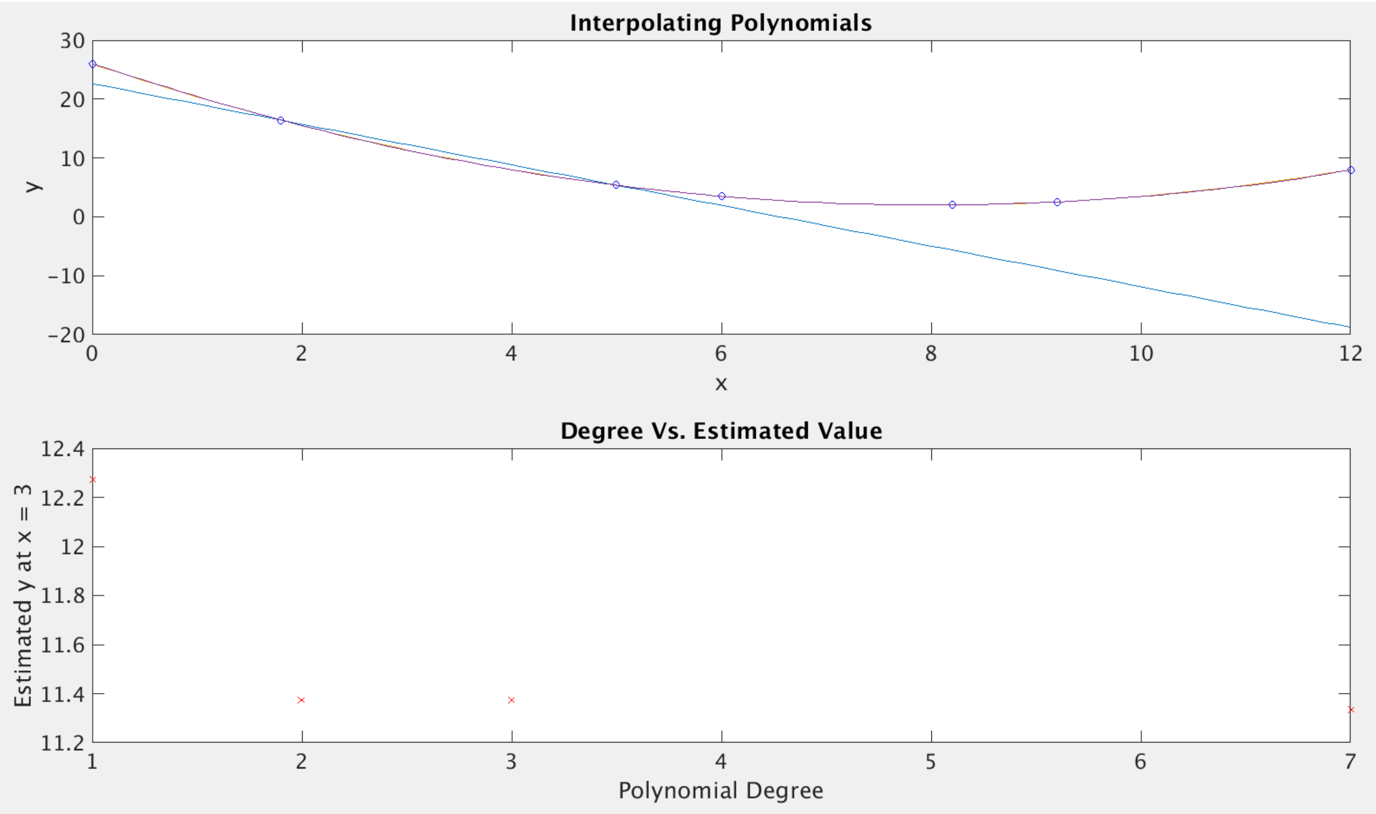
Practical Answers 9 – Interpolation

1. Find an interpolating polynomial using matrices to determine y at x = 3. Try linear, quadratic, and cubic polynomials between appropriate data points first, then try a polynomial that interpolates the entire data set. Make one plot with the data along with the 4 interpolating polynomials, then another plot with degree of interpolating polynomial on the horizontal axis and the estimated value of y at x = 3 on the vertical axis.

x	0	1.8	5	6	8.2	9.2	12
y	26	16.415	5.375	3.5	2.015	2.54	8



2. Use Newton's interpolating polynomial to determine y at x = 3.5. Make a plot of the data along with the polynomial.

x	0	1	2.5	3	4.5	5	6
y	2	5.4375	7.3516	7.5625	8.4453	9.1875	12

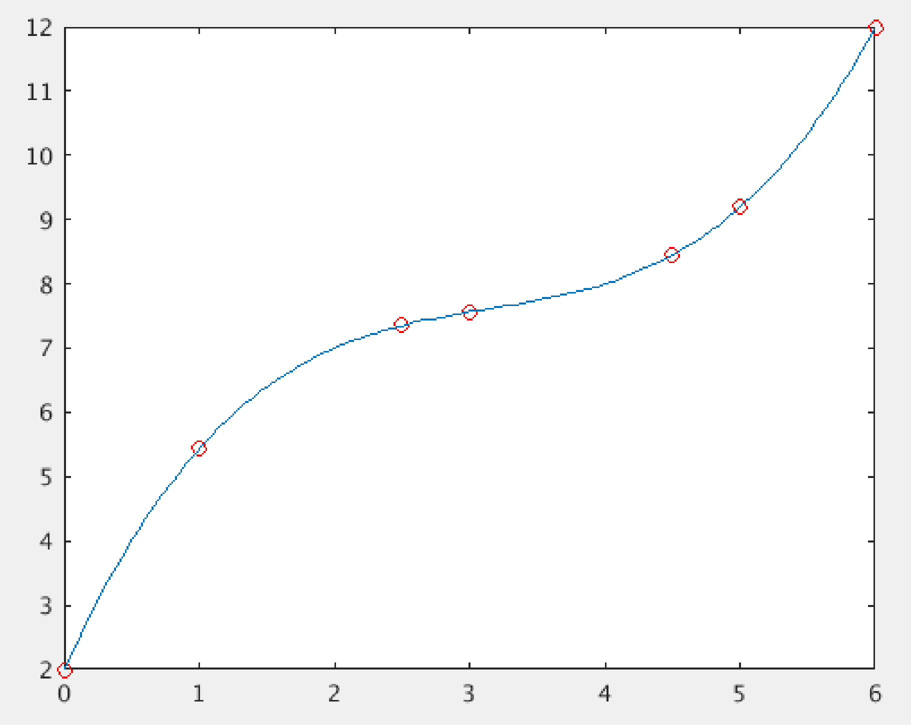
The fd table is:

fd =

2.0000	3.4375	-0.8646	0.1458	0.0000	-0.0000	0.0000
5.4375	1.2761	-0.4271	0.1459	-0.0000	0.0000	0
7.3516	0.4218	0.0834	0.1458	-0.0000	0	0

7.5625	0.5885	0.4479	0.1458	0	0	0
8.4453	1.4844	0.8854	0	0	0	0
9.1875	2.8125	0	0	0	0	0
12.0000	0	0	0	0	0	0

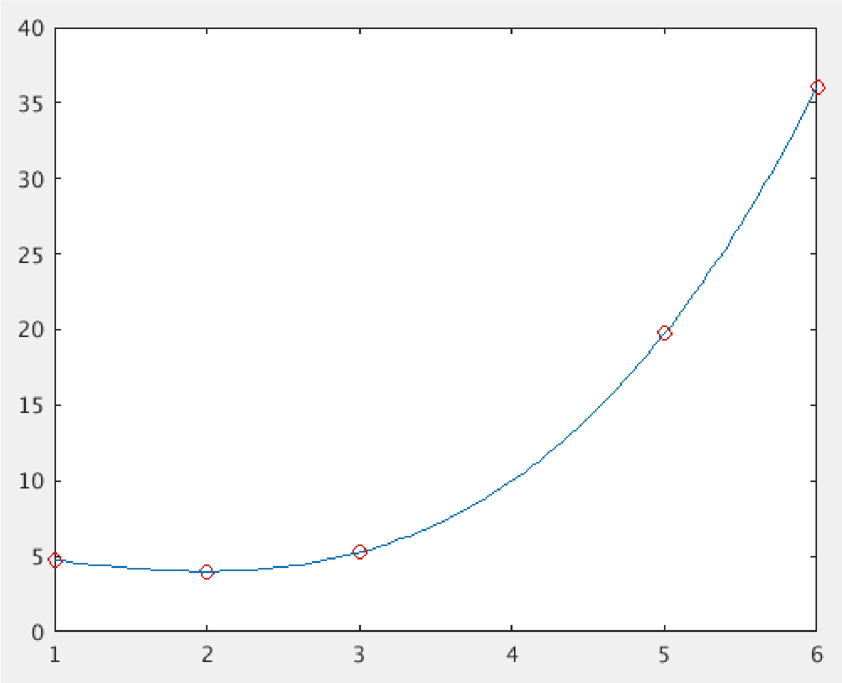
The estimated value of y at x = 3.5 is 7.7422.



3. Use a Lagrange interpolating polynomial to determine y at x = 4. Make a plot of the data along with the polynomial.

x	1	2	3	5	6
$f(x)$	4.75	4	5.25	19.75	36

The estimated function value at x = 4 is 10.



4. Use any built-in Matlab functions to do inverse interpolation to estimate the value of x at which f(x) = 1.7.

x	1	2	3	4	5	6	7
$f(x)$	3.6	1.8	1.2	0.9	0.72	1.5	0.51429

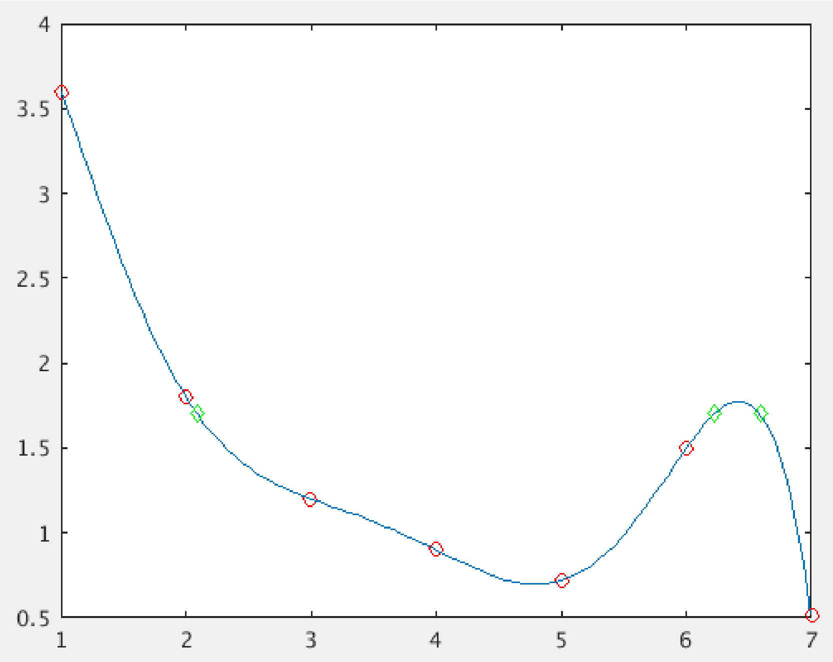
Using all data points to do an n-1 degree interpolating polynomial results in 3 values of x where f(x) = 1.7:

x =

6.5928

6.2216

2.0909



5. Use the portion of the given steam table for superheated water at 200 MPa to find:

   (a) the corresponding -entropy *s* for a specific volume *υ* of 0.118 with linear interpolation,

   (b) the same corresponding entropy using quadratic interpolation, and

   (c) the volume corresponding to an entropy of 6.45 using inverse interpolation of both the linear and quadratic cases.

υ, m^3/kg	0.10377	0.11144	0.12547
s, kJ/(kg K)	6.4147	6.5453	6.7664

s_est_lin =

    6.6487

s_est_quad =

    6.6515

v_est_lin =

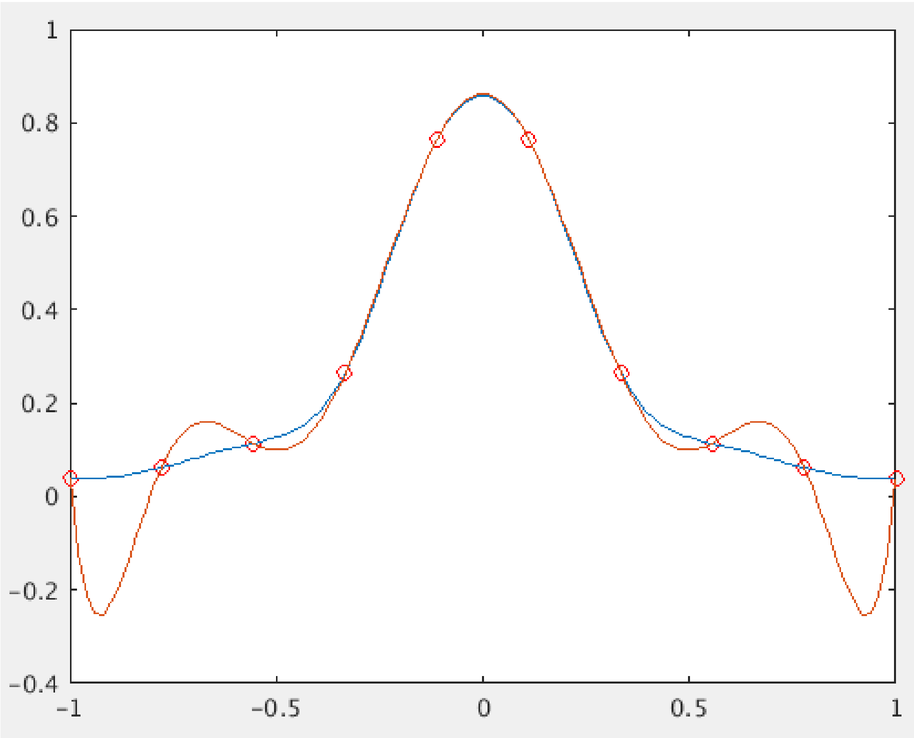
    0.1054

v_est_quad =

    0.1058

6. Sample the function, f = 1 / (1 + 25x^2), 10 times equally spaced from -1 to 1 and store the results as data y, with x being the sample points. Use Matlab's spline function to fit a curve to this data (note that Newton and Lagrange polynomials do not work for this).

   The blue curve is the correct one with splines. I also plotted the incorrect polyfit function in orange for comparison.



7. Interpolate the following data using cubic splines with knot-a-not end condition and also with clamped end condition with first derivatives set equal to 0. Plot both curves along with the data on the same graph. Use both interpolating methods to estimate the y-value when x = 1.5.

x	0	2	4	7	10	12
y	20	20	12	7	6	6

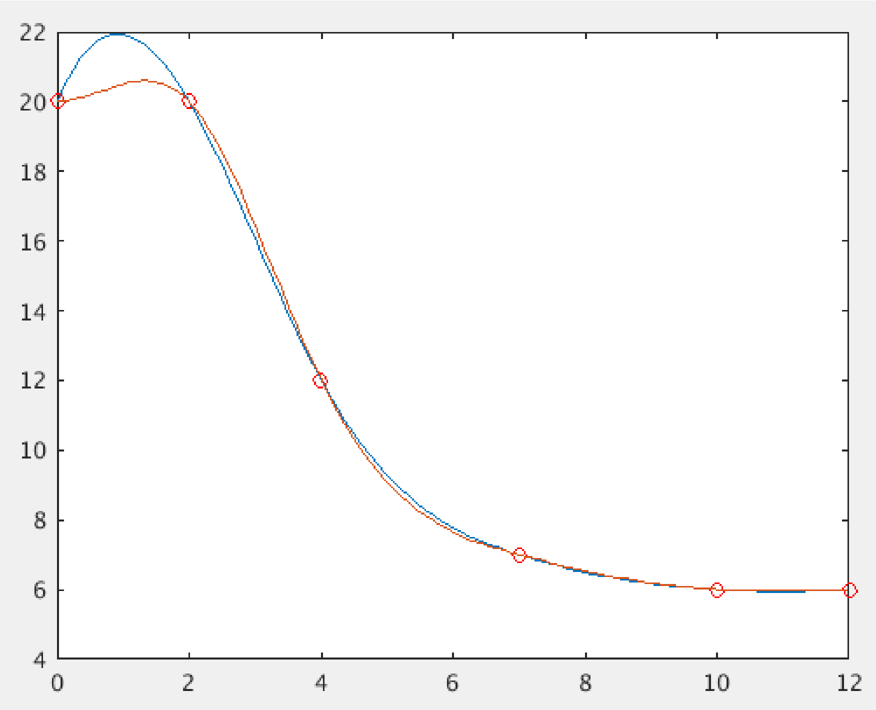
y_est_not =

  21.3344

y_est_clamped =

  20.5701

In the graph below the blue curve is not-a-knot and the orange curve is with the derivatives set to 0.



8. The following grid represents 6 pixels of an image and their corresponding colour value (one of RGB for example).

10	15	30
30	35	50

We zoom in and create more pixels which we need to interpolate in order to smooth out the image.

10			15			30
30			35			50

Use linear interpolation to find the missing values (store the values as a matrix containing the colour value at each of the coordinate locations).

Zint =

10.0000	11.6667	13.3333	15.0000	20.0000	25.0000	30.0000
16.6667	18.3333	20.0000	21.6667	26.6667	31.6667	36.6667
23.3333	25.0000	26.6667	28.3333	33.3333	38.3333	43.3333

30.0000   31.6667   33.3333   35.0000   40.0000   45.0000   50.0000

Challenge Problem

9. Repeat question 8 but using splines with clamped ends with first derivatives equal to 3 and -3 alternately (you will have to do each direction separately).