# Practical Answers 11 – Further Numerical Integration

1. Use Matlab's integral2 function to evaluate the following integrals.

a. $$\int_{-2}^{2}\int_{0}^{4}(x^2 - 3y^2 + xy^3)\,dx\,dy$$

21.3333

b. $$\int_{0}^{1}\int_{3x}^{5-2x} 10 - 4x - 2y\,dy\,dx$$

8.3333

2. Use Matlab's integral3 function to evaluate the following integrals.

a. $$\int_{-4}^{4}\int_{0}^{6}\int_{-1}^{3}(x^3 - 2yz)\,dx\,dy\,dz$$

960

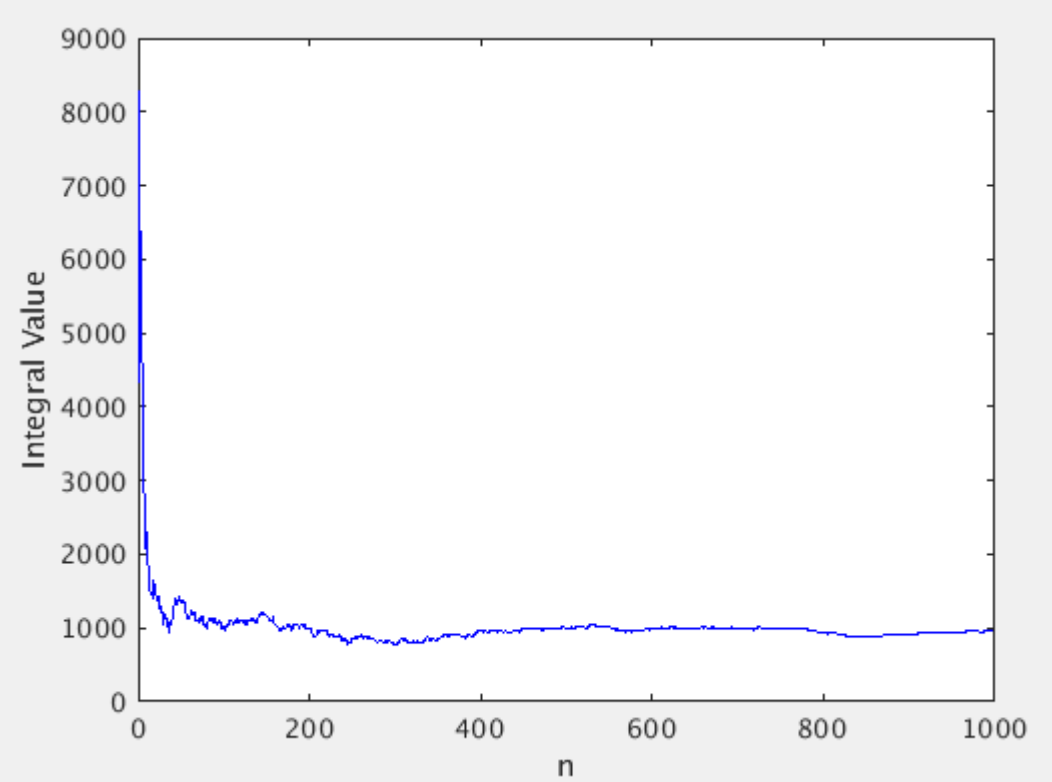
b. $$\int_{0}^{\frac{5}{2}}\int_{0}^{10-4x}\int_{0}^{5-2x-\frac{y}{2}} 6z^2\,dz\,dy\,dx$$

312.5

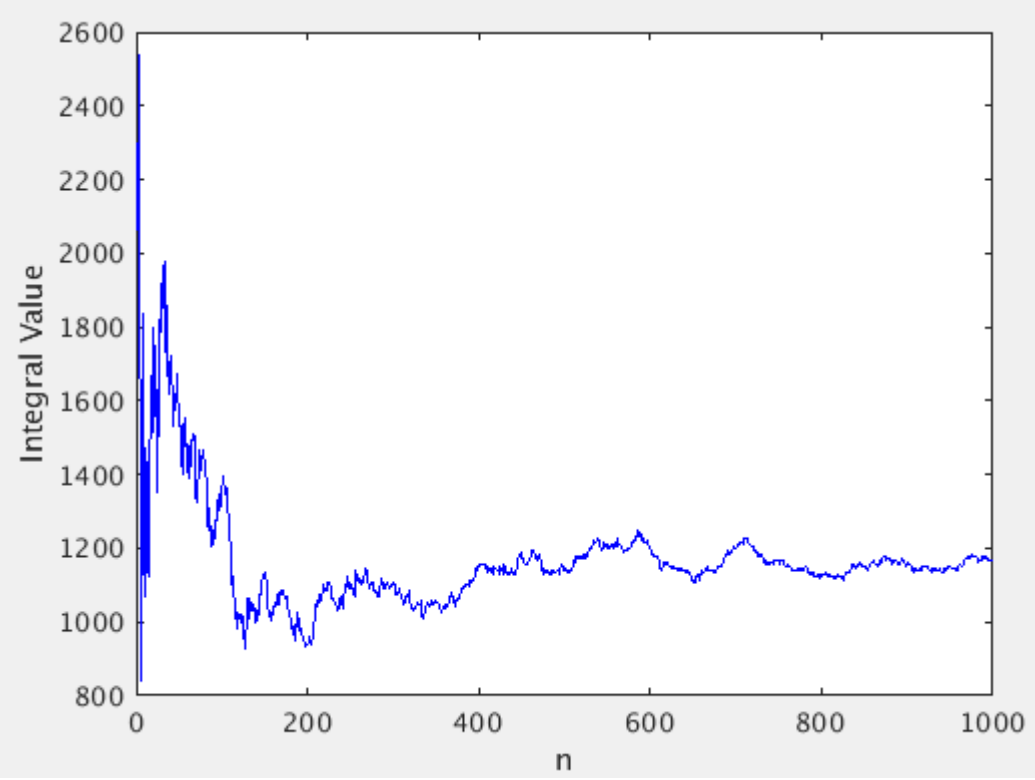
3. Apply Monte-Carlo integration to the triple integral in 2a using sample points from 1 to 1000. Make a plot with the number of sample points on the horizontal axis and the integral estimate on the vertical axis.

   This answer depends on your random numbers which are generated at runtime. Using Matlab's basic rand() function can generate different results such as the ones shown below:

   Converged to 967.2087




Or,
Converged to 1171.1263




etc.

4. Write a function that performs Monte-Carlo integration on a function of 2 variables. The inputs should be the function to integrate, x and y limits, and the desired accuracy.

   N/A

5. Write a function file that performs **Romberg integration** to the desired accuracy. You may use the Matlab built-in function trapz or your own trapezium rule function to calculate the approximation at each step size. The inputs should be the function you wish to integrate, the start and end points, and the desired accuracy (use estimated accuracy).

   N/A

6. Test your function from Question 5 by integrating the following to within 0.5% accuracy then compare with Matlab's integral function.

$$F = \int_0^H 200 \left(\frac{z}{5+z}\right) e^{-2z/H} dz$$

   where $H = 30$.

   1476.8

7. Write a function file that approximates integrals using **adaptive quadrature** with Simpson's 1/3 rule. You should use your function file for Simpson's 1/3 rule that you made from the previous problem sheet. The inputs should be the function to be integrated, starting and end points, and the desired accuracy.

   N/A

8. Test your function from Question 7 with the following integral and compare with Matlab's integral function.

$$\int_0^2 \frac{e^x \sin x}{1+x^2} dx$$

   Matlab's integral function gives 1.9401.
   Using the solution to question 7 with a 1% error criterion gives 1.9438.
   Using the solution to question 7 with a 0.1% error criterion gives 1.9402.
   etc.