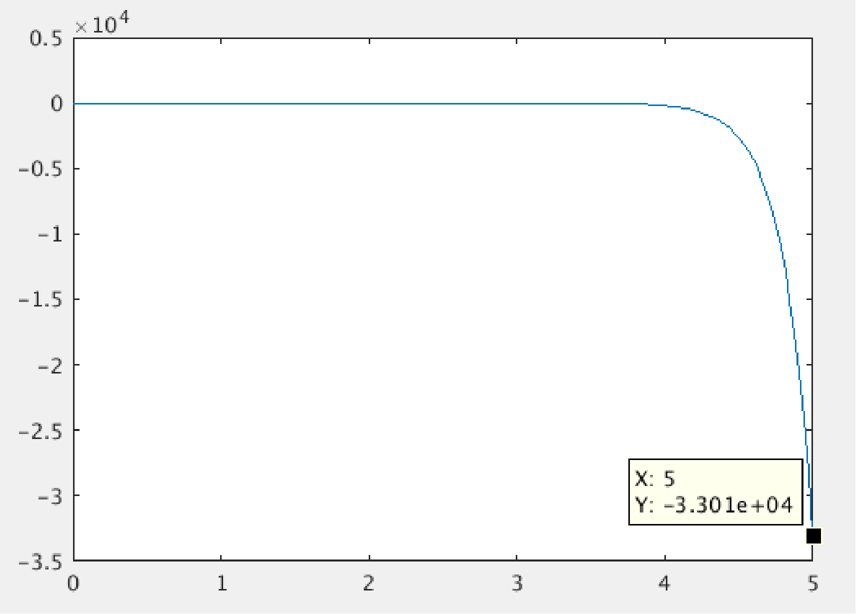
Practical Answers 14 – Adaptive Methods for ODEs

1. Solve the ODE,

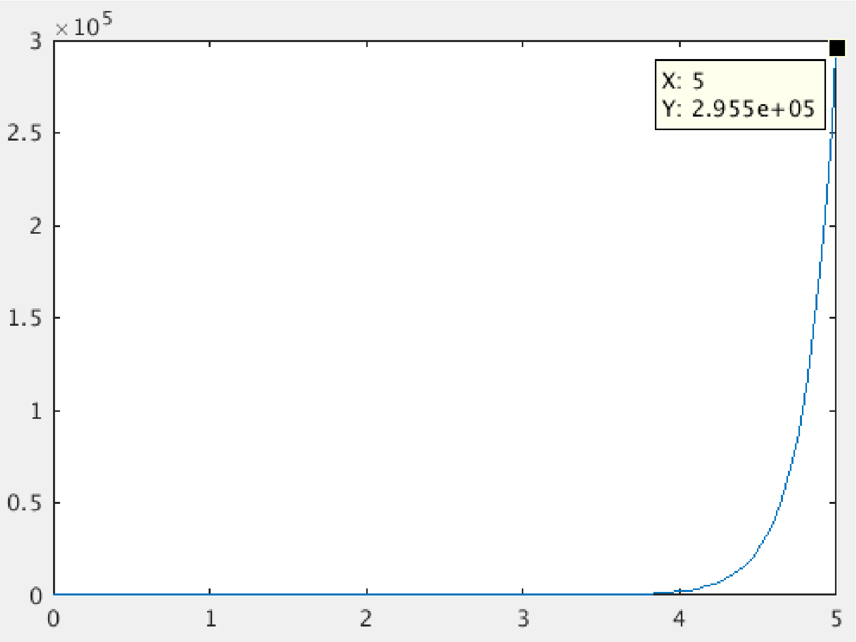
$$\frac{dy}{dt} = 5(y - t^2)$$

from t = 0 to 5 with y(0) = 0.08 using:

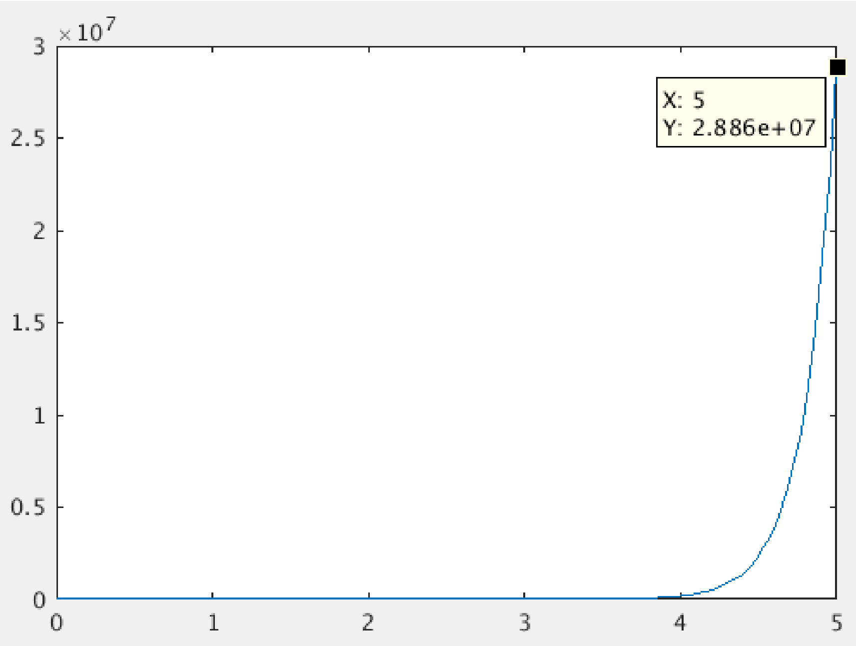
a. 4th order RK with constant step size of 0.03125.



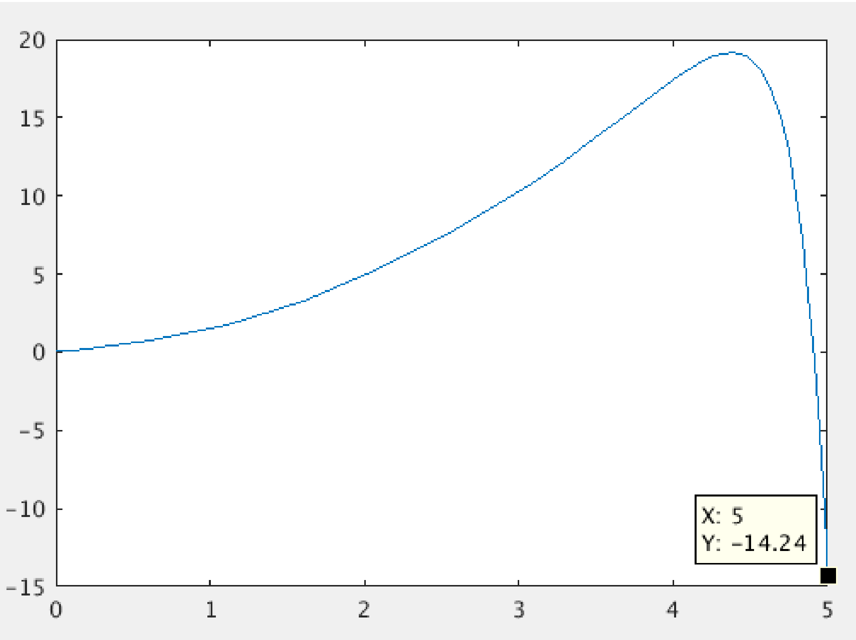
b. Using Matlab's ode45.



c. Using Matlab's ode23s.



d. Using Matlab's ode23tb.



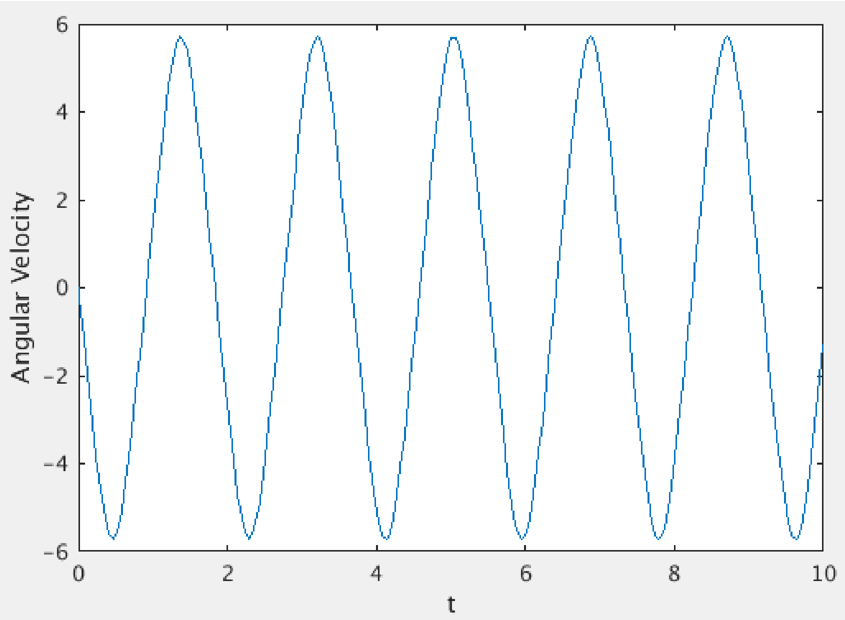
Note that all solutions diverge eventually.

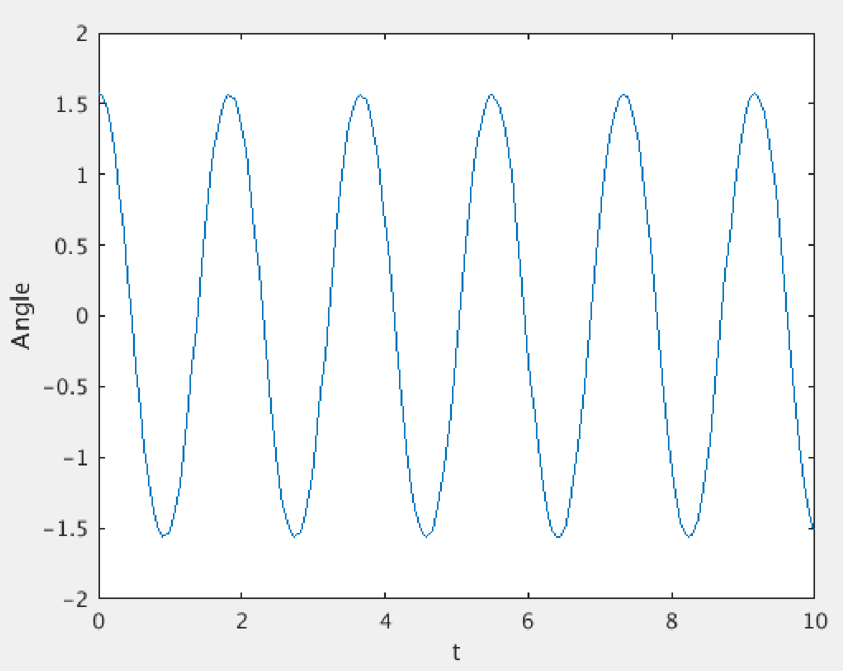
Use the stats parameter of the Matlab functions as well as tic toc to determine which one was more efficient at solving the equation then present your results in graphical form.

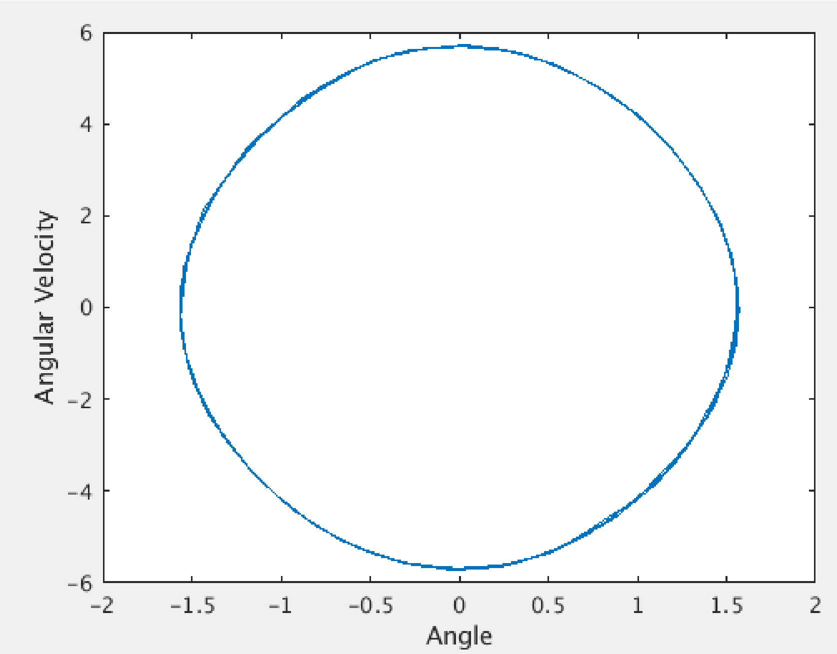
2. Convert the pendulum equation into a system of ODEs then solve using ode45.

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

where *g* is the gravitational constant = 9.81 m/s^2, l is the length of the pendulum = 0.6 m, θ and *t* are the angle the pendulum makes with the horizontal and time respectively. Take the initial condition to be θ = π/2 and dθ/dt = 0 and solve for the first 10 seconds. Plot your results in a phase diagram with angle on the horizontal axis and angular velocity on the vertical axis.







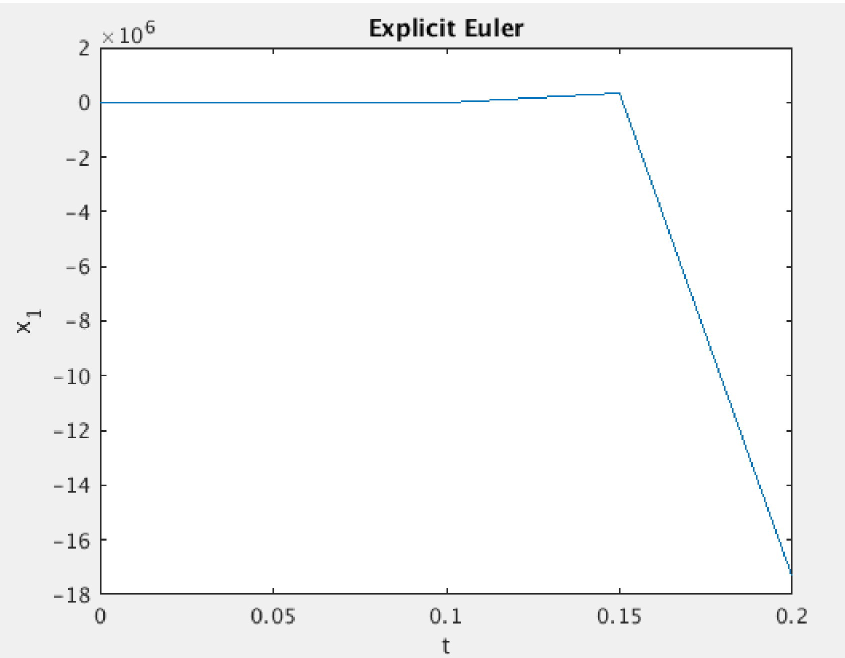
3. Solve and plot the solution of the system (independent vs. dependent variables),

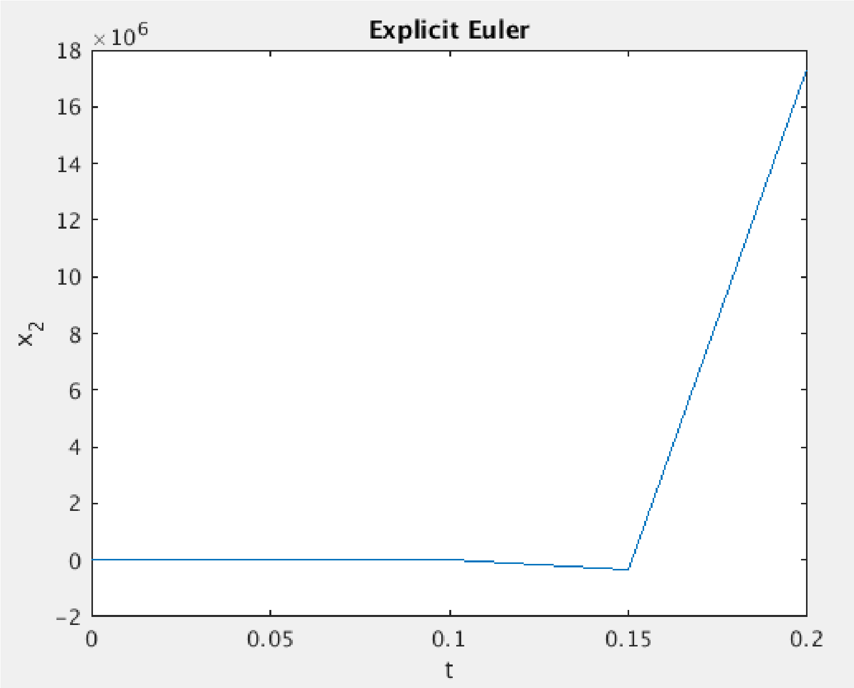
$$\frac{dx_1}{dt} = 999x_1 + 1999x_2$$

$$\frac{dx_2}{dt} = -1000x_1 - 2000x_2$$

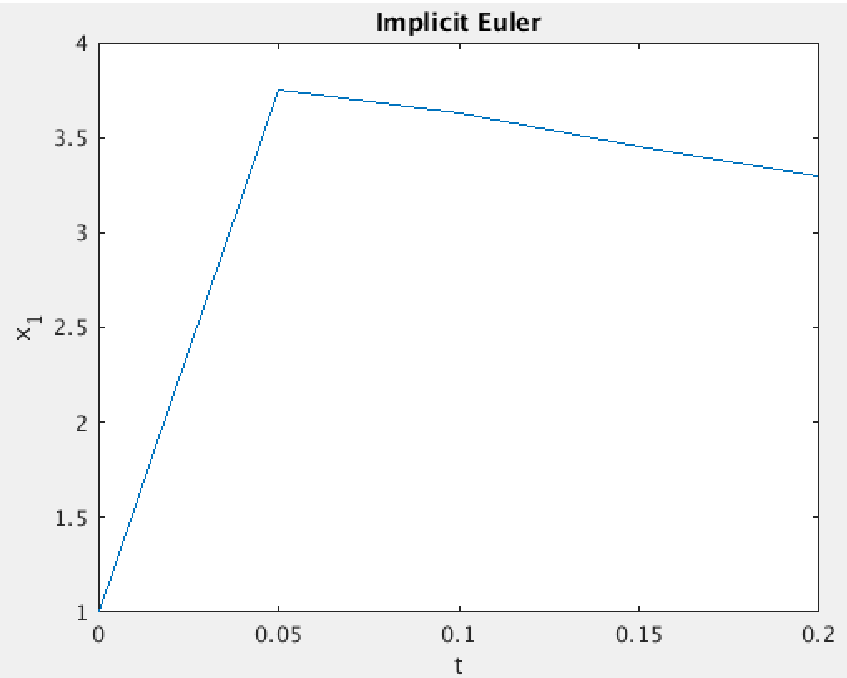
with x1(0) = x2(0) = 1 from t = 0 to 0.2 using a step size of h = 0.05 with:

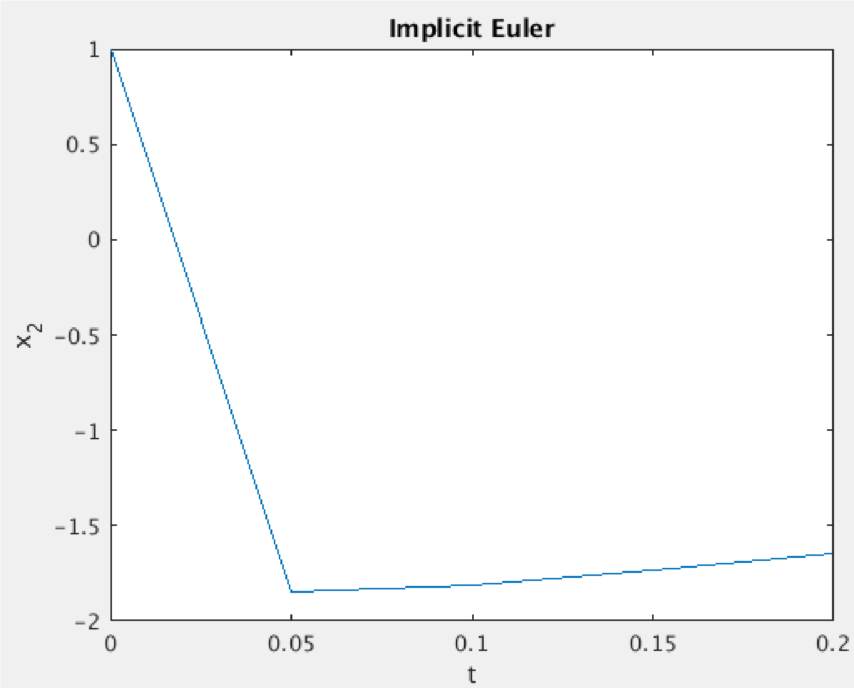
a. The explicit (original) Euler method.





b. The implicit Euler method.

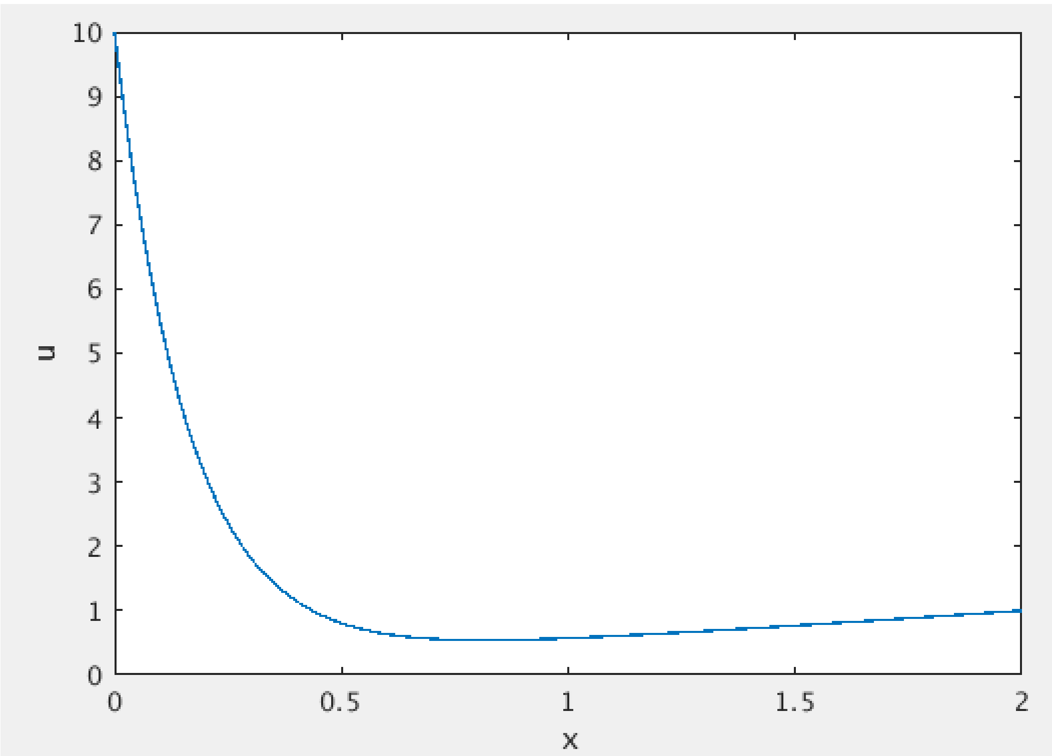




4. Use bvp4c to solve the following boundary-value ordinary differential equation:

$$\frac{d^2u}{dx^2} + 6\frac{du}{dx} - u = 2$$

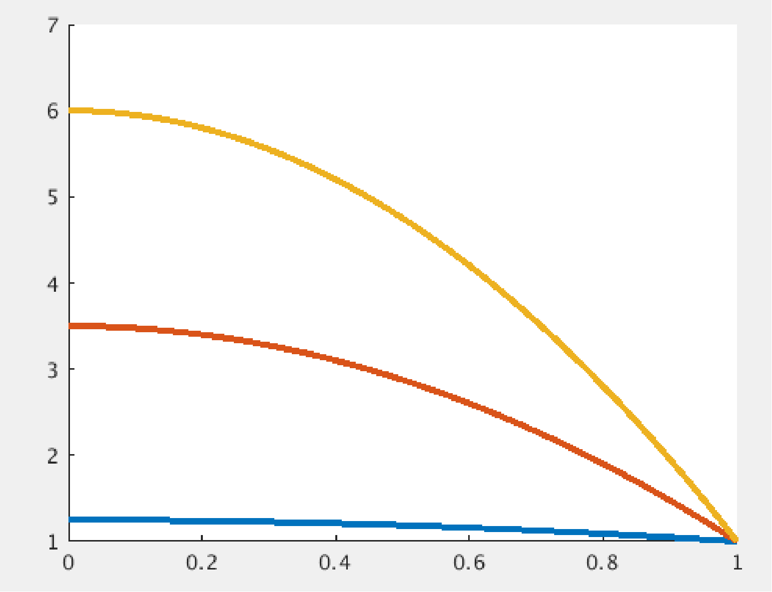
with u(0) = 10 and u(2) = 1. Plot *x* against *u*.



5. Use bvp4c to solve the following boundary-value ordinary differential equation which represents the heat distribution in a cylindrical rod with an internal heat source, S:

$$\frac{d^2 T}{d r^2} + \frac{1}{r}\frac{dT}{dr} + S = 0 \, ,\qquad \begin{array}{l} T(1) = 1 \\ \frac{dT}{dr}(0) = 0 \end{array}$$

Solve for S = 1, 10 and 20. Plot r against T in all 3 cases.

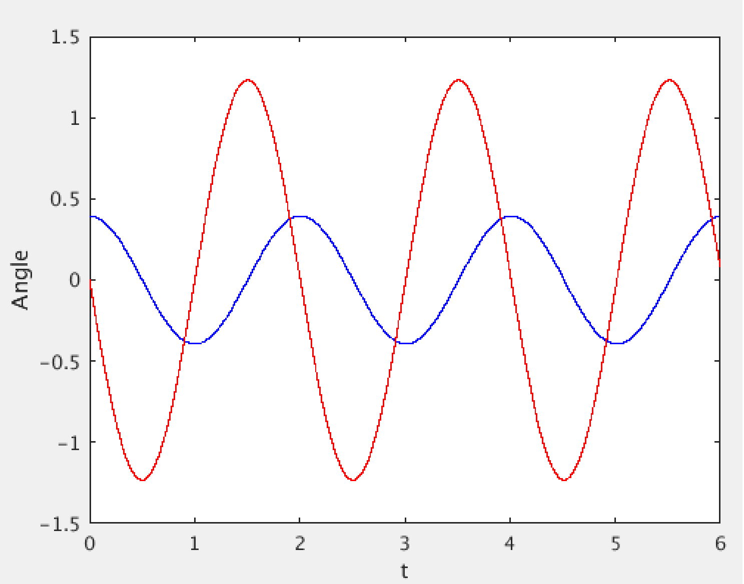


6. The linearised equation of motion of a pendulum is given by,

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

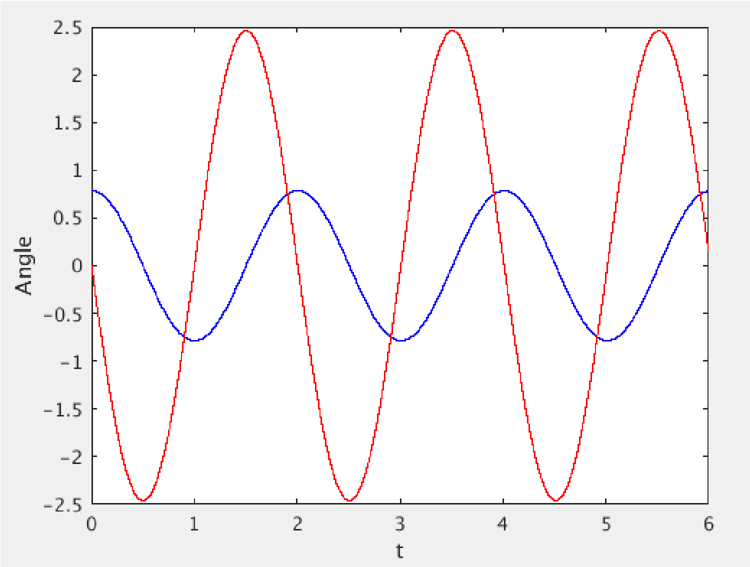
with initial angular velocity of 0. Taking *l* = 1 m, *g* = 9.81 m/s^2, use the events parameter in Matlab to determine the period of the oscillation for:

a. θ = π/8



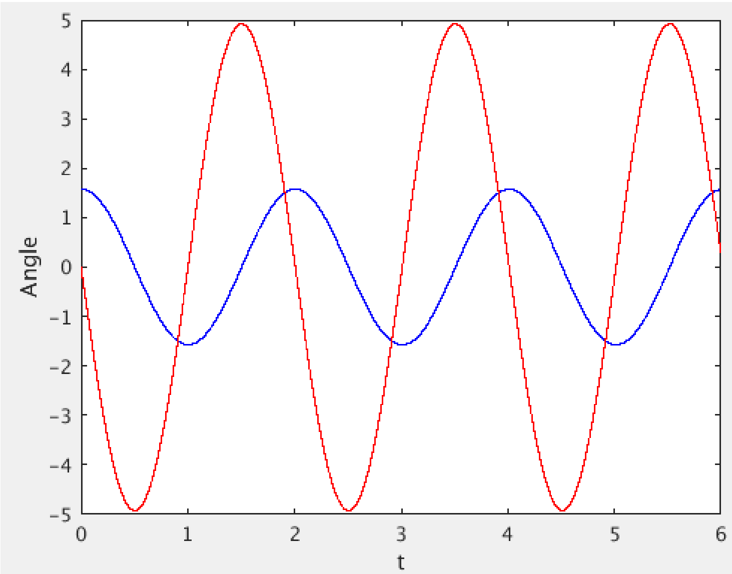
Period is approximately 2.005

c. $\theta = \pi/4$



Period is approximately 2.005

d. $\theta = \pi/2$



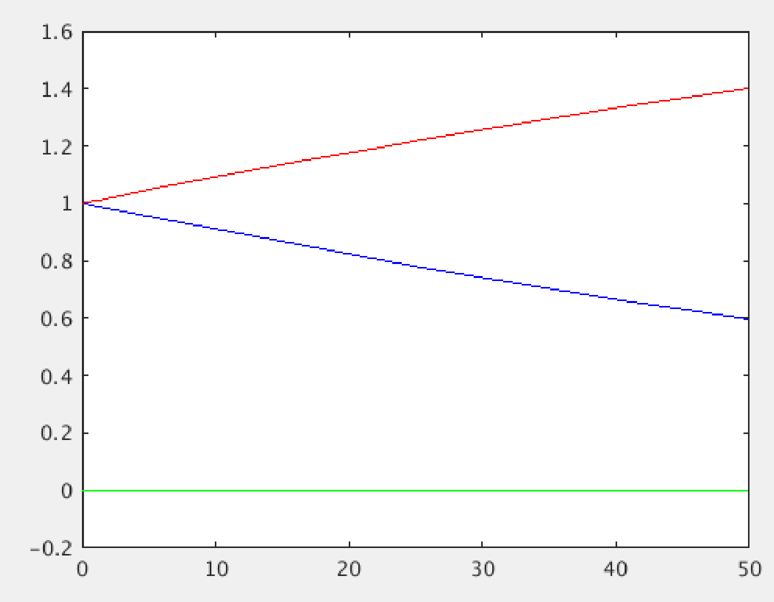
Period is approximately 2.005

7. Solve the following system of 3 ODEs representing chemical reaction rates from $t = 0$ to 50 with $c_1(0) = c_2(0) = 1$, $c_3(0) = 0$. Plot your results in a 3 dimensional phase diagram.

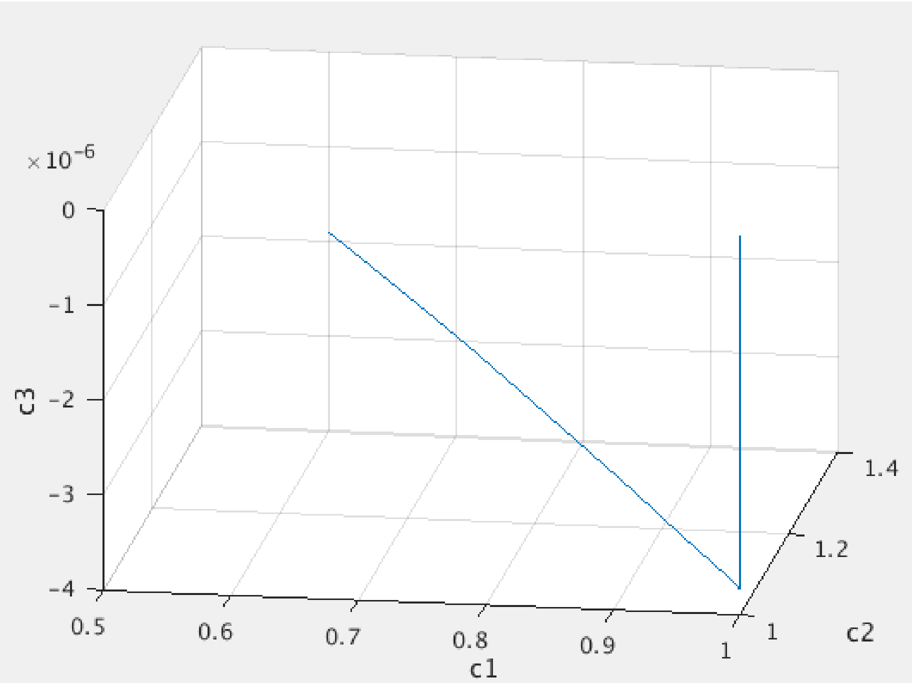
$$\frac{dc_1}{dt} = -0.013c_1 - 1000c_1c_3$$

$$\frac{dc_2}{dt} = -2500c_2c_3$$

$$\frac{dc_3}{dt} = -0.013c_1 - 1000c_1c_3 - 2500c_2c_3$$



Blue is c1, red is c2 and green is c3.

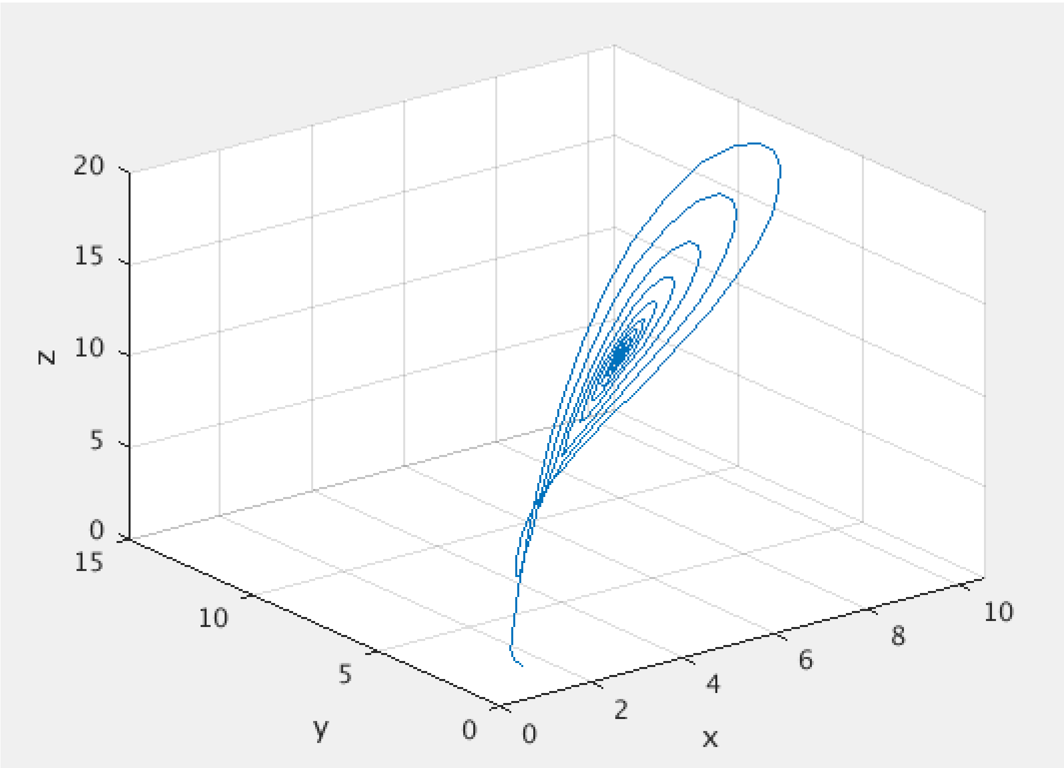


8. Solve the Lorenz system,

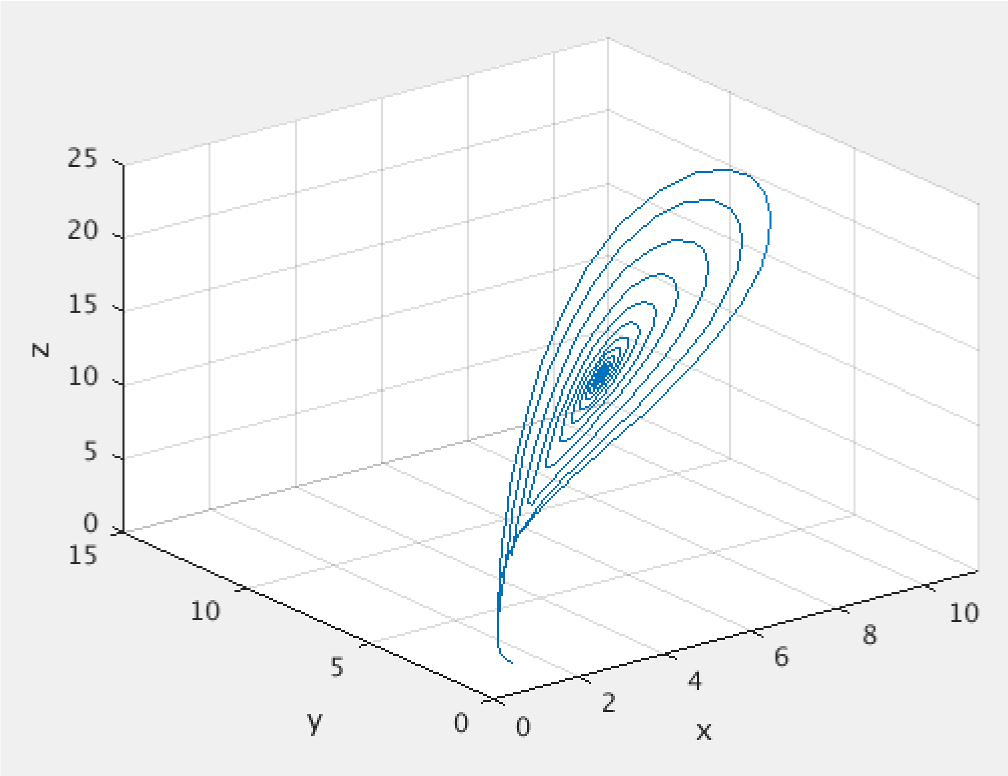
$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x),$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y,$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z.$$

for σ = 10, β = 8/3 and,

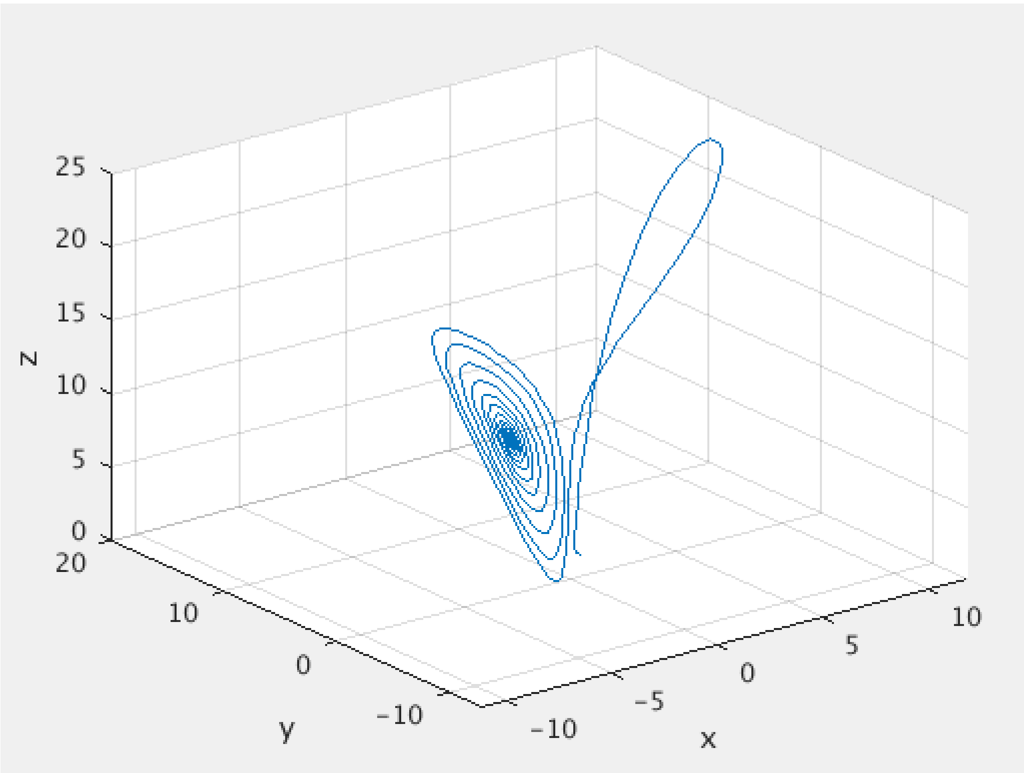
a. ρ = 13



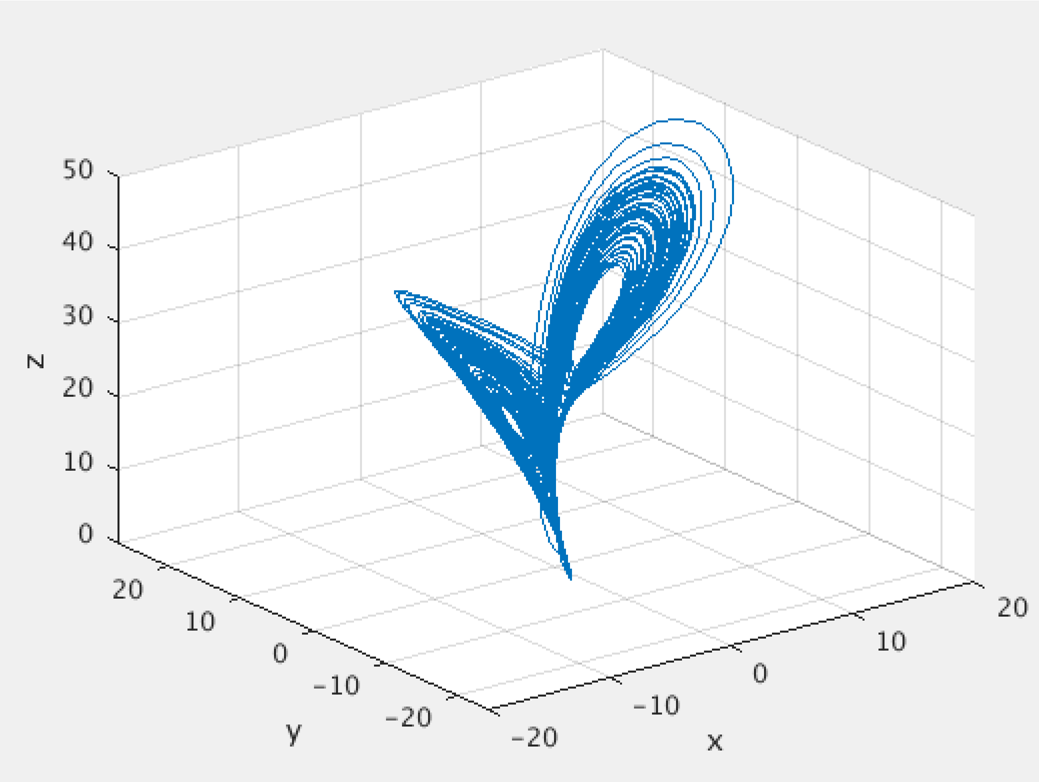
b. ρ = 14

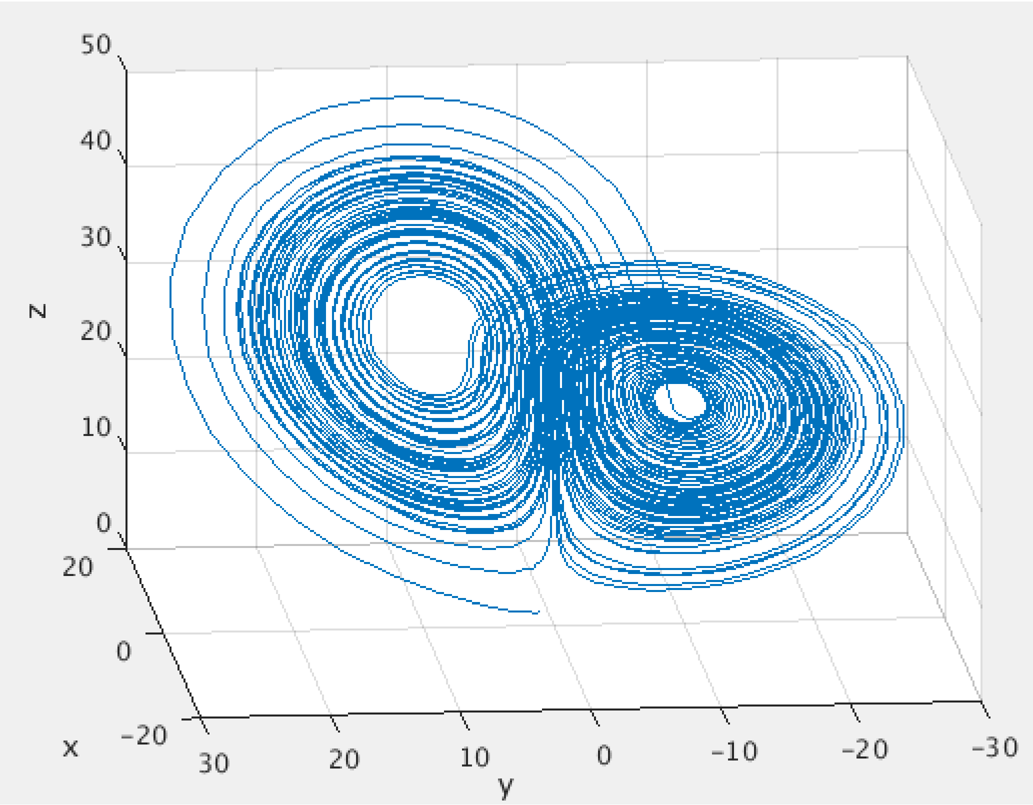


c. ρ = 15



d. ρ = 28





Find the equilibrium points by **<u>choosing starting points</u>** manually and **<u>solving both forwards and backwards in time until the solution value stops changing</u>**.

Once found, plot the equilibrium points and several trajectories in a 3D phase plot that represents the behaviour of the system.