

Digital World (2018)

Week 9, SI: State Machines

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WHAT WAS IT?

“lunch today was exquisite!”

it



“the weather was atrocious”

it



“I just watched Stephen King’s IT”

it



State machines

- state machines model systems for which the output depends on their history of inputs
- idea: find a set of states that capture the essential properties of the history of inputs
 - => use them to determine the current output...*
 - => ...as well as the next state*
- state machines perform a transduction from a stream of inputs to a stream of outputs

Abusing your torchlight



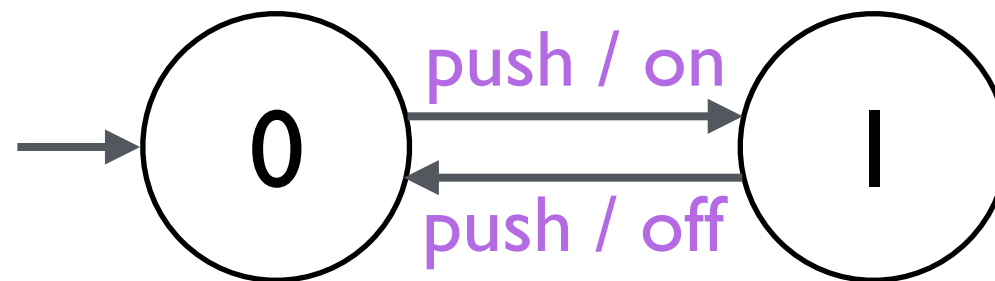
push push push push push push push



transduce

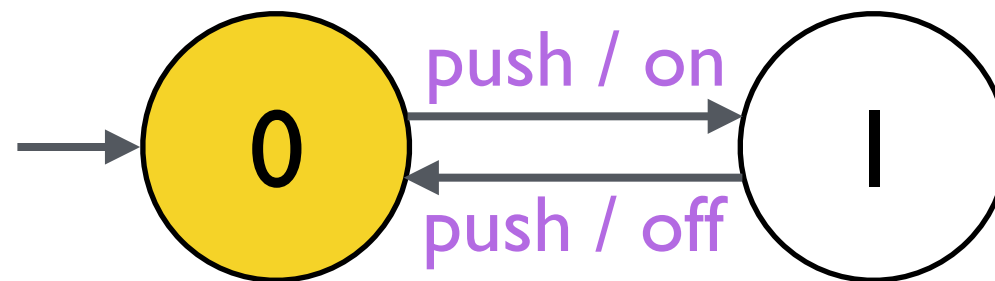
on off on off on off on

Abusing your torchlight



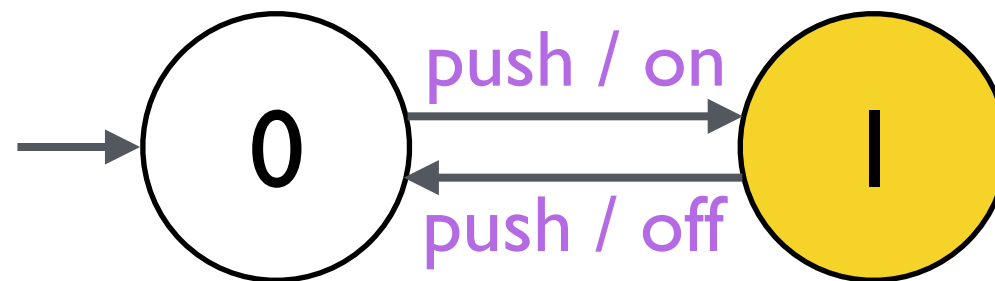
<i>input:</i>	push	push	push	push	push	push	push
<i>output:</i>							

Abusing your torchlight



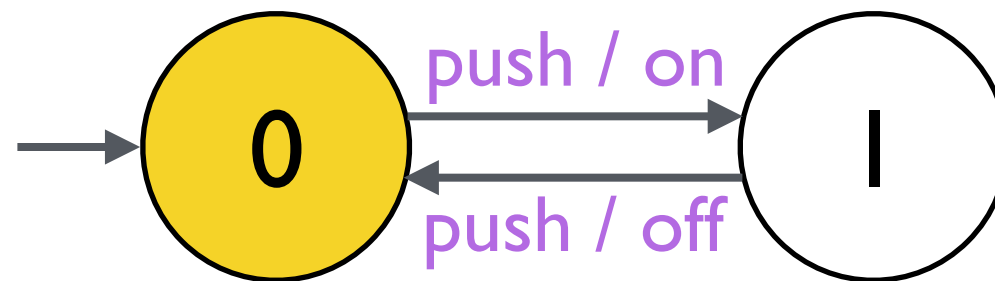
<i>input:</i>	push	push	push	push	push	push	push
<i>output:</i>							

Abusing your torchlight



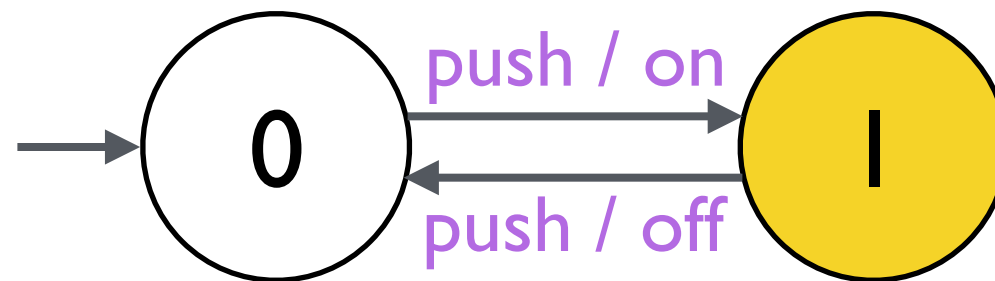
<i>input:</i>	push	push	push	push	push	push	push
<i>output:</i>	on						

Abusing your torchlight



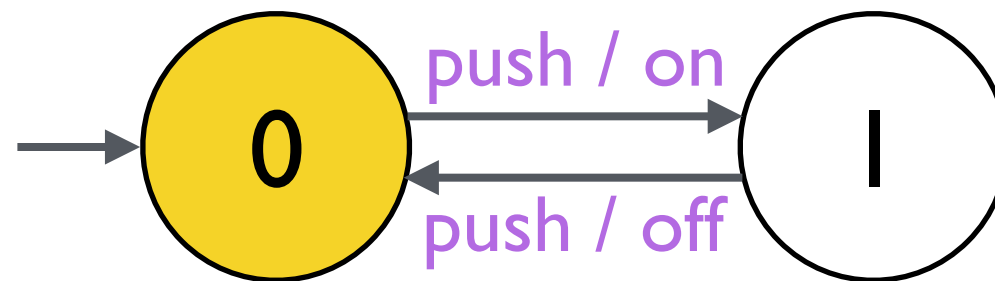
<i>input:</i>	push	push	push	push	push	push	push
<i>output:</i>	on	off					

Abusing your torchlight



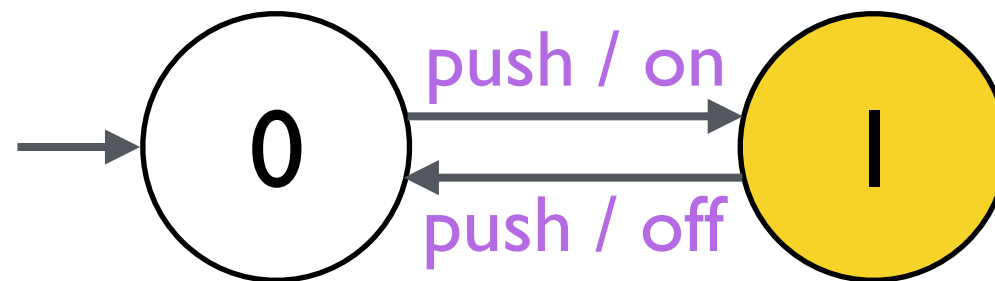
<i>input:</i>	push	push	push	push	push	push	push
<i>output:</i>	on	off	on				

Abusing your torchlight



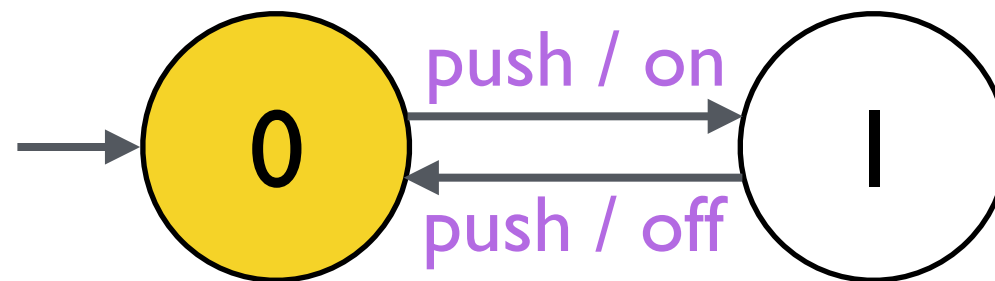
<i>input:</i>	push	push	push	push	push	push	push
<i>output:</i>	on	off	on	off			

Abusing your torchlight



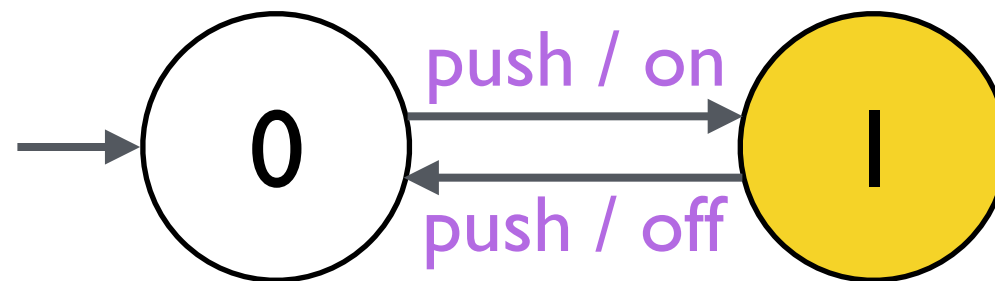
<i>input:</i>	push	push	push	push	push	push	push
<i>output:</i>	on	off	on	off	on		

Abusing your torchlight



<i>input:</i>	push	push	push	push	push	push	push
<i>output:</i>	on	off	on	off	on	off	

Abusing your torchlight



<i>input:</i>	push	push	push	push	push	push	push
<i>output:</i>	on	off	on	off	on	off	on

Language recogniser: abcabcabcabc...

● abcabcabcabc ✓

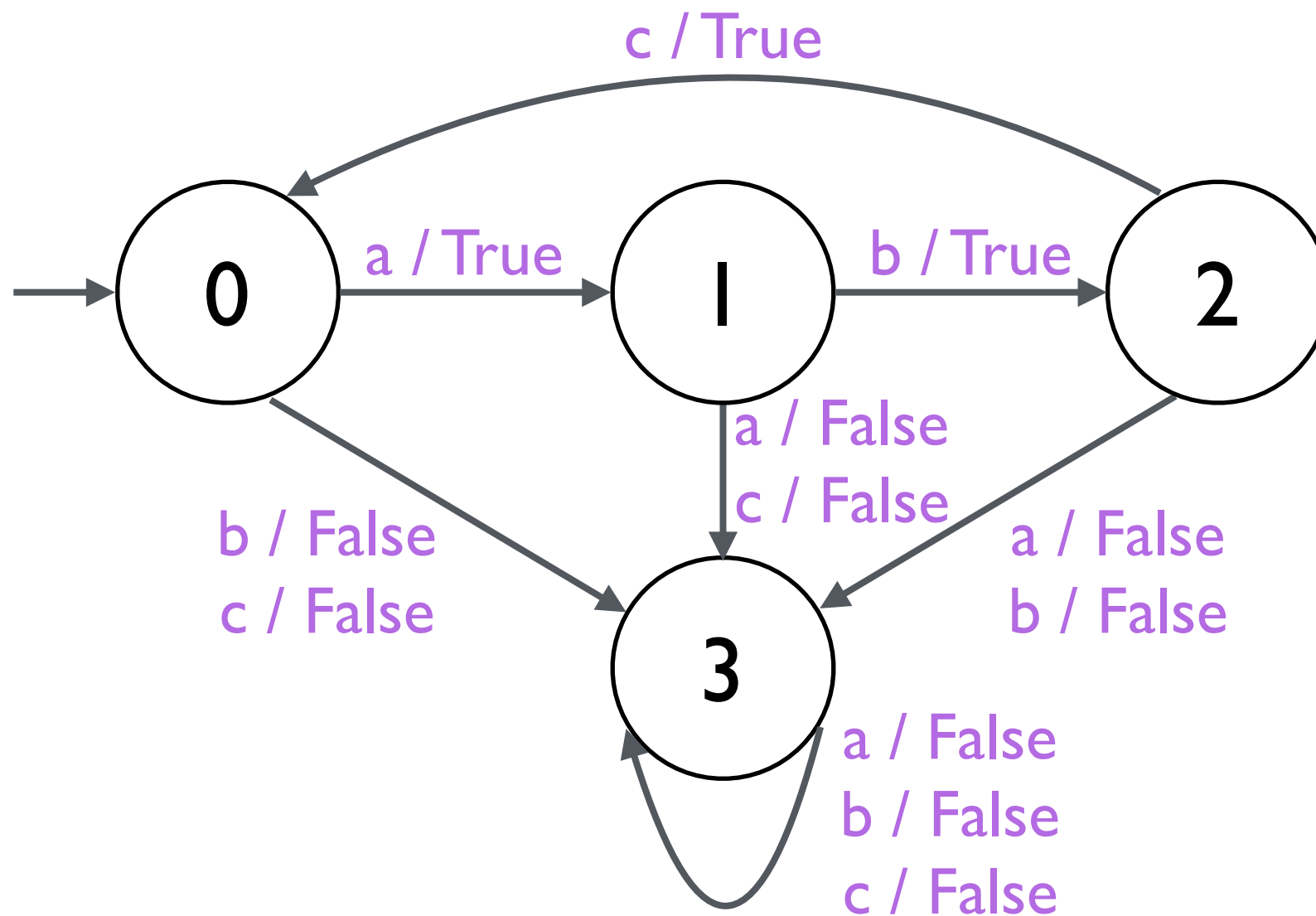
● aabcabcabcabc ✗

● abcabcaaaaaaa ✗

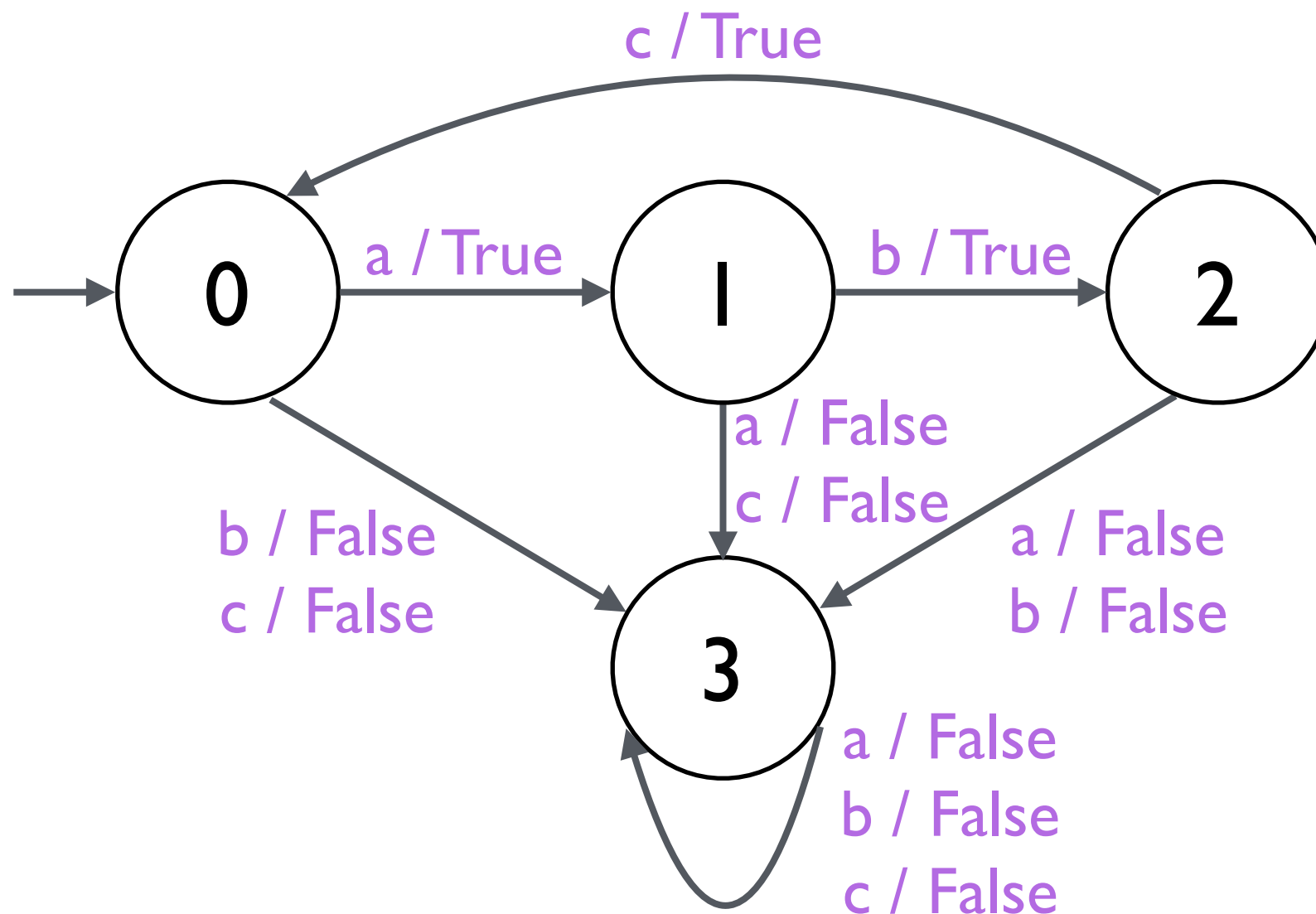
● abccccccccc ✗

a	b	c	c	c
		↓ <i>transduce</i>		
True	True	True	False	False

Language recogniser: *abcabcabcabc...*

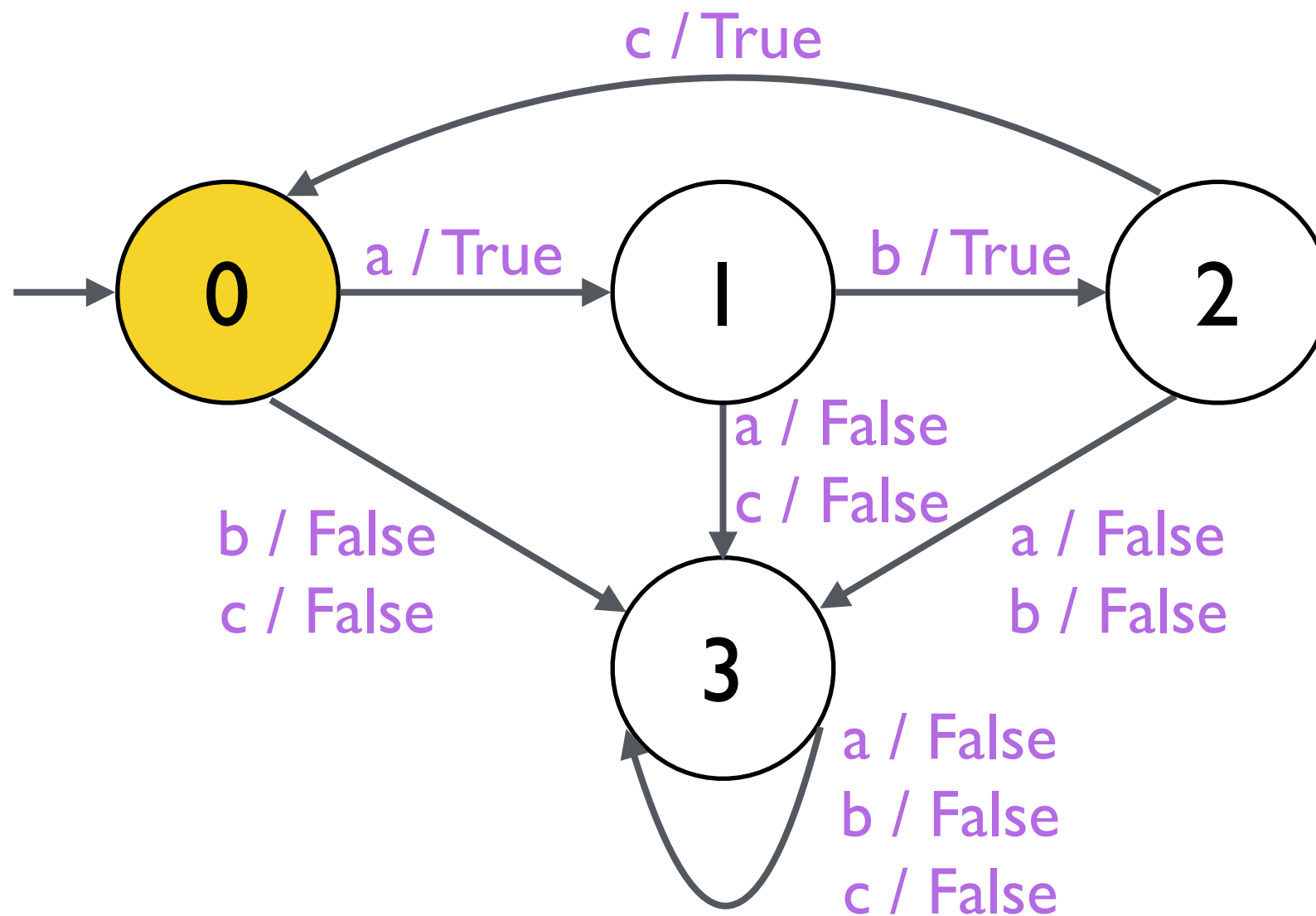


Language recogniser: *abcabcabcabc...*



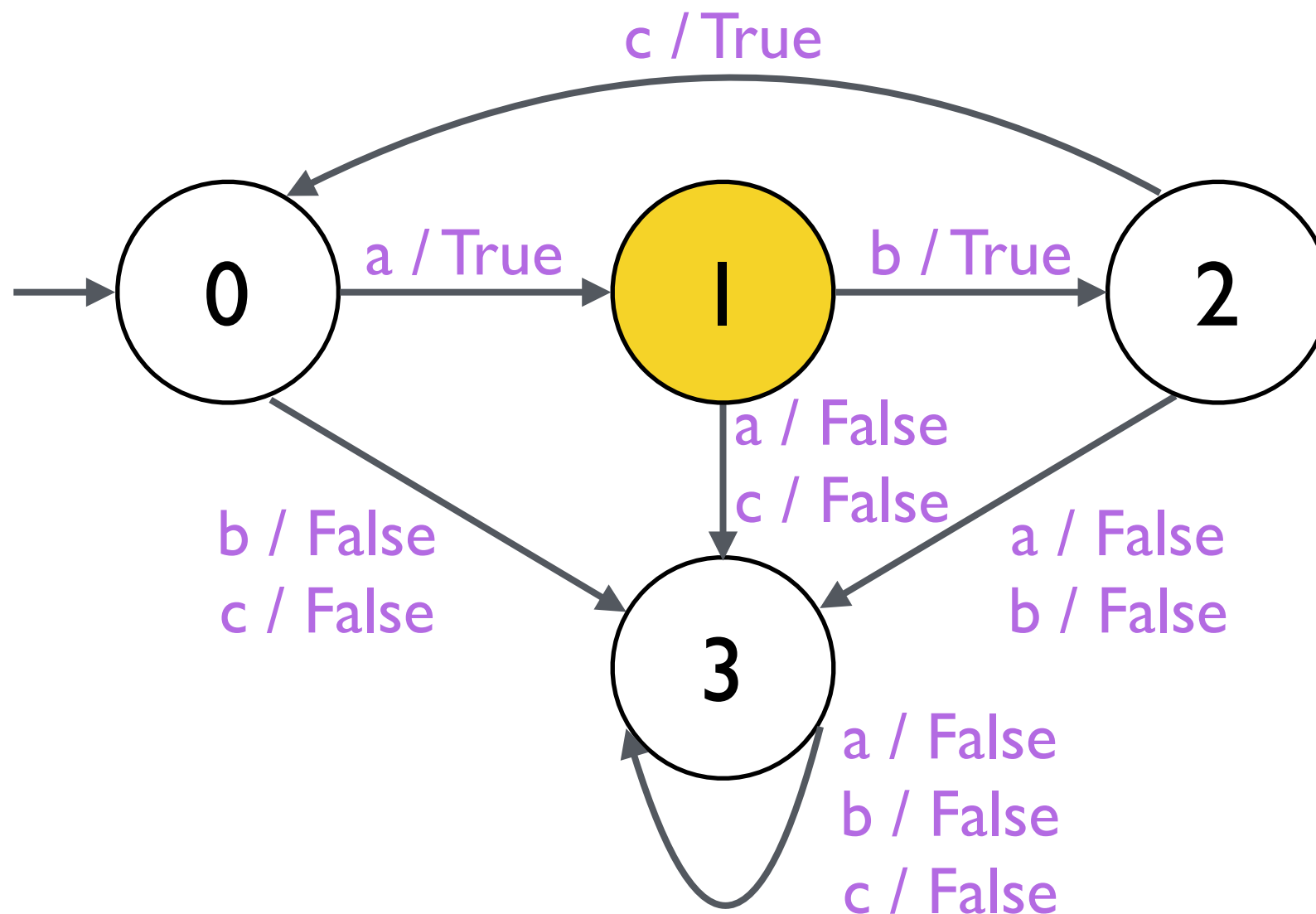
<i>input:</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>c</i>	<i>c</i>
<i>output:</i>					

Language recogniser: *abcabcabcabc...*



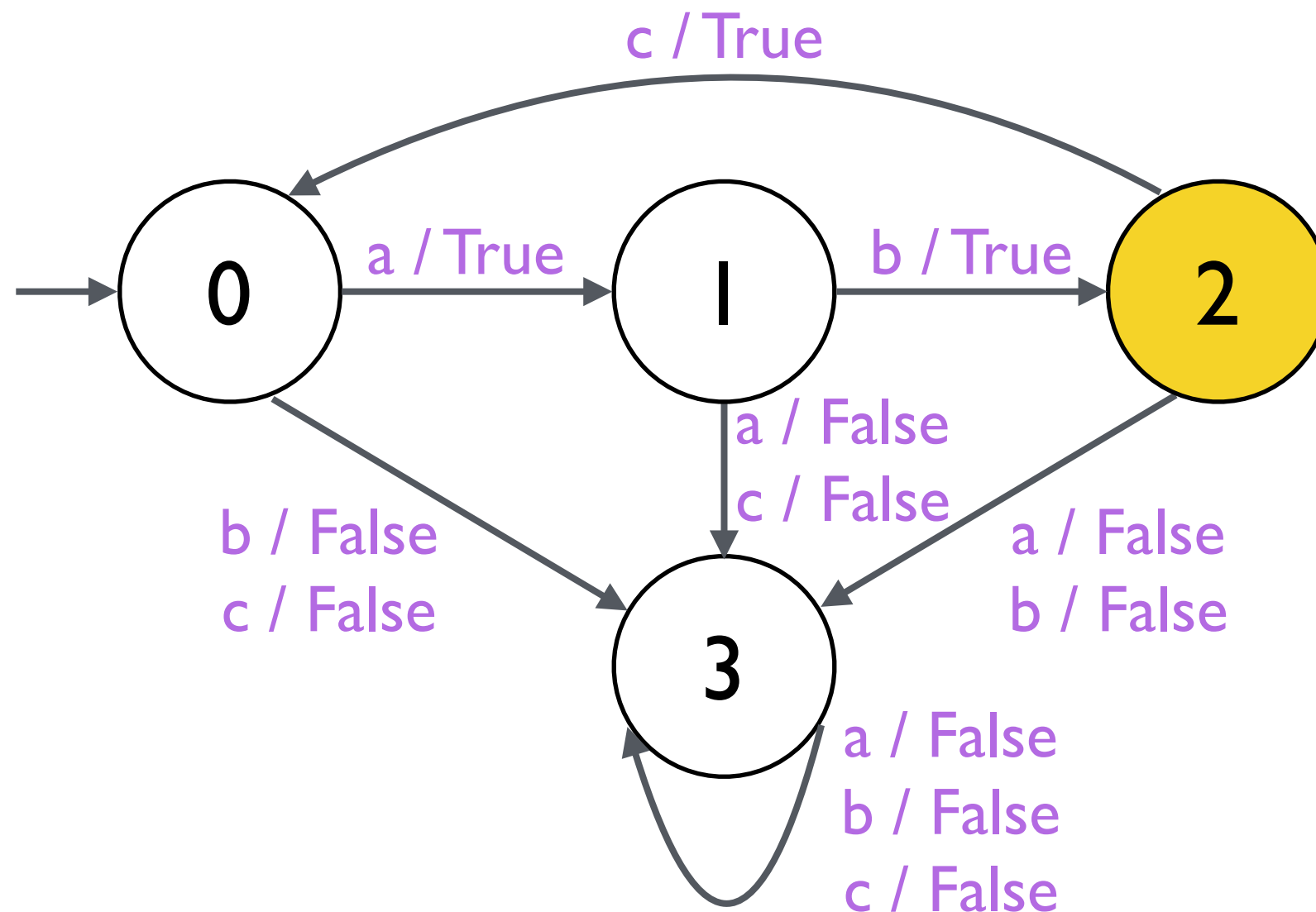
<i>input:</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>c</i>	<i>c</i>
<i>output:</i>					

Language recogniser: *abcabcabcabc...*



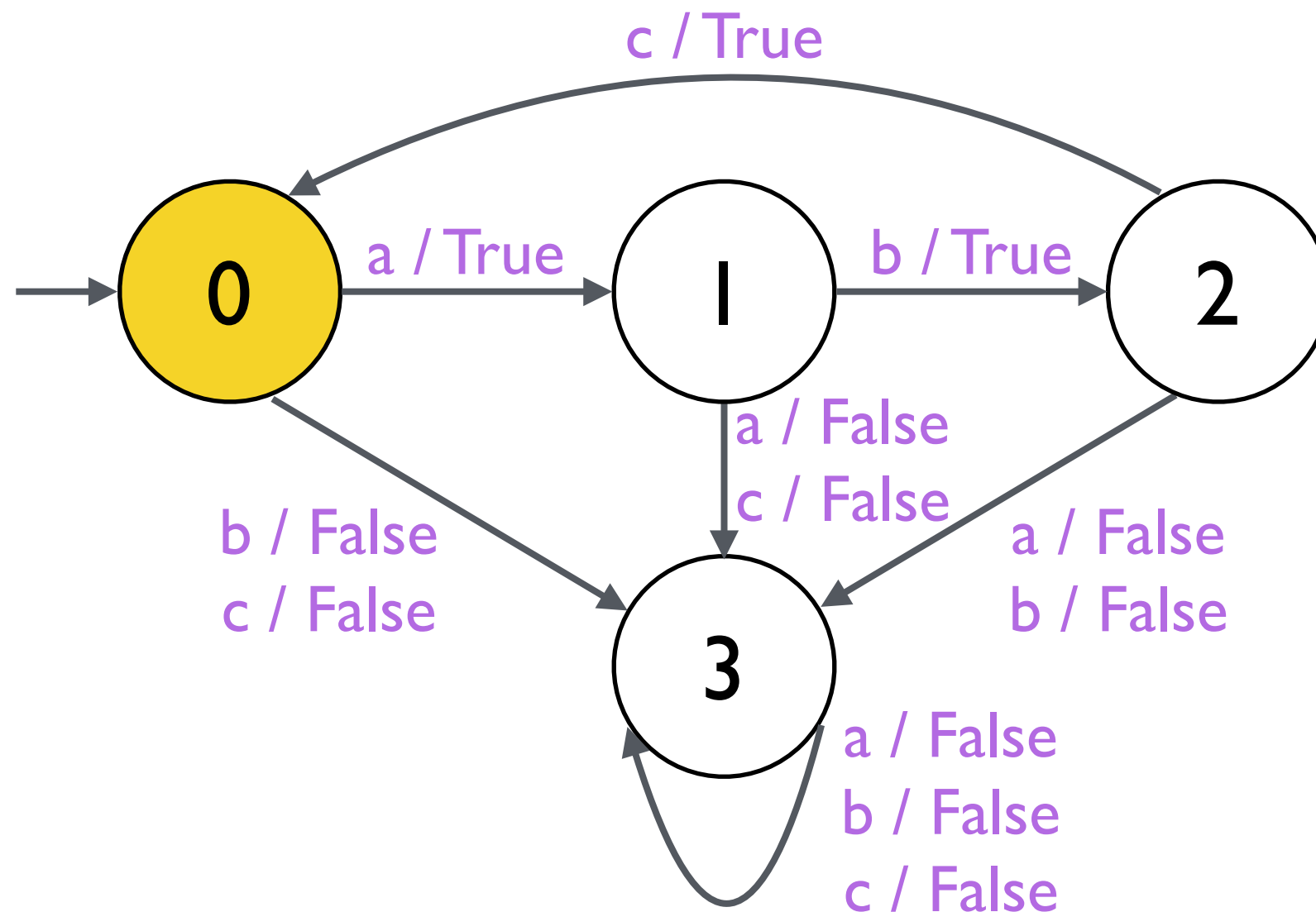
<i>input:</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>c</i>	<i>c</i>
<i>output:</i>	<i>True</i>				

Language recogniser: *abcabcabcabc...*



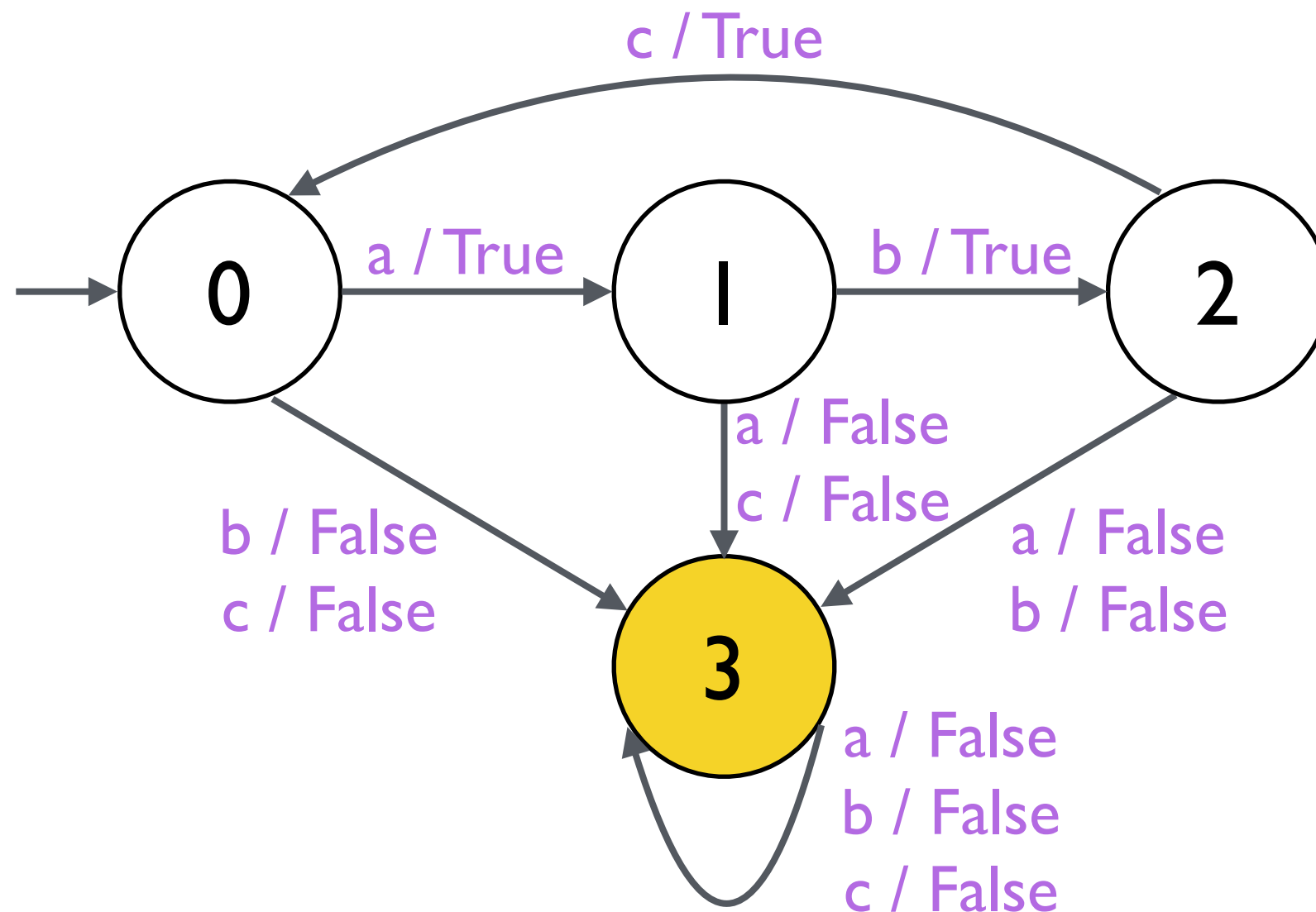
<i>input:</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>c</i>	<i>c</i>
<i>output:</i>	<i>True</i>	<i>True</i>			

Language recogniser: *abcabcabcabc...*



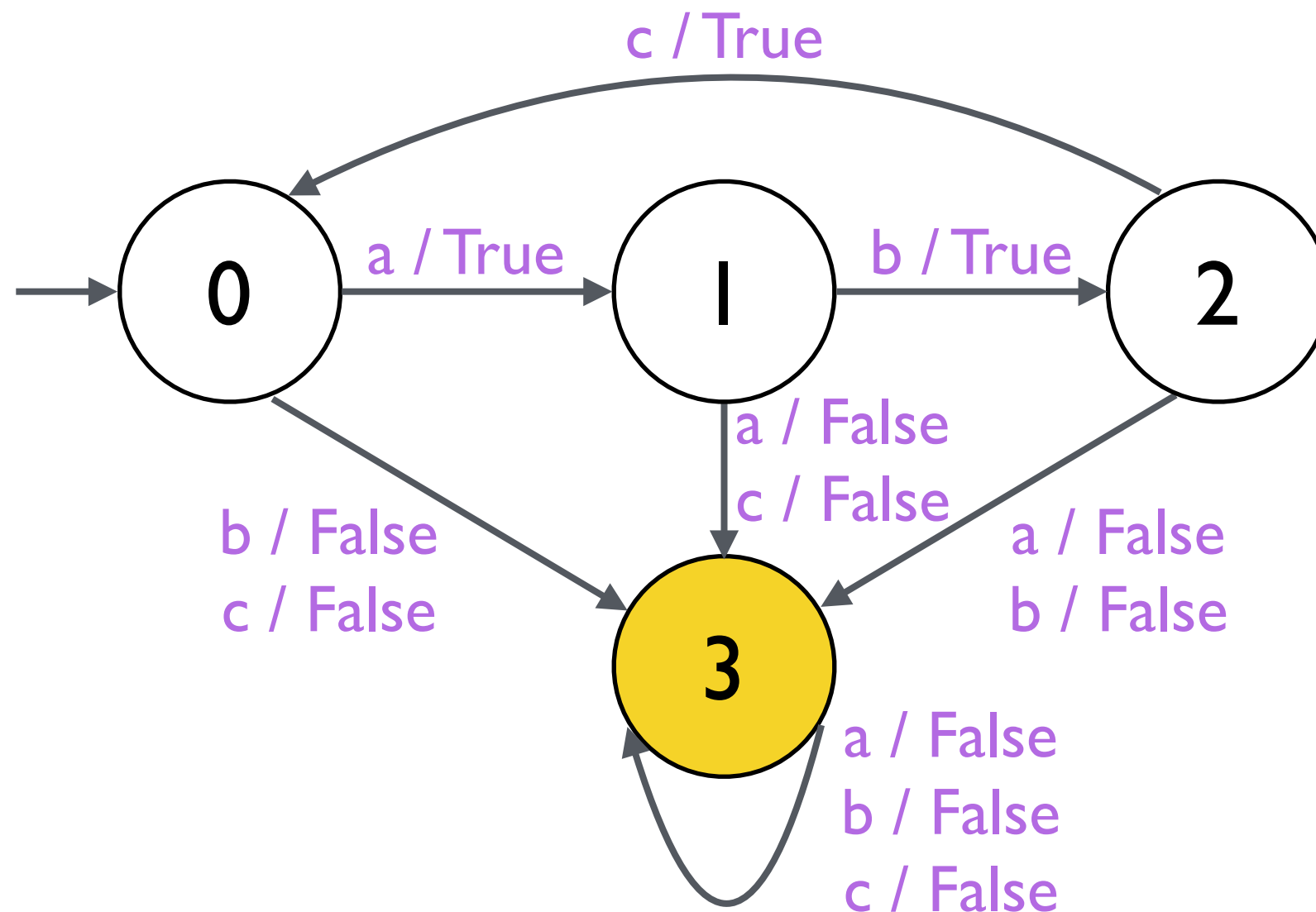
<i>input:</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>c</i>	<i>c</i>
<i>output:</i>	<i>True</i>	<i>True</i>	<i>True</i>		

Language recogniser: *abcabcabcabc...*



<i>input:</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>c</i>	<i>c</i>
<i>output:</i>	<i>True</i>	<i>True</i>	<i>True</i>	<i>False</i>	

Language recogniser: *abcabcabc...*



<i>input:</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>c</i>	<i>c</i>
<i>output:</i>	<i>True</i>	<i>True</i>	<i>True</i>	<i>False</i>	<i>False</i>

Infinite state machine: *up and down*

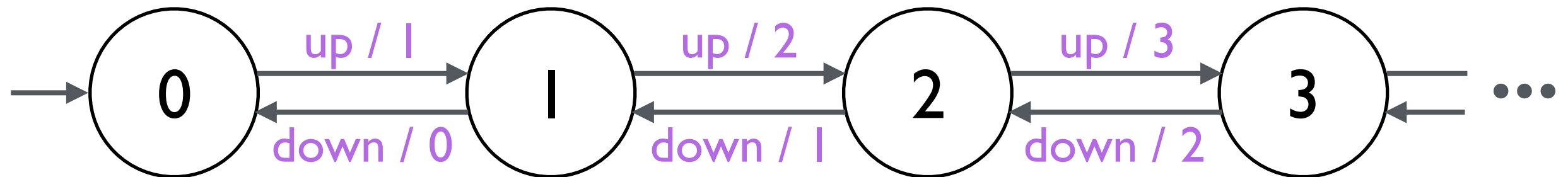


up down up up down

↓ *transduce*

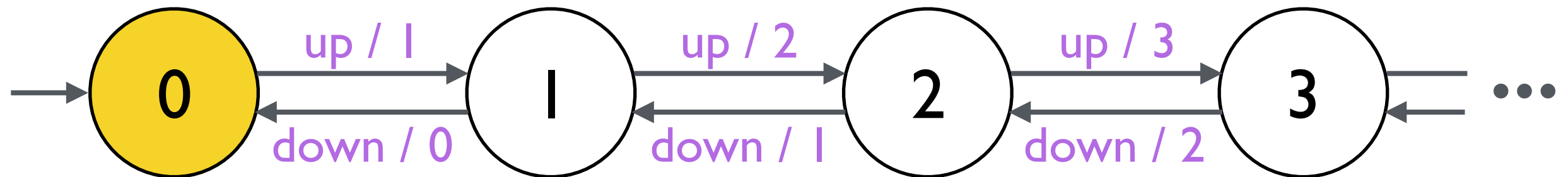
1 0 1 2 1

Infinite state machine: *up and down*



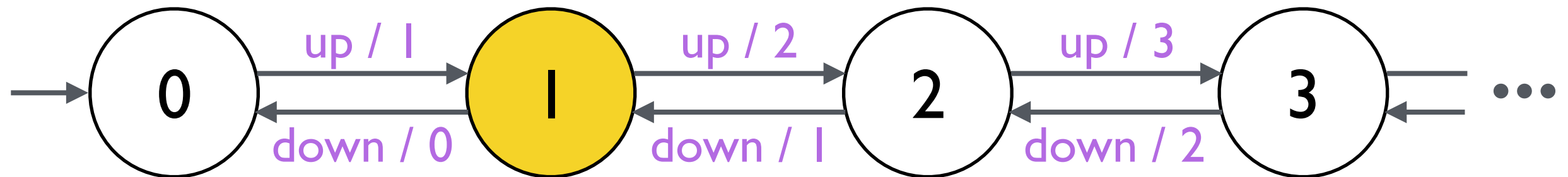
<i>input:</i>	up	up	up	down	down	up
<i>output:</i>						

Infinite state machine: *up and down*



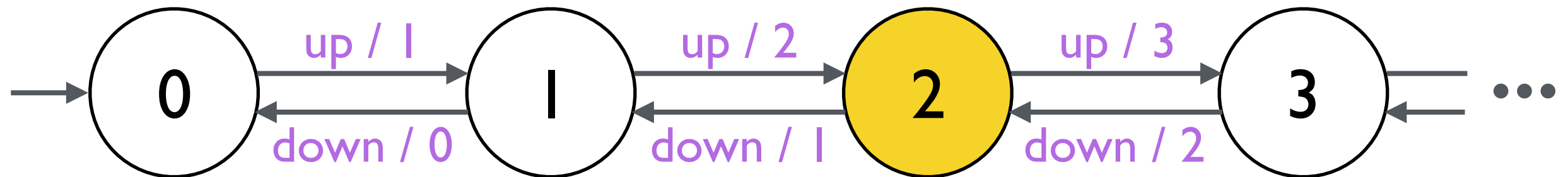
<i>input:</i>	up	up	up	down	down	up
<i>output:</i>						

Infinite state machine: *up and down*



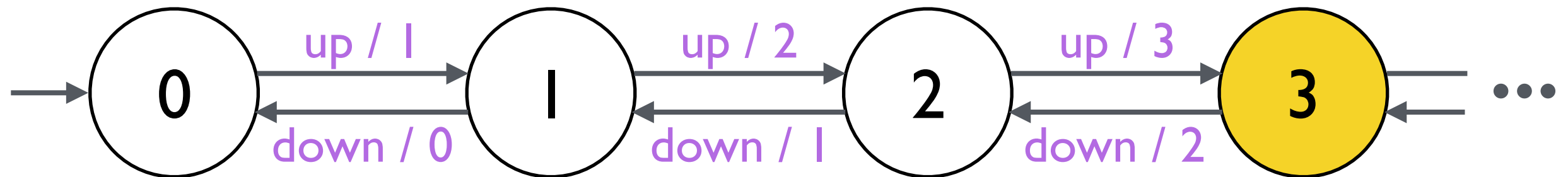
<i>input:</i>	up	up	up	down	down	up
<i>output:</i>	1					

Infinite state machine: *up and down*



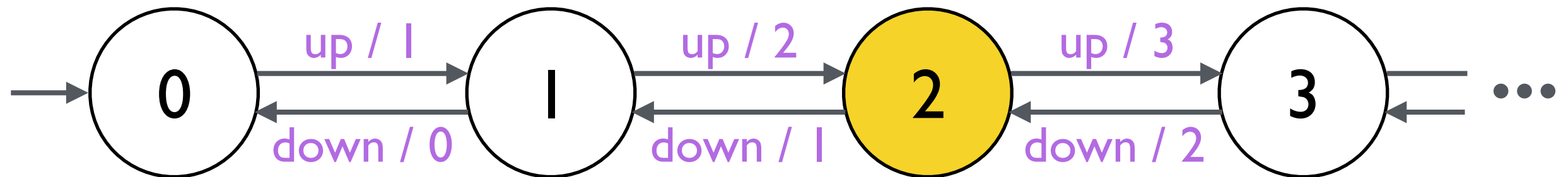
<i>input:</i>	up	up	up	down	down	up
<i>output:</i>	1	2				

Infinite state machine: *up and down*



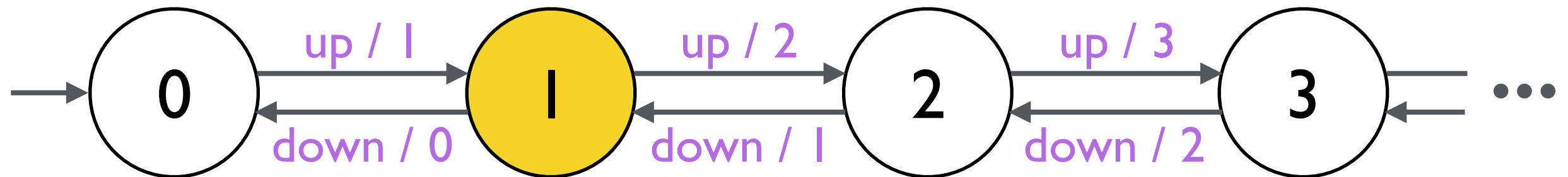
<i>input:</i>	<i>up</i>	<i>up</i>	<i>up</i>	<i>down</i>	<i>down</i>	<i>up</i>
<i>output:</i>	<i>1</i>	<i>2</i>	<i>3</i>			

Infinite state machine: *up and down*



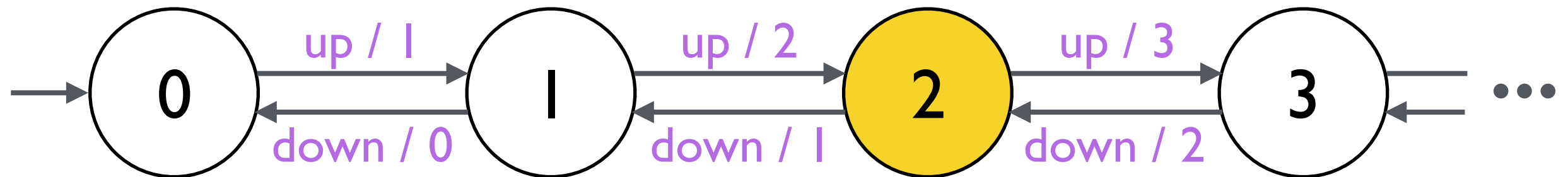
<i>input:</i>	<i>up</i>	<i>up</i>	<i>up</i>	<i>down</i>	<i>down</i>	<i>up</i>
<i>output:</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>2</i>		

Infinite state machine: *up and down*



<i>input:</i>	<i>up</i>	<i>up</i>	<i>up</i>	<i>down</i>	<i>down</i>	<i>up</i>
<i>output:</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>2</i>	<i>1</i>	

Infinite state machine: *up and down*

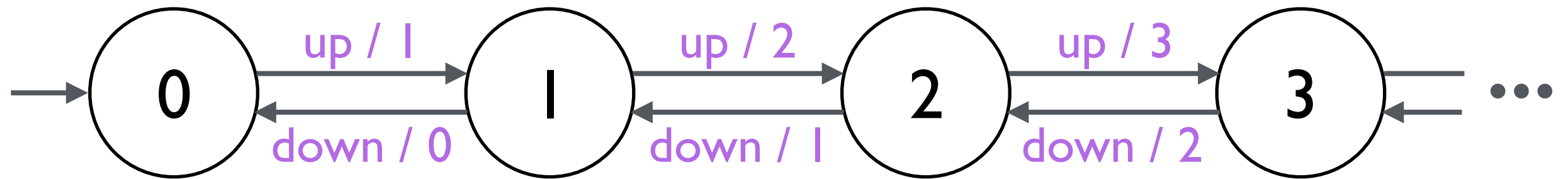


<i>input:</i>	up	up	up	down	down	up
<i>output:</i>	1	2	3	2	1	2



*the outputs come from the transitions,
not the state IDs (even though they match in our model)*

Infinite state machine: *up and down*

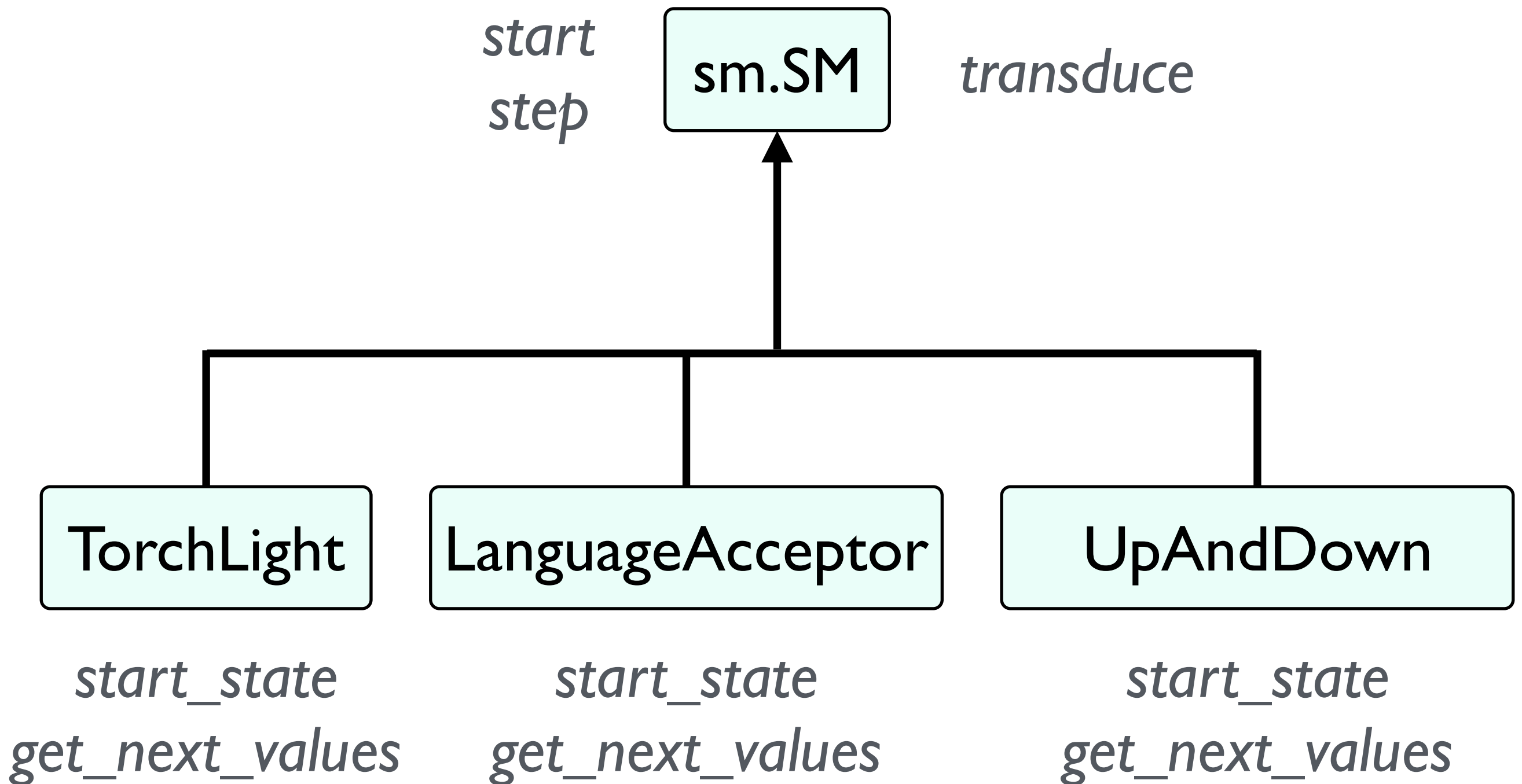


$$\begin{aligned}
 S &= \mathbb{N}_0 \\
 I &= \{\text{up, down}\} \\
 O &= \mathbb{N}_0 \\
 n(s, i) &= \begin{cases} s+1 & \text{if } i == \text{up} \\ s-1 & \text{if } i == \text{down and } s > 0 \end{cases} \\
 o(s, i) &= n(s, i) \\
 s_0 &= 0
 \end{aligned}$$

State machines in Python

- we will implement state machines using the `libdw` library
- `libdw` provides a class called `sm.SM` that we inherit from
 - => inherit several useful methods: `s.start()`, `s.step(i)`, `s.transduce(list)`*
- we then define the `start state` via attribute `start_state`, and the `transitions` via query `get_next_values(self, state, inp)`
 - => these are used by the inherited methods (`start`, `step`, `transduce`, ...) to realise the state machine*

Inheritance hierarchy for state machines



Summary

- **state machines** model systems for which the **output depends** on their **history of inputs**
- they perform a **transduction** from **input streams** to **output streams**
- we can express state machines in Python as classes that **inherit** from **sm.SM** (in the **libdw** library)
- we **define transitions** in those subclasses, **inheriting general methods** that fire/transduce them from sm.SM