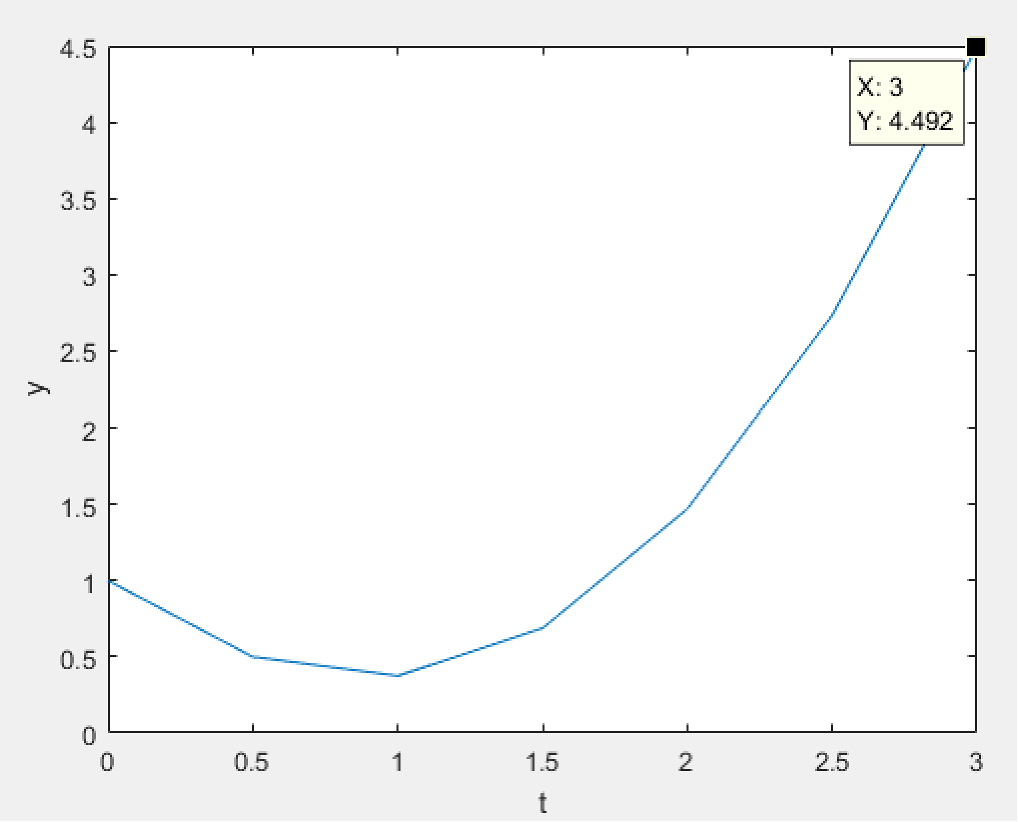
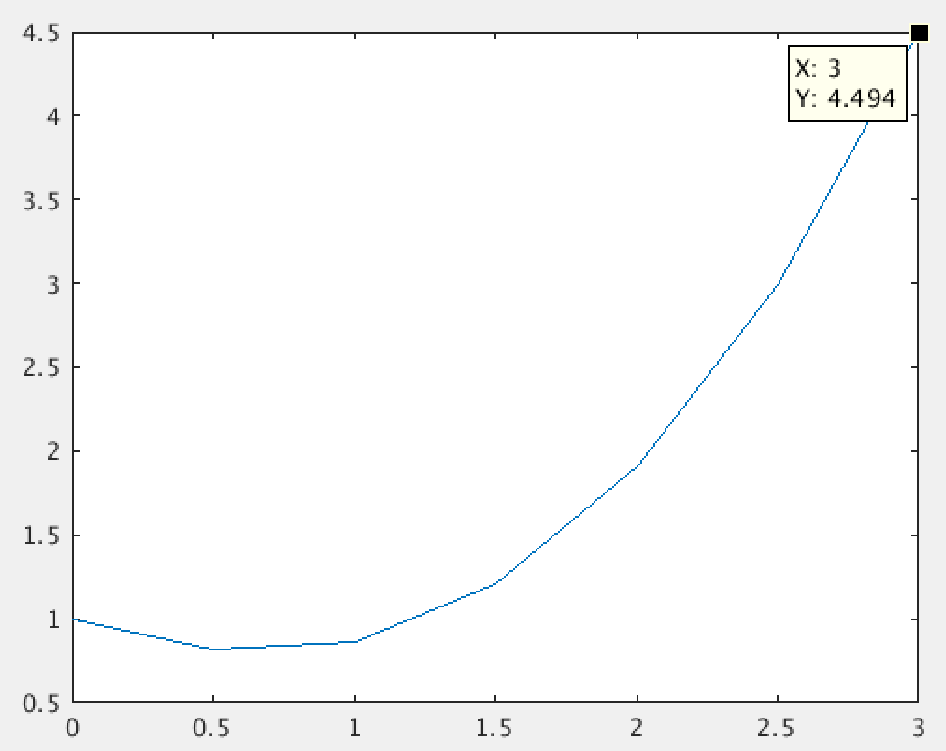
Practical Answers 13 – Ordinary Differential Equations

1. Write a script file that solves the following ODE using Euler's method from t = 0 to 3 with a step size of h = 0.5 where y(0) = 1. Plot the results.

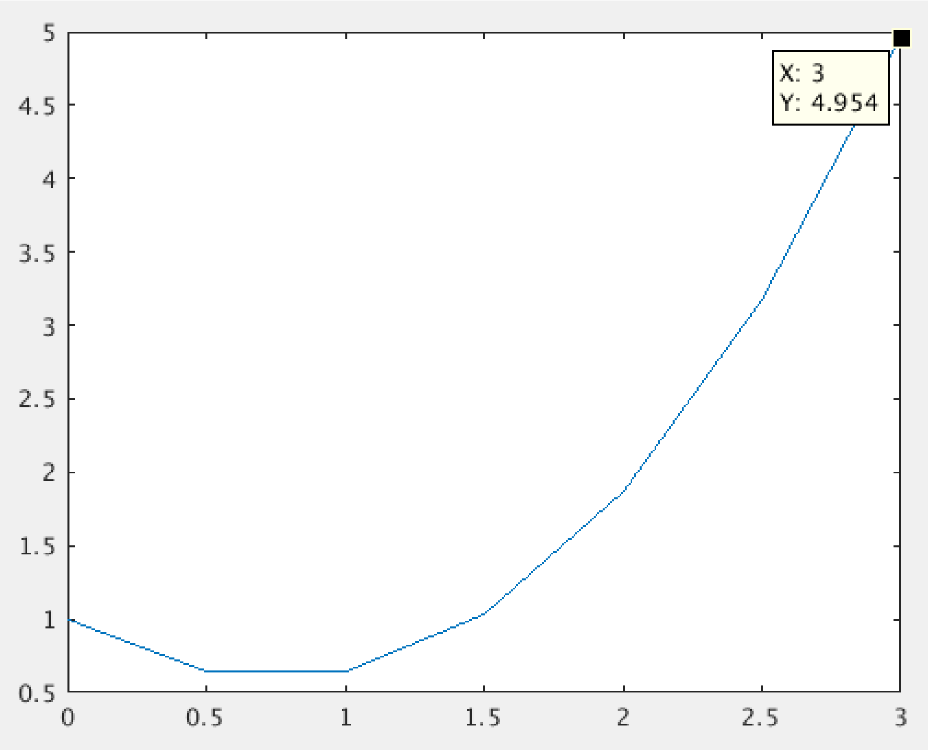
$$\frac{dy}{dt} = -y + t^2$$



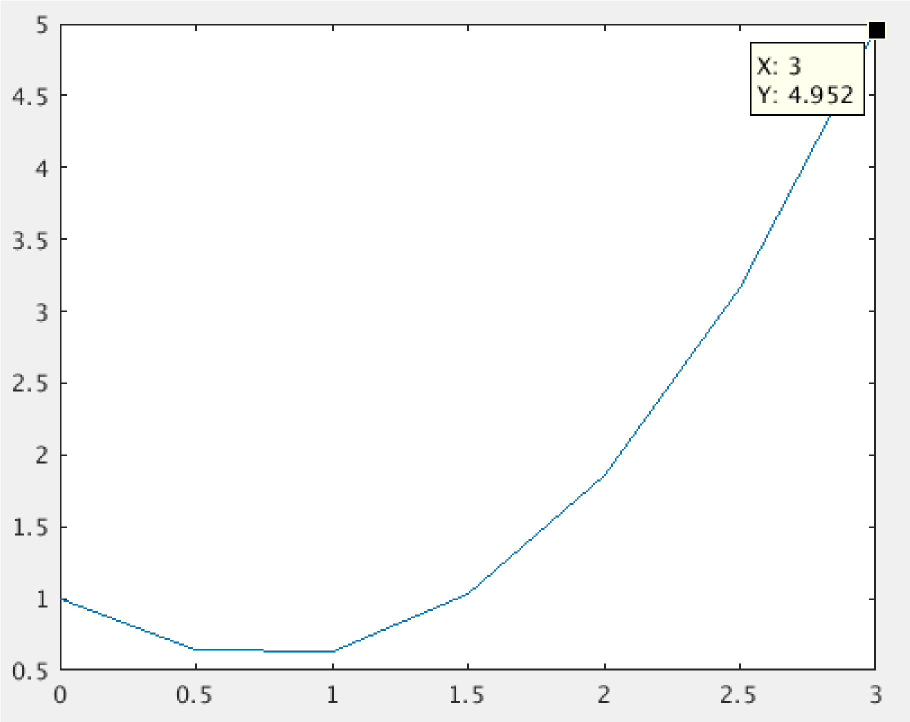
2. Repeat Question 1 using Heun's method without iteration.



3. Repeat Question 1 using Heun's method with iteration until the approximate error is less than 0.1%.



4. Repeat Question 1 using 4th order RK.

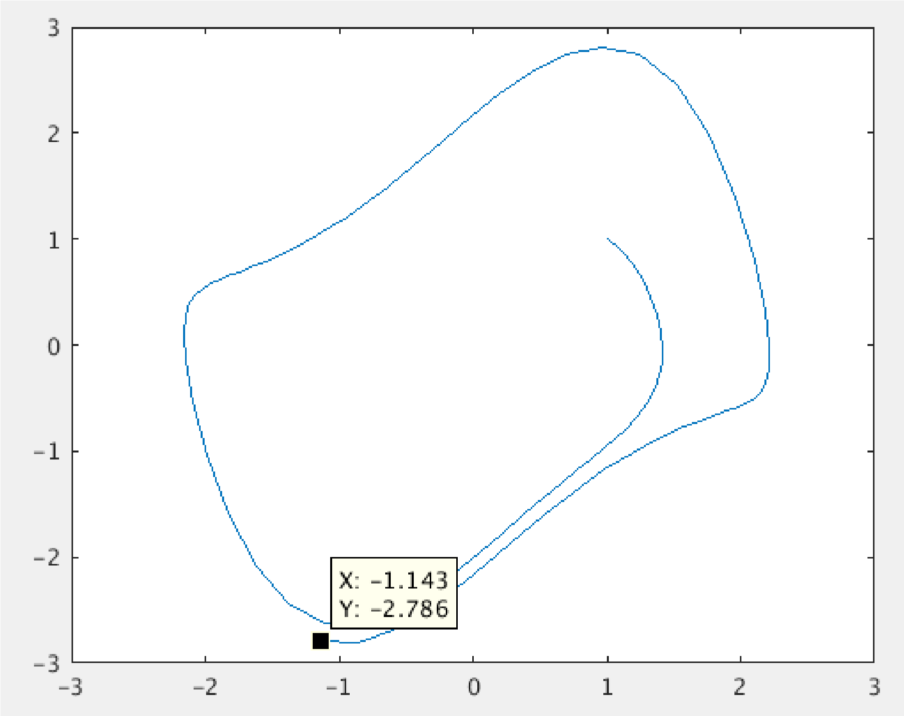


5. Use the scripts you created in Questions 2-4 to write function files that solve a 1st order ODE using each method. The inputs to the functions should be the 1st derivative function, f(t,y), the end time, an initial point, and the step size for all methods except the Heun method with iteration. For the Heun method with iteration replace the step size input with the required tolerance. Test your functions with the ODE from question 1.

   N/A

6. Convert the following 2nd order ODE into a system of 1st order ODEs and then write a script file that solves it from t = 0 to 10 using Euler's method using a step size of h = 0.1. The initial conditions are y(0) = y'(0) = 1. Plot the results.

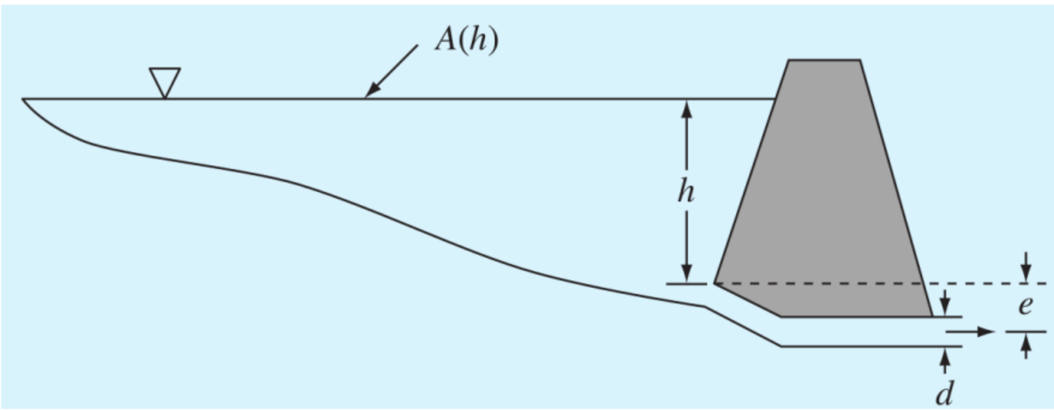
$$\frac{d^2y}{dt^2} - (1 - y^2)\frac{dy}{dt} + y = 0$$



7. Write a function file that solves a system of 1st order ODEs using Euler's method. The inputs should be the same as in question 5 except that the first input is a vector of functions representing each variable's derivative, and the initial conditions are also a vector with the initial value of each variable.

   N/A

8. A pond drains through a pipe as shown in the figure on the next page. Under a number of simplifying assumptions, the following differential equation describes how depth changes with time:



$$\frac{dh}{dt} = -\frac{\pi d^2}{4A(h)}\sqrt{2g(h+e)}$$

where h = depth (m), t = time (s), d = pipe diameter (m), $A(h)$ = pond surface area as a function of depth (m^2), g = gravitational constant (= 9.81 m/s^2), and e = depth of pipe outlet below the pond bottom (m). Based on the following area-depth table, solve this differential equation to determine how long it takes for the pond to empty, given that $h(0)$ = 6 m, d = 0.25 m, e = 1 m.

***h*, m**	6	5	4	3	2	1	0
***A*(*h*), 10^4 m^2**	1.17	0.97	0.67	0.45	0.32	0.18	0

Time to drain is approximately t = 67455.4 seconds or 18.7376 hours.