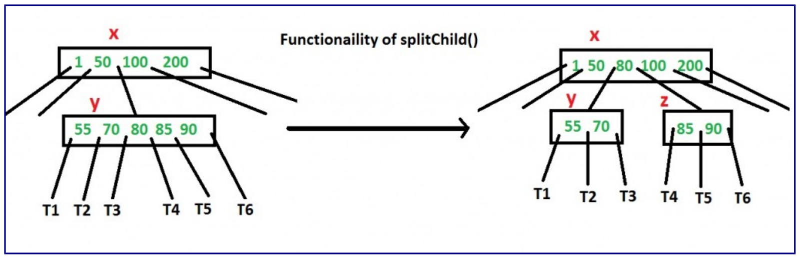
B-Tree | Set 2 (Insert)

In the previous post, we introduced B-Tree. We also discussed search() and traverse() functions.
In this post, insert() operation is discussed. A new key is always inserted at leaf node. Let the key to be inserted be k. Like BST, we start from root and traverse down till we reach a leaf node. Once we reach a leaf node, we insert the key in that leaf node. Unlike BSTs, we have a predefined range on number of keys that a node can contain. So before inserting a key to node, we make sure that the node has extra space.
How to make sure that a node has space available for key before the key is inserted? We use an operation called splitChild() that is used to split a child of a node. See the following diagram to understand split. In the following diagram, child y of x is being split into two nodes y and z. Note that the splitChild operation moves a key up and this is the reason B-Trees grow up unlike BSTs which grow down.



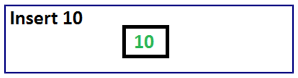
As discussed above, to insert a new key, we go down from root to leaf. Before traversing down to a node, we first check if the node is full. If the node is full, we split it to create space. Following is complete algorithm.

Insertion
1) Initialize x as root.
2) While x is not leaf, do following
..**a)** Find the child of x that is going to to be traversed next. Let the child be y.
..**b)** If y is not full, change x to point to y.
..**c)** If y is full, split it and change x to point to one of the two parts of y. If k is smaller than mid key in y, then set x as first part of y. Else second part of y. When we split y, we move a key from y to its parent x.
3) The loop in step 2 stops when x is leaf. x must have space for 1 extra key as we have been splitting all nodes in advance. So simply insert k to x.

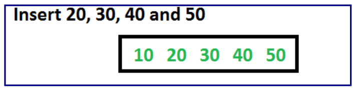
Note that the algorithm follows the Cormen book. It is actually a proactive insertion algorithm where before going down to a node, we split it if it is full. The advantage of splitting before is, we never traverse a node twice. If we don't split a node before going down to it and split it only if new key is inserted (reactive), we may end up traversing all nodes again from leaf to root. This happens in cases when all nodes on the path from root to leaf are full. So when we come to the leaf node, we split it and move a key up. Moving a key up will cause a split in parent node (because parent was already full). This cascading effect never happens in this proactive insertion algorithm. There is a disadvantage of this proactive insertion though, we may do unnecessary splits.

Let us understand the algorithm with an example tree of minimum degree 't' as 3 and a sequence of integers 10, 20, 30, 40, 50, 60, 70, 80 and 90 in an initially empty B-Tree.

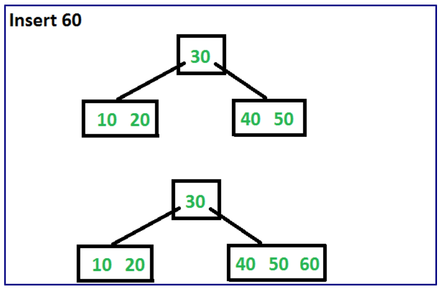
Initially root is NULL. Let us first insert 10.



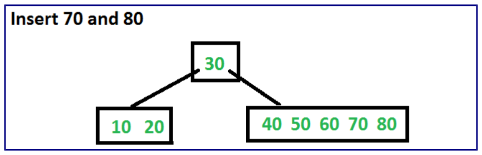
Let us now insert 20, 30, 40 and 50. They all will be inserted in root because maximum number of keys a node can accommodate is 2*t – 1 which is 5.



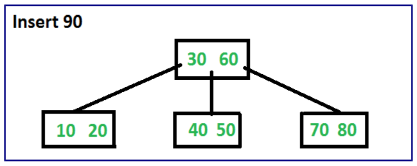
Let us now insert 60. Since root node is full, it will first split into two, then 60 will be inserted into the appropriate child.



Let us now insert 70 and 80. These new keys will be inserted into the appropriate leaf without any split.



Let us now insert 90. This insertion will cause a split. The middle key will go up to the parent.



See this for more examples.

Following is C++ implementation of the above proactive algorithm.

```
// C++ program for B-Tree insertion
#include<iostream>
using namespace std;

// A BTree node
class BTreeNode
{
    int *keys;  // An array of keys
    int t;      // Minimum degree (defines the range for number of
keys)
    BTreeNode **C; // An array of child pointers
    int n;     // Current number of keys
    bool leaf; // Is true when node is leaf. Otherwise false
public:
    BTreeNode(int _t, bool _leaf);   // Constructor

    // A utility function to insert a new key in the subtree rooted
with
    // this node. The assumption is, the node must be non-full when
this
    // function is called
    void insertNonFull(int k);

    // A utility function to split the child y of this node. i is
index of y in
    // child array C[].  The Child y must be full when this function
is called
    void splitChild(int i, BTreeNode *y);

    // A function to traverse all nodes in a subtree rooted with this
node
    void traverse();

    // A function to search a key in subtree rooted with this node.
    BTreeNode *search(int k);   // returns NULL if k is not present.

// Make BTree friend of this so that we can access private members of
this
// class in BTree functions
friend class BTree;
};

// A BTree
class BTree
{
    BTreeNode *root; // Pointer to root node
    int t;  // Minimum degree
public:
    // Constructor (Initializes tree as empty)
    BTree(int _t)
    {  root = NULL;  t = _t; }

    // function to traverse the tree
    void traverse()
    {  if (root != NULL) root->traverse(); }

    // function to search a key in this tree
    BTreeNode* search(int k)
    {  return (root == NULL)? NULL : root->search(k); }

    // The main function that inserts a new key in this B-Tree
    void insert(int k);
};

// Constructor for BTreeNode class
BTreeNode::BTreeNode(int t1, bool leaf1)
{
    // Copy the given minimum degree and leaf property
    t = t1;
    leaf = leaf1;

    // Allocate memory for maximum number of possible keys
    // and child pointers
    keys = new int[2*t-1];
    C = new BTreeNode *[2*t];

    // Initialize the number of keys as 0
    n = 0;
}

// Function to traverse all nodes in a subtree rooted with this node
void BTreeNode::traverse()
{
    // There are n keys and n+1 children, travers through n keys
    // and first n children
    int i;
    for (i = 0; i < n; i++)
    {
        // If this is not leaf, then before printing key[i],
        // traverse the subtree rooted with child C[i].
        if (leaf == false)
            C[i]->traverse();
        cout << " " << keys[i];
    }

    // Print the subtree rooted with last child
    if (leaf == false)
        C[i]->traverse();
}

// Function to search key k in subtree rooted with this node
BTreeNode *BTreeNode::search(int k)
{
    // Find the first key greater than or equal to k
    int i = 0;
    while (i < n && k > keys[i])
        i++;

    // If the found key is equal to k, return this node
    if (keys[i] == k)
        return this;

    // If key is not found here and this is a leaf node
    if (leaf == true)
        return NULL;

    // Go to the appropriate child
    return C[i]->search(k);
}

// The main function that inserts a new key in this B-Tree
void BTree::insert(int k)
{
    // If tree is empty
    if (root == NULL)
    {
        // Allocate memory for root
        root = new BTreeNode(t, true);
        root->keys[0] = k;  // Insert key
        root->n = 1;  // Update number of keys in root
    }
    else // If tree is not empty
    {
        // If root is full, then tree grows in height
        if (root->n == 2*t-1)
        {
            // Allocate memory for new root
            BTreeNode *s = new BTreeNode(t, false);

            // Make old root as child of new root
            s->C[0] = root;

            // Split the old root and move 1 key to the new root
            s->splitChild(0, root);

            // New root has two children now.  Decide which of the
            // two children is going to have new key
            int i = 0;
            if (s->keys[0] < k)
                i++;
            s->C[i]->insertNonFull(k);

            // Change root
            root = s;
        }
        else // If root is not full, call insertNonFull for root
            root->insertNonFull(k);
    }
}

// A utility function to insert a new key in this node
// The assumption is, the node must be non-full when this
// function is called
void BTreeNode::insertNonFull(int k)
{
    // Initialize index as index of rightmost element
    int i = n-1;

    // If this is a leaf node
    if (leaf == true)
    {
        // The following loop does two things
        // a) Finds the location of new key to be inserted
        // b) Moves all greater keys to one place ahead
        while (i >= 0 && keys[i] > k)
        {
            keys[i+1] = keys[i];
            i--;
        }

        // Insert the new key at found location
        keys[i+1] = k;
        n = n+1;
    }
    else // If this node is not leaf
    {
        // Find the child which is going to have the new key
        while (i >= 0 && keys[i] > k)
            i--;

        // See if the found child is full
        if (C[i+1]->n == 2*t-1)
        {
            // If the child is full, then split it
            splitChild(i+1, C[i+1]);

            // After split, the middle key of C[i] goes up and
            // C[i] is splitted into two.  See which of the two
            // is going to have the new key
            if (keys[i+1] < k)
                i++;
        }
        C[i+1]->insertNonFull(k);
    }
}

// A utility function to split the child y of this node
// Note that y must be full when this function is called
void BTreeNode::splitChild(int i, BTreeNode *y)
{
    // Create a new node which is going to store (t-1) keys
    // of y
    BTreeNode *z = new BTreeNode(y->t, y->leaf);
    z->n = t - 1;

    // Copy the last (t-1) keys of y to z
    for (int j = 0; j < t-1; j++)
```

```
        z->keys[j] = y->keys[j+t];

    // Copy the last t children of y to z
    if (y->leaf == false)
    {
        for (int j = 0; j < t; j++)
            z->C[j] = y->C[j+t];
    }

    // Reduce the number of keys in y
    y->n = t - 1;

    // Since this node is going to have a new child,
    // create space of new child
    for (int j = n; j >= i+1; j--)
        C[j+1] = C[j];

    // Link the new child to this node
    C[i+1] = z;

    // A key of y will move to this node. Find location of
    // new key and move all greater keys one space ahead
    for (int j = n-1; j >= i; j--)
        keys[j+1] = keys[j];

    // Copy the middle key of y to this node
    keys[i] = y->keys[t-1];

    // Increment count of keys in this node
    n = n + 1;
}

// Driver program to test above functions
int main()
{
    BTree t(3); // A B-Tree with minium degree 3
    t.insert(10);
    t.insert(20);
    t.insert(5);
    t.insert(6);
    t.insert(12);
    t.insert(30);
    t.insert(7);
    t.insert(17);

    cout << "Traversal of the constucted tree is ";
    t.traverse();

    int k = 6;
    (t.search(k) != NULL)? cout << "\nPresent" : cout << "\nNot
Present";
```

```
    k = 15;
    (t.search(k) != NULL)? cout << "\nPresent" : cout << "\nNot 
Present";

    return 0;
}
```
Output:

```
Traversal of the constucted tree is  5 6 7 10 12 17 20 30
Present
Not Present
```

References:

Introduction to Algorithms 3rd Edition by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest
http://www.cs.utexas.edu/users/djimenez/utsa/cs3343/lecture17.html