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Expert Blind Spot Among Preservice Teachers

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This study (N = 48) examined the relationship between preservice secondary teachers' subject-matter expertise in mathematics and their judgments of students' algebra problem-solving difficulty. As predicted by the "expert blind spot" hypothesis, participants with more advanced mathematics education, regardless of their program affiliation or teaching plans, were more likely to view symbolic reasoning and mastery of equations as a necessary prerequisite for word equations and story problem solving. This view is in contrast with students' actual performance patterns. An examination across several subject areas, including mathematics, science, and language arts, suggests a common pattern. This article considers how teachers' developmental views may influence classroom practice and professional development, and calls into question policies that seek to streamline the licensure process of new teachers on the basis of their subject-matter expertise.

Keywords: algebra, expertise, pedagogical content knowledge, teacher cognition.

Prior knowledge is essential for subsequent learning. It has been identified as critical for directing the subsequent learning of others (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992; Ma, 1999; Shulman, 1986;

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Vygotsky, 1978). The view that subject-matter expertise is critical for effective teaching, especially in secondary and postsecondary education, is widely accepted. Yet few studies examine the potential pitfalls for instruction that may be ascribed to expert subject-matter knowledge. One concern is that teachers' subject-matter expertise often overshadows their pedagogical knowledge about how their novice students learn and develop intellectually in the domain of interest.

In this article we investigate the "expert blind spot" hypothesis—the claim that educators with advanced subject-matter knowledge of a scholarly discipline tend to use the powerful organizing principles, formalisms, and methods of analysis that serve as the foundation of that discipline as guiding principles for their students' conceptual development and instruction, rather than being guided by knowledge of the learning needs and developmental profiles of novices (Koedinger & Nathan, 1997; Nathan, Koedinger, & Alibali, 2001). The "blind spot" metaphor draws on past research in visual perception that demonstrates that people's perceptions of the world are influenced by their expectations (Ramachandran, 1992). Literally, the blind spot is a section of the retina where the axons that make up the optic nerve exit the eye. This small area lacks visual receptors. If you close one eye and focus the open eye properly, as objects register on the blind spot they will seem to disappear. This area is mentally filled in with its immediate surroundings so that, instead of seeing a black spot as a camera would, we experience a complete image and have no awareness that the filling-in process has occurred. The existence of an expert blind spot (hereafter, EBS) in education raises the concern that expertise in a subject area may make educators blind to the learning processes and instructional needs of novice students and that educators with such expertise often are entirely unaware of having such a blind spot.

We first review the literature that examines the connections between expert subject-matter knowledge and pedagogical content knowledge. We then present data on the expectations for beginning algebra students' mathematical reasoning and development held by preservice mathematics and science teachers with greater and lesser levels of mathematics education. In exploring the EBS construct, we do not contend or imply that highly developed subject-matter knowledge is bad for teaching. On the contrary, it is clearly essential (e.g., Ingersoll, 1999). Rather, we present evidence suggesting that educators who have advanced knowledge of a subject but lack concomitant knowledge of how novices actually learn that subject tend toward views of student development that align more closely with the organization of the discipline than with the learning processes of students. Documenting this phenomenon is vital to understanding how teachers' implicit theories of student development influence curricular decision making and instructional practice. Such an understanding has implications for teacher education and professional development. It also calls into question policies that seek to streamline the licensure process of new teachers on the basis of their subject-matter expertise. These issues are discussed in the final section.

Theoretical Framework

The Nature of Expertise

The present study draws on and extends the expert–novice paradigm in cognitive science (Chi, Feltovich, & Glaser, 1981; Simon & Chase, 1973), which shows that experts and novices exhibit very different knowledge organization, perception, and problem-solving processes, despite a common cognitive architecture (Ericsson & Smith, 1991). Studies of expert performance have shown that it is based on vast amounts of well-organized, domain-specific knowledge, or schemas; intense, long-term practice within a narrow field; psychological and physiological adaptations; and the exploitation of regularities found in familiar tasks (Simon & Chase; Ericsson & Lehmann, 1996).

Expertise is not without its shortcomings, however. Studies have shown that people with a large amount of domain knowledge may actually be at a disadvantage in comparison with novices on certain tasks. For example, instructional designs that are effective for novices can be ineffective and even detrimental for experts (Kalyuga, Ayres, Chandler, & Sweller, 2003). Experts also can show impaired performance relative to novices on search-intensive language tasks, such as forming remote associations among disparate concepts (Wiley, 1998). Wiley argued that the reason for such impaired performance is that expert knowledge tends to be highly schema-based; thus improbable events or disparately related concepts may elude the expert. In other words, expert subject-matter knowledge can act as a mental set, fixating experts on unproductive solution paths during creative problem solving, whereas novices may behave more flexibly. Verbal "think aloud" reports have shown that experts are less likely than novices to have access to memory traces of their cognitive processes when engaged in tasks within their areas of expertise. Highly practiced cognitive and perceptual processes become automatized so that there is nothing in memory for experts to "replay," verbalize, and reflect upon (Ericsson & Simon, 1993).

Expertise in Teaching

Expert teaching is a complex phenomenon comprised of expertise in multiple domains, including curriculum subject matter, student behavior and development, and pedagogy (Shulman, 1987). Expert teaching also appears to substantiate many of the claims made about experts in general (e.g., Berliner, 1986; Borko & Livingston, 1989; Chi et al., 1981; Ericsson & Lehmann, 1996; Leinhardt & Greeno, 1986). Expert teachers differ from novices along several dimensions: (a) They notice different things about the classroom environment; (b) they do more planning and plan differently from novices; and (c) they organize their knowledge of subject matter, students, and pedagogy more deeply, in ways that readily facilitate lesson planning and teaching (Borko & Livingston, 1989).

Subject-Matter Knowledge and Pedagogical Content Knowledge in Teaching

Expert teaching behavior is highly dependent on efficient access to vast, well-managed knowledge structures, including domain-specific or content knowledge. Behaviors associated with expert teaching, as with expertise in general, have also been shown to be quite fragile; they generally are limited to familiar and well-practiced teaching situations (Borko & Livingston, 1989). Shulman's (1988) Knowledge Growth in Teaching Project described the practices of a beginning English teacher, Colleen, whose knowledge of literature was far better developed than her knowledge of grammar:

> In teaching literature, she conducted open-ended discussions, welcoming student questions and alternative interpretations of the text. When teaching a grammar lesson, Colleen looked like a very different teacher. She raced through a homework check at the speed of light, avoiding eye contact, and later admitted that she didn't want to give students the chance to ask questions she couldn't answer. (p. 15)

Although the importance of subject-matter knowledge for teaching has long been acknowledged, only in the past 15 years has the educational community become concerned with the specific knowledge that effective teachers possess on how to teach subject matter to novices. Shulman (1987) introduced the term *pedagogical content knowledge* to describe the "blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of learners for instruction" (p. 8). An example of pedagogical content knowledge is knowledge of specific algebra story problem-solving tasks that serve as effective scaffolds for learners. This is contrasted with subject-matter knowledge (e.g., how to solve a story problem), on the one hand, and general pedagogical knowledge (e.g., that it is helpful to get the attention of every student), on the other.

In many cases, teachers with high levels of subject-matter knowledge also have high pedagogical content knowledge. However, subject-matter knowledge can be viewed as developing independently from pedagogical content knowledge (e.g., Borko & Livingston, 1989). Their independence was evident in a study in which people of varying levels of teaching experience openly analyzed a video of classroom events (Copeland, Birmingham, DeMeulle, D'Emidio-Caston, & Natal, 1994). Regardless of their level of subject-matter knowledge, noneducators focused on surface-level characteristics of teaching behaviors; in contrast, educators with varying degrees of classroom experience tended to focus on the central purposes of the instruction and the connections between the teacher's actions and goals and the students' responses.

Not all teachers with high subject-matter knowledge necessarily have high pedagogical content knowledge. As suggested in one study of science teaching, knowledge of the subject matter appears to be a prerequisite for well-developed pedagogical content knowledge; however, pedagogical content knowledge appears to develop out of classroom teaching experiences that also draw on subject-matter knowledge (van Driel, Beijaard, & Verloop, 2001). Readily accessible pedagogical content knowledge is a principal component of effective teaching.

With this background on expertise and the role of knowledge in teaching, we now introduce the EBS hypothesis—the claim that well-developed knowledge of subject matter can lead people to assume that learning should follow the structure of the subject-matter domain rather than the learning needs and developmental profiles of novices. This hypothesis certainly has some face validity, as many college students who have sat through impenetrable lectures can attest. The EBS hypothesis has particular relevance to the pedagogical decisions made by K–12 teachers. We review findings about the knowledge and beliefs of mathematics teachers prior to presenting our study. In the final section we explore competing hypotheses. We then examine EBS in domains other than mathematics and discuss its implications for teacher education. We also discuss its implications for recent state and national policies that seek out professionals with advanced subject-matter knowledge to address the educational needs of communities.

Prior Work and the EBS Hypothesis

Prior research (Koedinger & Nathan, in press; Nathan, 2003b; Nathan & Koedinger, 2000a) has demonstrated that high school mathematics teachers expect to promote algebraic development by emphasizing symbolic reasoning and notation prior to the use of verbal reasoning and representations. Of note here is not only high school teachers' experience with student learning but also their relatively high level of subject-matter knowledge; all high school teachers who participated in the studies were mathematics majors or received the equivalent training. High school teachers defended this pedagogical approach because they viewed symbolic reasoning as "pure mathematics," more parsimonious, and a necessary prerequisite for more advanced verbal "applications." This has been termed the *symbol precedence view*.

Contrary to this view of development, a *verbal precedence view* of development has been found to be statistically more consistent with the performance of most students (Koedinger, Alibali, & Nathan, 2000; Koedinger & Nathan, in press; Nathan & Koedinger, 2000b; Nathan, Stephens, Masarik, Alibali, & Koedinger, 2002). Students in a number of studies at different grade levels solved verbally presented story and word-equation problems more readily than matched symbolic problems. An advantage of about 20 percentage points was evident for ninth-grade students, based on two samples of urban ninth graders who completed a year of algebra, $n_1 = 76$, $n_2 = 171$ (Koedinger & Nathan, in press). In model-fitting analyses the symbol precedence model of algebraic development accounted for 46% of students ($n = 171$); the verbal precedence model accounted for 88% (Nathan & Koedinger, 2000b). This verbal advantage has been replicated with middle school students (Nathan et al., 2002) and with high- and low-performing college students (Koedinger, Alibali, & Nathan, 2000).

In contrast with the high school teachers, middle school teachers who participated in the study had substantially less formal mathematics education ($n = 30$; all had elementary licensure, none had been mathematics majors) and gave greater homage to students' verbal reasoning abilities. The middle school teachers' predictions of students' difficulties were statistically more accurate for predicting student problem-solving performance than were the predictions made by high school teachers. A similar pattern of results was found among preservice teachers in Belgium (Van Dooren, Verschaffel, & Onghena, 2002). Those who taught high school preferred the use of algebraic methods for solving arithmetic and algebraic problems, both for themselves and when evaluating students' solution methods, even when arithmetic methods were more straightforward. In contrast, those who taught primary school provided evaluations that were more adapted to the specific demands of the problem-solving tasks.

The markedly different expectations expressed by middle school and high school teachers of varying mathematics educational backgrounds led Koedinger and Nathan to hypothesize that high school teachers' expertise in the area of mathematics may influence their views and lead them to think about their algebra students through a math-centric lens. They proposed the EBS hypothesis, speculating that teachers with greater mathematics knowledge tend to expect students to follow a normative process of development that mirrors the structure of the domain of mathematics (Koedinger & Nathan, 1997). That domain structure places formal representations (e.g., algebraic equations) as primary and verbal forms of mathematics as extensions and applications (cf. Kline, 1973). From a symbol precedence point of view, reasoning about verbally presented problems depends directly on one's abilities to translate the linguistic information into a formal symbolic equation (typically an equation or set of equations) and then to manipulate the resulting formalisms.

Prior research established the tendency for teachers at the high school level with high subject-matter knowledge to favor a symbol precedence view of algebraic development, in contrast to student performance data. However, these results may be attributed to influences within schools, such as the structure of textbooks used by teachers (Nathan, Long, & Alibali, 2002), and the demands placed on teachers by school districts and mathematics departments. Furthermore, the prior study of high school and middle school teachers confounded grade level of instruction with the mathematics education of the teachers studied. Thus teaching affiliation and prior mathematics education could not be separated out as influential factors.

In the present investigation we set out to study the expectations of preservice teachers with advanced and basic levels of mathematics education. These individuals did not meet one of the commonly accepted criteria for expertise because they had not accumulated approximately 10 years of preparation in one area of study, a commitment that corresponds to several thousands of hours of targeted practice (Ericsson & Charness, 1994). However, the participants in this study did approximately represent the essential differences observed among teaching professionals, enabling us to make comparisons between levels of subject-matter knowledge, on the one hand, and grade level affiliation, on the other. Our goal was to use this investigation of preservice teachers to help explain a phenomenon that had previously been observed among practicing teachers with mathematics expertise.

Method

Participants

All participants ($N = 48$) were preservice teachers enrolled in a nationally acclaimed teacher education program at an established research and teaching university. Participants' subject-matter knowledge in mathematics was rated high if they had completed calculus or above, and low if they had not completed pre-calculus. The participants can be thought of as "developing experts" in mathematics. Many of those categorized as relatively high in mathematics knowledge had gone well beyond a first course in calculus, completing majors in fields of mathematics and the physical sciences.

The mathematics education criterion follows prior research on retention of mathematics knowledge that showed that knowledge and retention of a given level of mathematics is most highly developed after learners go on to subsequent levels of mathematical study (Bahrick & Hall, 1991). In their investigation of the lifetime maintenance of high school mathematics knowledge, Bahrick and Hall examined the problem-solving performances of 1,050 individuals who took an algebra test up to 50 years after completing their high school algebra courses. In their analysis, Bahrick and Hall state:

> Even in the absence of further rehearsal activities, individuals who take college level mathematics courses at or above the level of calculus have minimal losses of high school algebra for half a century. Individuals who performed equally well in the high school course but took no college mathematics courses reduce performance to near chance levels during the same period. In contrast, the best predictors of test performance (e.g., Scholastic Aptitude Test scores and grades) have trivial effects on the rate of performance decline.

Of the 35 participating preservice teachers with advanced mathematics knowledge, 16 were in a specialized program for mathematics and science majors (the MathSci condition in this study) seeking secondary licensure in mathematics or science education. The remaining 19 with advanced mathematics knowledge (the HiMathK group) were from the general population of teacher education students and were seeking licensure at the elementary grade levels, as were a group of 13 who had basic mathematics knowledge (the BasicMath group).

Participants' Teacher Education Program

The preservice teachers' program in teacher education built on contemporary research that was centered on student learning, standards-based curriculums, the study of effective classroom interactions, the development of models of teaching, and equity issues. The teacher education curriculum used a novice-to-expert paradigm (Goldman, Petrosino, & Cognition and Technology Group at Vanderbilt, 1999) and drew on publications from the National Academies that summarized the body of current educational theory (e.g., National

Research Council, 1996, 2000, 2001). Discipline-specific courses integrated mastery of subject matter with the use of modern technology to develop instructional methods emphasizing inquiry-based, project-based, and problem-based learning (e.g., Petrosino, Lehrer, & Schauble, 2003). As part of their program, preservice teachers were assigned field placements in urban schools, where they observed classes and conducted clinical interviews of students. In addition, there was extensive use of video cases from the Third International Mathematics and Science Study (TIMSS) as well as from local classrooms, which the preservice teachers analyzed, critiqued, and used to reflect on pedagogical techniques. These experiences culminated in the design of innovative, technology-enhanced project-based curriculum units (Petrosino & Cunningham, 2003).

Materials and Procedure

All participants performed a ranking task that compared problems with the same underlying mathematical relations (Figure 1). Participants were asked to rank six problems in accordance with their expectations of the ease or difficulty that beginning-level algebra students would experience when solving them. We believed that this task would be more effective for eliciting participants' true beliefs about curriculum and student learning than asking them about student development or problem solving in the abstract. Implicit in the task is the 2×3 structure shown in Table 1. As shown along one dimension of the table, participants decided whether students would find problems more accessible if they were symbolic (algebra equations) or verbal (story problems or word equations). As shown on the other dimension, the participants also decided whether the students' performance would be higher for arithmetic or algebraic problems. However, the participants were not made aware of this underlying structure of the six problems.

After completing the ranking task, participants responded to a 47-item beliefs survey. They rated the degree to which they agreed or disagreed with Likert scale, with 1 representing *strongly agree* and 6 representing *strongly disagree*. The 47 statements were based on six constructs that broadly addressed current reform issues in pedagogical practice, mathematical learning, problem solving, student prior knowledge, the implications of alternative problem-solving strategies invented by students, and the role of algebra in complex problem solving. Included were statements supporting or challenging the symbol precedence view of algebra knowledge development. The constructs, briefly explained in Table 2, were "Algebra Is Best," "Student-Centered Pedagogy," "Symbol Precedence View," "Learning Through Intuition," "Product Over Process," and "Alternative Solutions Imply Gaps." The percentage of participants who agreed with each construct is presented in Table 2 along with reliability measures. For each construct the survey included items that were worded positively (affirming the construct) and negatively (negating the construct). Construct statements were ordered randomly across the survey form.

A Survey

Below are six problems that represent a broader set of problems that typically are given to public school students at the end of an Algebra 1 course—usually eighth- or ninth-grade students. My colleagues and I would like you to help us by answering this brief (10 min) survey. We will have an opportunity to discuss these problems later.

What we would like you to do:

Rank these problems, from the ones you think will be easiest for students to solve to the ones you think will be hardest for them to solve. You can have ties if you like. For example, if you think the fourth problem was the easiest, the third was the most difficult, and the rest were about the same, you would write:

4 (easiest)
2 1 5 6
3 (hardest)

Please provide an explanation and any assumptions you made in the space below.

Problems:

1) $(68.36 - 25)/4 = P$

2) Starting with 68.36, if I subtract 25 and then divide by 4, I get a number. What is it?

3) After buying a basketball with her four daughters, Ms. Jordan took the \$68.36 they all paid and subtracted out the \$25 she contributed. She then divided the remaining amount by 4 to see how much each daughter contributed. How much did each daughter pay?

4) Solve for D: $D \times 4 + 25 = 68.36$

5) Starting with some number, if I multiply it by 4 and then add 25, I get 68.36. What number did I start with?

6) After buying a basketball with her daughters, Ms. Jordan multiplied the amount each daughter had paid by 4 (because all four sisters paid the same amount). Then Ms. Jordan added the \$25 she had contributed and found the total cost of the ball to be \$68.36. How much did each daughter pay?

Please include any explanations and assumptions:

Figure 1. **Ranking task given to participants.**

Hypotheses

Following the EBS hypothesis, we expected that preservice teachers with high subject-matter knowledge (MathSci and HiMathK) would tend to base their expectations of student performance difficulty on their knowledge of and familiarity with algebraic formalisms. We therefore predicted that their rankings would correlate better with the symbol precedence view than would the rankings provided by BasicMath preservice teachers. Operationally, that view is

Table 1
Hidden Structure of the Task for Ranking Problem Difficulty

Level of mathematical difficulty	Verbal presentation		Symbolic presentation
	Story problem	Word equation	Symbol equation
Arithmetic	Problem 3	Problem 2	Problem 1
Algebra	Problem 6	Problem 5	Problem 4

exhibited by the following problem ranking (using the problem numbers shown in Table 1): 1 2 3 4 5 6. This ranking predicts that arithmetic problems are easiest within each level of representational format (symbolic or verbal) and that the ability to solve symbolic forms strictly precedes the ability to solve story and word problems.

Table 2
Percentages of Participants ($N = 48$) in Agreement With Each Construct When Presented on a 6-Point Likert Scale Survey (6 = Strongly Disagree)

Construct (number of statements)	Description	α	MathSci ($n = 16$)	HiMathK ($n = 19$)	BasicMath ($n = 13$)
Algebra Is Best (12)	Algebraic procedures are the single most effective solution method.	.79	54.2	47.8	37.8*
Student-Centered Pedagogy (7)	Teachers can effectively build on students' invented methods.	.89	36.1	39.8	65.8*
Symbol Precedence View (4)	Symbolic reasoning precedes story problem solving.	.54	79.7	85.5	66.2*
Learning Through Intuition (8)	Students enter the classroom with intuitive methods for reasoning algebraically.	.85	56.3	64.5	81.7*
Product Over Process (4)	Correct answers are more important than the method used.	.71	4.7	8	2
Alternative Solutions Imply Gaps (6)	Students' alternative problem-solving methods indicate gaps in their knowledge.	.76	34	32.3	27.5

Note. Numbers in parentheses at left indicate how many of the statements to which participants responded were linked with each construct.
*$p < .05$.

We also predicted that all preservice teachers would generally agree with reform-based views of student-centered learning and instruction in the survey because of their enrollment in a reform-oriented teacher education program that emphasized constructivist views of learning and student-centered instruction. Thus it was expected that participants across the three groups would tend to reject views that emphasize getting the correct answer (product) over the specific solution methods used (process) and views that disregard the importance of students' invented methods as a basis for subsequent learning. We further predicted that, regardless of their specific teacher education program affiliation, participants with higher mathematical knowledge (MathSci and HiMathK) would tend to agree most strongly with statements that specifically reflected symbol precedence view of algebraic development, while exhibiting reform-based views of learning and pedagogy more generally.

Results and Conclusions

Problem Difficulty Ranking

We first look at the expectations provided by the MathSci participants—those with high mathematics knowledge who were seeking licensure in secondary mathematics and science education. The average ranking across all MathSci participants ($n = 16$), ordered from easiest-to-solve to most difficult, was 1 2 4 3 5 6. This ranking was virtually indistinguishable from that predicted by the symbol precedence view, $r = .92$, $p < .005$. Analyses of individual rankings of each participant showed an average correlation with that view of 0.72, $SE = .18$. To correct the distribution, a Fisher transformation was applied to each participant's rank correlation. Mean transformed scores had a 95% confidence interval that included 1.0 ($1.19 \leq X \leq .64$), making this statistically indistinguishable from a perfect correlation with the symbol precedence ranking (Figure 2). A t test failed to reject the null hypothesis that the correlation was equal to 1.0, $t(14) < 1.0$. Thus the difficulty rankings of participants majoring in mathematics and science who intended to go into mathematics or science teaching at the secondary level paralleled the hypothetical ranking predicted from the symbol precedence view.

The HiMathK group—preservice teachers who were not pursuing certification as secondary mathematics or science teachers but reported advanced mathematics education ($n = 19$)—also exhibited an average ranking of 1 2 4 3 5 6, which strongly correlated with the symbol precedence ranking, $r = .94$, $p < .005$. The average Pearson's correlation was 0.81. The mean Fisher-transformed correlations were indistinguishable from $r = 1.0$ (see Figure 2).

The BasicMath group—preservice teachers with relatively limited mathematics education ($n = 13$)—showed an average ranking of 1, 2, and 4 as the easiest problems (they considered the three problems to be about equally difficult), 3 and 5 as being of middle difficulty, and 6 as the most difficult. This ranking correlated only moderately with the symbol precedence ranking, $r = 0.48$. The average individual Pearson's correlation was 0.48. The average ranking of BasicMath participants had structural similarities with those of the

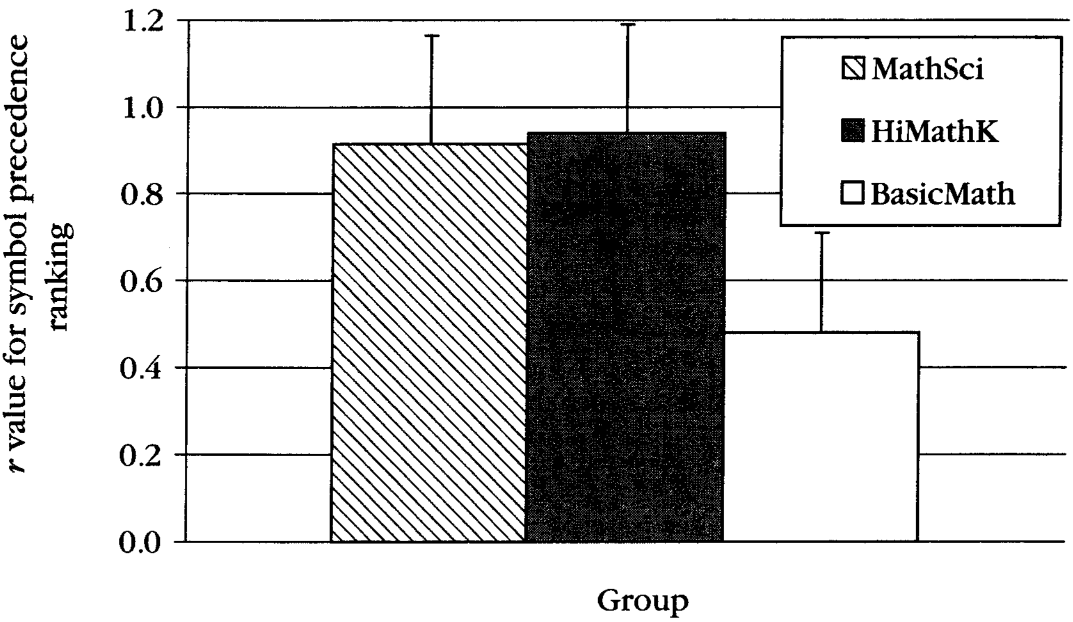

Figure 2. **Mean correlation (using adjusted Fisher transformation) of participants' ranking (by group) with predicted symbol precedence view ranking. Error bars indicate upper levels of the confidence interval.**

other groups. For example, all three participant groups expected algebra story problems (Problem 6) to be most difficult for students. However, individually Fisher-transformed ranking correlations with the symbol precedence ranking for BasicMath participants produced a 95% confidence interval that did not include 1.0 ($0.71 \leq X \leq .35$), indicating that their ranking was different from the ranking predicted from the symbol precedence view (see Figure 2). A *t* test showed that the similarity measure with the symbol precedence view was reliably less than 1.0, $t(12) = 5.3$, $p < .001$.

Pairwise comparisons of the transformed correlations showed that the ranking predicted from the symbol precedence view was significantly more dissimilar to the ranking generated by BasicMath participants than to the rankings of MathSci ($p < 0.01$) and HiMathK ($p < .0001$) participants. The MathSci and HiMathK groups did not differ statistically.

The ranking data support the hypothesis that preservice teachers with more advanced mathematics education (in our sample, those in the MathSci and HiMathK groups) are far more likely to follow the symbol precedence view of algebra development than are other preservice teachers (the BasicMath group in our sample). This holds as well for preservice teachers studying to be mathematics and science teachers at the secondary level (where algebra is typically taught and applied) as it does for those with high mathematics knowledge pursuing teaching careers in other areas, such as elementary education.

Several MathSci and HiMathK participants defended their rankings with comments that underscored their notions of the primacy of symbolic reasoning in algebra development. One proclaimed that the arithmetic equation "sets up the problem exactly as [students] need to do it, in familiar notation. . . . [The arithmetic story problem] provides a scenario that seems more likely to distract or confuse students, who tend to fear word problems." Fear of word problems was typical in justifying the symbol precedence ranking: "Words scare students, and they will struggle. And [algebra] problems where the variable is not isolated are harder still." Other MathSci and HiMathK participants focused on the greater demands of solving problems that are presented first with words, suggesting that they believed algebraic equations were necessary for finding solutions to such problems: "Word problems require the students to set it up themselves, and the scenario might make it even more difficult to interpret." "Word problems confuse me. . . . [Symbolic problems] are easiest because they're just straightforward." Only one MathSci participant explained why symbolic problems might be more difficult: "[The algebra equation] has notation [that students] may be unfamiliar with." These comments are very similar to the justifications provided by inservice teachers in prior studies (Nathan, 2003b; Nathan & Koedinger, 2000a, 2000b).

The broader pattern of results is consistent with the EBS hypothesis that it is advanced mathematics knowledge per se, rather than algebra teaching, that mediates educators' views of algebra development. Highly developed subject-matter knowledge appears to make fledgling teachers blind to the actual developmental processes of beginning algebra students, a finding that parallels previous data on practicing teachers.

The Beliefs Survey

The rating data from the 47-item beliefs survey showed that the six hypothesized constructs were well formed (Cronbach's alpha between .54 and .89, with 6 items removed on the basis of the reliability analyses). Thus, when we consider the participants' levels of agreement, we have high certainty that agreement with specific items is reflective of each construct.

Many of the participants' views were in line with current mathematics reform. However, there were also significant differences between participants at the 5% level of certainty, and those differences paralleled differences in the participants' levels of mathematics education. The percentage of participants in each group who agreed with each construct is shown in Figure 3. All of the differences reported in the following discussion are significant.

A greater percentage of HiMathK and MathSci participants believed that using algebraic formalisms is best ("Algebra Is Best" in Figure 3) for solving complex problems (an average of 51% agreed) than did BasicMath participants (38%). BasicMath participants were more likely to agree (82%) that learning is fostered through intuitive discovery ("Learning Through Intuition") than were HiMathK (64%) and MathSci (56%) participants. BasicMath participants were also more likely (66%) to believe that instruction should build on students' intuitions and invented methods ("Student-Centered Pedagogy") than were participants with more mathematics education (averaging 38%). Furthermore, consistent with the EBS hypothesis, both groups with greater mathematics

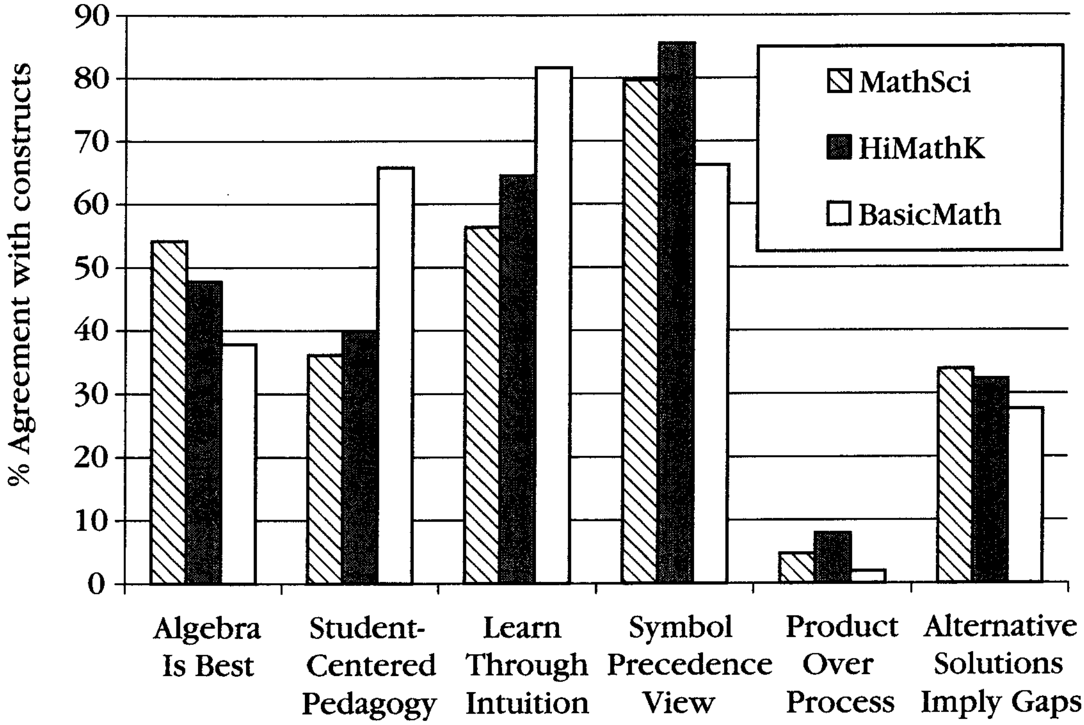

Figure 3. **Percentage agreement with belief constructs by condition.**

education were significantly more likely to agree (average of 83% across the two groups with high mathematics knowledge) than were BasicMath participants (66%) with the view that students' symbolic reasoning precedes, and serves as a necessary prerequisite to, their verbal reasoning and story problem-solving abilities ("Symbol Precedence View"). However, it must be noted that this view was still widely held among the preservice teachers in all three groups. Members of all three groups responded similarly to the remaining two constructs. All groups rejected the view that gives primacy to students' answers over their solution methods ("Product Over Process," 5% agreed), as well as the notion that students' alternative problem-solving approaches signal gaps in their knowledge ("Alternative Solutions Imply Gaps," 30% agreed).

Discussion

Future teachers with more advanced mathematics education expected algebra students to be best at solving equations and worst at solving story problems with the same underlying mathematical relations, although studies show that, in fact, students exhibit the opposite pattern. On the basis of these and previous findings with practicing teachers (Nathan & Koedinger, 2000a), it appears that educators with greater subject-matter knowledge tend to view student development through a domain-centric lens and consequently tend to make judgments about student problem-solving performance and mathematical development that differ from actual performance patterns in predictable ways.

The current study extends prior results by showing that the tendency for educators with high mathematics knowledge to conceptualize mathematics development as following a symbol-precedence trajectory is evident among preservice teachers regardless of their affiliation with secondary mathematics or influences from within-school settings such as choice of curriculum. Current evidence supports the EBS hypothesis that it is well-developed subject-matter knowledge per se that leads educators to inaccurately predict student problem-solving difficulty.

The results of these studies are best viewed in the context of a growing body of literature that examines the influences of prior knowledge on judgments of learner difficulty. Research on the "curse of knowledge" (Camerer, Loewenstein, & Weber, 1989) shows that adults and children who know the solution to a problem tend to overestimate how easy it is for someone else to solve that problem (Birch & Bloom, 2003). A related body of research considers how judgment is affected by people's subjective experiences when they have solved a problem themselves. Adults can exhibit a type of egocentrism when they rely on their own subjective experience of the difficulty of a task to predict difficulty level for others, even when the ease with which the problem was solved was manipulated by researchers (Kelley & Jacoby, 1996). For example, participants' prior exposure to solutions for scrambled words led them to rate the tasks as easier than when they had not previously seen the unscrambled words. Faster response times and correlations between speed and difficulty ratings implied that these participants relied largely on impressions in making their difficulty ratings, rather than on the more deliberative, analytical method evident among participants who judged difficulty but did not solve the problems. Warning participants of the egocentric effect, along with requiring them to recognize that they had previously seen some of the solution words, eliminated the effect, leading them to use their subjective experiences less often. The implications of this and other findings for teacher education and professional development are considered in a later section.

Alternative Hypotheses

It is important in evaluating the contributions of this body of work to consider alternative explanations to those suggested by EBS that could account for our findings. One competing hypothesis is that domain-centric views of development such as the symbol precedence view are societal and that the responses of our participants merely reflected the societal tendency toward such views. Though the broader societal influence cannot be ignored, if it were the dominant source of educators' expectations we would not expect to see such marked differences tied to levels of mathematics education as reported in the current study, or among teachers in different grade levels as previously reported (Nathan & Koedinger, 2000a; Van Dooren, Verschaffel, & Onghena, 2002).

A second hypothesis is that participants base their views largely on previous subjective experience in solving algebra problems. As was mentioned earlier, in this account there is no deliberation about what makes a problem easy or difficult; judgment is based on impressions of difficulty rather than on careful consideration of the elements of the task or of the intended student population. In some cases, subjective experiences can be more accurate than analytical approaches (Kelley & Jacoby, 1996). This view has some appeal because deliberations would make many aspects of teachers' decision making overwhelming, given time demands and many interacting factors.

However, there are reasons to challenge this hypothesis as an account of EBS as well. If participants who were strong in mathematical problem solving were simply drawing on their own experiences, it would seem likely that symbolic equations and story problems composed of the same quantitative relations would be ranked as similar in difficulty level. Yet we see story problems consistently ranked as most difficult by those with greater mathematics expertise. Participants across several studies (Nathan, 2003b; Nathan & Koedinger, 2000a, 2000b), including the present one, also provide cogent justifications for their rankings—a condition that is at odds with subjective experiences. Furthermore, these justifications show remarkable similarity across studies where participants varied by grade level of instruction, level of teaching experience, and geographical location.

A third view is that judgments may be reached in a more deliberative, analytic manner, such as drawing explicitly on a theory of problem difficulty or student development. In this account, people have awareness and control over the factors that influence their expectations and can consciously choose how to weigh factors. From a theory-based standpoint, preservice teachers possess both subject-matter knowledge and pedagogical content knowledge for this task, but their knowledge may be inadequate or even include conflicting elements. The effect of inadequate pedagogical content knowledge on instruction has been well documented (e.g., Grossman, 1990; Shulman, 1987). At first blush this is certainly a plausible account of EBS, because educators with a symbol precedence view make some inaccurate predictions of student problem-solving performance. But this account does not tell the whole story. In the survey, all three groups showed strong agreement with the "Product Over Process" and "Alternative Solutions Imply Gaps" constructs, as well as with others (see Table 2). We also note that the middle and high school teachers studied by Nathan and Koedinger (2000a) accurately predicted student performance differences in arithmetic and algebraic problems, even though the teachers differed in their views about symbolic and verbal problems. Inadequate pedagogical content knowledge is not likely to be the central cause of EBS.

Our final consideration is that educators who exhibit EBS may have the requisite subject-matter knowledge and pedagogical content knowledge for the general topic at hand, but as they apply that knowledge to a specific area of mathematics, such as algebra instruction, those bodies of knowledge come into conflict. The conflicting ideas may lead to a view of student development and performance that is influenced by, but different from, the view derived from the prevailing knowledge of the profession. As with the "curse of knowledge" mentioned above, knowledge of the power of formalisms leads those with advanced mathematics training to regard symbol-based problems more favorably than is warranted from the empirical studies of students.

This theory-based account seems closer to explaining EBS than the other alternatives. It allows for participants' articulate and consistent justifications of their problem difficulty rankings, and for demonstrations of pedagogical content knowledge in other facets of student performance. Thus it seems to do a reasonable job of explaining experts' preferences for symbolic reasoning. Yet this account raises an important question. If, in fact, educators with requisite subject-matter knowledge and pedagogical content knowledge can make inaccurate predictions of problem difficulty along the lines of the EBS, then what does it mean to have the proper pedagogical content knowledge and make knowledge-based decisions? At issue is what we mean in theory and in practice about the development and application of educators' pedagogical content knowledge.

There is by now a well-established body of literature, both empirical and theoretical, that establishes the existence and value of pedagogical content knowledge for effective teaching. But within the confines of a teacher education program, even one organized around contemporary educational reform principles, preservice teachers will not encounter every possible instructional scenario relevant to their teaching. Consequently, they will not proactively form pedagogical content knowledge about everything they teach. Preservice and practicing teachers can have only a limited set of experiences from which to induce effective principles of pedagogy. From these limited instances and from more general principles, they must make inferences about ways to facilitate learning in specific circumstances. Their inferences are affected by myriad factors. Among subject-matter experts, deep knowledge of the discipline and its structure appears to exert an influence on their theories of learning and instruction in a way that is not apparent among nonexperts. Instead of drawing on general principles of intellectual development when forming a model of student mathematical development, educators with high levels of mathematics knowledge seem to draw on the ontological structure of the discipline of mathematics, which places command of formalisms before areas of application. Teachers with expertise in mathematics who exhibit EBS need not have erroneous pedagogical content knowledge; they may simply be fleshing out underspecified pedagogical content knowledge by drawing on subject area knowledge.

EBS More Broadly Considered

Our position regarding teacher preparation is that well-developed subject-matter knowledge is vital for effective instruction. However, subject-matter expertise across disciplines can, if unchecked, lead teachers to be blind to certain developmental needs of novice learners. Domain experts may forget what students find easy and difficult to learn (National Research Council, 2000). In the following selective review, discipline-centric views of conceptual development that place the learning of formal concepts before their application are shown to be prevalent in subject areas other than mathematics, and such views may similarly misrepresent student learning.

In medical training, expert nurse clinicians, rather than professional medical school educators, customarily fill faculty positions. These expert clinicians typically demonstrate a lack of awareness of the learning needs of their students (Krisman-Scott, Kershbaumer, & Thompson, 1998). In college-level physics education, there is a recognized tension between educators who advocate "top down" instruction, starting from scientific principles and moving to technological applications, and educators with a "bottom up" approach that uses technology as the foundation from which to induce general scientific principles (Nathan, 2003a). The author of a physics textbook designed for undergraduates with majors outside the natural sciences articulates this dichotomy clearly:

> While this book starts with objects and looks inside them for scientific principles, most physics texts instead choose to develop the principles of physics first and to delay the search for real-life examples until later. . . . While a methodological and logical development of scientific principles can be very satisfying *to the seasoned physicist,* it can appear alien to an individual who isn't familiar with the language being used. (Bloomfield, 1998; vii, italics added)

Additional evidence for the EBS hypothesis can be found in Pamela Grossman's (1990) comparative case study of six beginning teachers of secondary-school English, all strong in subject-matter area knowledge but different in their teacher preparation. Three were graduates of a professional teacher education program, and three had received their degrees in academic programs such as literature. Grossman's study provides a valuable comparison of teachers' pedagogical content knowledge and instructional views while holding subject-matter knowledge constant at a relatively high level.

The case studies of English teachers who had no formal teacher education revealed how they tended to promote a *text-centered* view of English instruction that followed the "formal criticism" and "new criticism" approaches, which emphasize detailed textual analysis (*explication de texte*) as the means to understand literature (Kessey, 1987; Rosenblatt, 1991). Although this perspective offered a great deal of depth to the study of language arts, pedagogically it was a poor match for most high school students. The lessons that were developed from this critical view were often too analytical, insufficiently engaging, and quite disconnected from students' own personal experiences and their preconceptions about reading. For example, one of the teachers in Grossman's (1990) study used themes with roots in his understanding of literature, rather than any specific understanding of ninth graders. Grossman noted that, although this teacher tried to rethink his teaching "to make it more accessible to ninth graders, his disciplinary knowledge and interests seemed to overwhelm his emerging pedagogical instincts" (p. 39).

In contrast to text-centered views found among language arts teachers who had not received professional teacher training, those who graduated from a formal teacher education program (and who had a similarly high level of subject-matter knowledge) emphasized student-centered approaches in their instruction. Their lessons demonstrated greater sensitivity to their students' prior knowledge, interests, and preconceptions about their English classes, even though literary analysis and grammar still played a central role in these classrooms (Grossman, 1990).

In our view, the English teachers from the academic program found themselves in the same predicament as many of the high school mathematics teachers who have been studied. The English teachers' well-developed subject-matter knowledge, based in this case on formal principles from linguistics and literary analysis, served as valuable organizing principles for themselves and dominated their instructional approaches, irrespective of the developmental needs of their students.

Within mathematics education, EBS takes the form of the symbol precedence view of development because of the primacy and enormous utility of symbolic formalisms within the field of mathematics. Although we share the goal of advancing learners' understanding of and facility with formal representations, previous findings have shown that students do not necessarily develop these formal representations first. Symbolic reasoning may trail and even depend on the development of verbal representations and procedures (Kalchman, Moss, & Case, 1999; Nathan et al., 2002).

Implications for Educational Research and Teacher Education

The present study does not report directly on classroom instruction. However, we claim that the data presented here provide insight on the models of student development that educators embrace and on how those models influence educators' judgments about student learning and performance. The existence of EBS should be a central concern to teacher–educators because teachers' beliefs about the goals of teaching in their subject areas act as a "conceptual map for instructional decision making, serving as the basis for judgments about textbooks, classroom objectives, assignments, and evaluation of students" (Grossman, 1990, p. 86). EBS research provides a foundation against which instructional practices and curricular designs may be interpreted as researchers try to understand how teachers' knowledge, beliefs, and practice relate to each other and how they lead to successes or failures in the classroom.

The existing research highlights the need to better understand the preexisting views and expectations that preservice and practicing teachers have about student learning, and how these views may interact with empirically based theories of student development that educators will encounter throughout their careers. The existence of EBS underscores the need to balance subject-matter knowledge with well-developed pedagogical content knowledge and an understanding of how students' subject-matter-specific knowledge develops. The presence of EBS among nascent teachers in a reform-based education program points out the limitations of imparting pedagogical content knowledge that is meant to transfer broadly to novel learning situations. Its prevalence among practicing teachers suggests that we do not yet understand pedagogical content knowledge well enough to predict how it influences decision making and instruction.

Researchers must be willing to reexamine the influences of prior conceptions in the learning processes of teachers, just as they have done for the learning processes of students. If subjective experiences are responsible for EBS, in part or entirely, the challenges for professional development are great. Tacit views such as these often exist outside practitioner awareness and generally are not subject to direct inspection. Thus their initial avoidance and later reformation pose serious challenges to the educational community. Theory-based accounts of EBS seem to be more encouraging, given the state of our understanding of knowledge and belief change. If educators are working from explicit theories that can be verbalized, these can be brought to the fore with relative ease. Discursive and reflective methods that are already commonplace in professional development and teacher education programs can serve as the basis for interventions aimed at aligning teachers' views with accurate models of student reasoning and development.

One positive finding comes from research on subjective experiences. The effect of egocentrism was eliminated when participants were warned and explicitly directed to examine their prior knowledge (Kelley & Jacoby, 1996). Methods such as this, as part of larger, reflection-based programs of teacher education and professional development, hold promise for addressing inaccurate views of students, such as the symbolic precedence view that can be attributed to advanced subject-matter knowledge.

Recent Policies on Teacher Licensure Based on Subject-Matter Expertise

In the current zeitgeist of educational reform, many see subject-matter preparation as paramount and place pedagogical knowledge in a distant second place (e.g., Holmes Group, 1986). This view has been echoed in the rhetoric of the "math wars" between proponents of calculation-centered (or "back to basics") curriculums and those who emphasize the situated nature and social construction of mathematical knowledge. It has led at least one education-oriented foundation president to push for higher standards for teachers' subject-matter knowledge and to recommend that the American mathematics research community ("Number one in the world") lead the reform of our national mathematics curriculums (Goldman, 1997).

This discipline-centered approach to mathematics curriculum development echoes the so-called New Math movement. The New Math curriculum was designed by mathematicians to highlight the formal structure of mathematics, particularly of set theory and number theory, with less regard to how that subject matter was to be learned by children, understood by teachers, or taught in classrooms by nonmathematicians (Loveless, 1997). Opponents of New Math criticized what they saw as an overemphasis on formal structure and notation at the expense of good instructional practices, meaning making, and connections to areas of application (e.g., Kline, 1973; National Council of Teachers of Mathematics, 1970).

Research on learning and teaching, as well as policies regarding teacher preparation, must include the drawbacks as well as the merits of teachers' subject area knowledge. Recent reports have made much of the deficits in teacher subject-matter knowledge and its apparent impact on student learning and performance on high-stakes assessments (e.g., Educational Trust, 2002; Gonzales et al., 2000). Some have used these findings to argue that teacher education and professional development programs spend too much time on pedagogy and on the understanding of students' prior knowledge and experiences, and too little time on improving teachers' subject-matter knowledge (e.g., No Child Left Behind Act of 2001). Recent federal programs such as Transition to Teaching even seek to streamline the licensure process of new teachers on the basis of their subject-matter expertise. EBS research suggests that teacher education and professional development programs must keep sight of the importance of pedagogical content knowledge in teaching. This emphasis must not be traded for attention to subject-matter preparation that can contribute to teachers' holding inaccurate views of students' intellectual development.

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