Practical Answers 5 – Open Root Finding Methods

Plotting the function is a good idea to get an idea of where the roots are and provide initial guesses, then the methods can be used to get an accurate estimation.
You can also substitute your answers back into the equation to see if it is correct (anonymous functions can help with that).

1. Use the **roots** function to find the roots of the polynomial.

$$p(x) = x^5 - 10x^3 + 21x$$

2. Create an anonymous function for p(x) and substitute the roots you found in question 1 into it to verify your answer.

```
Command Window
>> Q1_2_Sol
The roots are:
         0
   -2.6458
   -1.7321
    2.6458
    1.7321

Substituting the roots back into the function gives:
   1.0e-12 *

         0
   -0.1705
   -0.0568
   -0.0995
    0.0142
```

3. Use the **roots** function to find the roots of the polynomial (no template). Display only the real roots.

$$q(x) = -2x^6 - 1.5x^4 + 10x + 2$$

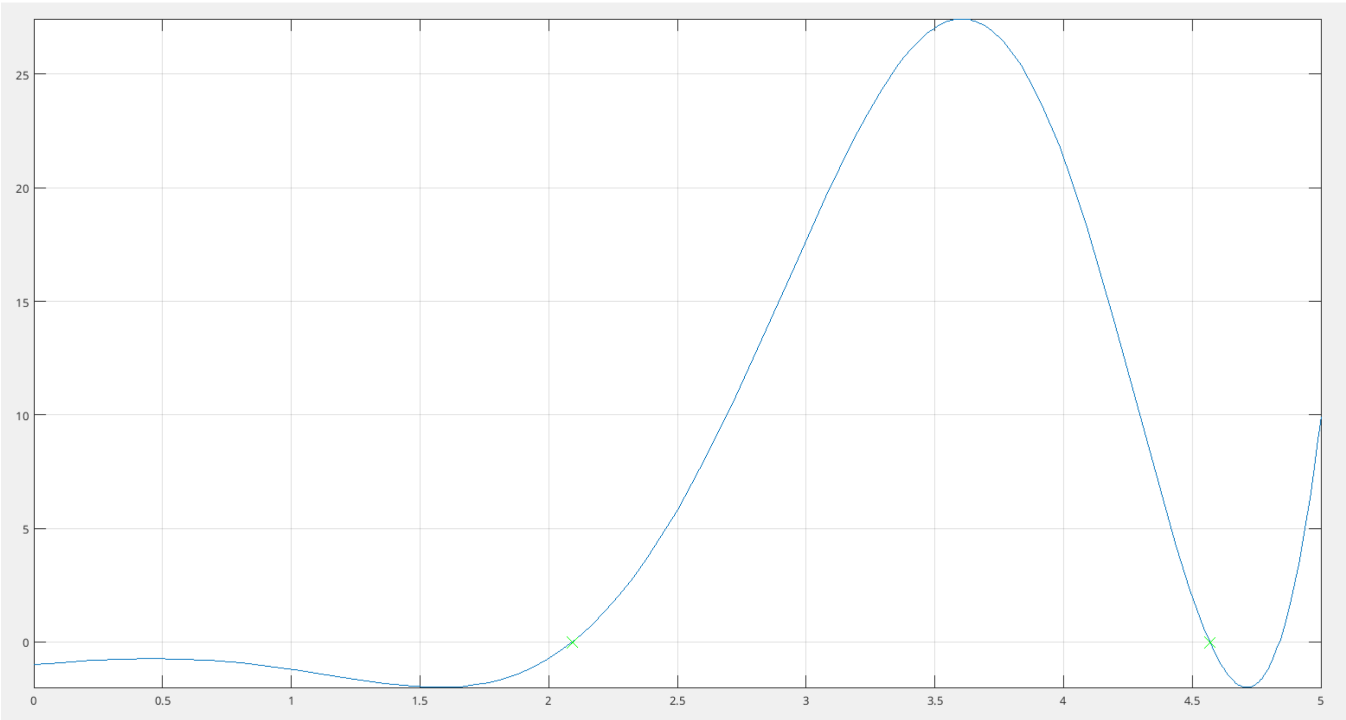
```
Command Window
>> Q3_Sol
Real roots are as follows.
Root 1:
    1.3213

Substituting back into the function gives:
   1.2434e-13

Root 2:
   -0.1997

Substituting back into the function gives:
  -4.4409e-16
```

4. Use the **fzero** function to locate all roots of

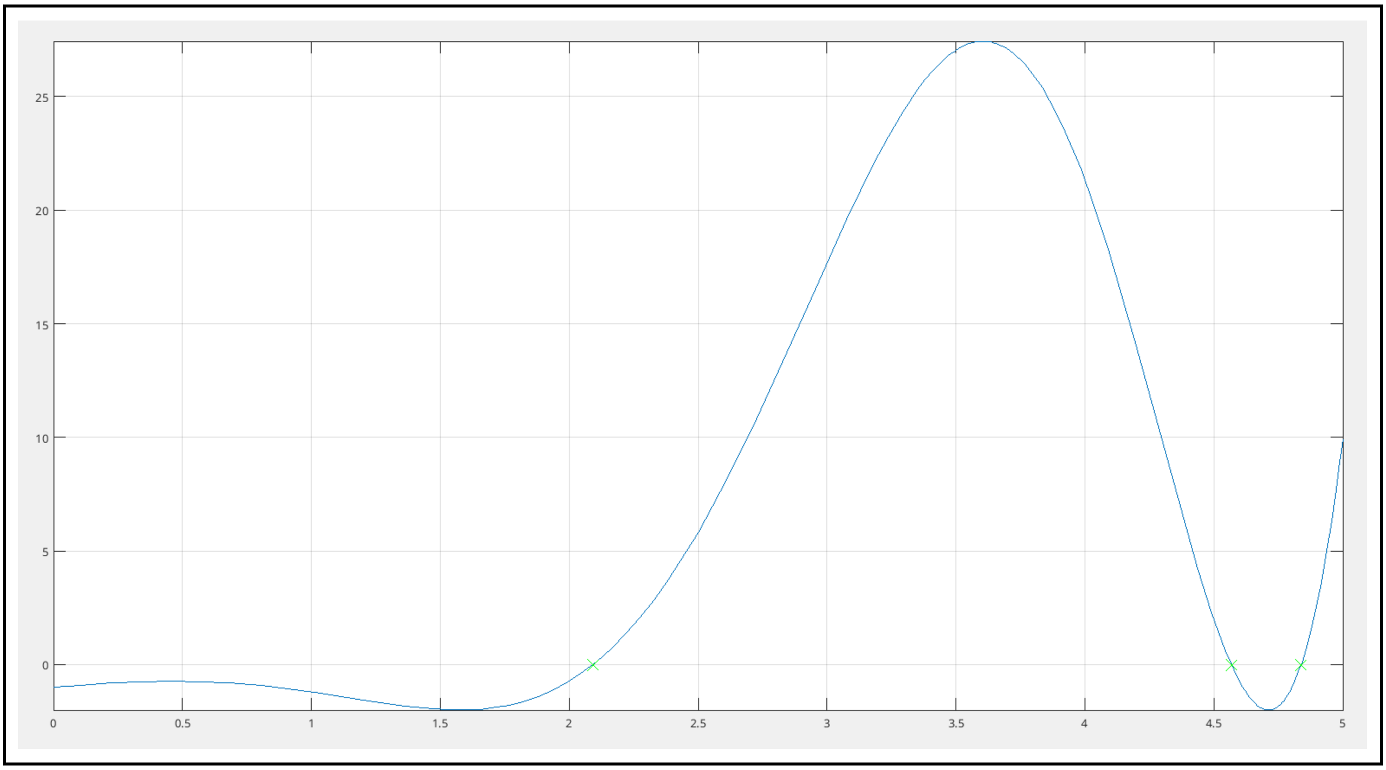
$$f(x) = e^x \cos^2(x) - 2$$

on the interval [0,5]. Plot the function with a grid and mark the roots with large green diamonds. For this question make a function file to hold the function. You will have to divide the interval into pieces using an appropriate step size, h. First try h = 2, then plot the function and roots to see if it found them all. If not, adjust your value of h and try again.

With h = 2:



With h = 1:



5. Write a script that solves

$$h(x) = x^2 - x - e^{-x}$$

using Fixed Point Iteration. Choose your own stopping criteria and keep track of the error at each iteration. Keep in mind that you can choose any of the following 4 formulas for your fixed point iteration:

$$g(x) = -e^{-x} + x^2$$
$$g(x) = \sqrt{e^{-x} + x}$$
$$g(x) = -\ln(x^2 - x)$$
$$g(x) = 1 + \frac{e^{-x}}{x}$$

Test all of them to see which work and which don't.

I used x = 1 as my initial guess with a tolerance of 0.0001.
Formula 1:

Command Window

```
>> Q5_Sol
Derivative larger than 1, stopping.
Number of iterations:
     1
```

Formula 2:

```
Command Window
>> Q5_Sol
Number of iterations:
     7

>> x

x =

    1.0000    1.1696    1.2166    1.2300    1.2338    1.2349    1.2352    1.2353
```

Formula 3:

```
Command Window
>> Q5_Sol
Derivative larger than 1, stopping.
Number of iterations:
     1
```

Formula 4:

```
Command Window
>> Q5_Sol
Number of iterations:
    11

>> x

x =

  Columns 1 through 10

    1.0000    1.3679    1.1862    1.2575    1.2261    1.2393    1.2337    1.2361    1.2350    1.2355

  Columns 11 through 12

    1.2353    1.2354
```

6. Write a function that takes an initial guess as input then locates a root of

$$r(x) = \sin(x)\cos(2x)$$

using the Newton-Raphson Method. Remember you will have to differentiate the function to get the formula. Choose your own convergence criterion.

When x0 = 1:

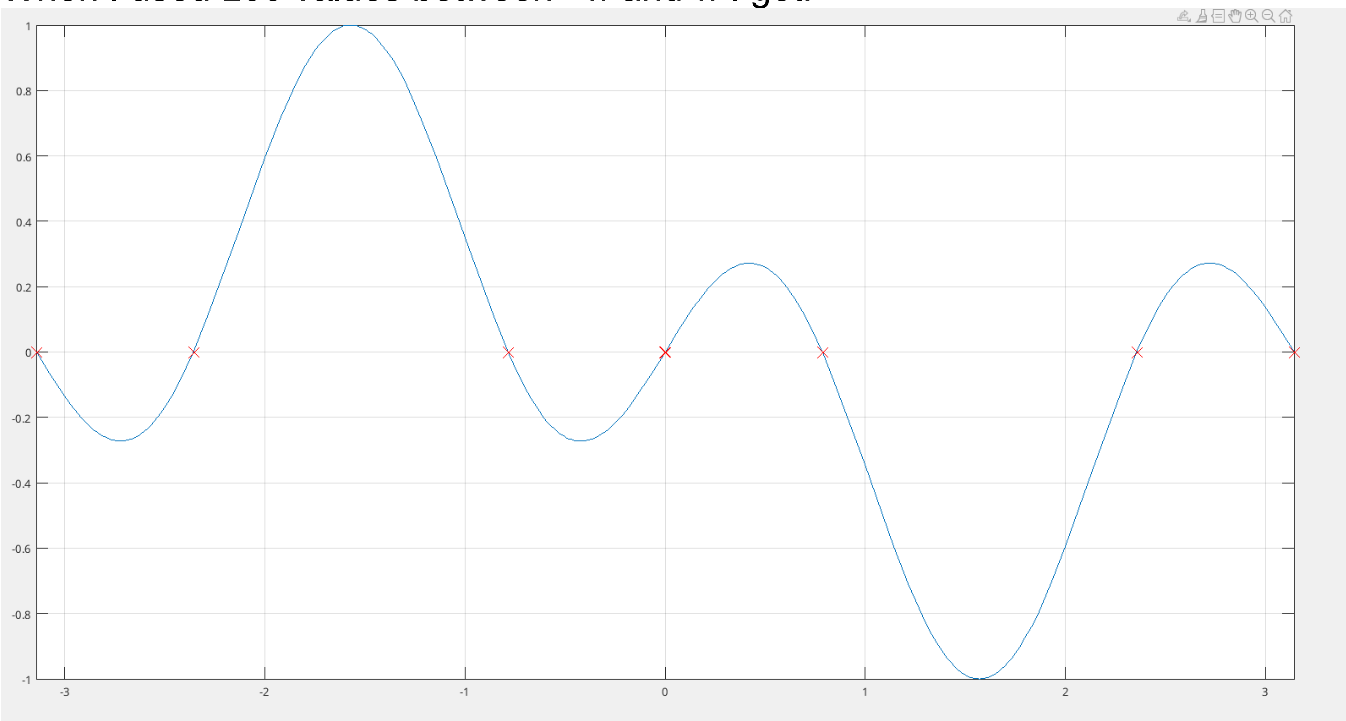
```
Command Window
>> [root, n] = Q6_Sol(1)

root =

    0.7854


n =

     4
```

When x0 = 5:

```
Command Window
>> [root, n] = Q6_Sol(5)

root =

    5.4978


n =

     5
```

7. Use your function from question 7 to find all roots of r(x) on the interval [–π,π].

When I used 100 values between –π and π I got:



8. Write a script that locates all roots of

$$w(x) = x^3 - 6x^2 + 10x - 4$$

using the Bisection Method. Locate brackets by using the incremental search method you wrote before.

Note below I checked whether an exact root was found by the incremental search method first instead of wasting computation time on the Bisection method where unneeded.

```
Command Window
>> Q8_Sol
A root was found by the incremental search method.

ans =

   1.0e-03 *

         0    0.1355   -0.1355

The roots are:

wroot =

    2.0000    0.5858    3.4142
```

9. The volume of liquid V in a hollow horizontal cylinder of radius r and length L is related to the depth of the liquid h by,

$$V = \left[r^2 \cos^{-1}\left(\frac{r-h}{r}\right) - (r-h)\sqrt{2rh - h^2}\right]L$$

Determine *h* given *r* = 2m, *L* = 5m, and *V* = 8m.

```
Command Window
>> Q9_Sol
h =
    0.7400
```

10. Determine the location of the horizontal tangents (turning points) of

$$m(x) = \cos(x^2/2) + \cos(2x)$$

on the interval [–π,π]. Plot the figure and mark the turning points with thick, large red circles with blue faces  .