# 5.3 Compound Interest

## Compound Interest

A common application for an exponential function is calculating compound interest. We are interested to know the future value, $A$, of an investment of $P$ dollars made today (called the present value) subject to compounding.

Before proceeding we need to distinguish between compounding which is performed $n$ times per year, such as monthly compounding where the compounding is performed 12 times a year, versus continuous compounding. As you review both formulas shown below, notice how each of these formulas represent exponential models.

### Compound Interest Formulas

Let $P$ be the principal (initial investment), $r$ be the annual interest rate expressed as a decimal, and $A(t)$ be the amount in the account at the end of $t$ years.

## Compounding $n$ times per year

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

where n is determined based on the frequency of the compounding as follows:

| Type | N |
|---|---|
| **Annual Compounding** | n=1 |
| **Semi-Annual Compounding** | n=2 |
| **Quarterly Compounding** | n=4 |
| **Monthly Compounding** | n=12 |
| **Daily Compounding** | n=365 |

**Types of Compounding**

## Compounded continuously

$$A(t) = P\,e^{rt}$$

If you're using this formula to find what an account will be worth in the future, $t > 0$ and $A(t)$ is called the **future value**.

If you're using the formula to find what you need to deposit today to have a certain value $P$ sometime in the future, $t < 0$ and $A(t)$ is called the **present value**.

## Example 1

Susan would like to start a college savings fund for her newborn son. She invests $20,000 in a savings account paying 3.5% yearly interest compounded on a monthly basis. The plan is to leave the money in the account for 20 years until her son needs the fund for college. How much will the savings account be worth in 20 years?

Summarizing the given information:

$P = \$20000$

$r = 3.5\% = 0.035$

n = 12 (based on monthly compounding)
t = 20

Substituting into the compound interest formula:

$$A = P(1 + \frac{r}{n})^{nt} = 20000(1 + \frac{0.035}{12})^{12 \cdot 20} = 40234.04$$

Thus the college saving account has grown from \$20,000 to \$40,234.04 over the course of 20 years based on monthly compounding. Notice the value of the account has doubled!

## Example 2

Let's repeat Example 1, but instead of monthly compounding let's assume that Susan invests in a savings account which pays 3.5% yearly interest based on continuous compounding.

How much will the savings account be worth in 20 years based on continuous compounding?

Summarizing the given information:
P = \$20000
r = 3.5% = 0.035
t = 20

Substituting into the continuous compound interest formula:

$$A = P e^{rt} = 20000e^{0.035 \cdot 20} = 40275.05$$

Thus the college saving account has grown from \$20,000 to \$40,275.05 over the course of 20 years based on continuous compounding. Notice the value of the account is slightly larger based on continuous compounding as compared to monthly compounding.