michelle beatles chords

Practical Answers 12 – Numerical Differentiation

1. Write a function file to obtain first derivative estimates for unequally spaced data. You may use any Matlab built-in functions. Test it with the following data:

x	0.6	1.5	1.6	2.5	3.5
$f(x)$	0.9036	0.3734	0.3261	0.08422	0.01596

A 3rd order polynomial fits the data well:

p =

-0.0178 0.2426 -1.0394 1.4438

The derivative is:

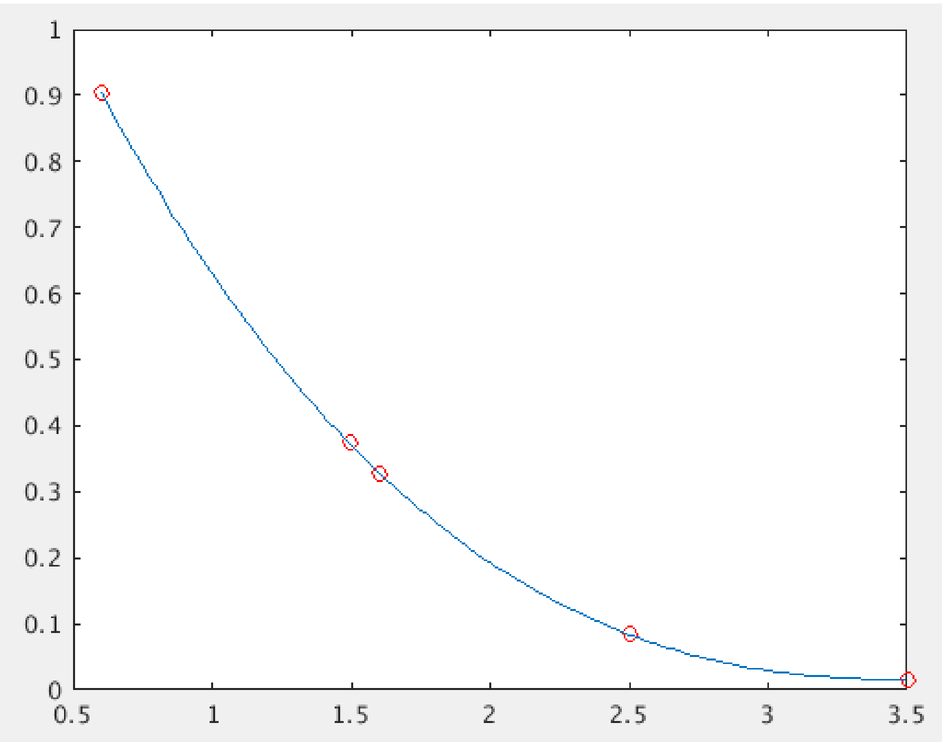
dp =

-0.0533 0.4851 -1.0394

The values of the derivatives at each x are:

df =

-0.7675 -0.4316 -0.3996 -0.1595 0.0059



2. Use the following data to find the velocity and acceleration at t = 10 seconds using second order (a) centred finite difference, (b) forward finite difference, and (c) backward finite difference methods.

Time, t, s	0	2	4	6	8	10	12	14	16
Position, x, m	0	0.7	1.8	3.4	5.1	6.3	7.3	8.0	8.4

```
v_fd =

   0.5000

v_bd =

   0.6000

v_cd =

   0.5500

a_fd =

  -0.0750

a_bd =

  -0.1250

a_cd =

  -0.0500
```

3. The following data were collected for the distance travelled versus time for a rocket. Use finite difference approximations to estimate the rocket's velocity at each time using both standard and improved formulas.

t, s	0	25	50	75	100	125
y, km	0	32	58	78	92	100

```
v_std =

   1.2800    1.1600    0.9200    0.6800    0.4400    0.3200

v_imp =

   1.4000    1.1600    0.9200    0.6800    0.4400    0.2000
```

4. Write a function file that calculates the second order centred finite difference approximation of an input function at a given point (improved first derivative). Use a

loop to decrease the step size until the relative iterative error becomes small enough (Use your discretion).

N/A

5. The velocity (m/s) of an object at time t seconds is given by the following function. Using one level of Richardson's extrapolation, find the acceleration of the particle at time t = 5 s initially using h = 0.5 and 0.25. Use the exact solution to compute the true percent relative error of each estimate.

$$v = \frac{2t}{\sqrt{1+t^2}}$$

Using a forward difference approximations:

D_worse =

0.0132

D_better =

0.0141

D_improved =

0.0144

err_worse =

12.7863 %

err_better =

6.7794 %

err_improved =

4.7771 %

6. Evaluate ∂f/∂x, ∂f/∂y and $\partial^2 f/\partial x^2$ for the following function at x = y = 1 numerically with Δx = Δy = 0.0001:

$$f(x, y) = 3xy + 3x - x^3 - 3y^2$$

fx =

3.0000

fy =

-3.0000

fxx =

   -6.0000

7. Write a function file to apply a **<u>Romberg algorithm</u>** to estimate the derivative of a given function using centred finite differences. The inputs should be the function, the point at which to evaluate the derivative, and the tolerance (use the same error formula as in the lecture notes for Romberg integration).

   N/A