Problem Set 2
Part II: Modeling Firms in Vietnam

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Course: Dynamic Macroeconomics

Modeling rms in Vietnam is not straightforward because most rms are small operations and/or are typically family-owned. In the latter case, households are both producers and consumers. For now, we will focus on their role as producers. Consider the dynamic prot-maximization problem for an infinitely-lived representative firm.

$$V(A_t, K_t, p_t) = \max_{x_t, K_{t+1}} \Big\{ R(A_t, K_t, x_t) - w_t x_t - \frac{\gamma}{2} C(K_{t+1}, A_t, K_t)$$
$$- p_t \left(K_{t+1} - (1-\delta)K_t\right) + \beta \mathbb{E}_{A_{t+1}, p_{t+1} | A_t, p_t} \left[V(A_{t+1}, K_{t+1}, p_{t+1})\right] \Big\},$$

subject to the stochastic processes for productivity and capital prices:

$$\log A_{t+1} = \rho_A \log A_t + \varepsilon_{A,t+1},$$
$$\log p_{t+1} = \rho_p \log p_t + \varepsilon_{p,t+1},$$

where:

- $R(\cdot)$ is the revenue function,
- $C(\cdot)$ is the adjustment cost of capital,
- w_t is the wage (cost of variable input),
- p_t is the price of investment in new capital,
- γ controls the size of adjustment costs,
- δ is the depreciation rate of capital,
- β is the discount factor,
- $\varepsilon_{A,t+1} \sim \mathcal{N}(0, \sigma_A^2)$ and $\varepsilon_{p,t+1} \sim \mathcal{N}(0, \sigma_p^2)$.

- Modify the codes on Canvas to obtain the baseline model above, assuming functional forms for revenue and adjustment costs. You must do the following.

Splitting firms by size:

Firms are split into **large** and **small** based on the median value of total annual sales (variable d2). The median value is calculated across the full dataset, and firms with total sales above this threshold are classified as **large**, while the rest are **small**.

Key production input variable x :

Labor input is measured by the number of **full-time employees** (l1), which directly reflects the scale of variable input used in production.

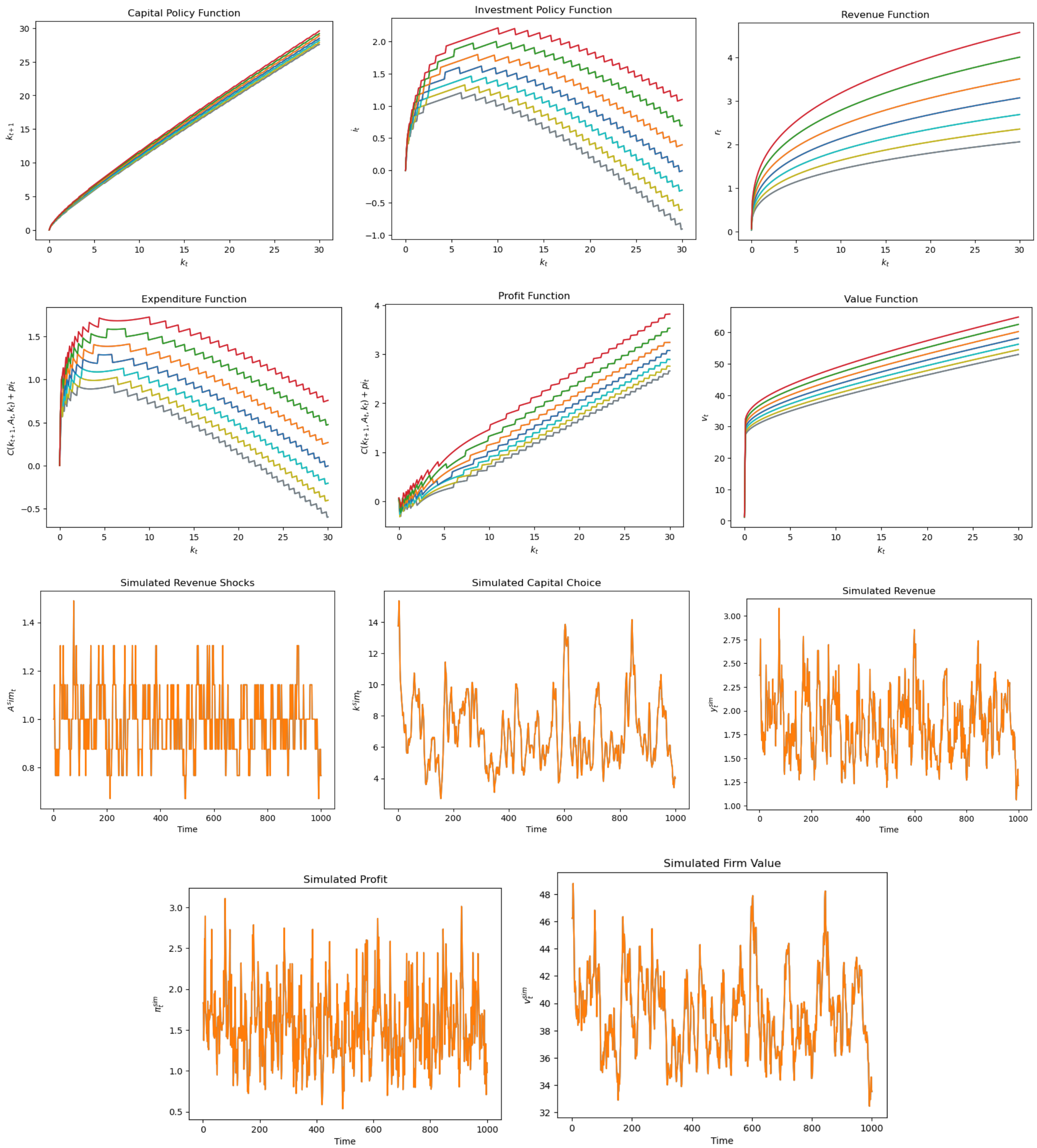
Variable cost w x :

This is approximated by combining several cost-related variables:

- **Total labor cost**: wages, salaries, bonuses, and social insurance if available (e.g., n2b, n2c)
- **Raw material & intermediate inputs**: annual cost of inputs like raw materials (n2e) and goods used in production (n2f)
- **Utilities**: total annual spending on electricity and fuel if available (e.g., n2g, n2h)
- **Capital investment price p :**
  This is approximated from reported expenditures on new equipment and machinery. The variable n5a and n5b report spending on new and used capital goods, which we sum or take a median from to estimate p .
- **Key determinant chosen for the model:**
  We choose **raw materials and intermediate goods cost** (n2e) as the main production determinant for modeling. This is supported by:
- Strong correlation with total output and firm scale
- It varies significantly across firms and is well-documented in the dataset
- **Empirical patterns:**
- There is a positive correlation between **debt levels** and **input spending**, suggesting firms use borrowing to finance intermediate inputs and expand capacity.
- Firms with higher debt also show **higher capital investment**, consistent with using credit for machinery upgrades or expansion.
- In contrast, small firms with less access to financing show flatter patterns in input scaling, aligning with higher sensitivity to variable cost pressures.
- **Decisions for model input by firm size:**
  Small firms: Typically face more constraints on formal credit, so we emphasize intermediate input cost and variable labor costs.
- **Large firms:** Can smooth shocks via access to debt and capital markets, so we also incorporate capital depreciation and adjustment cost more heavily.

- Simulate the model for small and large firms
- Plot the policy functions for each type of rm and discuss the patterns you see.
- Plot the simulated investment and capital for each type of firm over 1000 periods and discuss the patterns.

Large firms:

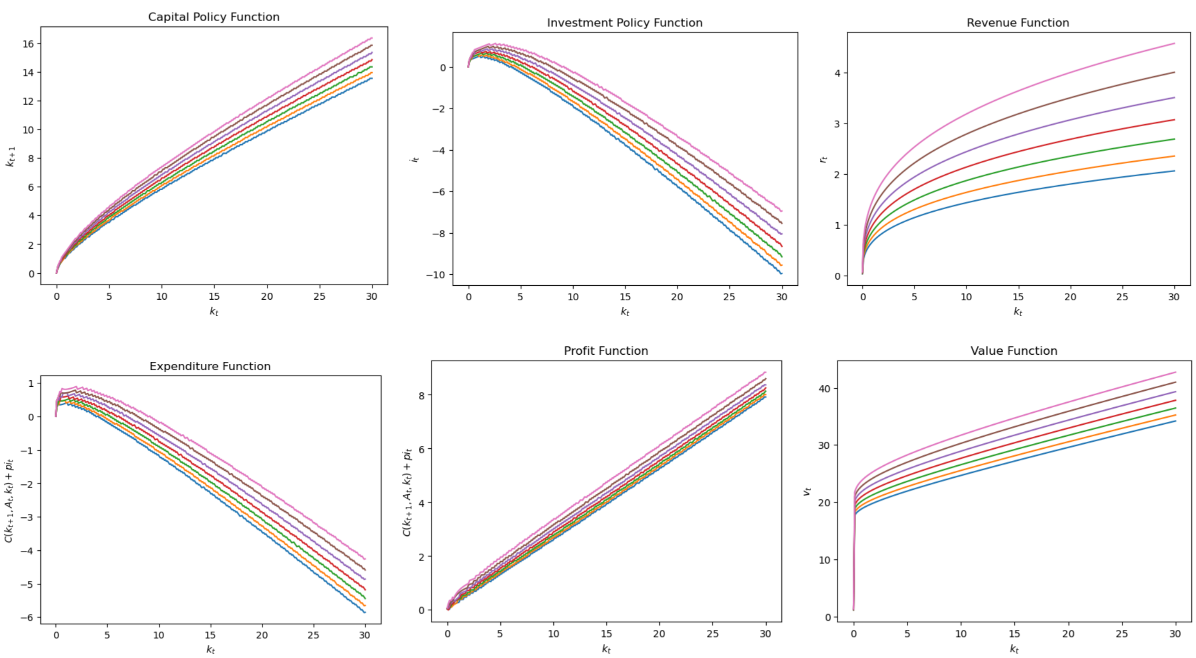


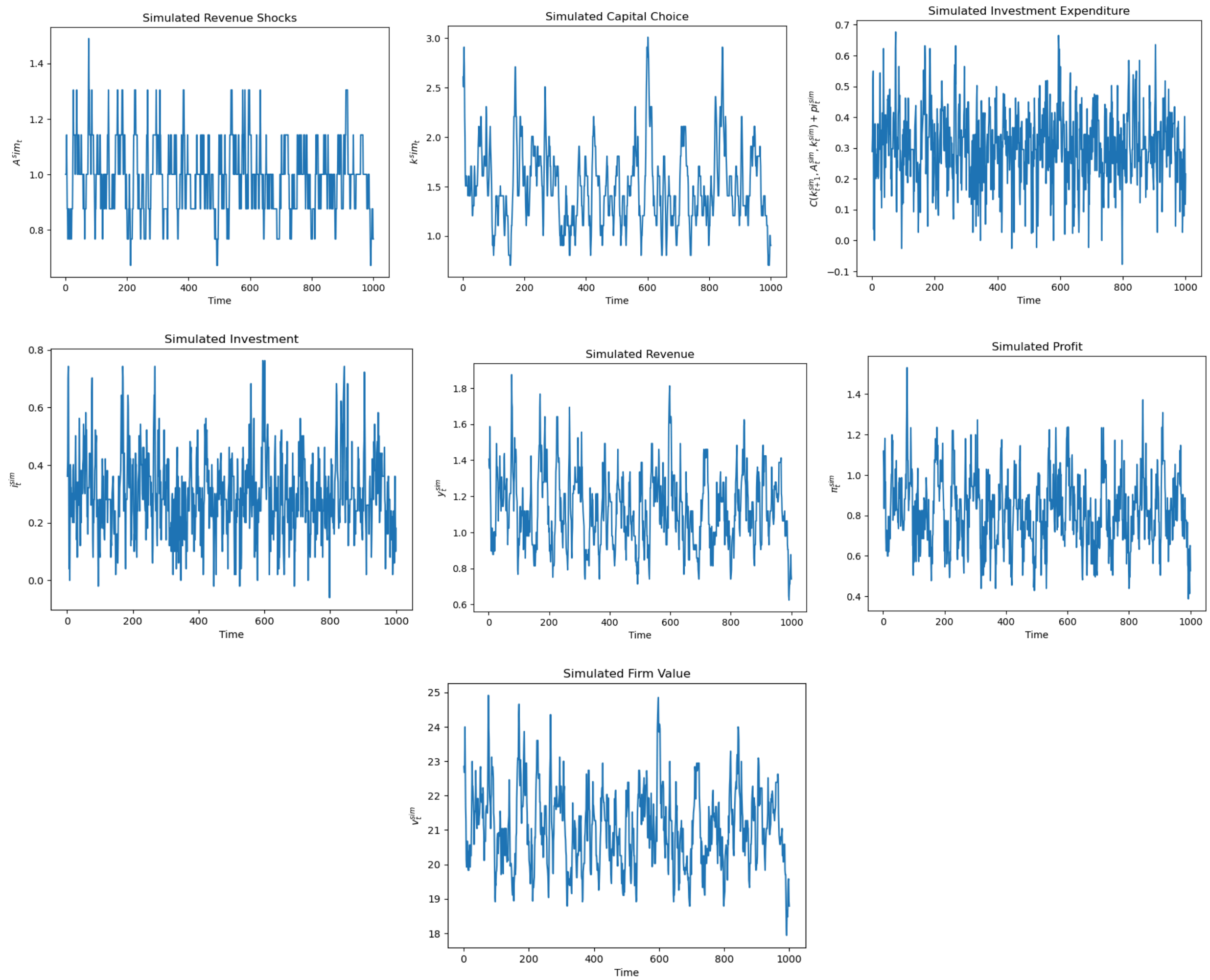
The policy and simulation graphs for large firms reveal clear and consistent patterns. Starting with the Capital Policy Function, the curve is smooth and concave, indicating that while firms increase future capital as current capital grows, the rate of increase slows down. This diminishing marginal return to capital is expected in larger, more established firms with already substantial resources. The Investment Policy Function has a distinct inverted-U shape: firms invest more aggressively at lower capital levels, but once capital passes a certain threshold, investment declines. This shows that firms are optimizing , they invest heavily early on, then taper off as additional capital becomes less productive. The Revenue Function is concave as well, showing diminishing returns to scale; revenue increases with capital but at a decreasing rate, likely reflecting market saturation or internal inefficiencies at higher scales.

The Expenditure Function (including adjustment costs and capital prices) rises quickly at low capital and flattens, and even bends slightly, at high capital levels, again showing concavity and signaling increasing investment costs. In the Profit Function, we see a steady, upward-sloping line with mild curvature. Profits grow with capital but reflect the influence of rising expenditures and flattening revenue, resulting in gradual rather than exponential gains. Lastly, the Value Function is also concave and increasing, firms with more capital are valued higher, but the marginal gain in value per unit of capital shrinks, consistent with risk aversion and adjustment frictions.

In the time-series simulations, large firms exhibit realistic fluctuations. Simulated Revenue Shocks show relatively frequent changes in productivity, while the Simulated Capital Choice graph reflects capital adjustments that respond to those shocks with inertia and smoothing. Simulated Revenue, Profit, and Firm Value all demonstrate some volatility but stay within a stable range, this reflects the stability and predictability often associated with larger firms that can better absorb shocks and maintain consistent performance. The firm value, in particular, fluctuates less drastically, suggesting long-run planning and resilience in their investment strategies.

Small firms:





The behavior of small firms in the model is quite different and more volatile compared to large firms. The **Investment Policy Function** shows a very steep and concave curve that peaks early and then drops sharply, even turning negative at high levels of capital. This indicates that small firms invest aggressively when capital is low but reduce or even disinvest as they accumulate more, a sign of tighter financial constraints or high marginal adjustment costs. The **Revenue Function** maintains a concave shape, but the gap between lines (representing different productivity levels) is narrower than in large firms, reflecting lower responsiveness of revenue to capital.

The **Expenditure Function** also drops steeply with capital, showing that investment costs rise quickly, and likely discourage further capital accumulation. The **Profit Function**, in contrast, is more linear and increases steadily, but the slope is lower than in large firms, implying smaller profit gains per additional unit of capital. The **Value Function** is again concave and increasing, but flattens earlier, which means the long-run value of small firms doesn't increase much after a point possibly due to limited scalability or higher risk.

Looking at the **simulated time-series**, small firms show much more fluctuation. **Simulated Capital Choice**, **Investment**, **Profit**, and **Firm Value** all exhibit jagged movements, with no strong trends. This reflects that small firms are more reactive and less stable, possibly because they are more exposed to shocks and lack buffers. The **Simulated Revenue Shocks** are just as frequent, but their effects are more visible in the firm's outcomes. Overall, the data portrays small firms as highly sensitive to their environment, adjusting sharply to conditions and facing more volatile performance.

- Simulate the model for different values of γ and δ for each type of firm:

γ = 0.10 and δ = {0.05, 0.06, 0.07, 0.08}

γ = {0.10, 0.15, 0.20, 0.25} and δ = {0.08}

γ = {0.10, 0.15, 0.20, 0.25} and δ = {{0.05, 0.06, 0.07, 0.08}**Average capital**

different values of γ / δ	δ = 0.05	δ = 0.06	δ = 0.07	δ = 0.08
γ = 0.10	8.066962742474916 0.3837127510702345	6.740545759197325 0.3892481807826091	5.7398133090301 0.3900297801872912	4.967942972240802 0.3881123919331102

Average investment

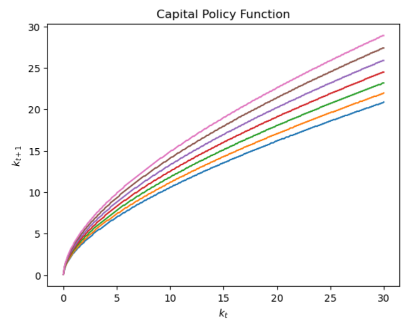
different values of γ / δ	γ = 0.10	γ = 0.15	γ = 0.20	γ = 0.25
δ = 0.08	3.886943231103679 0.3822829732441471	2.389356249832776 0.35533321359531783	1.6514991775919732 0.32853395511705674	1.2335072598662207 0.307248056187291

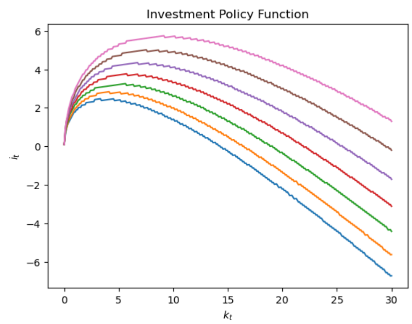
To understand how firms respond to different investment environments, we compare three parameter combinations with the baseline case. When both the adjustment cost and depreciation rate are low (γ = 0.10, δ = 0.05), firms tend to accumulate capital aggressively and smoothly over time. This is reflected in the capital policy function, where the curve is much steeper than in the baseline, suggesting that firms are more willing to invest in future production. Investment is also higher and more sustained, as shown in the corresponding investment policy function. Because adjustment is cheap and depreciation is minimal, firms experience higher revenue and lower expenditure, leading to consistently strong profits. The value function in this case also rises faster, indicating that the firm expects greater long-term benefits from its decisions.

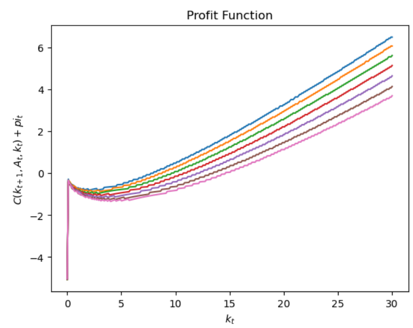
In contrast, when the depreciation rate increases to δ = 0.08 while keeping γ low (γ = 0.10, δ = 0.08), the firm becomes more cautious. Capital accumulation slows slightly, and investment levels drop, though still remain above the baseline. Revenues remain solid due to higher capital levels, but the faster depreciation eats into firm value and profitability. We still observe smoother transitions compared to the baseline, but the overall firm performance is more muted.

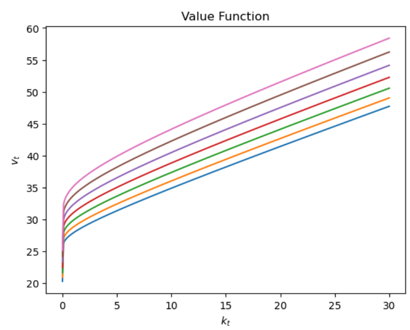
The third case, with high adjustment cost and high depreciation (γ = 0.25, δ = 0.08), paints a very different picture. Here, capital accumulation is much slower, and the firm heavily restricts investment due to the high penalty of adjustment. The investment policy curve peaks early and drops sharply. Despite similar revenue functions across all scenarios, high adjustment and depreciation costs inflate expenditures and squeeze profits. As a result, the value function grows more slowly and flattens earlier, suggesting that the firm anticipates limited gains from additional capital. Simulated dynamics further confirm this behavior, capital choices are more erratic, investment is more volatile, and profits are lower and less stable.

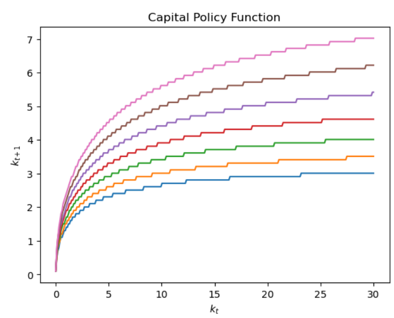
Together, these comparisons show that small changes in γ and δ have large effects on firm dynamics. Firms facing lower adjustment and depreciation costs not only invest more, but also benefit from higher stability and long-term value. This makes these parameters critical levers for understanding firm behavior in the model.

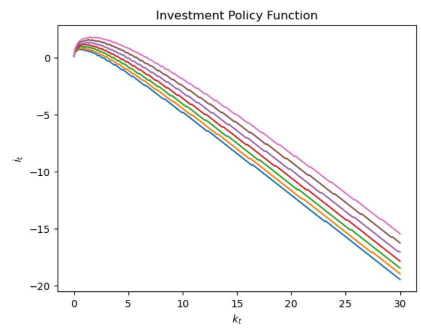


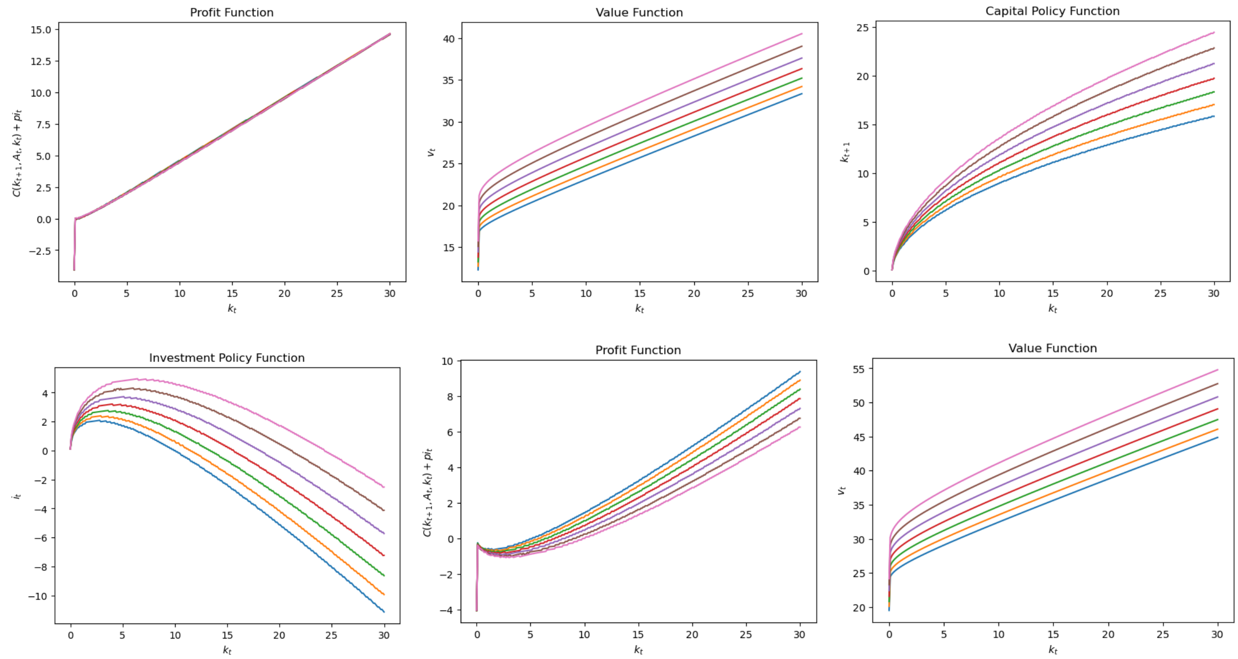












- Key determinants of production choices among Vietnamese firms and add at least one of these to the model.

There are several important factors that influence how Vietnamese firms make production decisions. These include access to capital, the cost of inputs, market uncertainty, regulatory requirements, and government policies. For small firms, especially family-run businesses, choices are often shaped by short-term constraints, such as limited financing options or fluctuating demand. Larger firms, in contrast, tend to make decisions based on long-term strategies, expected returns, and expansion opportunities. However, a common factor that affects both groups is **taxation**. In Vietnam, business taxes such as value-added tax (VAT) or corporate income tax play a significant role in shaping how firms operate. While large firms may be better equipped to manage tax planning, smaller firms are typically more sensitive to tax burdens due to their lower profit margins and limited capacity to absorb additional costs.

Given the wide-reaching impact of tax policy on firms of all sizes, I chose to incorporate a **proportional tax on revenue** into the model. This approach assumes that firms do not keep their full earnings from production; instead, they retain only a portion after paying taxes. This is captured in the model by multiplying the revenue term by $1 - \tau$, where τ represents the tax rate. By including this feature, the model becomes more realistic and allows us to analyze how changes in tax policy can affect firms' investment behavior, capital accumulation, and overall performance.

$$V(A_t, K_t, p_t) = \max_{x_t, K_{t+1}} \left\{ (1-\tau)R(A_t, K_t, x_t) - w_t x_t - \frac{\gamma}{2} C(K_{t+1}, A_t, K_t) \right.$$
$$\left. - p_t \left(K_{t+1} - (1-\delta) K_t \right) + \beta \mathbb{E}_{A_{t+1}, p_{t+1} | A_t, p_t} \left[V(A_{t+1}, K_{t+1}, p_{t+1}) \right] \right\}$$

- Write the optimization problem of the representative firm, given the determinant you have chosen, for large and small firms.

In this analysis, I focus on the role of **taxation** as a key determinant of firm behavior. Specifically, I introduce a **proportional tax on revenue** into the firm's dynamic optimization problem. This determinant is both theoretically significant and empirically relevant in the Vietnamese context, where tax policy can directly affect firms' production and investment decisions. The tax reduces the effective returns from output, thereby influencing how much labor to hire and how much capital to accumulate over time.

To account for the heterogeneity between firm types, I specify separate optimization problems for large and small firms. These groups often face different tax burdens in practice. Large firms are generally more formalized and better able to comply with tax obligations, while small firms, particularly informal or family-run ones, may either be subject to simplified tax schemes or face disproportionate burdens due to limited administrative capacity.

The optimization problem for each type of firm is outlined below. Both firms aim to maximize the expected present value of profits, but under different tax rates and possibly different structural parameters.

The corresponding optimization problems are as follows:

Optimization Problem for Large Firms

$$V_L(A_t, K_t, p_t) = \max_{x_t, K_{t+1}} \left\{ (1-\tau_L)R(A_t, K_t, x_t) - w_t x_t - \frac{\gamma_L}{2} C(K_{t+1}, A_t, K_t) \right.$$
$$\left. - p_t \left(K_{t+1} - (1-\delta_L) K_t \right) + \beta \mathbb{E}_{A_{t+1}, p_{t+1}} \left[V_L(A_{t+1}, K_{t+1}, p_{t+1}) \right] \right\}$$

Optimization Problem for Small Firms

$$V_S(A_t, K_t, p_t) = \max_{x_t, K_{t+1}} \left\{ (1-\tau_S)R(A_t, K_t, x_t) - w_t x_t - \frac{\gamma_S}{2} C(K_{t+1}, A_t, K_t) \right.$$
$$\left. - p_t \left(K_{t+1} - (1-\delta_S) K_t \right) + \beta \mathbb{E}_{A_{t+1}, p_{t+1}} \left[V_S(A_{t+1}, K_{t+1}, p_{t+1}) \right] \right\}$$

Where:

$R(A_t, K_t, x_t)$: revenue function depending on productivity, capital, and labor input

τ_L, τ_S: tax rates on revenue for large and small firms

γ, δ: adjustment cost and depreciation parameters (can differ by firm size)

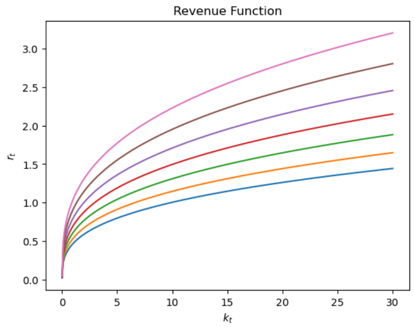
w_t: wage; p_t: price of investment

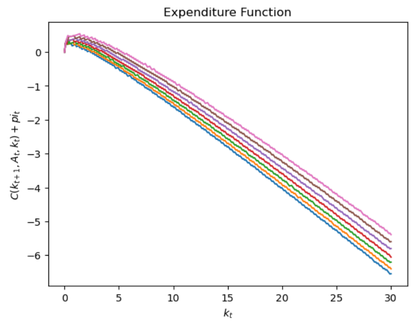
$C(\cdot)$: adjustment cost function for capital

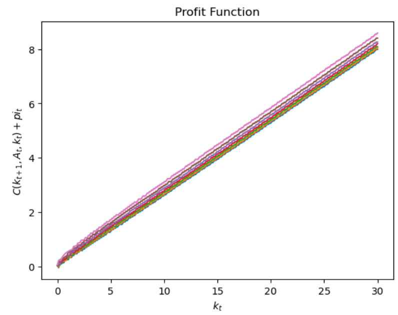
β: discount factor

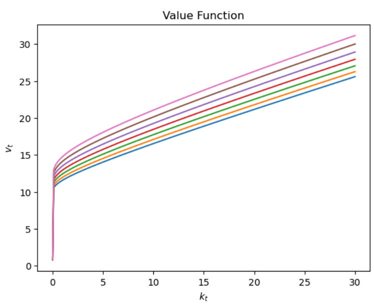
- Simulate your modified model and plot the average investment and capital for a given γ and δ

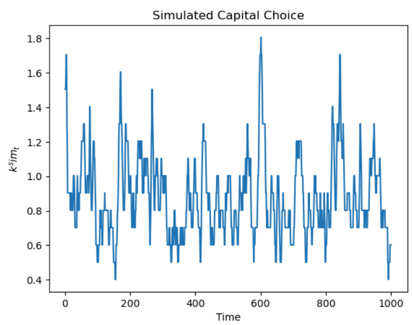
Small firm with tax:

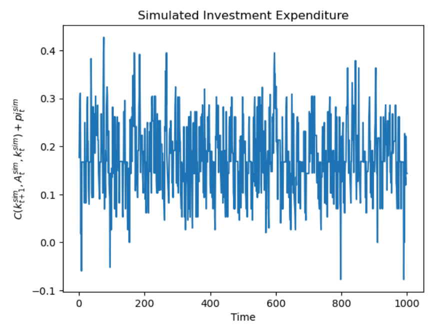


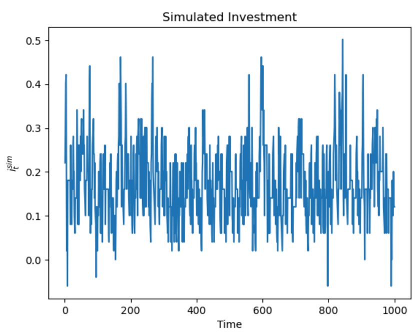


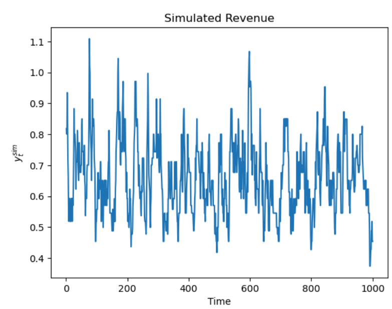


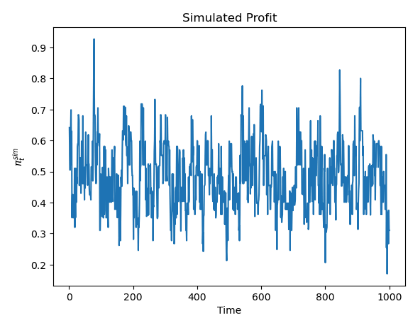


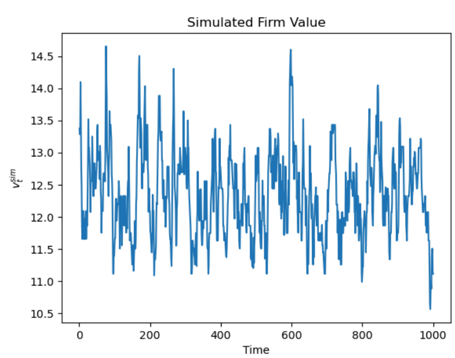












Small Firms After Taxation (vs. Baseline)

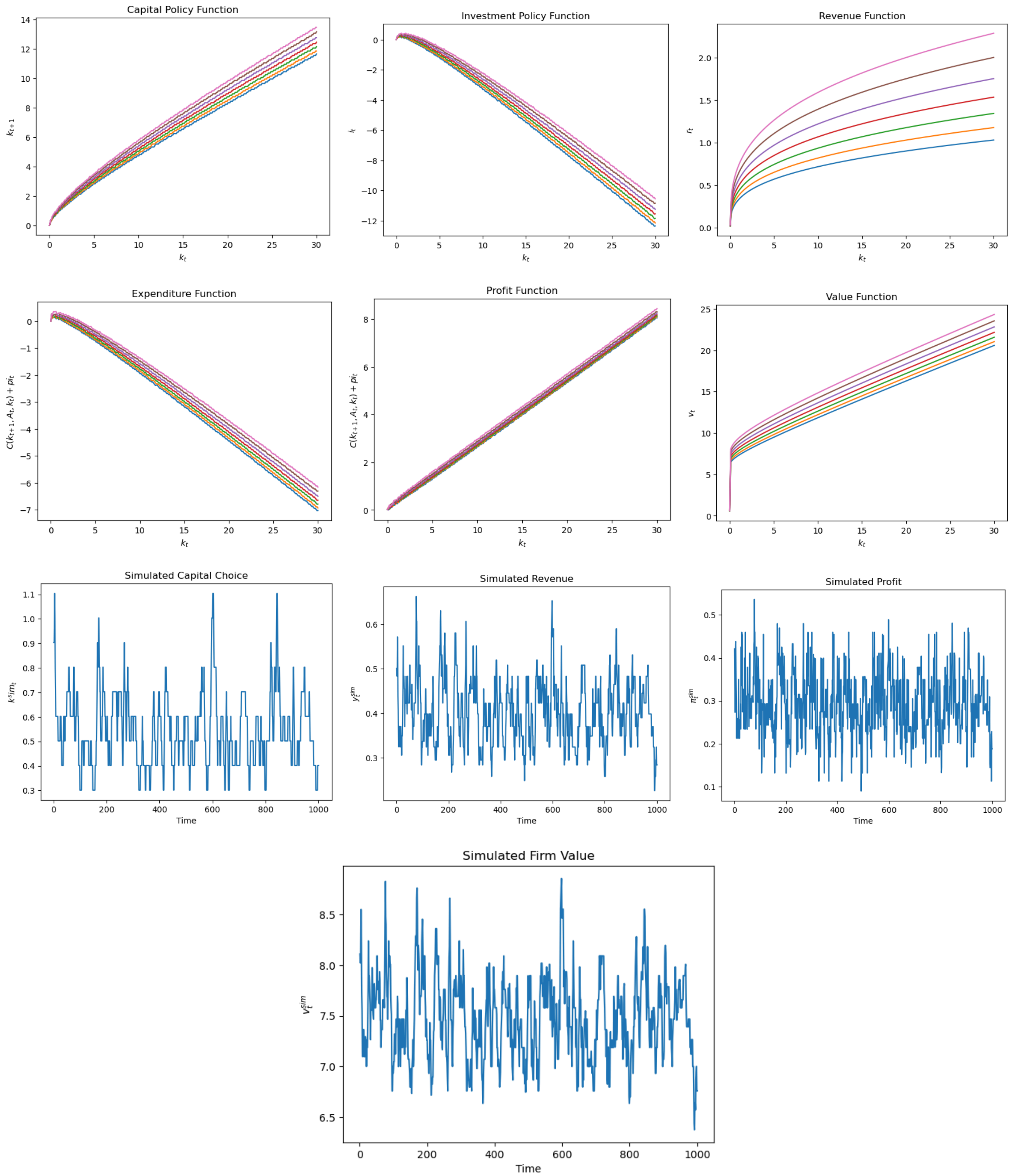
After incorporating a proportional tax on revenue into the model for **small firms**, several key changes emerge compared to the baseline version without taxation. Most clearly, the **revenue function** shifts downward across all levels of capital, as the tax reduces the effective returns from each unit of output. This directly lowers the firm's incentive to expand production, especially in capital-intensive regions.

The **expenditure function** becomes slightly flatter, and the **profit function** shows a slower rate of increase with capital, reflecting tighter margins. Because small firms typically operate with more constraints and less buffer, even a modest tax reduces their willingness to scale up. The **value function** is compressed relative to the untaxed model, meaning firms expect lower future profits under taxation.

Dynamic simulations show that **capital choices fluctuate around a lower level**, and investment is more conservative overall. In particular, the **simulated investment and expenditure paths** are less volatile but also more constrained, with fewer spikes in high-investment periods. The **simulated profit and revenue streams** confirm that taxation imposes a consistent drag on firm activity, while the **firm value** hovers at a lower average level throughout.

In sum, adding tax to the model makes small firms noticeably more cautious, lowers their expected value, and suppresses both investment and output. These results highlight how sensitive small firms are to policy changes and why uniform tax schemes may disproportionately affect them in settings like Vietnam's economy.

Large firm with tax:



After introducing a proportional tax on revenue into the model for large firms, the resulting behavioral adjustments differ in both scale and nature from those seen in small firms. While the revenue function also shifts downward across all capital levels due to taxation, the magnitude of change in firm behavior is more moderate. Large firms retain a stronger incentive to invest because they have greater financial buffers and more access to external capital, which helps them absorb the effects of reduced post-tax returns.

The profit function shows only a slight reduction in its slope, and the expenditure function shifts downward but maintains a similar curvature. This suggests that large firms continue to spend and invest, albeit slightly less aggressively. The value function, though marginally compressed compared to the baseline, remains relatively high, indicating that these firms still expect sizable future returns even after taxes.

Simulation results reveal that capital choices remain relatively stable and at higher levels compared to small firms. Investment activity becomes slightly smoother, but the average investment level does not fall as sharply. The simulated profit and revenue paths remain robust, with taxation leading to a manageable reduction in firm value rather than a sharp drop. These dynamics suggest that large firms adapt more flexibly to tax policy, adjusting margins and expenditures without fundamentally changing their growth trajectory.

In summary, while taxation does reduce the profitability and investment incentives for large firms, the impact is far less disruptive than for small firms. Their larger scale, diversified operations, and financial resilience allow them to maintain relatively high firm values and output levels even under policy constraints. This divergence underscores the importance of designing tax policy that accounts for firm size and capacity to adapt.