(6) Big omega notation. Prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

(5) $g(n) \geq c \cdot n^3$

$g(n) = n^3 + 2n^2 + 4n$

for finding constants c and n_0

$n^3 + 2n^2 + 4n \geq cn^3$

Divide both sides with n^3

$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$

here $\frac{2}{n}$ and $\frac{4}{n^2}$ approaches 0

$1 + 2/n + 4/n^2$ +d 1

Example $c = 1/2$

$1 + 2/n + 4/n^2 \geq \frac{1}{2}$

$1 + 2/n + 4/n^2 \geq 1$

$1 + 2/n + 4/n^2 \geq 1/2$

Thus, $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$

(7) Big theta notation: Determine whether $f(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not

$c_1 n^2 \leq h(n) \leq c_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

upper bound $(O(n^2))$:

$h(n) = 4n^2 + 3n$

$h(n) \le C_2 n^2$

$4n^2+3n \le C_2 n^2$

$4n^2+3n \le 5n^2$

lets $C_2 = 5$

Divide both sides by n^2

$4 + 3/n \le 5$

$h(n) = 4n^2+3n$ is $O(n^2)$ $(C_2=5, n_0=1)$

lower bound:

$h(n) = 4n^2+3n$

$h(n) \ge c_1 n^2$

lets $c_1=4 \Rightarrow 4n^2+3n \ge 4n^2$

Divide both sides by n^2

$4+3/n \ge 4$

$h(n)=4n^2+3n$

$h(n)=4n^2+3n$ is $O(n^2)$

(8) Let $f(n)=n^3-2n^2+n$ and $g(n)-n^2$ show whether $f(n) = \Omega(g(n))$ or false and justify your answer.

(5) $f(n) \ge c \cdot g(n)$

substituting $f(n)$ and $g(n)$ into this inequality we get

$n^3-2n^2+n \ge c(-n^2)$

find c and no holds $n \ge n_0$.

$n^3-2n^2+n \ge -cn^2$

$n^3-2n^2+n+cn^2 \ge 0$

$n^3+2(c-2)n^2+n \ge 0$

$n^3+(c-2)n^2+n \ge 0$

$n^3+(c-2)n^2+n = n^3-n^2+n \ge 0$

$f(n)=n^3-2n^2+n$ is $\Omega(g(n)) = \Omega(-n^2)$

$\therefore$ The statement $f(n) = \Omega(g(n))$ is True

(7) Determine whether $h(n) = n\log n + n$ is in $O(n\log n)$ prove for your conclusion

(S) $c_1 n\log n \le h(n) \le c_2 n\log n$

upper bound :

$h(n) \le c_2 n\log n$

$h(n) = n\log n + n$

$n\log n + n \le c_2 n\log n$

Divide both sides by $n\log n$

$1 + \frac{n}{n\log n} \le 2$

$1 + \frac{1}{\log n} \le c_2$

$1 + \frac{1}{\log n} \le 2$

then $h(n)$ is $O(n\log n)$

<u>lower</u> <u>bound</u>:-

$h(n) \ge c_1 n\log n$

$h(n) = n\log n + n$

Divide both sides by $n\log n$

$1 + \frac{n}{n\log n} \ge c_1$

$1 + \frac{1}{\log n} \ge c_1$

$1 + \frac{1}{\log n} \ge 1$

$\frac{1}{\log n} \ge 0$ for all $n > 1$

$h(n)$ is $\Omega(n\log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n\log n + n$ is $O(n\log n)$

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(10) Solve the following recurrence relations and find the order of growth

(b) $T(n) = 4T(n/2) + n^2,\ T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$a = 4,\ b = 2,\ f(n) = n^2$

Applying master theorem

$T(n) = aT(n/b) + f(n)$

$f(n) = O(n^{\log_b a - \epsilon})$

$f(n) = \Theta(n^{\log_b a})$ then $T(n) = \Theta(n^{\log_b a} \log n)$

$f(n) = \Omega(n^{\log_b a + \epsilon})$ then $T(n) = f(n)$

calculating $\log_b a$:

$\log_b a - \log_2 4 = 2$

$f(n) = n^2 = \Theta(n^2)$

$f(n) = \Theta(n^2) = \Theta(n^{\log_b a})$

$T(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$

order of growth

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$ is $\Theta(n^2 \log n)$