AI VIET NAM
@aivietnam.edu.vn

Text Classification with Mamba - Project

TA Minh-Duc Bui
STA Khai-Xuan Trinh

Outline

1. Motivation
2. State Space Models
3. Mamba
4. Coding

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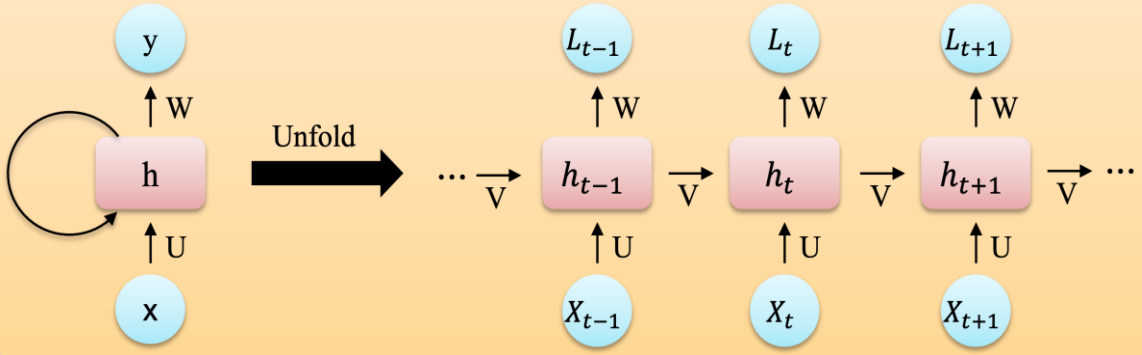
RNNs vs. Transformers

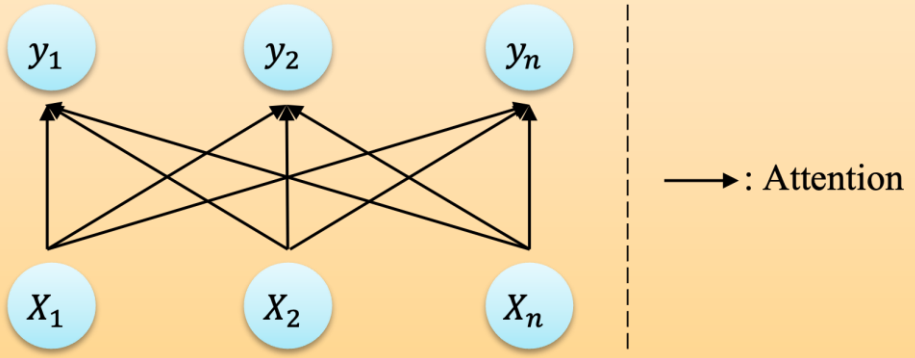
RNNs

- train stage: $O(N)$, **non-parallelable**
- inference stage: $O(N)$
- $O(1)$ per token



Transformers

- train stage: $O(N^2)$, **parallelable**
- inference stage: $O(N^2)$
- $O(N)$ per token

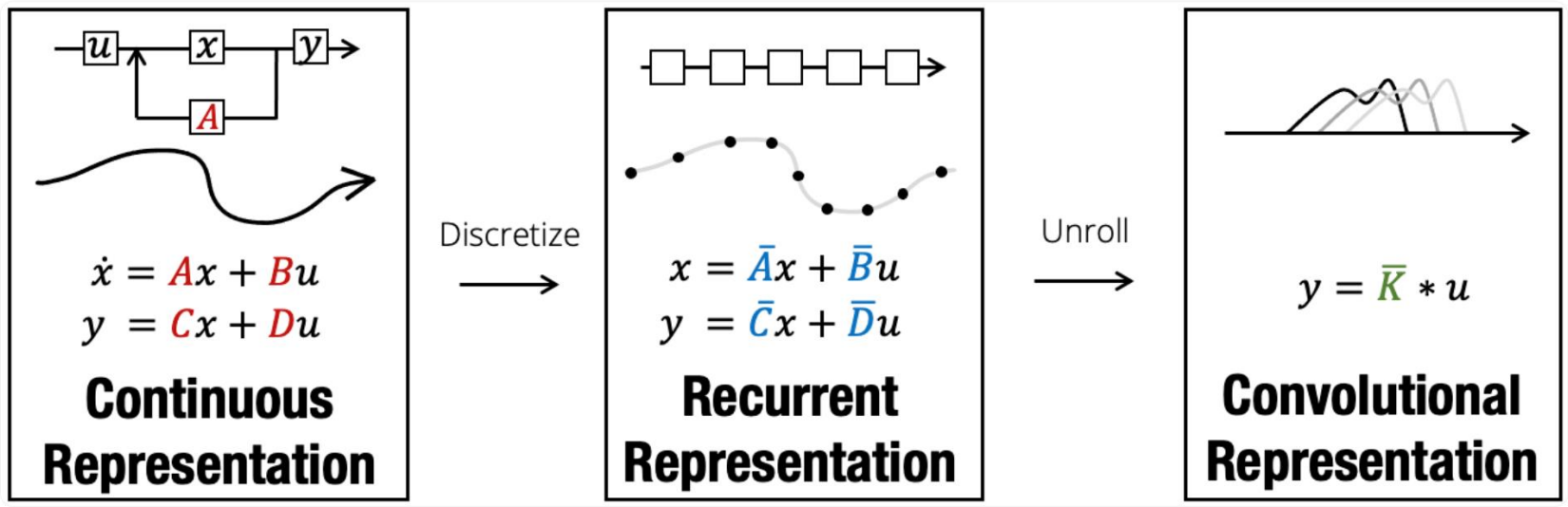


Ideal Models

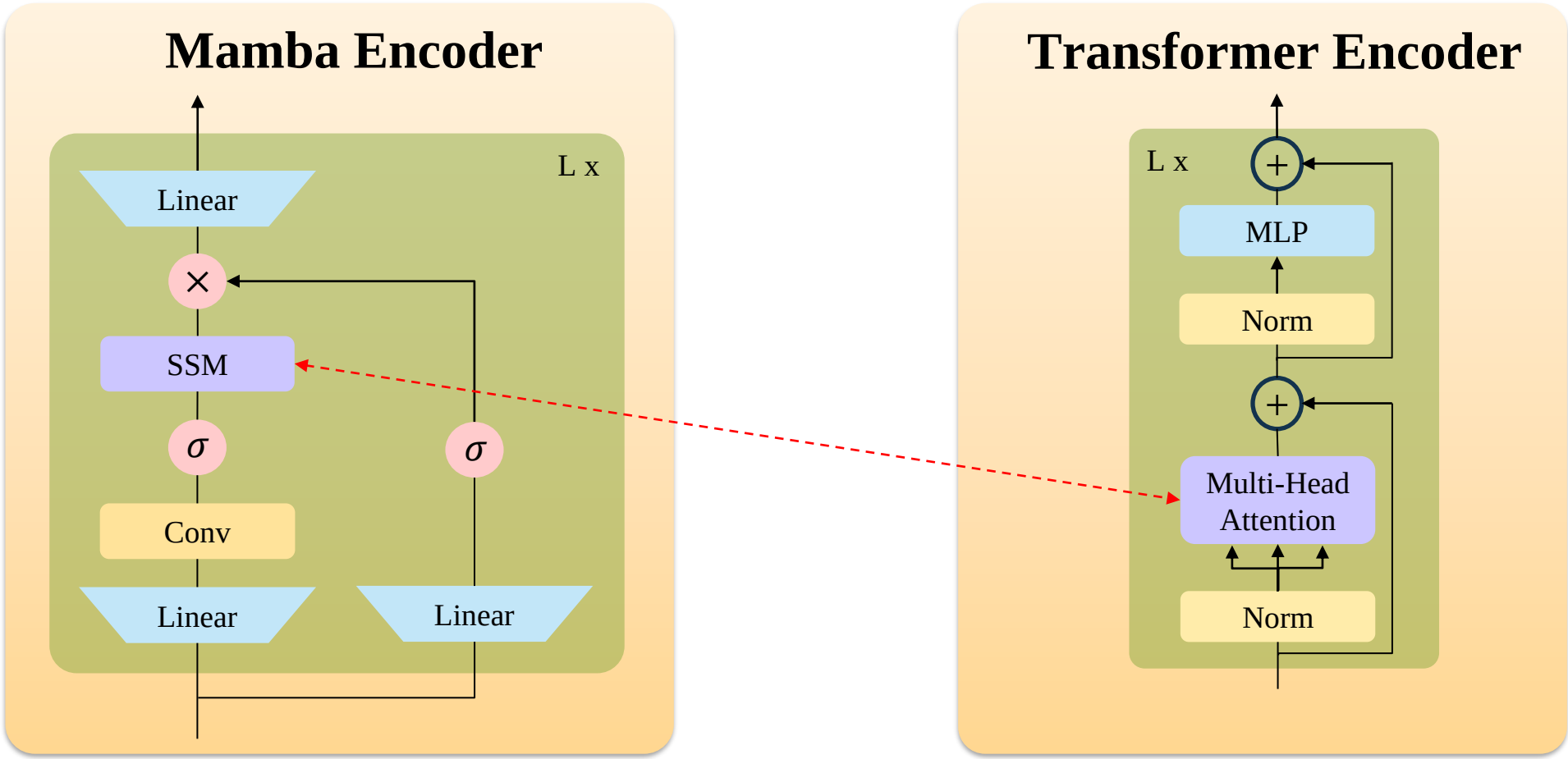
- train stage: $O(N)$, **parallelable**
- inference stage: $O(N)$
- $O(1)$ per token

RNNs vs. Transformers

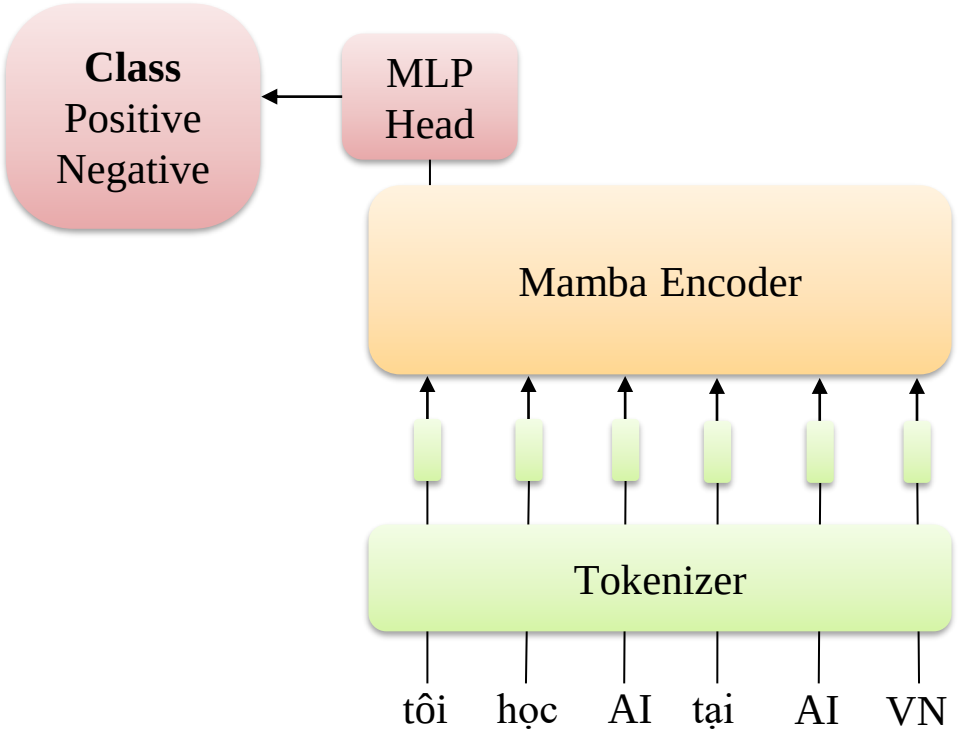
	RNN	Transformer	Ideal Model
Parallelizable	no	yes	yes
Training Stage	$O(n)$	$O(n^2)$	$O(n)$
Inference Stage	$O(n)$	$O(n^2)$	$O(n)$
Inference per Token	$O(1)$	$O(n)$	$O(1)$

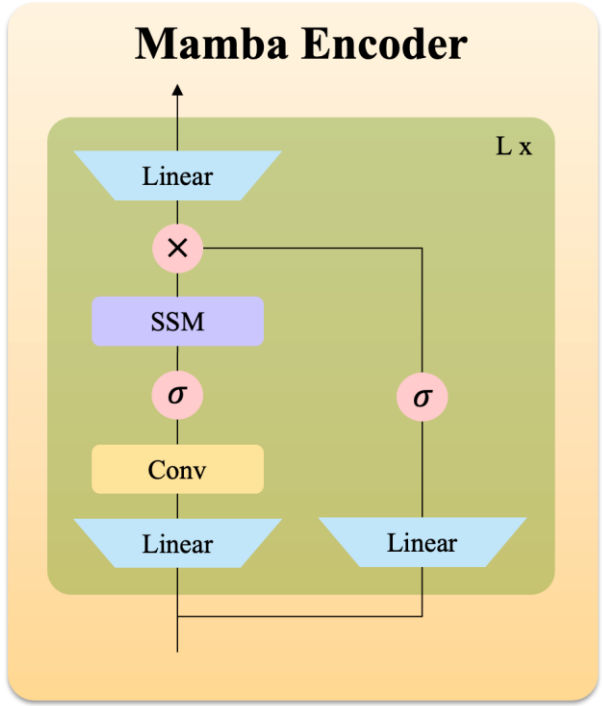


Mamba Block


Mamba Encoder
L x
Linear
SSM
Conv
Linear
Linear
Transformer Encoder
L x
MLP
Norm
Multi-Head Attention
Norm

Mamba Architecture





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State Space Models (SSMs)

What

The SSMs are traditionally used in control theory to model a dynamic system via state variables.

How

$$h'(t) = \mathbf{A}h(t) + \mathbf{B}x(t)$$
$$y(t) = \mathbf{C}h(t) + \mathbf{D}x(t)$$

Why

- train stage: $O(N)$, **parallelable**
- inference stage: $O(N)$
- $O(1)$ per token

Benefits

- suitable for long-range tasks
- faster training and inferencing
- low computational cost
- less memory

Discretize SSMs

Continuous

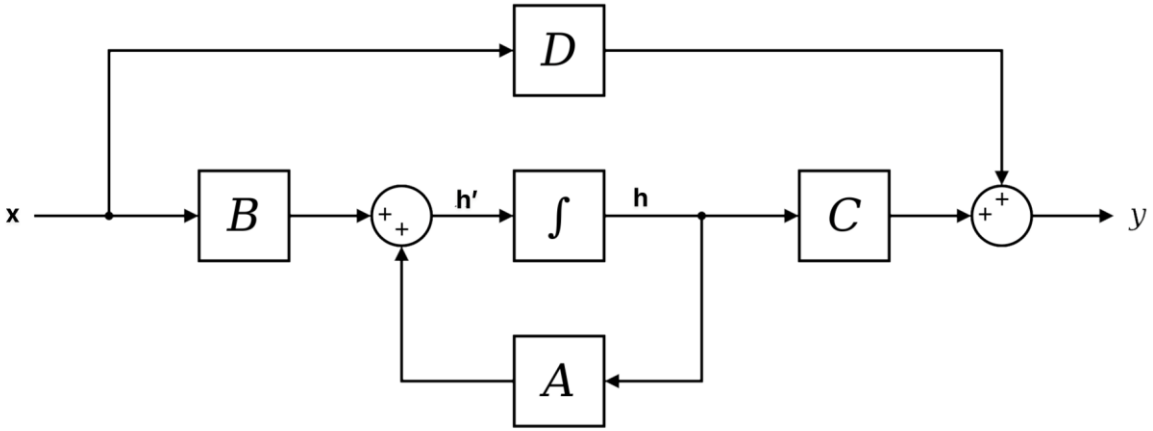
$$h'(t) = \mathbf{A}h(t) + \mathbf{B}x(t)$$
$$y(t) = \mathbf{C}h(t) + \mathbf{D}x(t)$$

Discretize

Recurrent

$$h_t = \overline{\mathbf{A}}h_{t-1} + \overline{\mathbf{B}}x_t$$
$$y_t = \mathbf{C}h_t + \mathbf{D}x_t$$

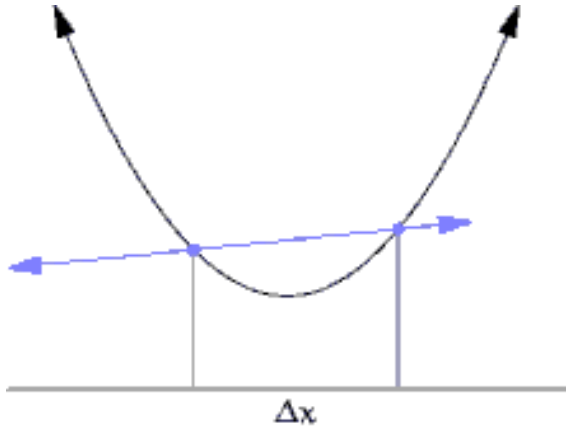
$$\overline{\mathbf{A}}, \overline{\mathbf{B}} = \text{Bilinear}(\mathbf{A}, \mathbf{B}, \Delta) \quad \text{hoặc} \quad \overline{\mathbf{A}}, \overline{\mathbf{B}} = \text{ZOH}(\mathbf{A}, \mathbf{B}, \Delta)$$



Discretize SSMs

1

$$h'(t) = \lim_{\Delta \to 0} \frac{h(t+\Delta) - h(t)}{\Delta}$$



$$h'(t) = \mathbf{A}h(t) + \mathbf{B}x(t)$$

when Δ is small

2

$$h'(t) \approx \frac{h(t+\Delta) - h(t)}{\Delta}$$
$$h'(t)\Delta \approx h(t+\Delta) - h(t)$$
$$h(t+\Delta) \approx h'(t)\Delta + h(t).$$

3

$$h_{t+\Delta} \approx (\mathbf{A}h_t + \mathbf{B}x_t)\Delta + h_t$$
$$h_{t+\Delta} \approx \Delta\mathbf{A}h_t + \Delta\mathbf{B}x_t + h_t$$
$$h_{t+\Delta} \approx (\Delta\mathbf{A} + \mathbf{I})h_t + \Delta\mathbf{B}x_t$$
$$h_{t+\Delta} \approx \overline{\mathbf{A}}h_t + \overline{\mathbf{B}}x_t,$$

Example of Recurrent Representation

Recurrent Representation

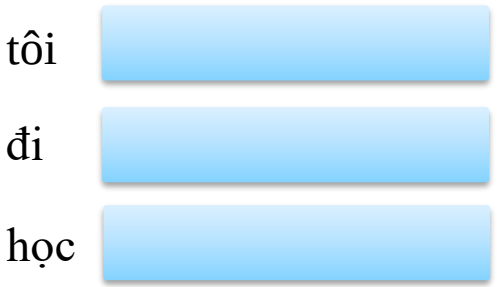
$$h_t = \overline{\mathbf{A}} h_{t-1} + \overline{\mathbf{B}} x_t$$
$$y_t = \mathbf{C} h_t + \mathbf{D} x_t$$

$$\overline{\mathbf{A}}, \overline{\mathbf{B}} = \text{Bilinear}(\mathbf{A}, \mathbf{B}, \Delta) \quad \text{hoặc} \quad \overline{\mathbf{A}}, \overline{\mathbf{B}} = \text{ZOH}(\mathbf{A}, \mathbf{B}, \Delta)$$

$$(\textbf{ZOH}) \quad \overline{\mathbf{A}} = \exp(\Delta \mathbf{A})$$
$$\overline{\mathbf{B}} = (\Delta \mathbf{A})^{-1}(\exp(\Delta \cdot \mathbf{A}) - \mathbf{I}) \cdot \Delta \mathbf{B}.$$

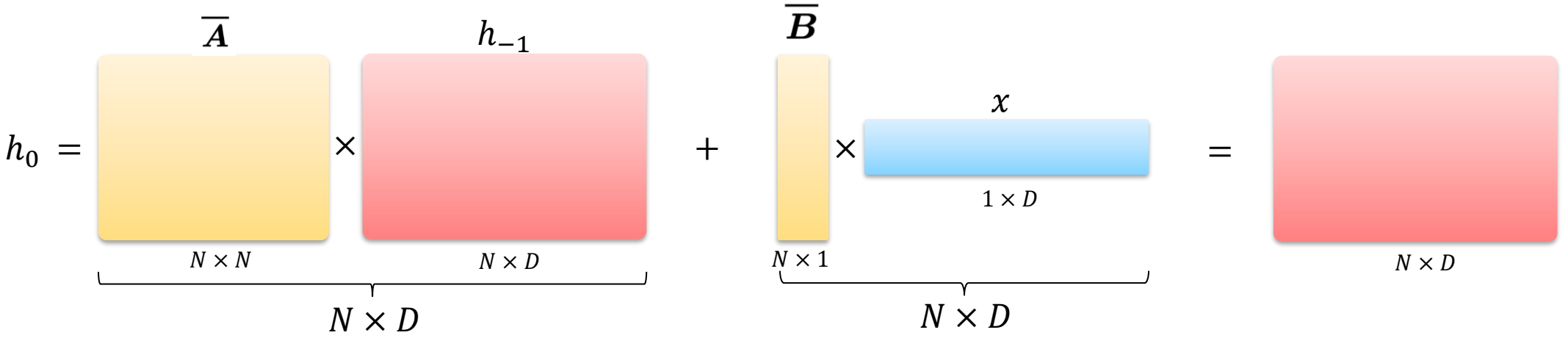
$$(\textbf{Bilinear}) \quad \overline{\mathbf{A}} = (\mathbf{I} - \Delta/2\mathbf{A})^{-1}(\mathbf{I} + \Delta/2\mathbf{A})$$
$$\overline{\mathbf{B}} = (\mathbf{I} - \Delta/2\mathbf{A})^{-1} \cdot \Delta \mathbf{B}$$

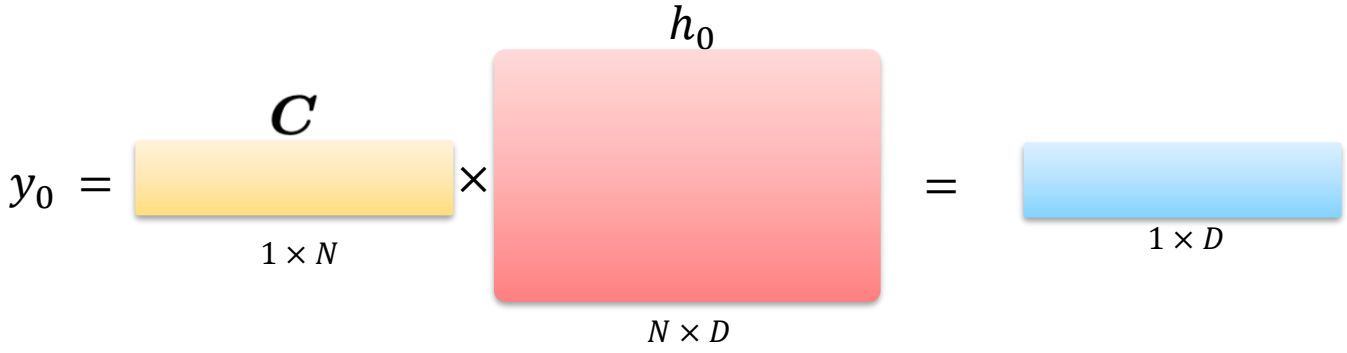
Example of Recurrent Representation



L: sequence len
D: hidden state
N: SSM dimension

$$h_t = \overline{\mathbf{A}} h_{t-1} + \overline{\mathbf{B}} x_t$$
$$y_t = \mathbf{C} h_t + \mathbf{D} x_t$$





Example of Recurrent Representation

tôi	1	2	1	4
đi	5	1	7	3
học	4	3	2	4

input: 3×4

L: sequence len
D: hidden state
N: SSM dimension

$$h_t = \overline{\mathbf{A}} h_{t-1} + \overline{\mathbf{B}} x_t$$
$$y_t = \mathbf{C} h_t + \mathbf{D} x_t$$

A

0.5	1	6
1	2	3
2	5	4

3×3

B

1
5
3

3×1

Δ 1

C

1	3	5

1×3

(ZOH)
$$\overline{\mathbf{A}} = \exp(\Delta \mathbf{A})$$
$$\overline{\mathbf{B}} = (\Delta \mathbf{A})^{-1}(\exp(\Delta \cdot \mathbf{A}) - \mathbf{I}) \cdot \Delta \mathbf{B}.$$

(Bilinear)
$$\overline{\mathbf{A}} = (\mathbf{I} - \Delta/2\mathbf{A})^{-1}(\mathbf{I} + \Delta/2\mathbf{A})$$
$$\overline{\mathbf{B}} = (\mathbf{I} - \Delta/2\mathbf{A})^{-1} \cdot \Delta \mathbf{B}$$

$\overline{A}$

1	2	1
2	3	2
3	4	1

3×3

$\overline{B}$

2
3
1

3×1

C

1	3	5

1×3

Example of Recurrent Representation

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học	4	3	2	4

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$$h_t = \overline{\mathbf{A}} h_{t-1} + \overline{\mathbf{B}} x_t$$
$$y_t = \mathbf{C} h_t + \mathbf{D} x_t$$

$$h_0 = \underbrace{\underbrace{\overline{A}}_{\begin{bmatrix}1&2&1\\2&3&2\\3&4&1\end{bmatrix}_{3\times3}} \times \underbrace{h_{-1}}_{\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}_{3\times4}}}_{3\times4} + \underbrace{\underbrace{\overline{B}}_{\begin{bmatrix}2\\3\\1\end{bmatrix}_{3\times1}} \times \underbrace{x_0\ (tôi)}_{\begin{bmatrix}1&2&1&4\end{bmatrix}_{1\times4}}}_{3\times4} = \underbrace{\begin{bmatrix}2&4&2&8\\3&6&3&12\\1&2&1&4\end{bmatrix}}_{3\times4}$$

$$y_0 = \underbrace{C}_{\begin{bmatrix}1&3&5\end{bmatrix}_{1\times3}} \times \underbrace{h_0}_{\begin{bmatrix}2&4&2&8\\3&6&3&12\\1&2&1&4\end{bmatrix}_{3\times4}} = \underbrace{\begin{bmatrix}16&32&16&64\end{bmatrix}}_{1\times4}$$

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Example of Recurrent Representation

tôi	1	2	1	4
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học	4	3	2	4

input: 3×4

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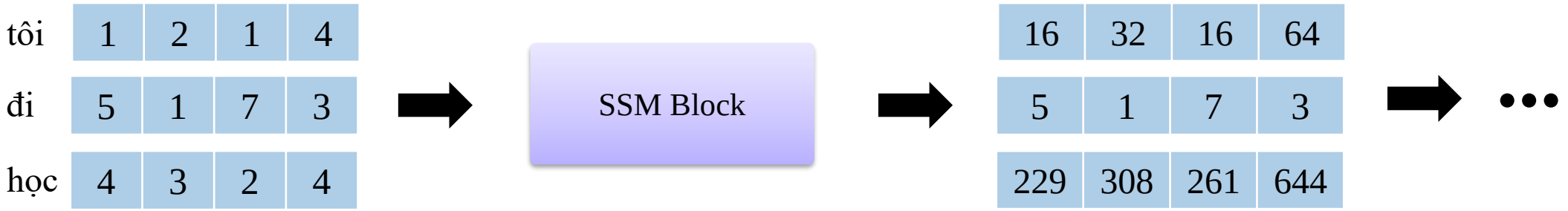
$$h_t = \overline{\mathbf{A}} h_{t-1} + \overline{\mathbf{B}} x_t$$
$$y_t = \mathbf{C} h_t + \mathbf{D} x_t$$

$$h_1 = \underbrace{\overbrace{\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & 4 & 1 \end{bmatrix}}^{\overline{A}\ (3\times 3)} \times \overbrace{\begin{bmatrix} 2 & 4 & 2 & 8 \\ 3 & 6 & 3 & 12 \\ 1 & 2 & 1 & 4 \end{bmatrix}}^{h_0\ (3\times 4)}}_{3\times 4} + \underbrace{\overbrace{\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}}^{\overline{B}\ (3\times 1)} \times \overbrace{\begin{bmatrix} 5 & 1 & 7 & 3 \end{bmatrix}}^{x_1\ (đi)\ (1\times 4)}}_{3\times 4} = \underbrace{\begin{bmatrix} 19 & 20 & 23 & 42 \\ 30 & 31 & 36 & 69 \\ 24 & 39 & 26 & 79 \end{bmatrix}}_{3\times 4}$$

$$y_1 = \overbrace{\begin{bmatrix} 1 & 3 & 5 \end{bmatrix}}^{C\ (1\times 3)} \times \overbrace{\begin{bmatrix} 19 & 20 & 23 & 42 \\ 30 & 31 & 36 & 69 \\ 24 & 39 & 26 & 79 \end{bmatrix}}^{h_1\ (3\times 4)} = \underbrace{\begin{bmatrix} 229 & 308 & 261 & 644 \end{bmatrix}}_{1\times 4}$$

Example of Recurrent Representation



Discretize SSMs

Recurrent Representation

$$h_t = \overline{\mathbf{A}} h_{t-1} + \overline{\mathbf{B}} x_t$$
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$$\overline{\mathbf{A}}, \overline{\mathbf{B}} = \text{Bilinear}(\mathbf{A}, \mathbf{B}, \Delta) \quad \text{hoặc} \quad \overline{\mathbf{A}}, \overline{\mathbf{B}} = \text{ZOH}(\mathbf{A}, \mathbf{B}, \Delta)$$

$$\textbf{(ZOH)} \quad \overline{\mathbf{A}} = \exp(\Delta \mathbf{A})$$
$$\overline{\mathbf{B}} = (\Delta \mathbf{A})^{-1}(\exp(\Delta \cdot \mathbf{A}) - \mathbf{I}) \cdot \Delta \mathbf{B}.$$

$$\textbf{(Bilinear)} \quad \overline{\mathbf{A}} = (\mathbf{I} - \Delta/2\mathbf{A})^{-1}(\mathbf{I} + \Delta/2\mathbf{A})$$
$$\overline{\mathbf{B}} = (\mathbf{I} - \Delta/2\mathbf{A})^{-1} \cdot \Delta \mathbf{B}$$

Learnable Params

- A, B, C, D, Δ

	Ideal Model
Parallelizable	yes
Training Stage	$O(n)$
Inference Stage	$O(n)$
Inference per Token	$O(1)$

Convolutional Representation of SSMs

$$h_t = \overline{\mathbf{A}} h_{t-1} + \overline{\mathbf{B}} x_t$$
$$y_t = \mathbf{C} h_t + \mathbf{D} x_t$$

t=0

$$h_0 = \overline{\mathbf{B}} x_0$$
$$y_0 = \mathbf{C} h_0 = \mathbf{C}\overline{\mathbf{B}} x_0$$

t=1

$$h_1 = \overline{\mathbf{A}} h_0 + \overline{\mathbf{B}} x_1 = \overline{\mathbf{AB}} x_0 + \overline{\mathbf{B}} x_1$$
$$y_1 = \mathbf{C} h_1 = \mathbf{C}(\overline{\mathbf{AB}} x_0 + \overline{\mathbf{B}} x_1) = \mathbf{C}\overline{\mathbf{AB}} x_0 + \mathbf{C}\overline{\mathbf{B}} x_1$$

t=2

$$h_2 = \overline{\mathbf{A}} h_1 + \overline{\mathbf{B}} x_2 = \overline{\mathbf{A}}(\overline{\mathbf{AB}} x_0 + \overline{\mathbf{B}} x_1) + \overline{\mathbf{B}} x_2 = \overline{\mathbf{A}}^2 \overline{\mathbf{B}} x_0 + \overline{\mathbf{AB}} x_1 + \overline{\mathbf{B}} x_2$$
$$y_2 = \mathbf{C} h_2 = \mathbf{C}(\overline{\mathbf{A}}^2 \overline{\mathbf{B}} x_0 + \overline{\mathbf{AB}} x_1 + \overline{\mathbf{B}} x_2) = \mathbf{C}\overline{\mathbf{A}}^2 \overline{\mathbf{B}} x_0 + \mathbf{C}\overline{\mathbf{AB}} x_1 + \mathbf{C}\overline{\mathbf{B}} x_2$$

... ...

t=k

$$y_k = \mathbf{C}\overline{\mathbf{A}}^k \overline{\mathbf{B}} x_0 + \mathbf{C}\overline{\mathbf{A}}^{k-1} \overline{\mathbf{B}} x_1 + \ldots + \mathbf{C}\overline{\mathbf{AB}} x_{k-1} + \mathbf{C}\overline{\mathbf{B}} x_k$$

Convolutional Representation of SSMs

t=k

$$y_k = \mathbf{C}\overline{\mathbf{A}}^k\overline{\mathbf{B}}x_0 + \mathbf{C}\overline{\mathbf{A}}^{k-1}\overline{\mathbf{B}}x_1 + \ldots + \mathbf{C}\overline{\mathbf{A}\mathbf{B}}x_{k-1} + \mathbf{C}\overline{\mathbf{B}}x_k$$

$$\overline{\mathbf{K}} = (\mathbf{C}\overline{\mathbf{B}}, \mathbf{C}\overline{\mathbf{A}\mathbf{B}}, \ldots, \mathbf{C}\overline{\mathbf{A}}^{L-1}\overline{\mathbf{B}}),$$
$$x = (x_0, x_1, \ldots, x_{L-1}),$$
$$y = \overline{\mathbf{K}} * x.$$

Example of Convolutional Representation

tôi
đi
học

input: $L \times D$

L: sequence len
D: hidden state
N: SSM dimension

$$\overline{\mathbf{K}} = (\mathbf{C}\overline{\mathbf{B}}, \mathbf{C}\overline{\mathbf{A}\mathbf{B}}, \ldots, \mathbf{C}\overline{\mathbf{A}}^{L-1}\overline{\mathbf{B}}),$$
$$x = (x_0, x_1, \ldots, x_{L-1}),$$
$$y = \overline{\mathbf{K}} * x.$$

	$C\overline{A}^2\overline{B}$	$C\overline{A}\overline{B}$	$C\overline{B}$

Kernel

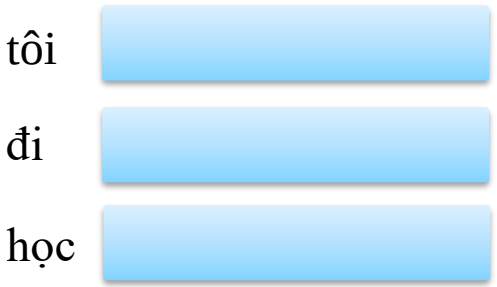
Padding

0	**0**	x_0	x_1	x_2

Input

Output

Example of Convolutional Representation



input: $L \times D$

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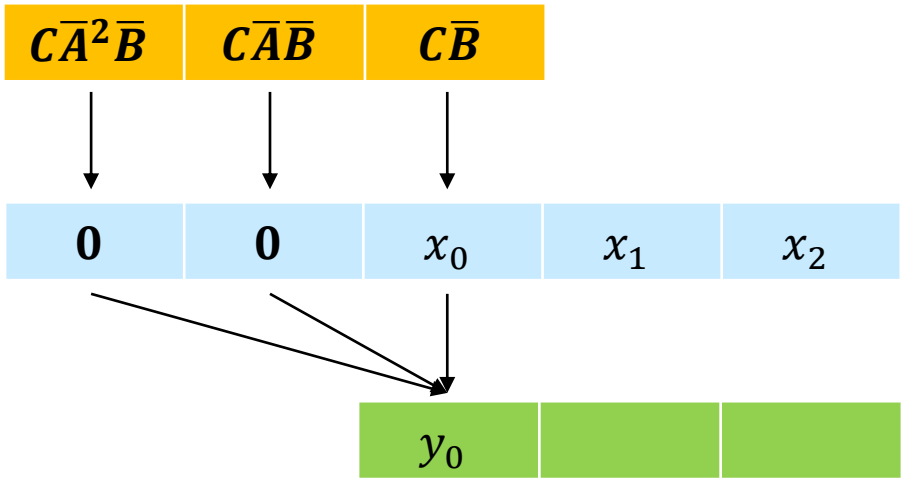
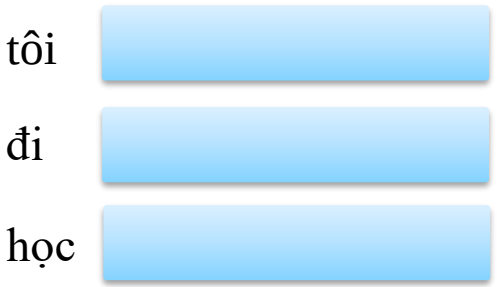
$$\overline{\mathbf{K}} = (\mathbf{C}\overline{\mathbf{B}}, \mathbf{C}\overline{\mathbf{A}\mathbf{B}}, \ldots, \mathbf{C}\overline{\mathbf{A}}^{L-1}\overline{\mathbf{B}}),$$
$$x = (x_0, x_1, \ldots, x_{L-1}),$$
$$y = \overline{\mathbf{K}} * x.$$

Step 1



$$y_0 = C\bar{B}x_0$$

Example of Convolutional Representation

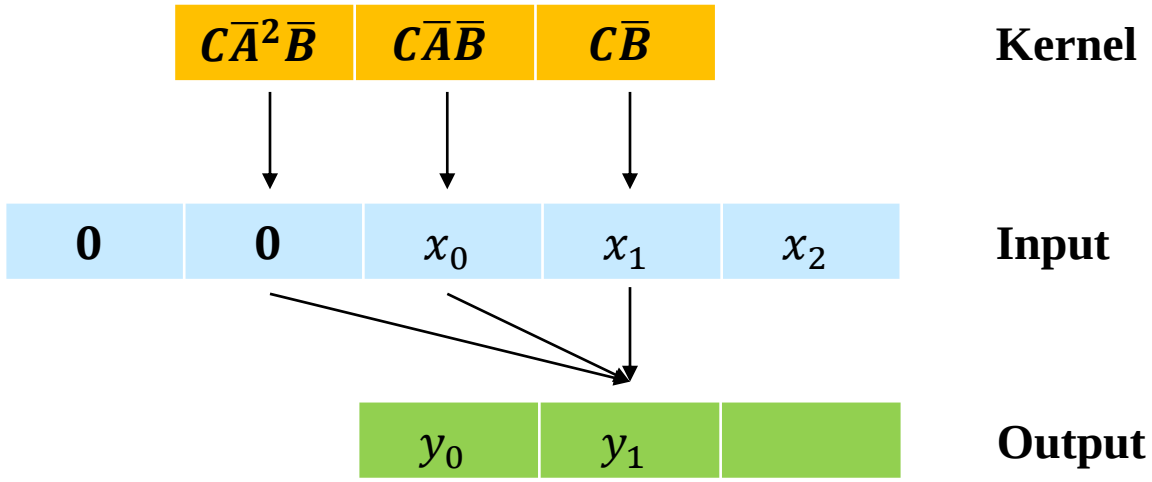


input: $L \times D$

L: sequence len
D: hidden state
N: SSM dimension

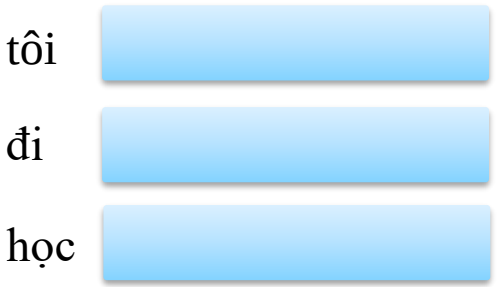
$$\overline{\mathbf{K}} = (\mathbf{C}\overline{\mathbf{B}}, \mathbf{C}\overline{\mathbf{A}\mathbf{B}}, \ldots, \mathbf{C}\overline{\mathbf{A}}^{L-1}\overline{\mathbf{B}}),$$
$$x = (x_0, x_1, \ldots, x_{L-1}),$$
$$y = \overline{\mathbf{K}} * x.$$

Step 2



$$y_1 = C\overline{AB}x_0 + C\bar{B}x_1$$

Example of Convolutional Representation

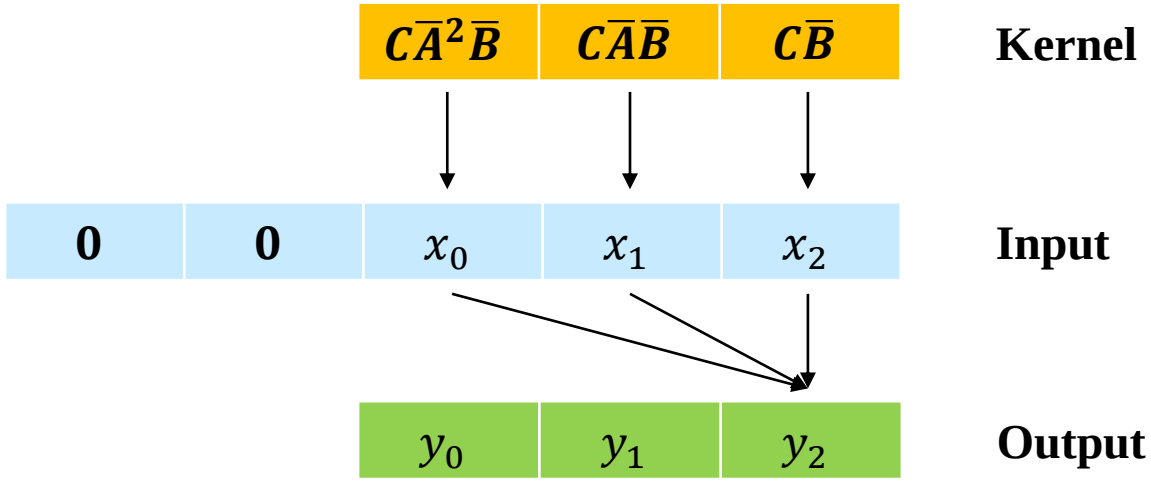


input: $L \times D$

L: sequence len
D: hidden state
N: SSM dimension

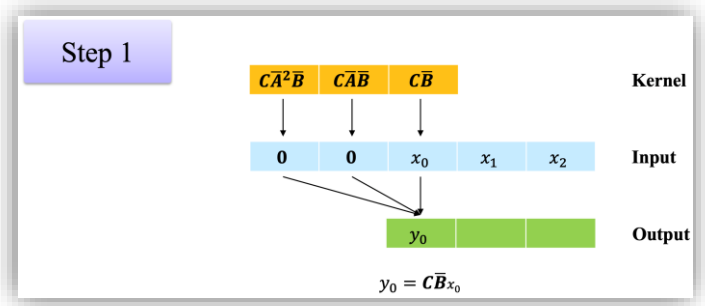
$$\overline{\mathbf{K}} = (\mathbf{C}\overline{\mathbf{B}}, \mathbf{C}\overline{\mathbf{A}\mathbf{B}}, \ldots, \mathbf{C}\overline{\mathbf{A}}^{L-1}\overline{\mathbf{B}}),$$
$$x = (x_0, x_1, \ldots, x_{L-1}),$$
$$y = \overline{\mathbf{K}} * x.$$

Step 3

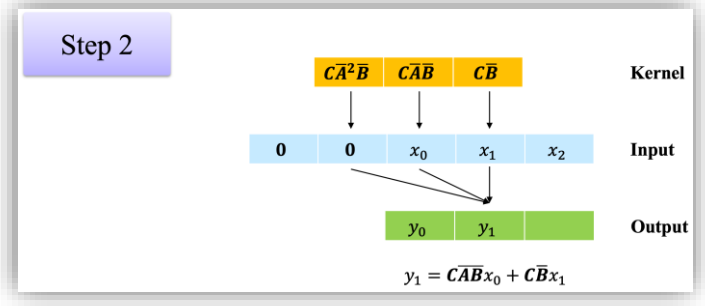


$$y_2 = C\bar{A}^2\bar{B}x_0 + C\bar{A}\bar{B}x_1 + C\bar{B}x_2$$

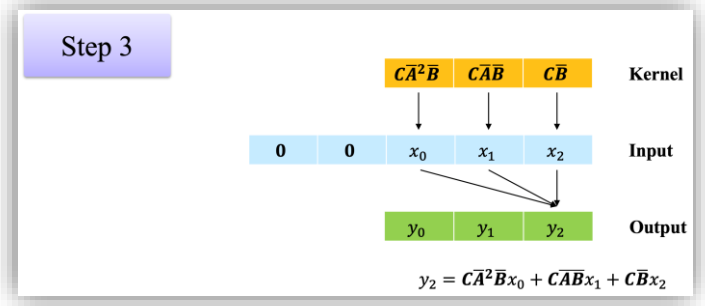
Example of Convolutional Representation



worker 1 → y_0



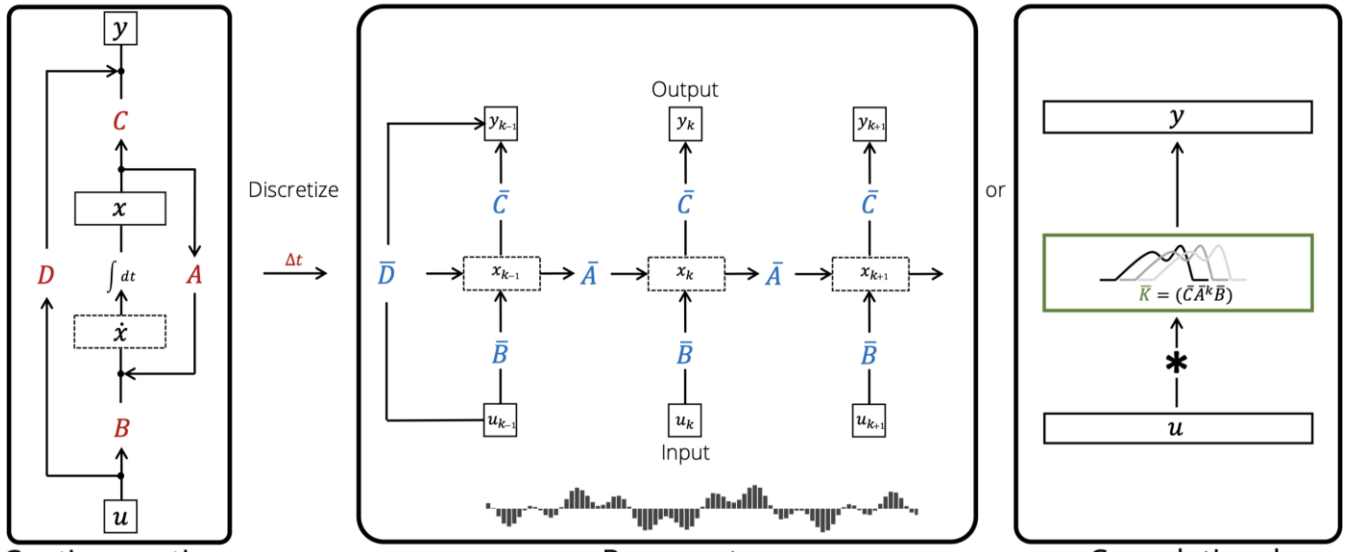
worker 2 → y_1



worker 3 → y_2

Can be computed in parallel!

SSMs is Ideal Models



✓ *continuous data*
✓ *irregular sampling*

✓ *unbounded context*
✓ *efficient inference*

✓ *local information*
✓ *parallelizable training*

Ideal Model	
Parallelizable	yes
Training Stage	$O(n)$
Inference Stage	$O(n)$
Inference per Token	$O(1)$

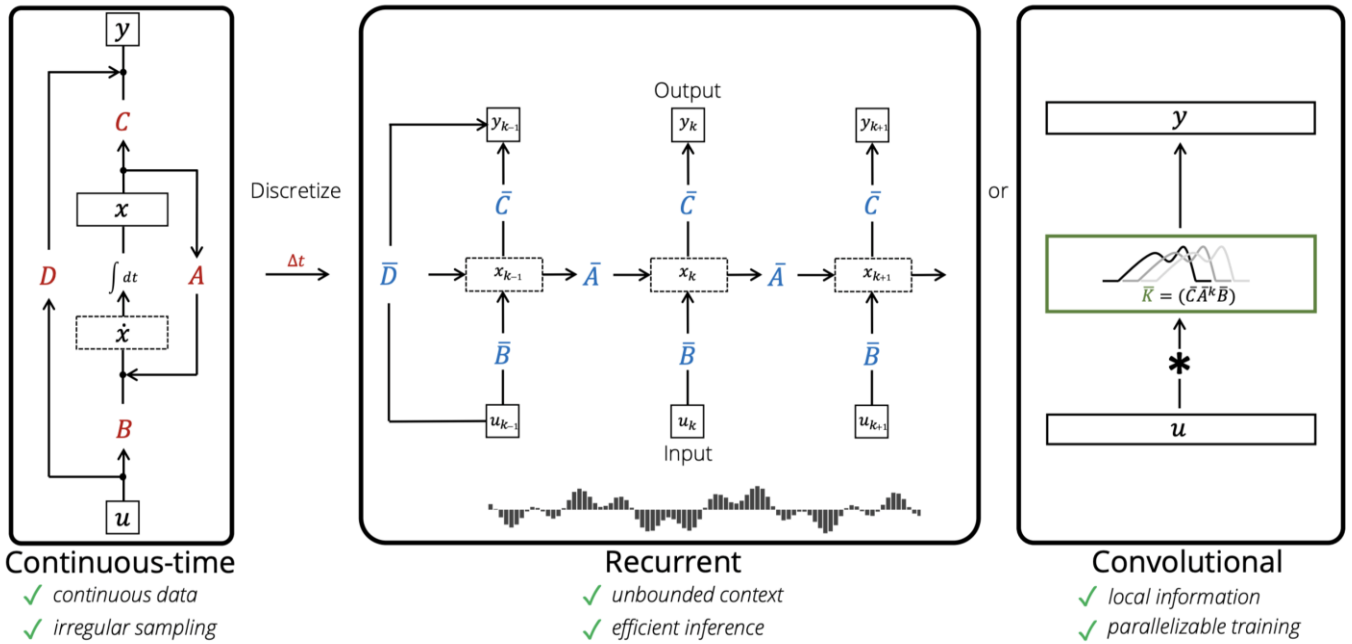
Recurrent

$$h_t = \overline{\mathbf{A}} h_{t-1} + \overline{\mathbf{B}} x_t$$
$$y_t = \mathbf{C} h_t + \mathbf{D} x_t$$

Convolutional

$$\overline{\mathbf{K}} = (\mathbf{C}\overline{\mathbf{B}}, \mathbf{C}\overline{\mathbf{A}\mathbf{B}}, \ldots, \mathbf{C}\overline{\mathbf{A}}^{L-1}\overline{\mathbf{B}}),$$
$$x = (x_0, x_1, \ldots, x_{L-1}),$$
$$y = \overline{\mathbf{K}} * x.$$

SSMs is Ideal Models



Recurrent

$$h_t = \overline{\mathbf{A}} h_{t-1} + \overline{\mathbf{B}} x_t$$
$$y_t = \mathbf{C} h_t + \mathbf{D} x_t$$

Convolutional

$$\overline{\mathbf{K}} = (\mathbf{C}\overline{\mathbf{B}}, \mathbf{C}\overline{\mathbf{A}\mathbf{B}}, \ldots, \mathbf{C}\overline{\mathbf{A}}^{L-1}\overline{\mathbf{B}}),$$
$$x = (x_0, x_1, \ldots, x_{L-1}),$$
$$y = \overline{\mathbf{K}} * x.$$

The importance of the A matrix

SSMs still have some of the same issues as RNNs, like vanishing/exploding gradients.

solution

(**HiPPO Matrix**) $$A_{nk} = -\begin{cases} (2n+1)^{1/2}(2k+1)^{1/2} & \text{if } n > k \\ n+1 & \text{if } n = k \\ 0 & \text{if } n < k \end{cases}$$

The performance in the Sequencial MNIST increased from 60% to 98%.

Recurrent

$$h_t = \overline{\mathbf{A}} h_{t-1} + \overline{\mathbf{B}} x_t$$
$$y_t = \mathbf{C} h_t + \mathbf{D} x_t$$

Convolutional

$$\overline{\mathbf{K}} = (\mathbf{C}\overline{\mathbf{B}}, \mathbf{C}\overline{\mathbf{A}\mathbf{B}}, \ldots, \mathbf{C}\overline{\mathbf{A}}^{L-1}\overline{\mathbf{B}}),$$
$$x = (x_0, x_1, \ldots, x_{L-1}),$$
$$y = \overline{\mathbf{K}} * x.$$

AI VIET NAM

@aivietnam.edu.vn

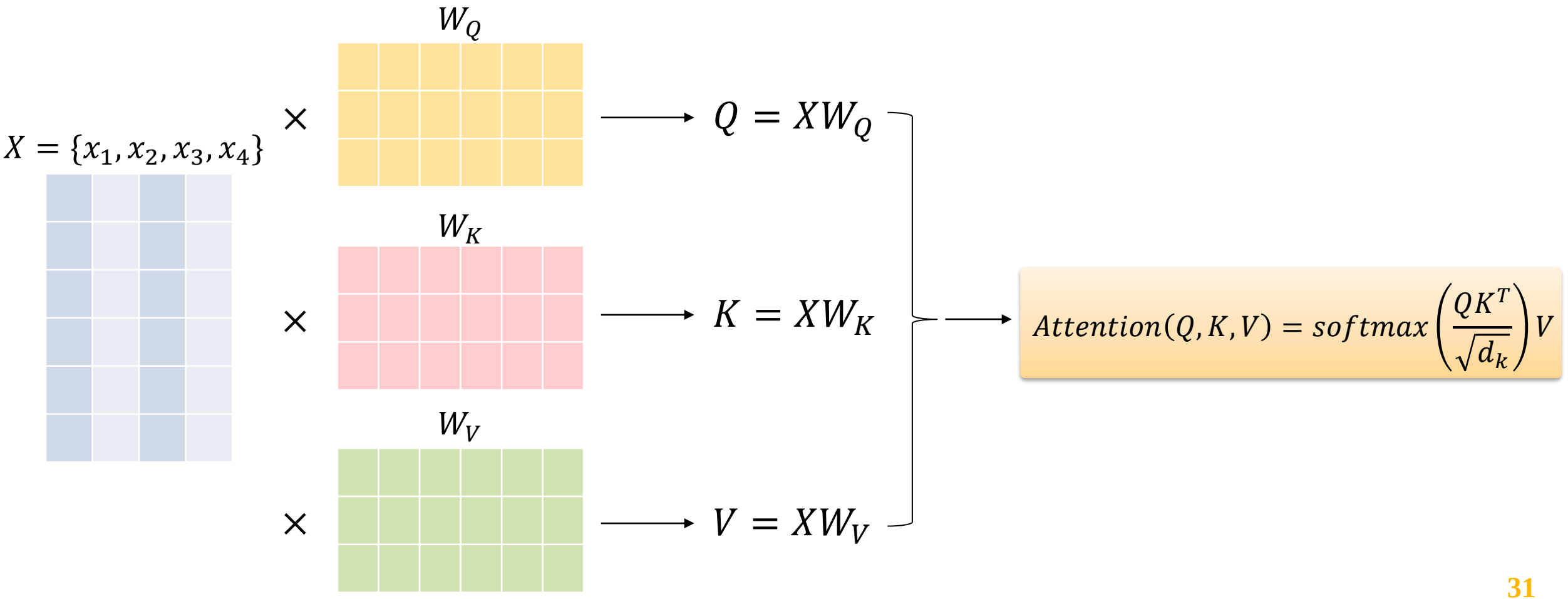
QUIZ! TIME

Outline

1. Motivation
2. State Space Models
3. **Mamba**
4. Coding

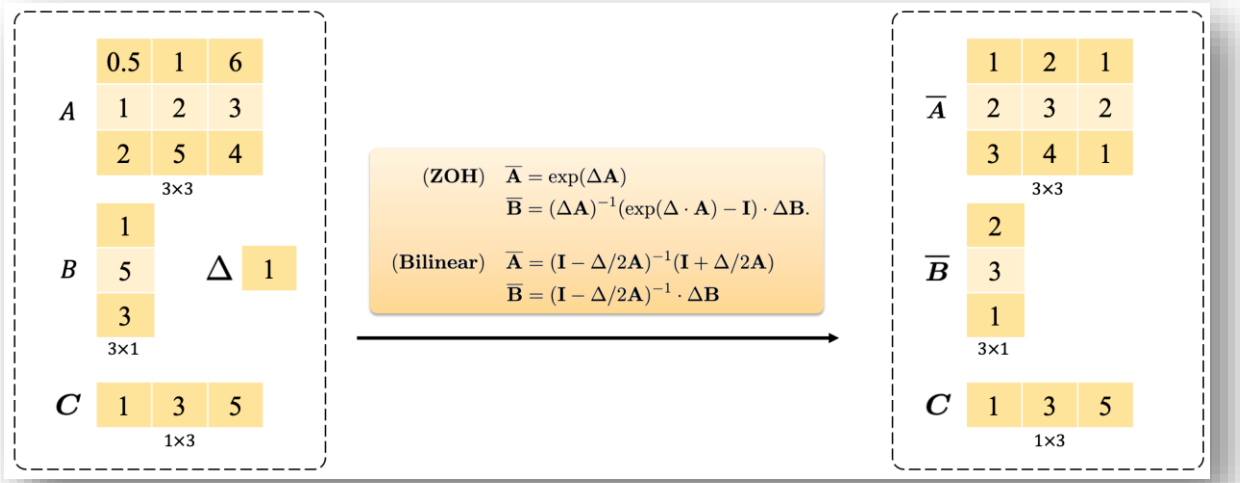
Mamba Motivation

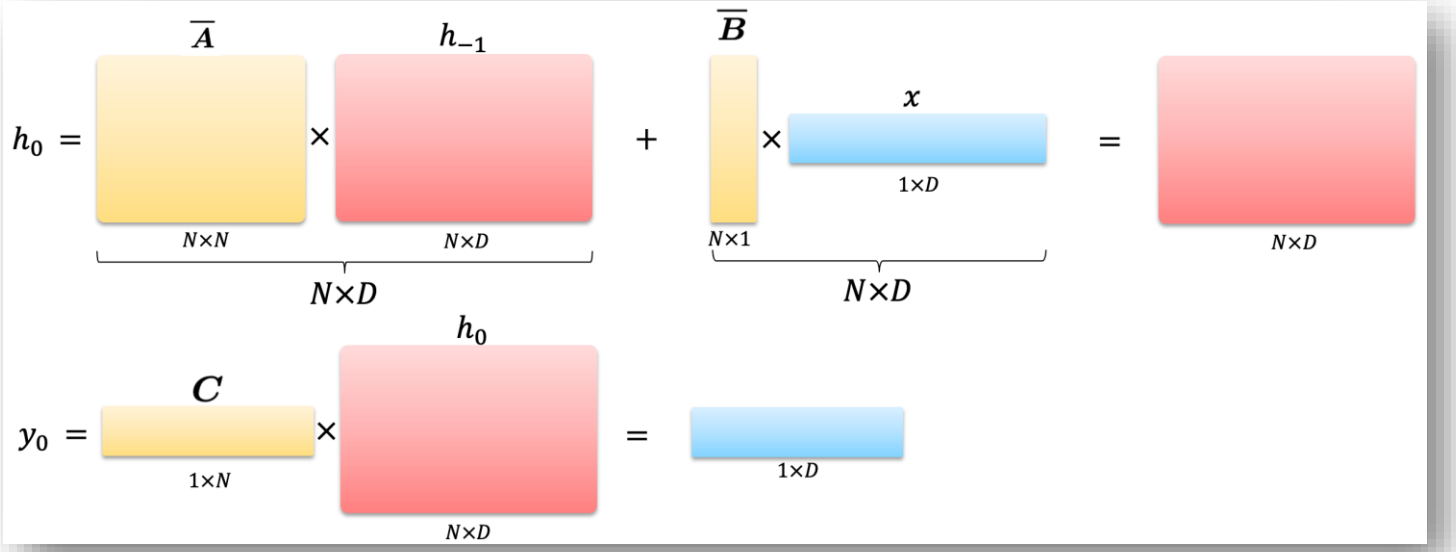
A step back: Attention in Transformer



Mamba Motivation

A step back: S4





Mamba Contribution #1: Selection Mechanism

A

0.5	1	6
1	2	3
2	5	4

$B = Linear_B$

$C = Linear_C$

$\Delta = Softplus(\Delta + Linear_\Delta(\Delta))$

(ZOH)

$$\overline{\mathbf{A}} = \exp(\Delta\mathbf{A})$$
$$\overline{\mathbf{B}} = (\Delta\mathbf{A})^{-1}(\exp(\Delta \cdot \mathbf{A}) - \mathbf{I}) \cdot \Delta\mathbf{B}.$$

(Bilinear)

$$\overline{\mathbf{A}} = (\mathbf{I} - \Delta/2\mathbf{A})^{-1}(\mathbf{I} + \Delta/2\mathbf{A})$$
$$\overline{\mathbf{B}} = (\mathbf{I} - \Delta/2\mathbf{A})^{-1} \cdot \Delta\mathbf{B}$$

$\overline{A}$

1	2	1
2	3	2
3	4	1

3×3

$\overline{B}$

2
3
1

3×1

C

1	3	5

1×3

Mamba Contribution #1: Selection Mechanism

Compare S4 and Mamba (S6)

Algorithm 1 SSM (S4)

Input: $x : (\mathtt{B}, \mathtt{L}, \mathtt{D})$
Output: $y : (\mathtt{B}, \mathtt{L}, \mathtt{D})$
1: $\boldsymbol{A} : (\mathtt{D}, \mathtt{N}) \leftarrow$ Parameter
▷ Represents structured $N \times N$ matrix
2: $\boldsymbol{B} : (\mathtt{D}, \mathtt{N}) \leftarrow$ Parameter
3: $\boldsymbol{C} : (\mathtt{D}, \mathtt{N}) \leftarrow$ Parameter
4: $\Delta : (\mathtt{D}) \leftarrow \tau_\Delta(\text{Parameter})$
5: $\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}} : (\mathtt{D}, \mathtt{N}) \leftarrow \text{discretize}(\Delta, \boldsymbol{A}, \boldsymbol{B})$
6: $y \leftarrow \text{SSM}(\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}, \boldsymbol{C})(x)$
▷ Time-invariant: recurrence or convolution
7: **return** y

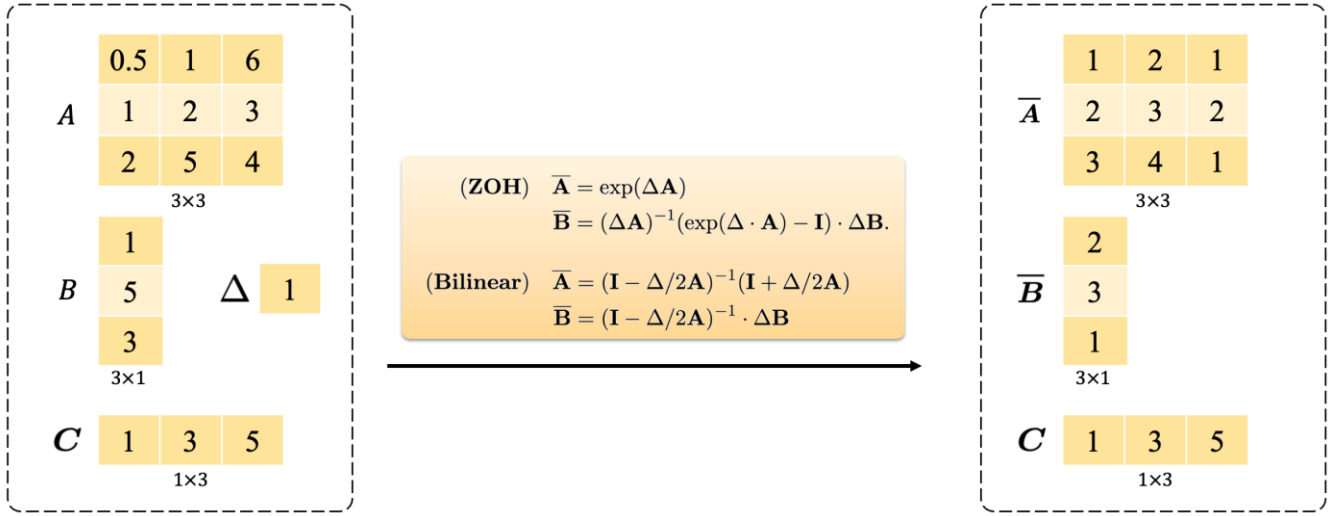
Algorithm 2 SSM + Selection (S6)

Input: $x : (\mathtt{B}, \mathtt{L}, \mathtt{D})$
Output: $y : (\mathtt{B}, \mathtt{L}, \mathtt{D})$
1: $\boldsymbol{A} : (\mathtt{D}, \mathtt{N}) \leftarrow$ Parameter
▷ Represents structured $N \times N$ matrix
2: $\boldsymbol{B} : (\mathtt{B}, \mathtt{L}, \mathtt{N}) \leftarrow s_B(x)$
3: $\boldsymbol{C} : (\mathtt{B}, \mathtt{L}, \mathtt{N}) \leftarrow s_C(x)$
4: $\Delta : (\mathtt{B}, \mathtt{L}, \mathtt{D}) \leftarrow \tau_\Delta(\text{Parameter} + s_\Delta(x))$
5: $\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}} : (\mathtt{B}, \mathtt{L}, \mathtt{D}, \mathtt{N}) \leftarrow \text{discretize}(\Delta, \boldsymbol{A}, \boldsymbol{B})$
6: $y \leftarrow \text{SSM}(\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}}, \boldsymbol{C})(x)$
▷ Time-varying: recurrence (*scan*) only
7: **return** y

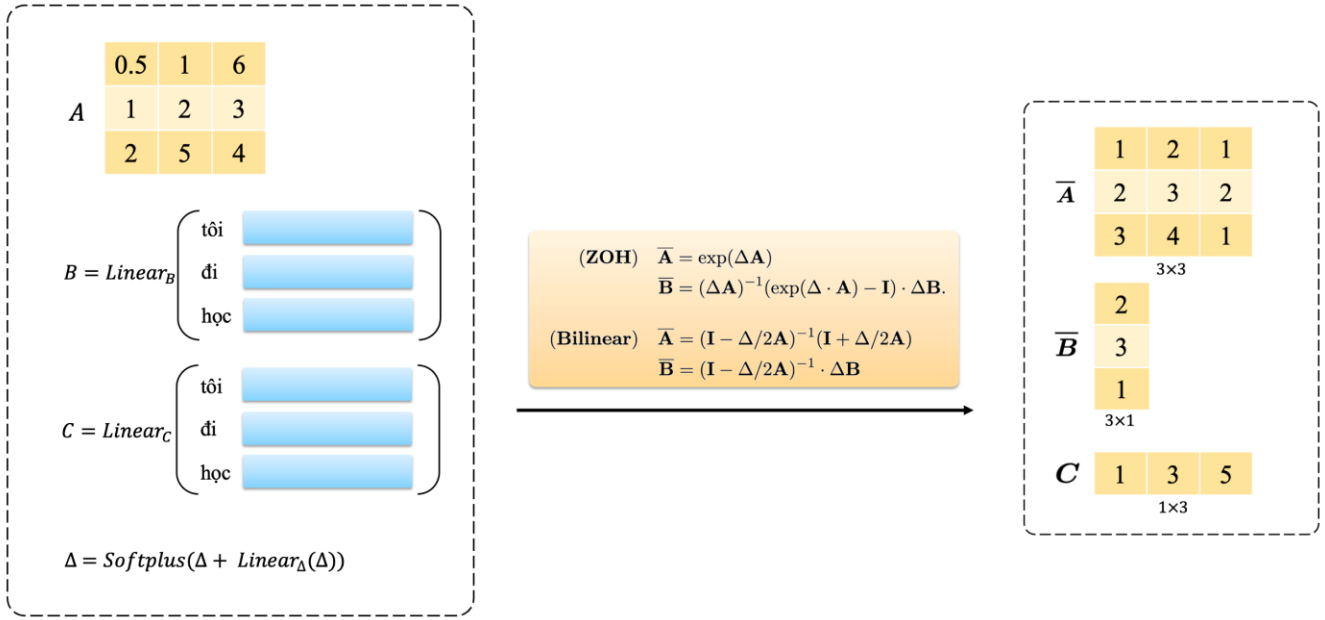
AI VIET NAM
@aivietnam.edu.vn

Mamba Contribution #1: Selection Mechanism

S4



Mamba



Mamba Contribution #1: Selection Mechanism

Mamba becomes time-varying

t=0

$$h_0 = \overline{\mathbf{B}}x_0$$
$$y_0 = \mathbf{C}h_0 = \mathbf{C}\overline{\mathbf{B}}x_0$$

t=1

$$h_1 = \overline{\mathbf{A}}h_0 + \overline{\mathbf{B}}x_1 = \overline{\mathbf{AB}}x_0 + \overline{\mathbf{B}}x_1$$
$$y_1 = \mathbf{C}h_1 = \mathbf{C}(\overline{\mathbf{AB}}x_0 + \overline{\mathbf{B}}x_1) = \mathbf{C}\overline{\mathbf{AB}}x_0 + \mathbf{C}\overline{\mathbf{B}}x_1$$

t=2

$$h_2 = \overline{\mathbf{A}}h_1 + \overline{\mathbf{B}}x_2 = \overline{\mathbf{A}}(\overline{\mathbf{AB}}x_0 + \overline{\mathbf{B}}x_1) + \overline{\mathbf{B}}x_2 = \overline{\mathbf{A}}^2\overline{\mathbf{B}}x_0 + \overline{\mathbf{AB}}x_1 + \overline{\mathbf{B}}x_2$$
$$y_2 = \mathbf{C}h_2 = \mathbf{C}(\overline{\mathbf{A}}^2\overline{\mathbf{B}}x_0 + \overline{\mathbf{AB}}x_1 + \overline{\mathbf{B}}x_2) = \mathbf{C}\overline{\mathbf{A}}^2\overline{\mathbf{B}}x_0 + \mathbf{C}\overline{\mathbf{AB}}x_1 + \mathbf{C}\overline{\mathbf{B}}x_2$$

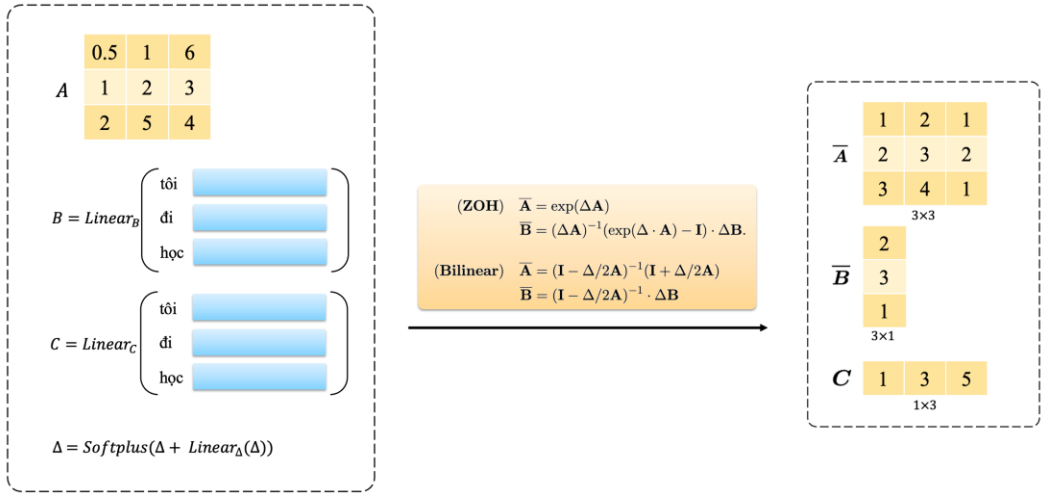
...

t=k

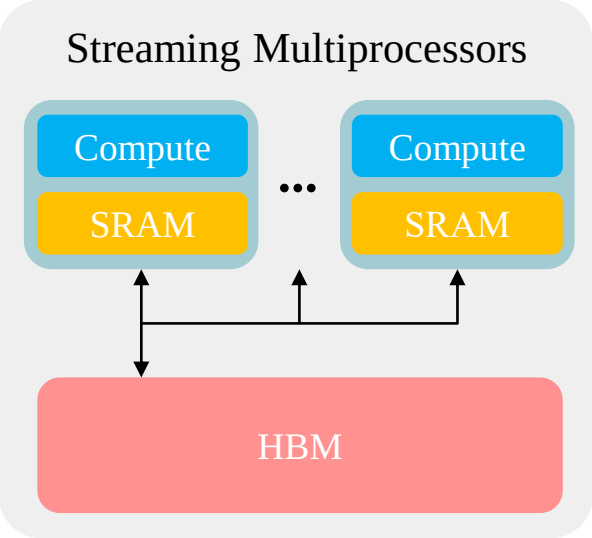
$$y_k = \mathbf{C}\overline{\mathbf{A}}^k\overline{\mathbf{B}}x_0 + \mathbf{C}\overline{\mathbf{A}}^{k-1}\overline{\mathbf{B}}x_1 + \ldots + \mathbf{C}\overline{\mathbf{AB}}x_{k-1} + \mathbf{C}\overline{\mathbf{B}}x_k$$

Convolutional

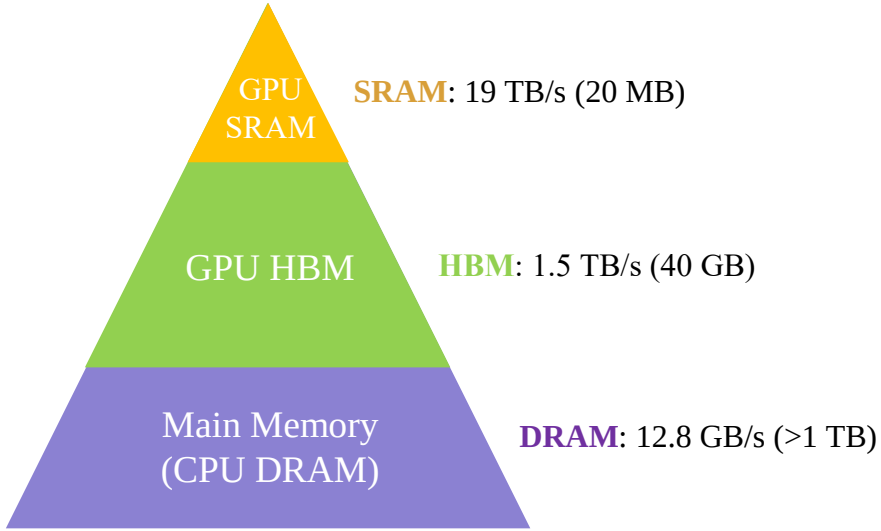
$$\overline{\mathbf{K}} = (\mathbf{C}\overline{\mathbf{B}}, \mathbf{C}\overline{\mathbf{AB}}, \ldots, \mathbf{C}\overline{\mathbf{A}}^{L-1}\overline{\mathbf{B}}),$$
$$x = (x_0, x_1, \ldots, x_{L-1}),$$
$$y = \overline{\mathbf{K}} * x.$$



How does GPU work?



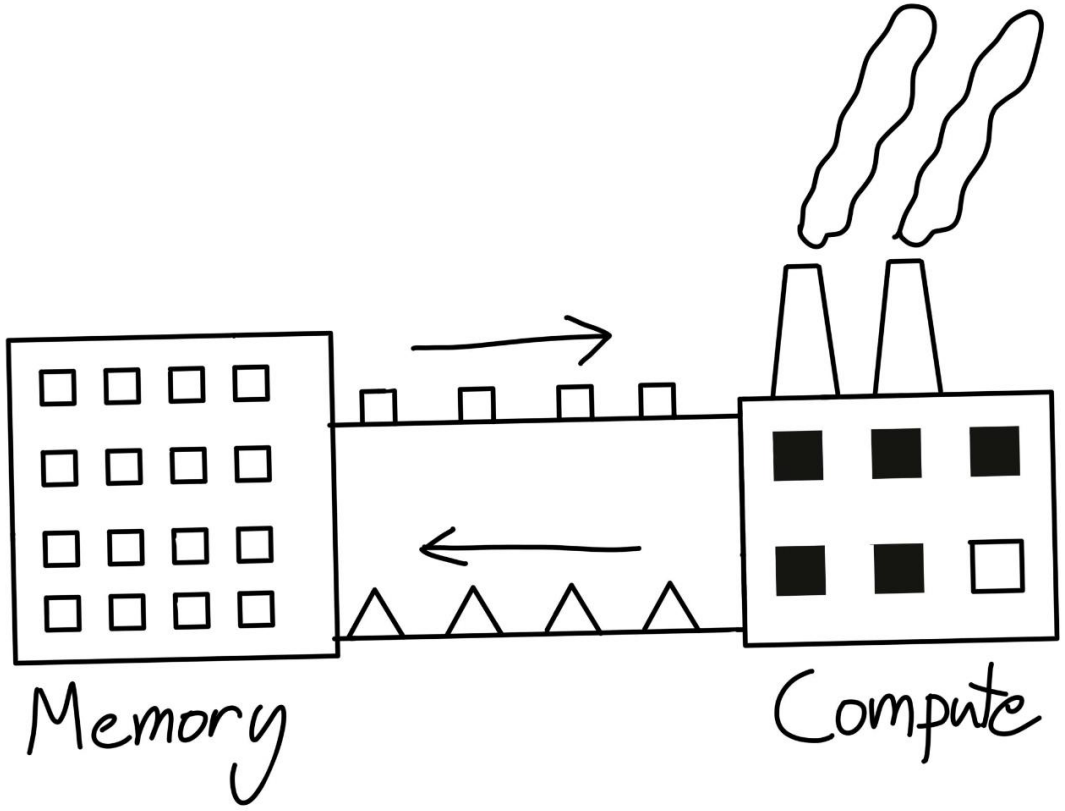
GPU

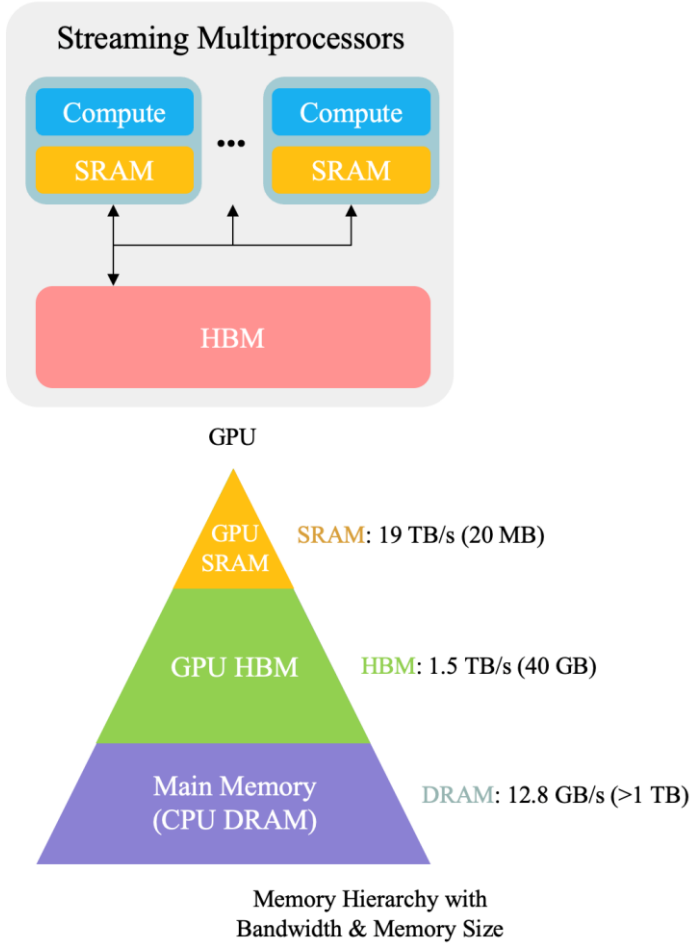


Memory Hierarchy with
Bandwidth & Memory Size

How does GPU work?

Memory & Compute



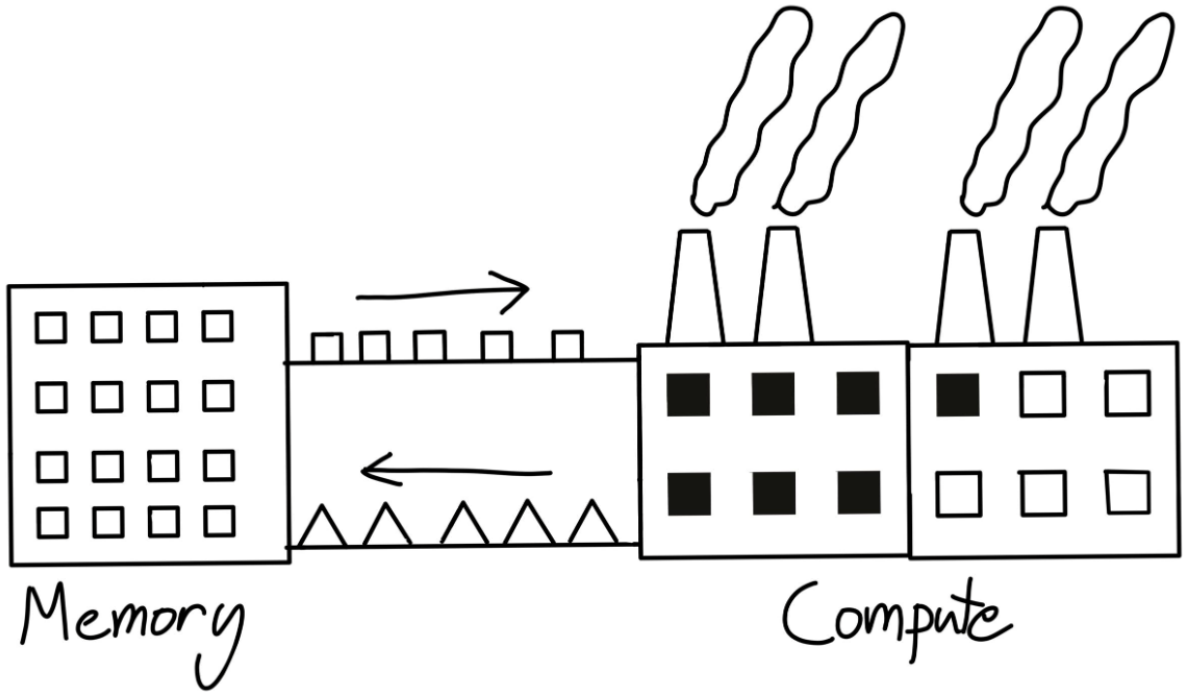


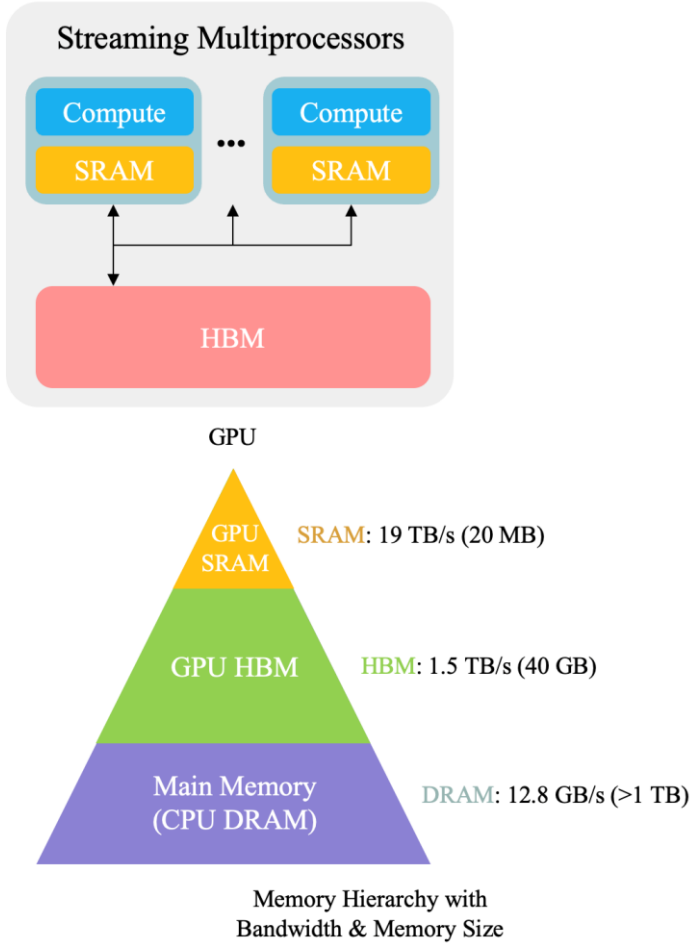
Memory Hierarchy with Bandwidth & Memory Size

Blog: Making Deep Learning Go Brrrr From First Principles

How does GPU work?

Memory & Compute





Memory Hierarchy with Bandwidth & Memory Size

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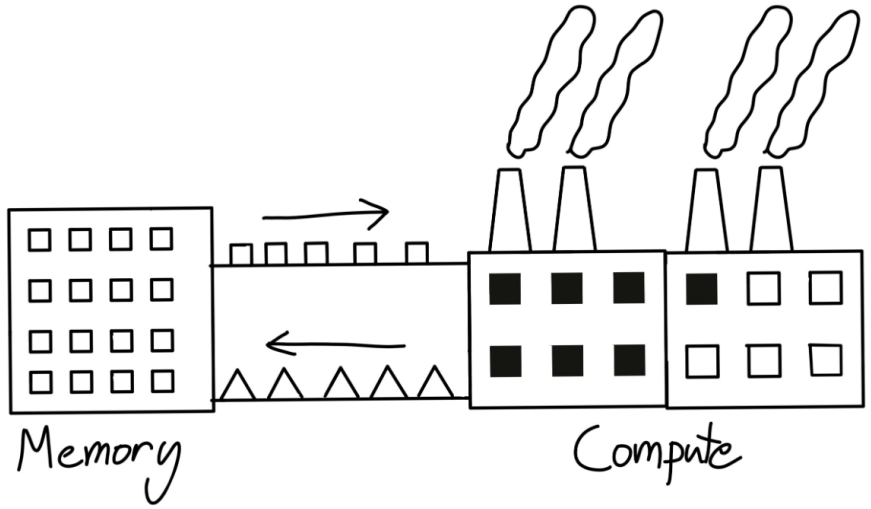
AI VIET NAM
@aivietnam.edu.vn

How does GPU work?

Memory & Compute

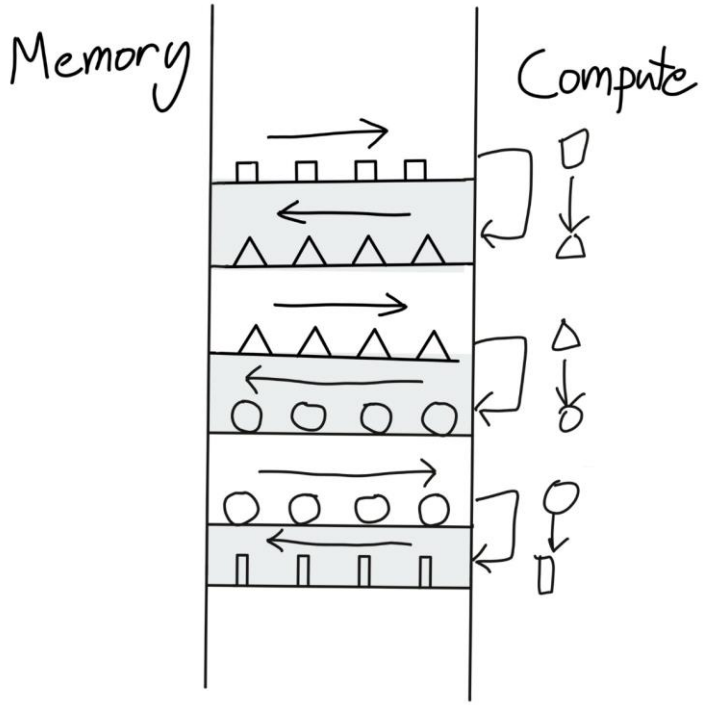


	A100 80GB PCIe	A100 80GB SXM
FP64	9.7 TFLOPS	
FP64 Tensor Core	19.5 TFLOPS	
FP32	19.5 TFLOPS	
Tensor Float 32 (TF32)	156 TFLOPS | 312 TFLOPS*	
BFLOAT16 Tensor Core	312 TFLOPS | 624 TFLOPS*	
FP16 Tensor Core	312 TFLOPS | 624 TFLOPS*	
INT8 Tensor Core	624 TOPS | 1248 TOPS*	
GPU Memory	80GB HBM2e	80GB HBM2e
GPU Memory Bandwidth	1,935 GB/s	2,039 GB/s
Max Thermal Design Power (TDP)	300W	400W ***

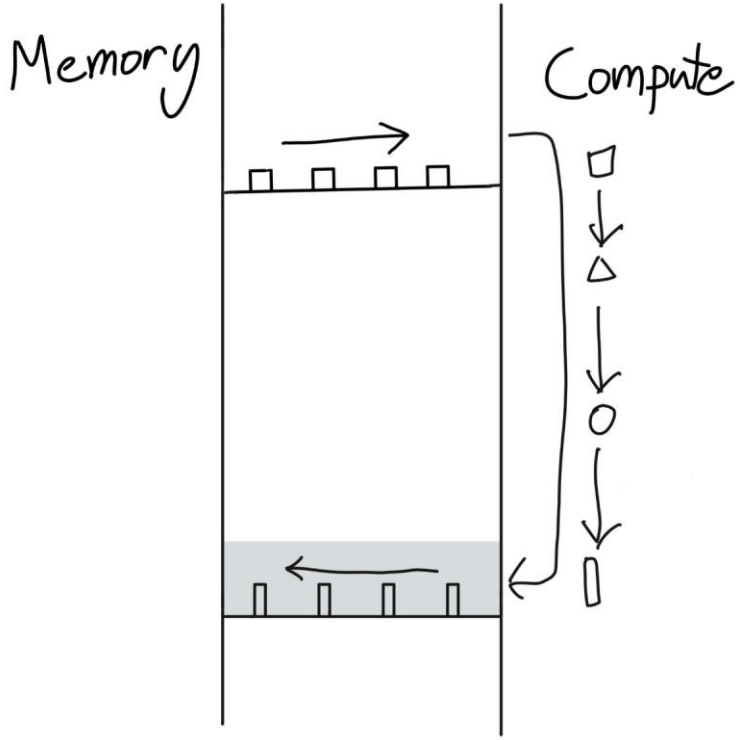
Blog: Making Deep Learning Go Brrrr From First Principles
NVIDIA A100 Tensor Core GPU

Mamba Contribution #2: Hardware-aware Algorithm

Kernel Fusion




```
x1 = x.cos()  # Read from x  in global memory, write to x1
x2 = x1.cos() # Read from x1 in global memory, write to x2
x3 = x2.cos() # Read from x2 in global memory, write to x3
```

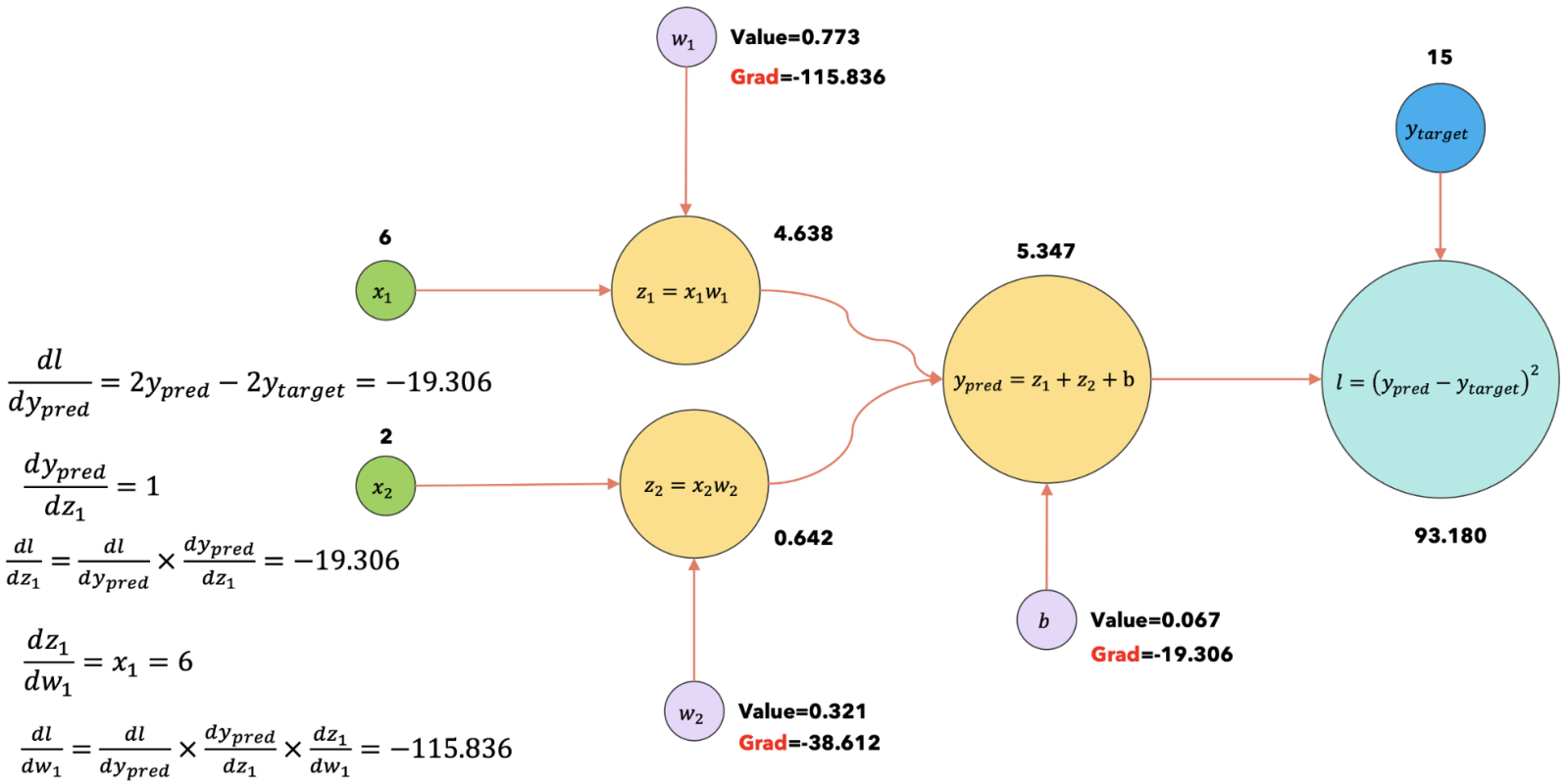



```
x3 = x.cos().cos().cos() # Read from x in global memory, write to x3
```


Blog: Making Deep Learning Go Brrrr From First Principles

Mamba Contribution #2: Hardware-aware Algorithm

Recomputation



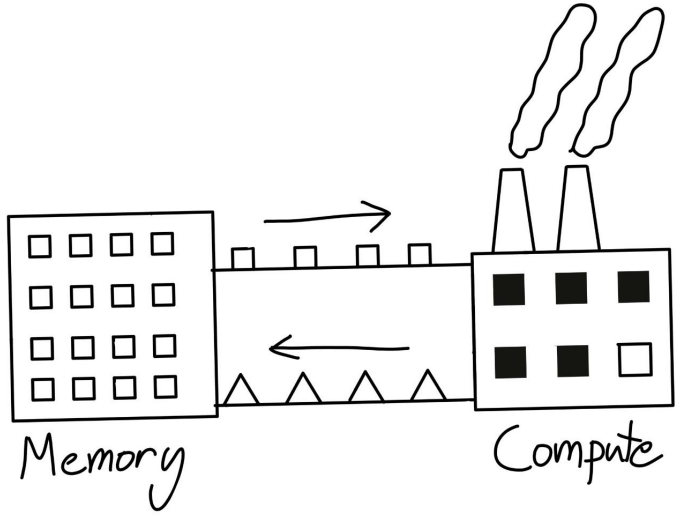
$$\frac{dl}{dy_{pred}} = 2y_{pred} - 2y_{target} = -19.306$$

$$\frac{dy_{pred}}{dz_1} = 1$$

$$\frac{dl}{dz_1} = \frac{dl}{dy_{pred}} \times \frac{dy_{pred}}{dz_1} = -19.306$$

$$\frac{dz_1}{dw_1} = x_1 = 6$$

$$\frac{dl}{dw_1} = \frac{dl}{dy_{pred}} \times \frac{dy_{pred}}{dz_1} \times \frac{dz_1}{dw_1} = -115.836$$



Mamba Contribution #2: Hardware-aware Algorithm

Parallel Scan

Prefix-sum

Initial array	9	6	7	10	8	7
Pre-fix sum	9	15	22	32	40	47

Parallel Scan

Model input	x_0	x_1	x_2	x_3	x_4	x_5
Scan output	h_0	h_1	h_2	h_3	h_4	h_5

Recurrent

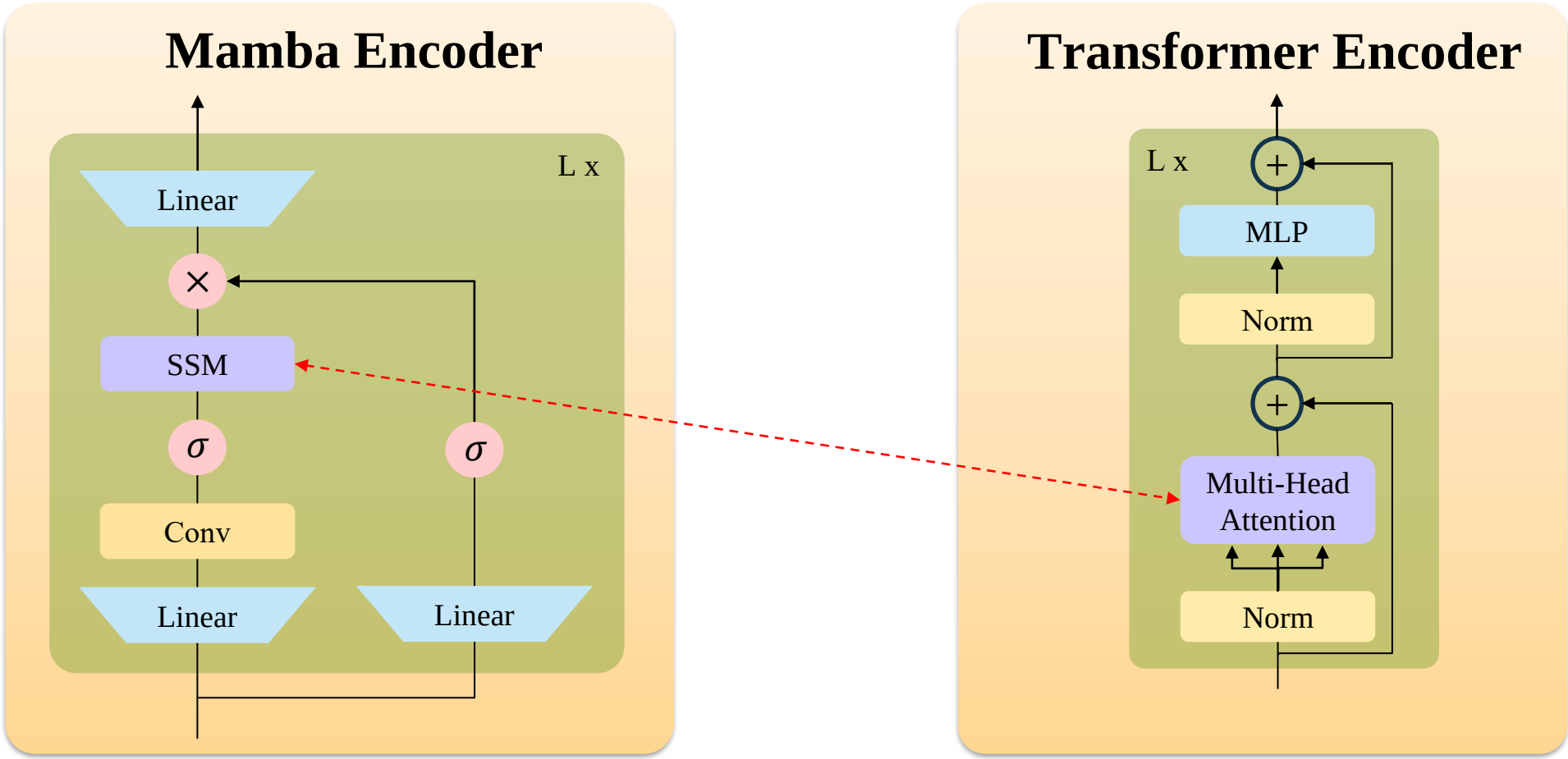
$$h_t = \overline{\mathbf{A}} h_{t-1} + \overline{\mathbf{B}} x_t$$
$$y_t = \mathbf{C} h_t + \mathbf{D} x_t$$

Mamba Contribution #2: Hardware-aware Algorithm

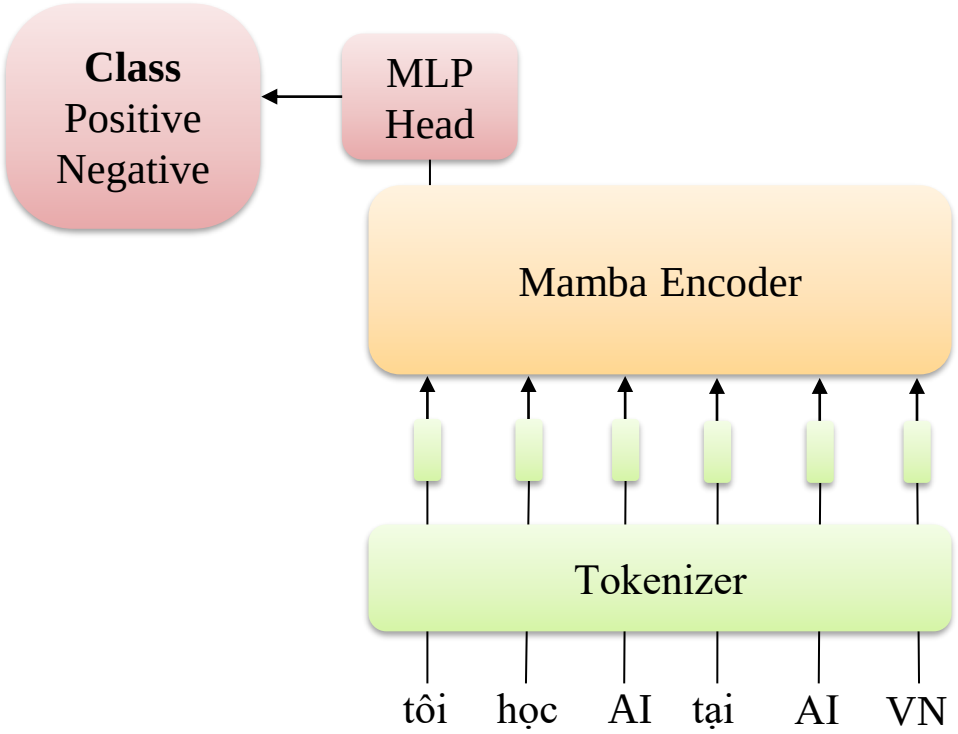
Parallel Scan

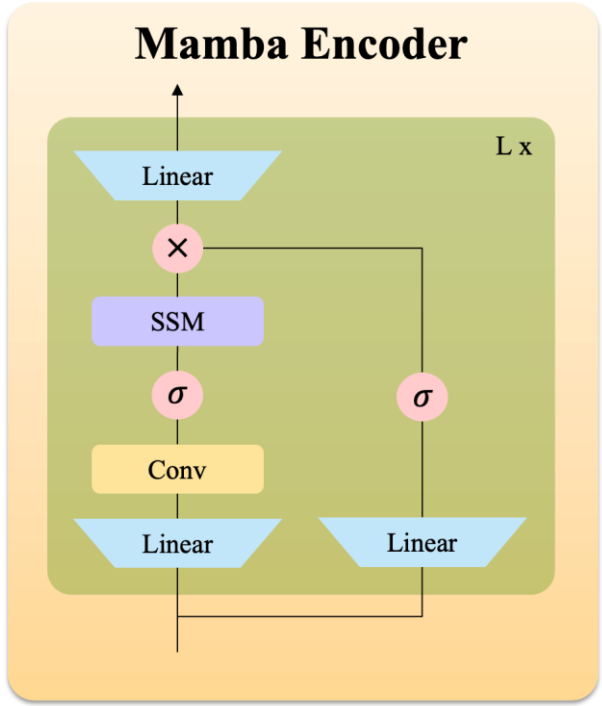
Position	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Sequential Prefix-Sum (what we expect as output from the parallel computation)	1	5	11	14	24	26	33	34	41	50	58	66	76	85	91	101
Array of values	1	4	6	3	10	2	7	1	7	9	8	8	10	9	6	10
		5		9		12		8		16		16		19		16
				14				20				32				35
								34								67
																101
												66				
						26				50				85		
			11		24		33		41		58		76		91	
Parallel Prefix-Sum (output of parallel computation)	1	5	11	14	24	26	33	34	41	50	58	66	76	85	91	101
Do the two computations match?	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES

Mamba Block


Mamba Encoder
L x
Linear
SSM
Conv
Linear
Linear
Transformer Encoder
L x
MLP
Norm
Multi-Head Attention
Norm

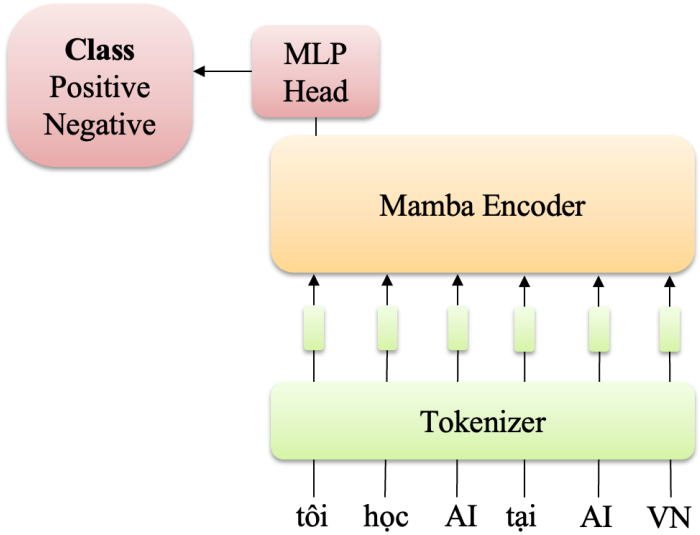
Mamba Architecture

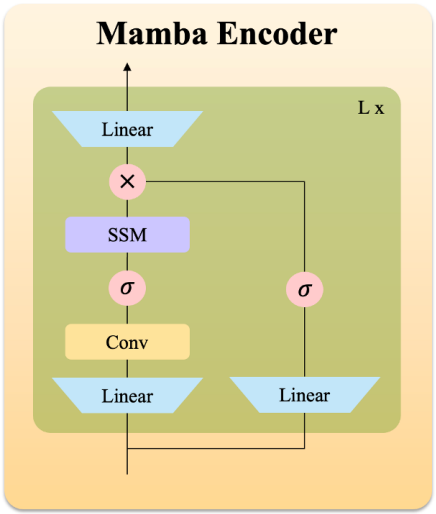




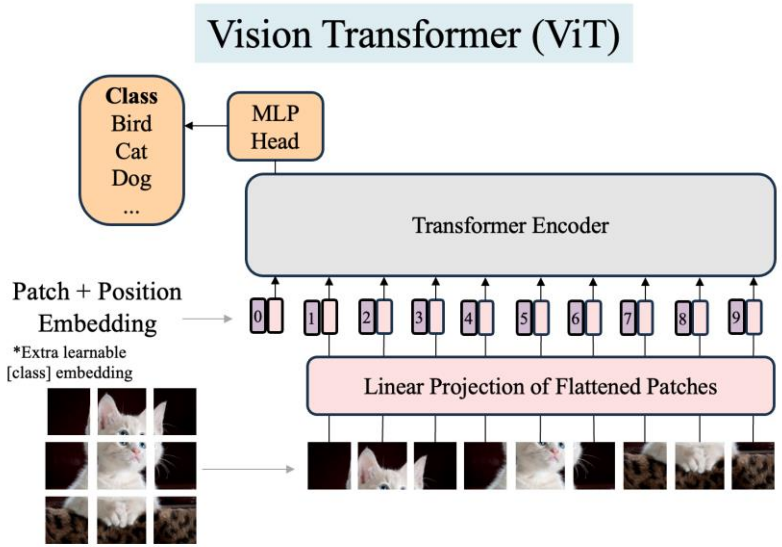
Mamba Arch. & Transformer Arch.

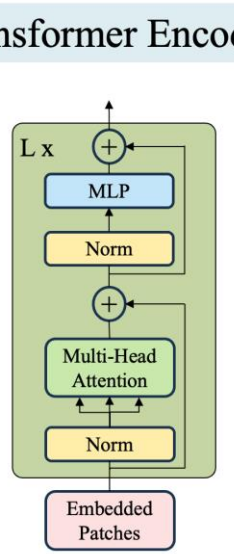
Mamba





ViT

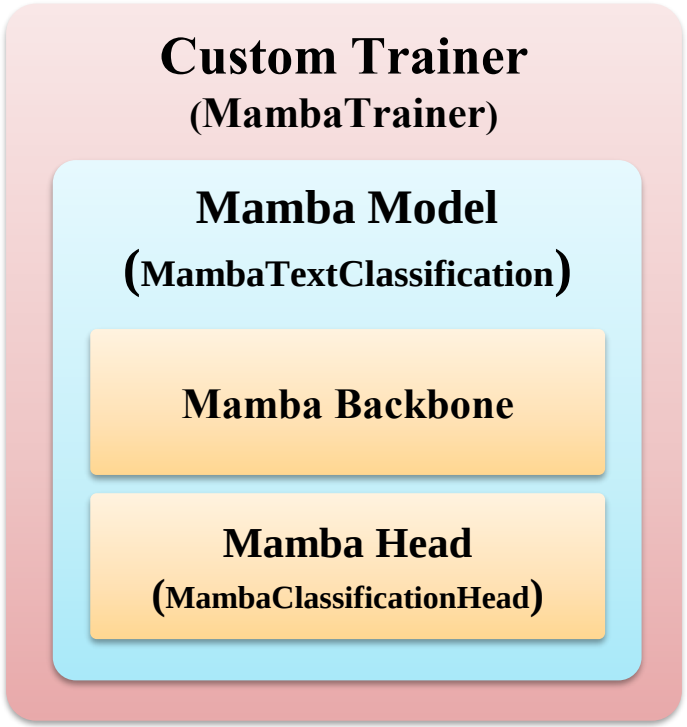




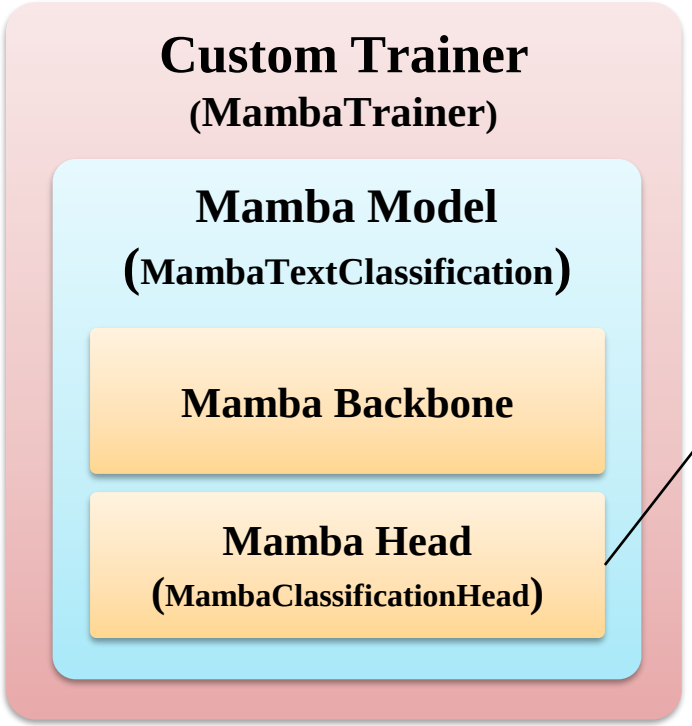
Outline

1. Motivation
2. State Space Models
3. Mamba
4. **Coding**

Coding

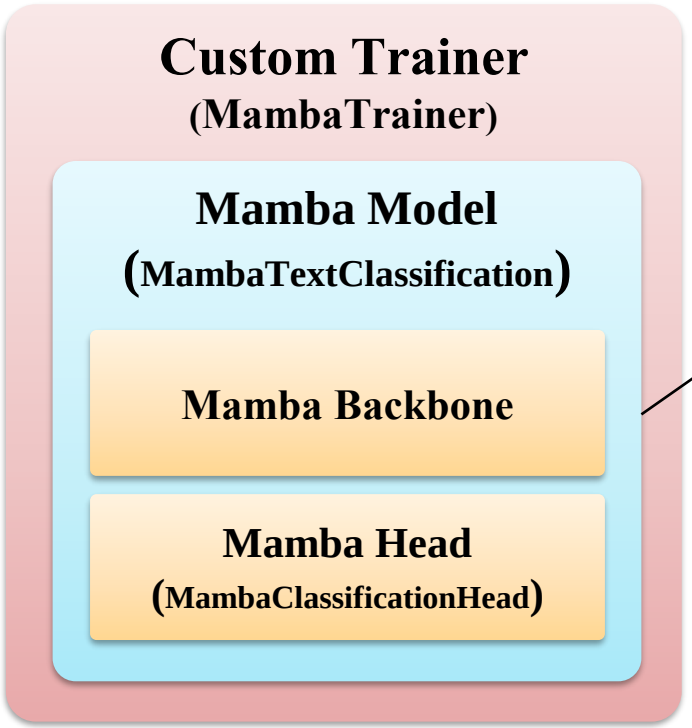

Custom Trainer
(MambaTrainer)
Mamba Model
(MambaTextClassification)
Mamba Backbone
Mamba Head
(MambaClassificationHead)

Coding



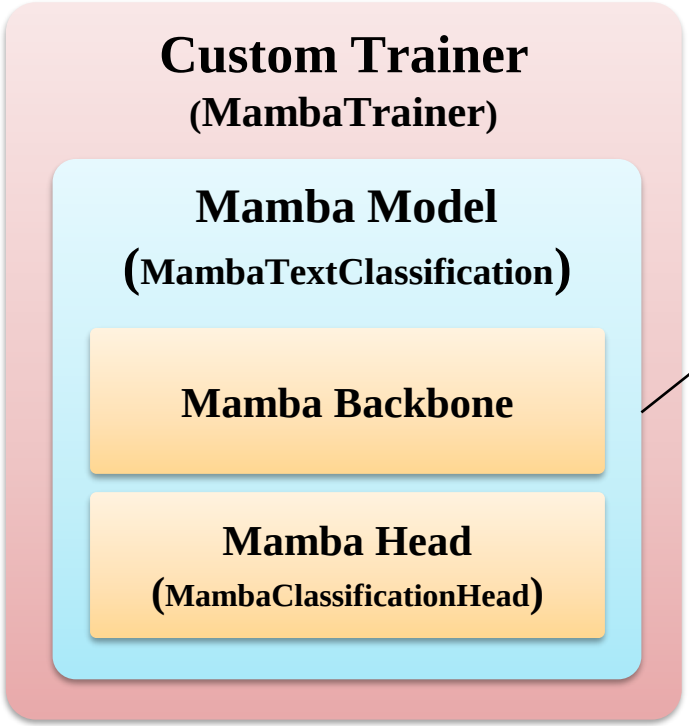

```python
# Định nghĩa lớp head để phân loại
class MambaClassificationHead(nn.Module):
    def __init__(self, d_model, num_classes, **kwargs):
        super(MambaClassificationHead, self).__init__()
        # Sử dụng một lớp tuyến tính để thực hiện phân loại dựa trên đầu vào
        # có kích thước d_model và num_classes cần phân loại.
        self.classification_head = nn.Linear(d_model, num_classes, **kwargs)

    def forward(self, hidden_states):
        return self.classification_head(hidden_states)
```

Coding



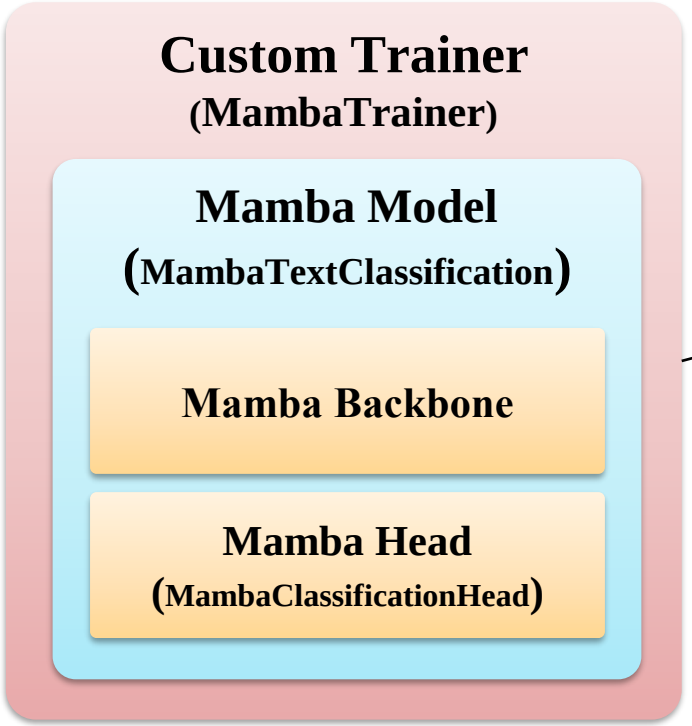

```python
class MambaTextClassification(MambaLMHeadModel):
    def __init__(
        self,
        config: MambaConfig,
        initializer_cfg=None,
        device=None,
        dtype=None,
    ) -> None:
        super().__init__(config, initializer_cfg, device, dtype)

        # Tạo một đầu phân loại sử dụng MambaClassificationHead với kích thước đầu vào là d_model và số lớp là 2.
        self.classification_head = MambaClassificationHead(d_model=config.d_model, num_classes=2)

        del self.lm_head

    def forward(self, input_ids, attention_mask=None, labels=None):
        # Truyền input_ids qua mô hình gốc để nhận hidden_states.
        hidden_states = self.backbone(input_ids)

        # Lấy trung bình của hidden_states theo chiều thứ 2 để tạo ra [CLS] feature đại diện
        mean_hidden_states = hidden_states.mean(dim=1)

        # Đưa mean_hidden_states qua đầu phân loại để nhận logits.
        logits = self.classification_head(mean_hidden_states)

        if labels is None:
          ClassificationOutput = namedtuple("ClassificationOutput", ["logits"])
          return ClassificationOutput(logits=logits)
        else:
          ClassificationOutput = namedtuple("ClassificationOutput", ["loss", "logits"])

          # Sử dụng hàm mất mát CrossEntropyLoss để tính loss.
          loss_fct = nn.CrossEntropyLoss()
          loss = loss_fct(logits, labels)

          return ClassificationOutput(loss=loss, logits=logits)
```

Coding




```python
class MambaTextClassification(MambaLMHeadModel):
    ...
    def predict(self, text, tokenizer, id2label=None):
        input_ids = torch.tensor(tokenizer(text)['input_ids'], device='cuda')[None]
        with torch.no_grad():
          logits = self.forward(input_ids).logits[0]
          label = np.argmax(logits.cpu().numpy())

        if id2label is not None:
          return id2label[label]
        else:
          return label

    @classmethod
    def from_pretrained(cls, pretrained_model_name, device=None, dtype=None, **kwargs):
        # Tải pretrained từ mô hình đã được huấn luyện trước đó.
        config_data = load_config_hf(pretrained_model_name)
        config = MambaConfig(**config_data)

        # Khởi tạo mô hình từ cấu hình và chuyển nó đến thiết bị và kiểu dữ liệu mong muốn.
        model = cls(config, device=device, dtype=dtype, **kwargs)

        # Tải trạng thái mô hình đã được huấn luyện trước đó.
        model_state_dict = load_state_dict_hf(pretrained_model_name, device=device, dtype=dtype)
        model.load_state_dict(model_state_dict, strict=False)

        # In ra các tham số embedding mới được khởi tạo.
        print("Newly initialized embedding:", set(model.state_dict().keys()) - set(model_state_dict.keys()))
        return model
```

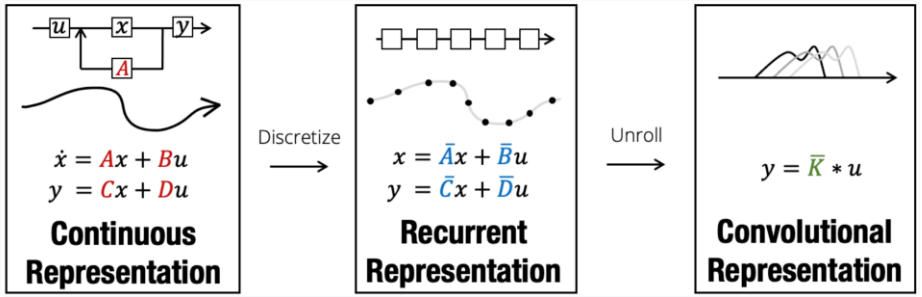
Coding

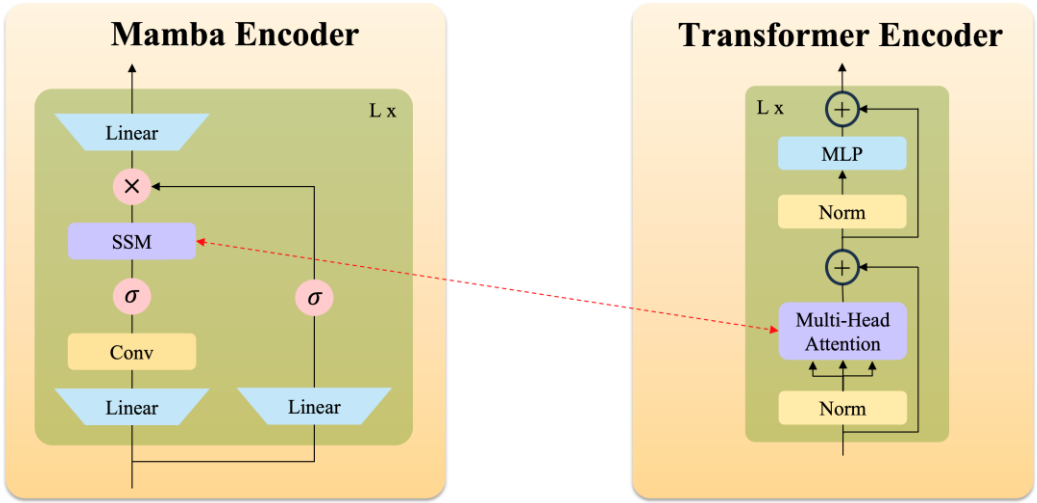


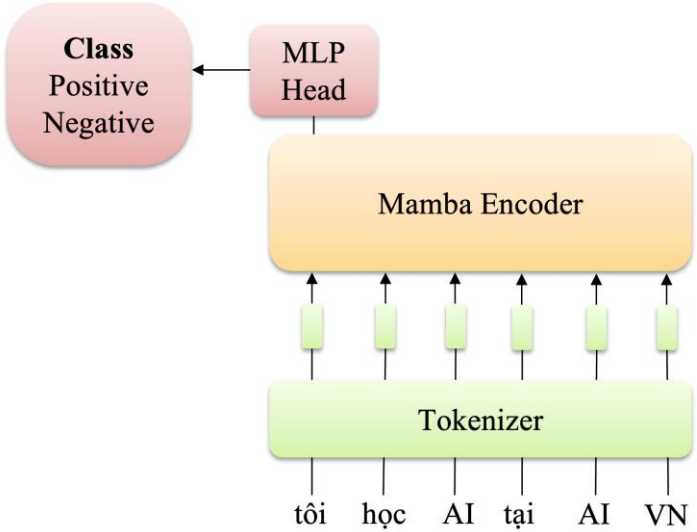

```python
# Định nghĩa một lớp con MambaTrainer kế thừa từ lớp Trainer.
class MambaTrainer(Trainer):

    # Định nghĩa hàm compute_loss để tính toán hàm mất mát trong quá trình huấn luyện.
    def compute_loss(self, model, inputs, return_outputs=False):
        # Lấy giá trị input_ids và labels từ inputs.
        input_ids = inputs.pop("input_ids")
        labels = inputs.pop('labels')

        # Gọi hàm forward của mô hình với input_ids và labels để nhận các kết quả.
        outputs = model(input_ids=input_ids, labels=labels)

        # Lấy giá trị loss từ kết quả của mô hình.
        loss = outputs.loss

        # Trả về cả loss và outputs nếu return_outputs là True, ngược lại chỉ trả về loss.
        return (loss, outputs) if return_outputs else loss

    # Định nghĩa hàm save_model để lưu model trong quá trình huấn luyện.
    def save_model(self, output_dir = None, _internal_call = False):
        # Kiểm tra nếu thư mục lưu trữ không được chỉ định, sử dụng thư mục mặc định từ đối số 'args'.
        if output_dir is None:
            output_dir = self.args.output_dir

        # Nếu thư mục đầu ra không tồn tại, tạo mới nó.
        if not os.path.exists(output_dir):
            os.makedirs(output_dir)

        # Lưu trạng thái của mô hình PyTorch vào file 'pytorch_model.bin' trong thư mục đầu ra.
        torch.save(self.model.state_dict(), f"{output_dir}/pytorch_model.bin")

        # Lưu trạng thái của tokenizer vào thư mục đầu ra.
        self.tokenizer.save_pretrained(output_dir)

        # Lưu cấu hình của mô hình vào file 'config.json' trong thư mục đầu ra.
        with open(f'{output_dir}/config.json', 'w') as f:
            json.dump(self.model.config.to_dict(), f)
```

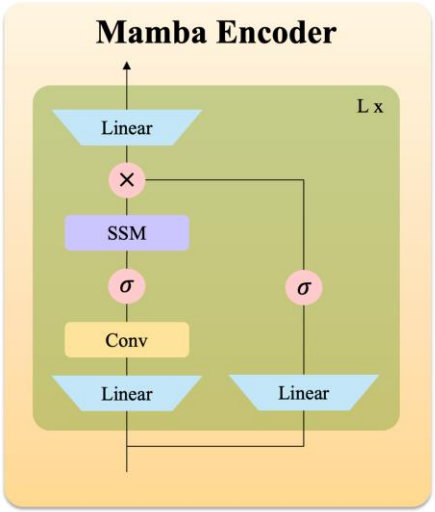
Summary

	RNN	Transformer	Ideal Model
Parallelizable	no	yes	yes
Training Stage	$O(n)$	$O(n^2)$	$O(n)$
Inference Stage	$O(n)$	$O(n^2)$	$O(n)$
Inference per Token	$O(1)$	$O(n)$	$O(1)$









AI VIET NAM

@aivietnam.edu.vn

Thanks!

Any questions?