DC Control design

0.1 Motivations

Almost every mechanical movement is accomplished by an **electric motor** : it is a mean of converting energy (eletrical $\rightarrow$ mechanical). Electric motors are used to power hundreds of devices from everyday life. Examples of small motor applications include motors used in automobiles, robots, hand power tools ....

Electric motors are broadly classified into two different categories : DC (Direct Current) and AC (Alternating Current). In most cases, regardless of type, electric motors consist of a stator (stationary field) and a rotor (the rotating field or armature) and operate through the interaction of magnetic flux and electric current to produce rotational speed and torque. **DC motors** are distinguished by their ability to operate from direct current.

They are widely used in robotics (**mobile robots**, drone, etc, ...).

From the previous sessions (TD 1 & 2), we have learned how to simulate analyse and identify the DC motor. Now the **aim of this session is to control the DC motor under different configurations**.

Various inputs, parameter variation, perturbation and using voltage within the DC motor opetating range [-12, +12] [V].

0.2 DC Motor modelling

A DC motor [1] consists of two parts

- a fixed one called "stator" consisting of either permanent magnets for motors of low power or electromagnets for the bigger ones,
- a rotating one called "rotor" consisting of a coil attached to the frame ; this frame is in contact with a brush that supplies it with voltage (v_a and thus a current i_a through the resistance R_a). Finally an output axis that allows to recover the rotating movement (angle θ_m).

Current i_a applied to the rotor generates a motor torque proportional to this current $\Gamma_m = K_t i_a$ leading to :

$$\begin{array}{l} J_m\ddot{\theta}_m + b\dot{\theta}_m = \Gamma_m - \Gamma_c \\ L_a\frac{di_a}{dt} + R_a i_a = v_a - K_e\dot{\theta}_m \end{array} \tag{1}$$

where J_m is the inertia motor, b is the friction coefficient, Γ_c is the load torque, L_a the motor inductance, R_a the motor resistor, v_a the supply voltage of the motor, $-K_e\dot{\theta}_m$ the back electromotive force (Back EMF) and K_e is the motor velocity constant alternatively called the back EMF constant. These two ODE are interconnected : the mechanical part with the electrical part through the term $\Gamma_m = K_t i_a$ (K_t is the **motor torque constant**) and vice versa (through the back EMF). Thus, one can produce the following figure showing this interconnections between the mechanical ODE part and the electrical one :

1. Here the DC Motor Lego 43362, but all details remain valid for other DC motors except the numerical values.

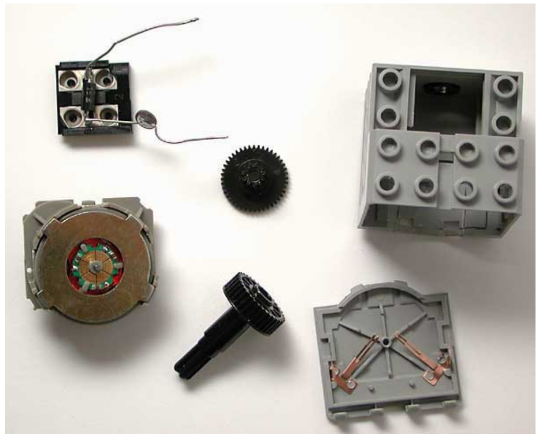

FIGURE 1 – DC motor Lego 43362.

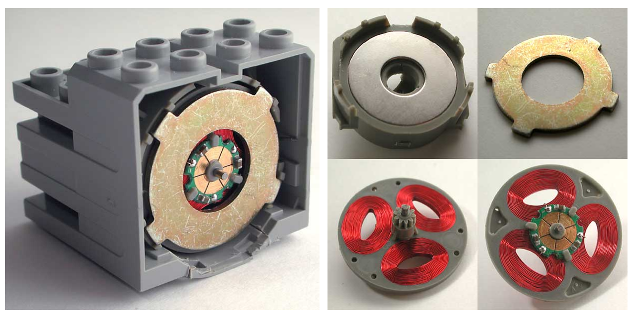

FIGURE 2 – DC motor Lego 43362.

Usually the electrical time constant is smaller than the mechanical one (because $L_a \ll 1$) : thus the mechanical equation reads as ($\Gamma_m = K_t i_a = \frac{K_t}{R_a}(v_a - K_e\dot{\theta}_m)$) :

$$J_m\ddot{\theta}_m + \left(b + \frac{K_t K_e}{R_a}\right)\dot{\theta}_m = \frac{K_t}{R_a}v_a - \Gamma_c \tag{2}$$

Numerical values of the DC motor parameters :
$R_a = 2\,[\Omega], L_a = 6.5\times10^{-5}\,[\mathrm{H}], J_m = 6\times10^{-7}\,[\mathrm{kg\,m^2}], K_t = K_e = 0.013\,[\mathrm{SI}], b = 0.008\,[\mathrm{SI}]$.

0.3 State feedback

In the first session (TD1), controllability of both models were shown. Now, we are in position to build the control.

Question 1. ***Dynamic selection :*** *To start with select a second order dynamic $p^2 + 2\zeta\omega_n p + \omega_n^2$:*

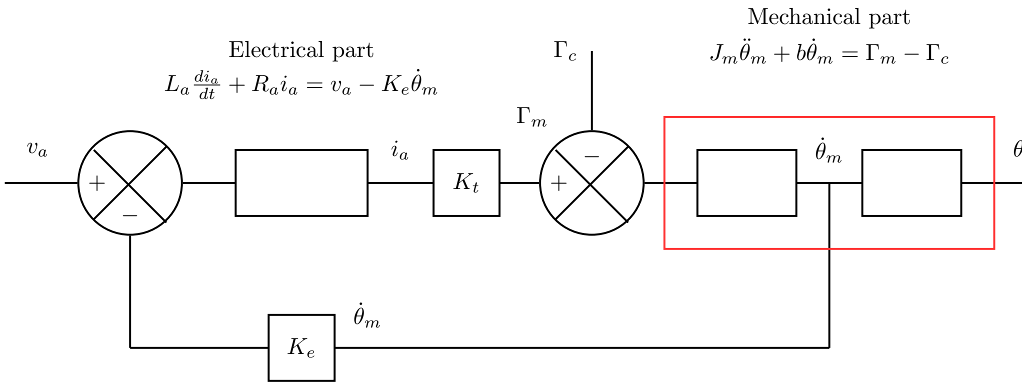

FIGURE 3 – DC motor diagram

this is choose ζ and ω_n. Remember that ζ can adjust damping (oscillations[2]) and ω_n can adjust the time reponse using abacus (see slides) or approximating formulas :

- *if $0.4 < \zeta < 0.7$ then $t_r = \frac{5}{\omega_n}$,*
- *if $\zeta = 0.7$ then $t_r = \frac{2.9}{\omega_n}$,*
- *if $0.7 < \zeta < 1$ then $t_r = \frac{4.3\zeta}{\omega_n}$,*
- *if $\zeta > 1.5$ then $t_r = \frac{6\zeta}{\omega_n}$.*

For the complete model select an additional first order dynamic.

Question 2. *Find the controllable canonical form (find matrix P).*

Question 3. *For both models (complete and simplified), design a state feedback $u = -Kx$:*

- *by hand using the original obtained state forms,*
- *by hand using the controllable canonical forms,*
- *using Matlab comand `place`.*

Compare solutions.

Question 4. *For both models, find the feedfoward gain (see slides $G_f = \left(D - (C - DK)(A - BK)^{-1}B\right)^{-1}$) which here reduces to*

$$G_f = -\left(C(A - BK)^{-1}B\right)^{-1}.$$

Question 5. *Implement the obtained feedbacks on Simulink. Test the solution when the reference is constant, parameters may vary (up to 50%) and under possible perturbation (load torque constant or try some time-varying ones)*

0.4 Robustness

In this section, we only consider the simpled model of the DC motor (2) recalled below :

$$J_m\ddot{\theta}_m + \left(b + \frac{K_t K_e}{R_a}\right)\dot{\theta}_m = \frac{K_t}{R_a}v_a - \Gamma_c \tag{3}$$

2. $0.7 < \zeta < 1$ or $\zeta > 1$ no oscillation, usefull for observer design

which has the following state-space form :

$$\dot{x} = Ax + Bu + w,$$
$$A = \begin{pmatrix} 0 & 1 \\ 0 & -\frac{1}{\tau} \end{pmatrix}, B = \begin{pmatrix} 0 \\ k \end{pmatrix}, w = \begin{pmatrix} 0 \\ \\ \frac{\Gamma_c}{J_m} \end{pmatrix}, \tag{4}$$

where $1/\tau = \frac{b}{J_m} + \frac{K_t K_e}{R_a J_m}, k = \frac{K_t}{R_a J_m}$.

Question 6. *Assume that for the nominal plant parameters A, B, C a state feedback K has been found. In particular $(A-BK)$ is Hurwitz (stable matrix : this is all eigenvalues are in the left hand side plane). For a given $k > 0$ find values of τ for which the system is unstable. Explain why in your simulation it is unstable for certain variation of the parameters.*

Question 7. *Adding an integrator, the control has the form :*

$$u = -Kx - k_I \int_0^t (y(v) - y_{ref})dv.$$

Prove that this will make the system robust if constant K k_I are well choosen.

Question 8. *Can we set, in the above given control, K as the gain computed in question 3 ? How to compute k_I ? Compute k_I (and K) using the Matlab command* `place`.

Question 9. *Implement the obtained integral feedback controller in Simulink and test robustness (perturbation and parameter variation).*