setosa |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 150 | 5.9 | 3.0 | 5.1 | 1.8 | virginica |

# Iris Flower Classification (Entropy)




```
dataset = load_iris()
X = dataset.data
y = dataset.target

classifier = tree.DecisionTreeClassifier(criterion="entropy",
                                         max_depth=4, min_samples_leaf=10)
classifier.fit(X,y)
fig, ax = plt.subplots(figsize=(10,10))
tree.plot_tree(classifier,ax=ax, feature_names=["sepal length", "sepal width",
                                                "petal length", "petal width"],
                                                 filled=True)

plt.show()
```

# Iris Flower Classification (Gini)




```
dataset = load_iris()
X = dataset.data
y = dataset.target

classifier = tree.DecisionTreeClassifier(criterion="gini",
                                         max_depth=4, min_samples_leaf=10)
classifier.fit(X,y)
fig, ax = plt.subplots(figsize=(10,10))
tree.plot_tree(classifier,ax=ax, feature_names=["sepal length", "sepal width",
                                                "petal length", "petal width"],
                                                 filled=True)

plt.show()
```