Navigation using IMU

The Principles of computer organisation course

Oleksandr Sobkovych   Volodymyr Fedynyak   Oleksa Hryniv

Lecturer: Oleg Farenyuk

Mentor: Oles Zaburannyi

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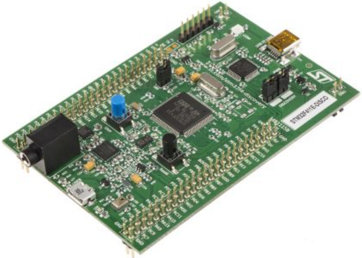
The goals of the project:

1. To determine the orientation of an object in space using an inertial measurement unit (accelerometer, gyroscope, magnetometer)
2. To determine the displacement of the object applying the mathematical tools
3. To improve the precision of the result applying the Madgwick and Kalman filters

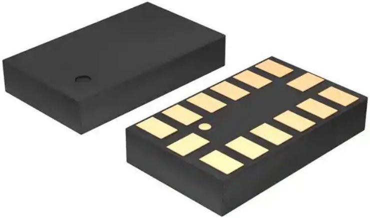
Hardware:

- STM32F411E-DISCO board with built in accelerometer and magnetometer LSM303DLHC and MEMS-gyroscope L3GD20:
- IMU Board, from SoftServe company:



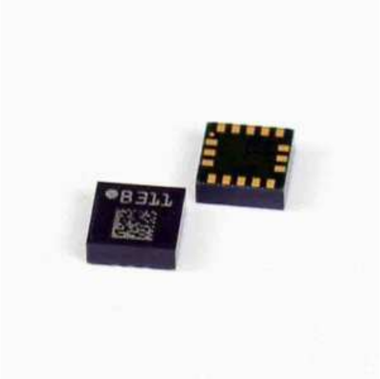
Accelerometer and Magnetometer:

- Both devices are 3-axis (XYZ) devices and are connected to I2C bus
- Accelerometer's measurement precision is $\pm 2g/\pm 4g/\pm 8g/\pm 16g$
- Magnetometer measures the vector of the Earth magnetic field with precision: $\pm 1.3/\pm 1.9/\pm 2.5/\pm 4.0/\pm 4.7/\pm 5.6/\pm 8.1$ Gauss; determines orientation with respect to Earth magnetic pole
- Output: 12-bit value for each axis



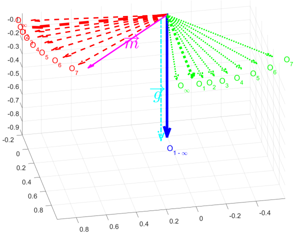
Gyroscope sensor:

- Device is 3-axis (XYZ) sesor and is connected to I2C and SPI bus
- Gyroscope measures rotation angles with respect to 3 axes with precision $\pm 250^\circ/s, \pm 500^\circ/s, \pm 2000^\circ/s$
- Output: 16-bit value for each axis



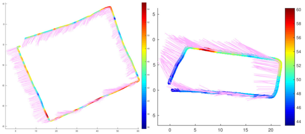
Working principles of accelerometer sensor:

- Accelerometer measures the (pseudo) acceleration w.r.t. 3 axes, fixed relative to sensor (but can change w.r.t. the absolute coordinate system)
- Accelerometer measures the (pseudo) acceleration in its own coordinate system, taking into account the Gravitational acceleration
- In the horizontal position accelerometer measurements are 0 on X and Y axes and $+g$ on the Z axis. To get the exact value of acceleration, we need to take into consideration the Gravitational acceleration g



Working principles of magnetometer sensor:

- Magnetometer measures the vector of Earth magnetic field w.r.t. its own coordinate system
- It is impossible to unambiguously determine the axes absolute orientation of accelerometer or magnetometer using only their indicators
- To this end, we assume that the device was still before the movement began; therefore, we can use its starting orientation as an absolute coordinate system (separately for accelerometer and magnetometer)



Project tasks:

- To localize the device using the gyroscope and accelerometer measurements
- To determine orientation and displacement
- To determine changes in orientation using the gyroscope
- To determine displacement in its own coordinate system using the accelerometer

Challenge:

Numerical integration of gyroscope and accelerometer measurements accumulate errors, which grow exponentially and amplify each other

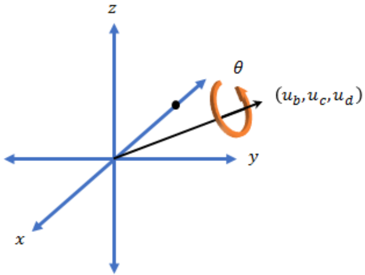
Solution:

Improve the results using the Madgwick filter (2010) for orientation and Kalman filter for displacements

Madgwick filter

- The idea: use the fact that the gravitational field vector $\vec{\mathbf{g}}$ and the magnetic $\vec{\mathbf{B}}$ are almost stable to improve the accuracy of orientation
- Speed of orientation change in its own coordinate system is given by the gyroscope measurements; integration gives an approximate orientation
- The Madgwick filter improves the orientation by synchronising with the accelerometer and magnetometer measurements $\vec{\mathbf{g}}$ and $\vec{\mathbf{B}}$, respectively
- Effectively uses quaternions to represent rotations in 3-dimensional space $\mathbb{R}^3$

- The device's orthogonal coordinate system can be aligned with the Earth coordinate system by rotation on an angle θ about an axis $\mathbf{u}$:



- Such a rotation can be represented by a quaternion

$$ {}^S_E\mathbf{q} = [\cos\frac{\theta}{2}, -u_b \sin\frac{\theta}{2}, -u_c \sin\frac{\theta}{2}, -u_d \sin\frac{\theta}{2}] $$

- Quaternions are an extension of complex numbers:

$$ q = a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k} $$

with similar addition and multiplication rules

Determining the orientation using gyroscope:

- Let $\omega_x, \omega_y, \omega_z$ be the device angular velocities in its own coordinate system given by the gyroscope; ${}^S\omega = [0, \omega_x, \omega_y, \omega_z]$
- Change in orientation is described by the differential equation

$$ {}^S_E\dot{\mathbf{q}} = \frac{1}{2} {}^S_E\mathbf{q} \otimes {}^S\omega, $$

  where $\otimes$ is the tensor product of quaternions, which describes the composition of rotations
- Numerically integrating this equation, we get the following system of two equations:

$$ \begin{aligned} {}^S_E\dot{\mathbf{q}}_t &= \frac{1}{2} {}^S_E\mathbf{q}_{t-\Delta t} \otimes {}^S\omega_t \\ {}^S_E\mathbf{q}_t &= {}^S_E\mathbf{q}_{t-\Delta t} + {}^S_E\dot{\mathbf{q}}_t \Delta t \end{aligned} $$

Ідея фільтру Маджвіка:

- If the device orientation ${}^S_E\mathbf{q}$ were found correctly, then the accelerometer measurements ${}^S\mathbf{a}$ (up to its own acceleration along the z-axis) should be consistent with ${}^E\mathbf{g}$, i.e.

$$ {}^E_S\mathbf{q} \otimes {}^E\mathbf{g} \otimes {}^S_E\mathbf{q} = {}^S\mathbf{a} $$

- To improve orientation determination, we minimize the residual:

$$ \mathbf{f}({}^E_S\mathbf{q}, {}^S\mathbf{a}) = {}^E_S\mathbf{q} \otimes {}^E\mathbf{g} \otimes {}^S_E\mathbf{q} - {}^S\mathbf{a} $$

- To this end, we use the *gradient descent method*:

$$ {}^E_S\mathbf{q}_{k+1} = {}^E_S\mathbf{q}_k - \mu \frac{\nabla \mathbf{f}({}^E_S\mathbf{q}, {}^S\mathbf{a})}{\|\nabla \mathbf{f}({}^E_S\mathbf{q}, {}^S\mathbf{a})\|}, \qquad k = 0, 1, 2, \ldots n $$

- Similarly, we synchronize the magnetometer measurements with the Earth magnetic field vector $\vec{\mathbf{B}}$

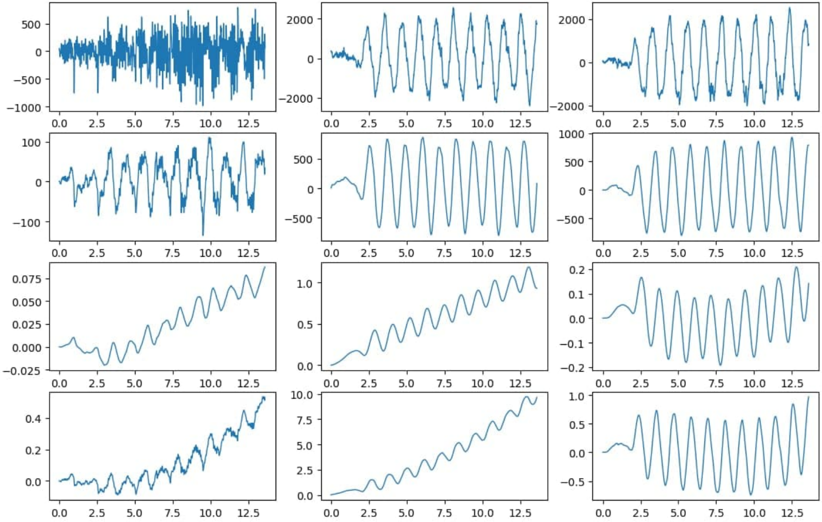
Determining the space orientation of the device

- We applied the Madgwick filter to accelerometer and gyroscope measurements to determine the device's orientation and conducted a series of experiments in order to check the method efficiency. The main experiment consists of a series of device rotations during 40 seconds.
- The results are quite precise. The Madgwick filter enabled us to detect all changes in device orientation with high precision: namely the predicted orientation only differed from the real one by rotation by at most 30 degrees w.r.t. the z-axis. This is caused by the absence of magnetometer; the measurements of the latter would enable to calibrate the orientation properly.

Determining the space location of the device

- Given the device orientation in the space, we can use quaternion operations to determine the acceleration vector w.r.t. the Earth coordinate system. Integrating twice the acceleration we will localize the device.
- Integration error is very substantial, as even small acceleration error amplifies drastically after integrating twice. To compensate the error in acceleration measurements as well as in velocity and position estimation, one could use the Kalman filter, however, this requires another measurement source (e.g. another accelerometer)
- Conducted the experiment on localizing the device without the Kalman filter and found the results quite satisfactory. In our experiments, the device moved along the circle with constant speed and its location in time was determined. Animated visualization of the results was created.

First result with Madgwick filter + simplified Kalman filter (average exponent weighted source from one accelerometer)

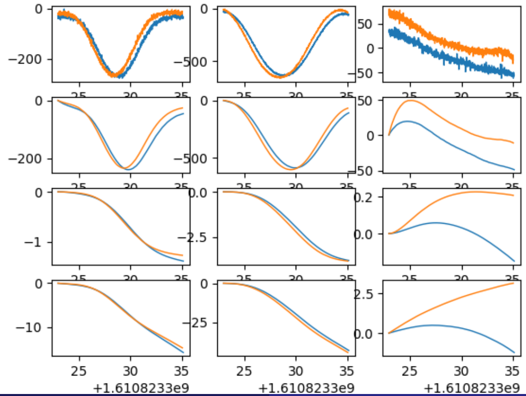


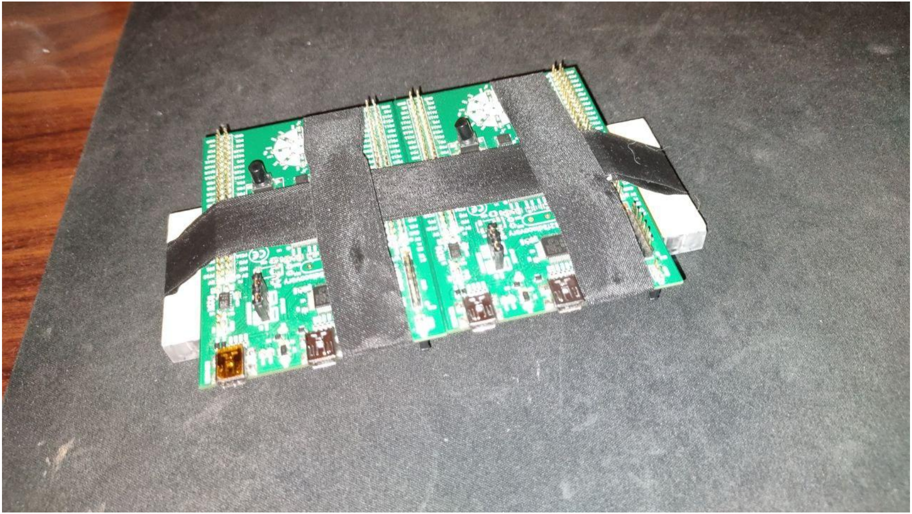
Determining the space position of the device

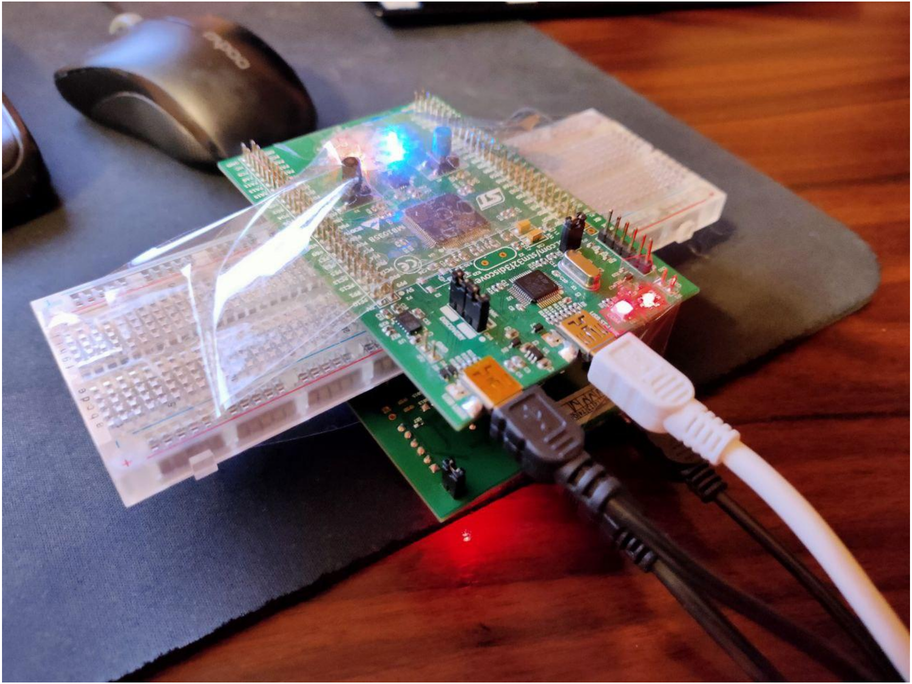
- We propose an interesting approach to solve the troubles with position computation and accelerometer error.
- Let us take two accelerometers and assume that their errors are the independent random variables! In such case we have two sources of information - so we can correlate between these two streams to estimate the real acceleration. Let us use the weighted average of two streams for that.

Determining the space position of the device

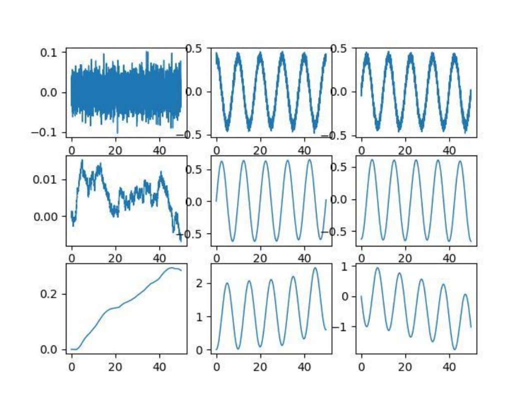
- Our solution were two identical MEMS blocks of two STM32F3DISCO boards. The problems are: fixing the positions of boards to make their axes coincide; synchronising the time between our boards (done by choosing two identical boards + threading); read the data in parallel mode (done by threading);







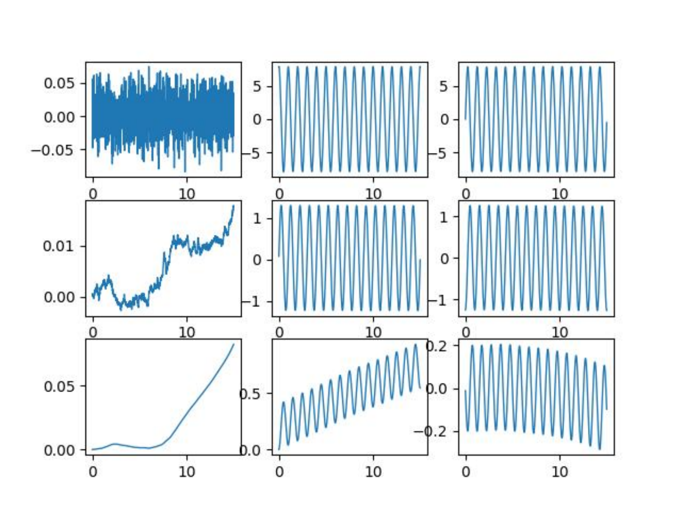
EXPERIMENTS

- We have done a sequence of some experiments to test our approach of double source of acceleration. The format - rotation with constant speed and angle rate. The orientation is fixed in space. $I_{rate} = |I_{end}$ - $I_{start}|/Radius$ - the criterio of total error
- Radius = 0.94 m; T = 10 s; => Xrate = 1; Yrate = 0.6; Zrate = 

0.4;

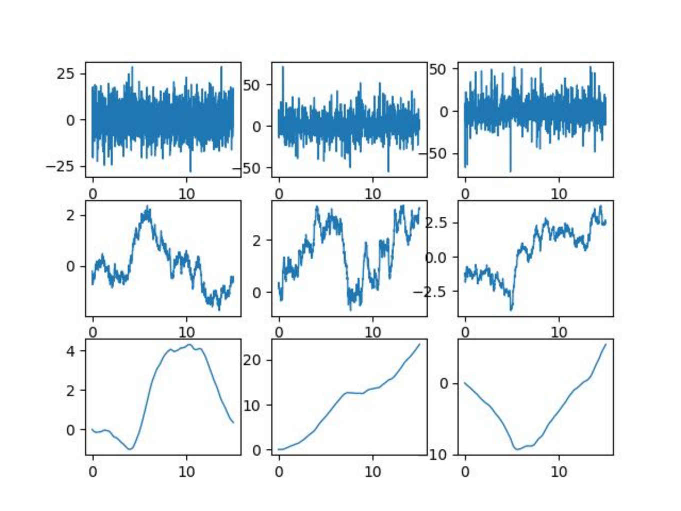
EXPERIMENTS

- Radius = 0.21 m;T = 1 s; => Xrate = 0.43; Yrate = 2.7; Zrate = 0.29;



EXPERIMENTS

- Just random walking with low speed;



Conclusion

- The Madgwick's approach of combining Accelerometer, Gyroscope and Magnetometer for orientation determination has been realized successfully - our device have amazing performance on that task and gives almost correct results for at least 60 s long experiment.
- The position determination task is far more difficult. There are no well-known approaches to do that only with one accelerometer. There exists a point to use GPS but that's not in our plate - we work with short distances.
- Our approach was to use double accelerometer to correlate between two independent (we assume) sources of acceleration information. In the result we have a performance far much better than the performance of a single acclerometer but not enough good for most of tasks. The slow and smooth ideal rotation leads us to not so bad error, but the typical task as walking doesn't seem to be great.