Practical Answers 8 – Nonlinear Regression & Symbolic Toolbox

1. Fit a cubic polynomial to the following data by extending the system of equations on lecture slide 3, then calculate the standard error and correlation coefficient. Graph the data and your fitting equation.

x	3	4	5	7	8	9	11	12
y	1.6	3.6	4.4	3.4	2.2	2.8	3.8	4.6

The polynomial should be:
a =

0.0467

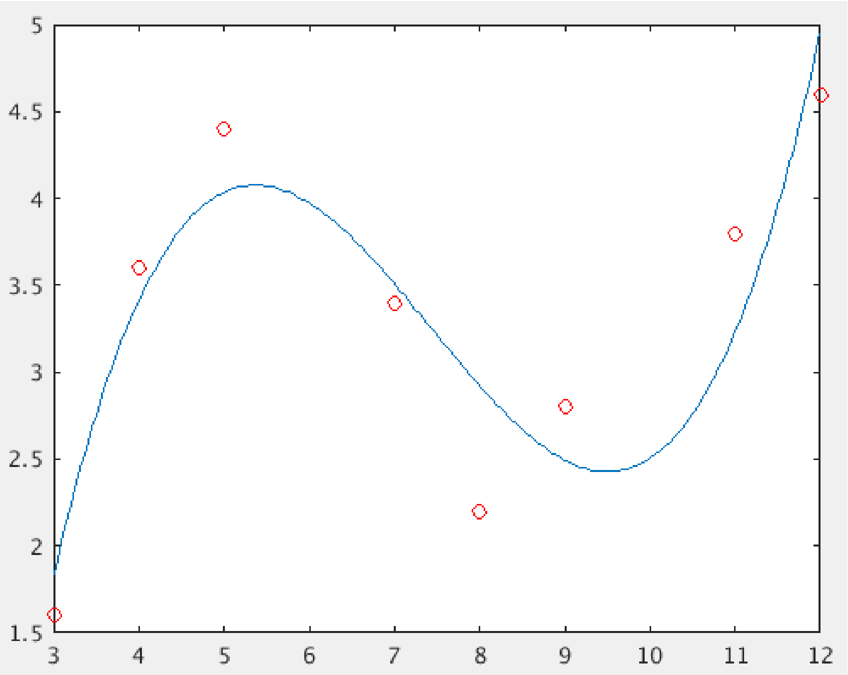
-1.0412

7.1438

-11.4887

r = 0.9105

Syx = 0.57



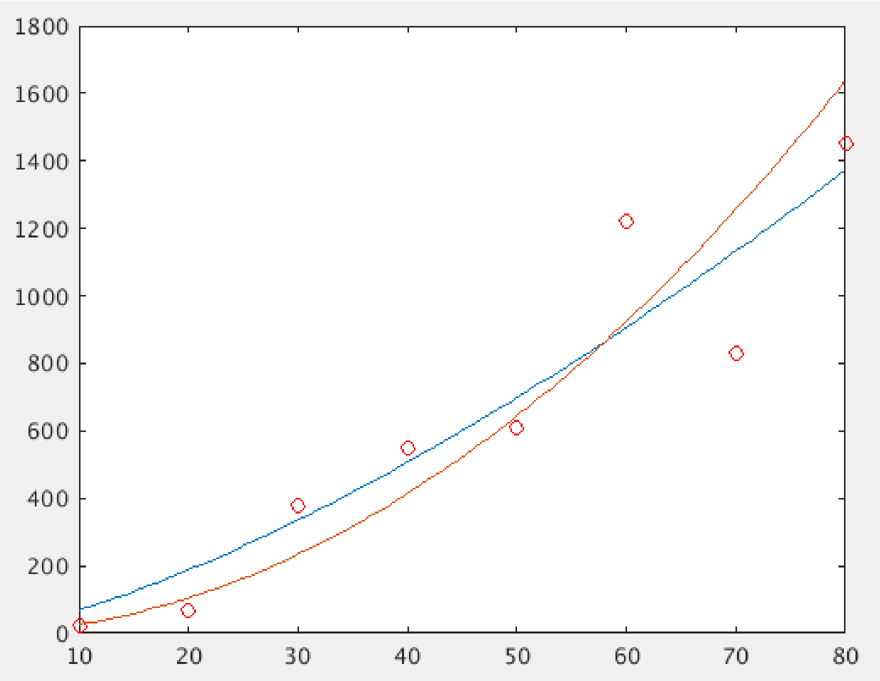
2. Do the same as in the Question 1 but using the general Z-matrix formulation given on slides 21-25. You should get the same result if you have done it correctly.

N/A

3. Calculate the standard error for the 2 power equation models from Example 6 in the lecture notes and decide if one equation is better than the other or not.

Syx_nonlinear = 192.6158

Syx_linearised = 240.0293

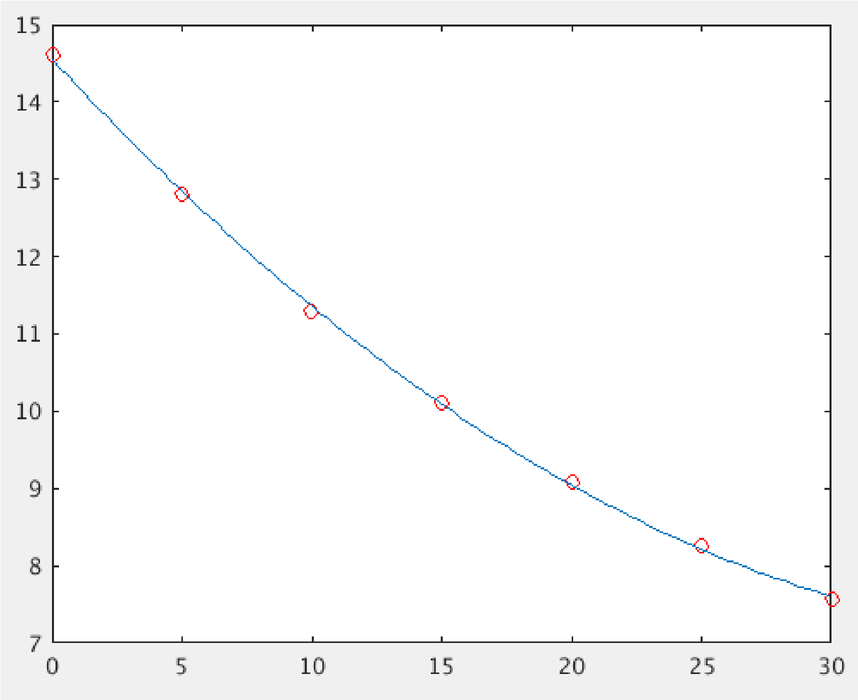


4. For the following data use polynomial regression to get an equation that predicts the dissolved oxygen concentration as a function of temperature for the case where the chloride concentration is 0. Graph the data and your fitting equation.

	Dissolved Oxygen (mg/L) for Temperature (°C) and Concentration of Chloride (g/L)		
T, °C	**c = 0 g/L**	**c = 10 g/L**	**c = 20 g/L**
0	14.6	12.9	11.4
5	12.8	11.3	10.3
10	11.3	10.1	8.96
15	10.1	9.03	8.08
20	9.09	8.17	7.35
25	8.26	7.46	6.73
30	7.56	6.85	6.20

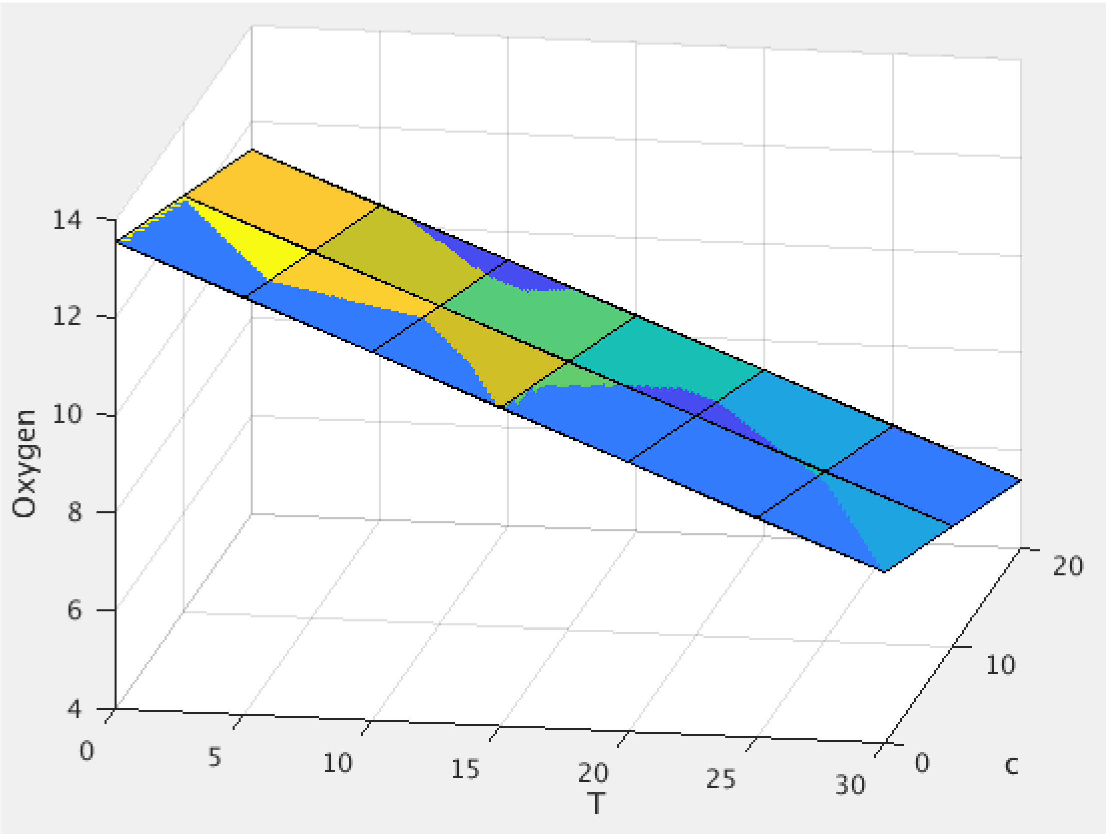
A 2nd degree polynomial can be used to fit the data:

p =
    0.0044   -0.3634   14.5519



5. For the data from Question 4 use multiple linear regression to predict the dissolved oxygen concentration as a function of temperature and chloride concentration (**Hint: Make a new table of values**). Plot the data and your equation then use your equation to predict the dissolved oxygen concentration when the chloride concentration is 15 g/L and temperature is 12^{o}C.

   Oxygen estimated to be 9.4543 mg/L



6. Use multiple linear regression to fit the following data. Also calculate the standard error and correlation coefficient.

x_1	0	1	1	2	2	3	3	4	4
x_2	0	1	2	1	2	1	2	1	2
y	15.1	17.9	12.7	25.6	20.5	35.1	29.7	45.4	40.2

The polynomial should be

a =

14.4609

9.0252

-5.7043

where the coefficients are given in the order of increasing powers of x.

Syx = 0.8888

r = 0.9978

7. Fluid flow was measured in concrete circular pipes of various sizes and angles (slopes). Use multiple linear regression (logarithm method) to fit the following equation to the data.

$$Q = \alpha_0 D^{\alpha_1} S^{\alpha_2}$$

Experiment	Diameter, m	Slope, m/m	Flow, m^3/s
1	0.3	0.001	0.04
2	0.6	0.001	0.24
3	0.9	0.001	0.69
4	0.3	0.01	0.13
5	0.6	0.01	0.82
6	0.9	0.01	2.38
7	0.3	0.05	0.31
8	0.6	0.05	1.95
9	0.9	0.05	5.66

The equation should be:

Qfit = 36.3813*D^2.6279*S^0.5320

8. Three disease-carrying organisms decay exponentially in seawater according to the following model equation. Use general least squares to estimate the

coefficients and estimate the initial concentration of each organism from the following measurements.

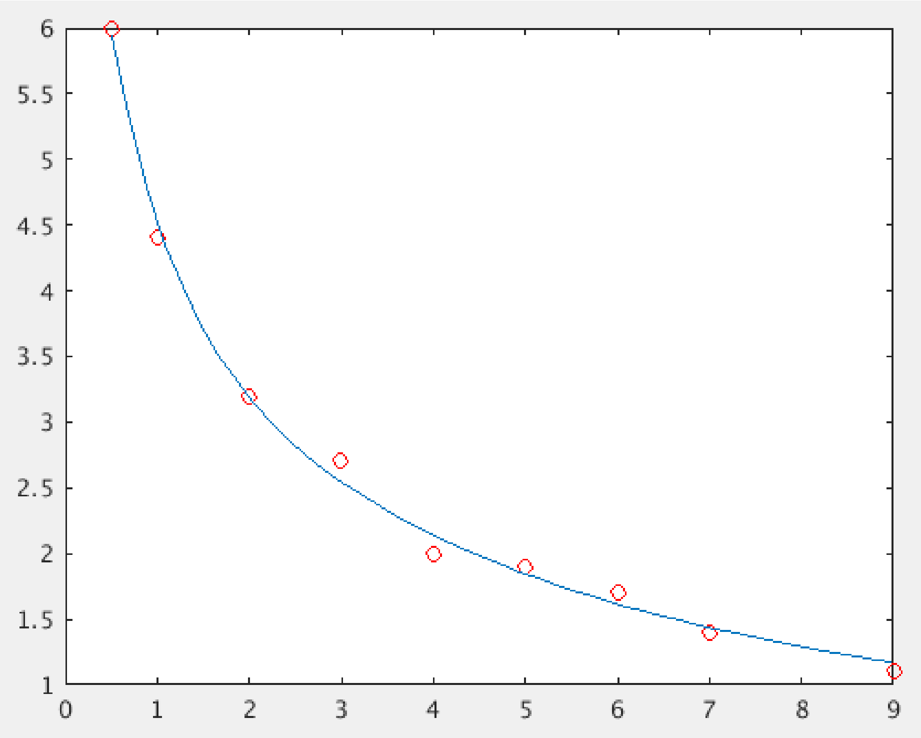
$$p(t) = Ae^{-1.5t} + Be^{-0.3t} + Ce^{-0.05t}$$

t	0.5	1	2	3	4	5	6	7	9
$p(t)$	6	4.4	3.2	2.7	2	1.9	1.7	1.4	1.1

A = 4.1375

B = 2.8959

C = 1.5349

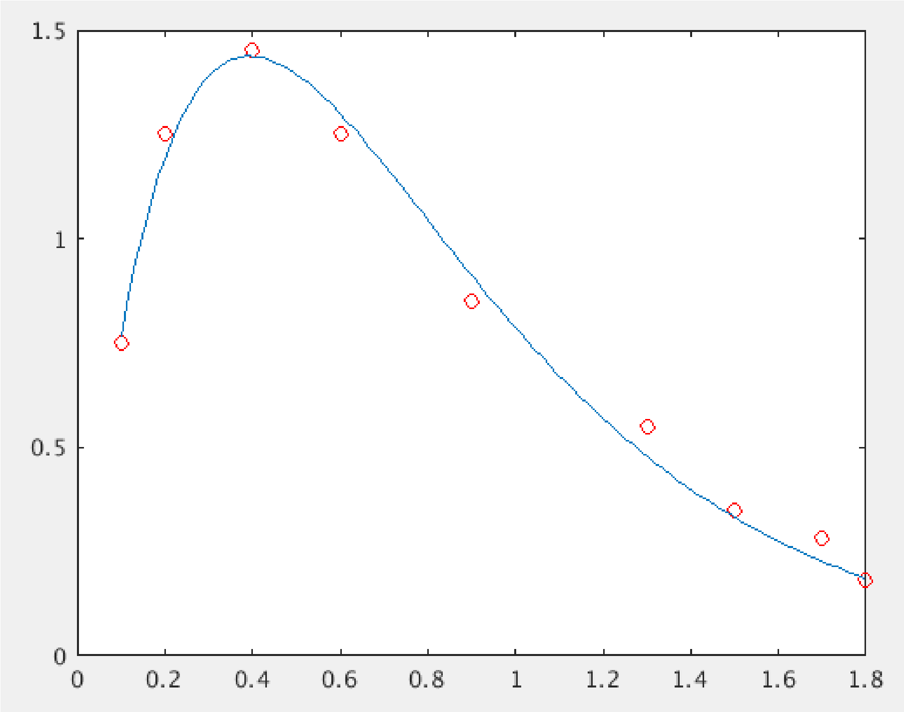


9. Use nonlinear regression to estimate α4 and β4 based on the following data. Develop a plot of your fit along with the data.

$$y = \alpha_4 x e^{\beta_4 x}$$

x	0.1	0.2	0.4	0.6	0.9	1.3	1.5	1.7	1.8
y	0.75	1.25	1.45	1.25	0.85	0.55	0.35	0.28	0.18

alpha4 = 9.8974
beta4 = -2.5319



10. Use Matlab's symbolic toolbox to factorise the following polynomial:

$$x^3 + 3x^2 - x - 3$$

(x+3)(x-1)(x+1)

11. Use Matlab's symbolic toolbox to find the 3rd order Taylor series expansion of the following function about the point x = –1.

$$y = e^{-3x^2} \sin(x^7 - 5x^2)$$

(17*cos(6)*exp(-3) - 6*exp(-3)*sin(6))*(x + 1) - exp(-3)*sin(6) - (x + 1)^2*(15*exp(-3)*sin(6) - 102*cos(6)*exp(-3) + exp(-3)*(26*cos(6) - (289*sin(6))/2))

12. Use Matlab's symbolic toolbox to solve the following system of equations for the variables x and y.

$$3x - y = 12$$
$$x + 3y = -7$$

x = 29/10

y = -33/10

13. Use Matlab's symbolic toolbox to differentiate the following function.

$$f(x) = \frac{x^2 \cos(2x^3)}{(x^2 - 1)^2 - \sin(x)}$$

(6*x^4*sin(2*x^3))/(sin(x) - (x^2 - 1)^2) - (2*x*cos(2*x^3))/(sin(x) - (x^2 - 1)^2) + (x^2*cos(2*x^3)*(cos(x) - 4*x*(x^2 - 1)))/(sin(x) - (x^2 - 1)^2)^2

14. Use Matlab's symbolic toolbox to solve the following indefinite integral.

$$\int -5x^2 \sin^3(2x^3) \cos(2x^3)\, dx$$

-(5*sin(2*x^3)^4)/24

15. Convert the antiderivative you found in Question 14 into a Matlab Function and use it to evaluate (using the Fundamental Theorem of Calculus):

$$\int_{-\pi/3}^{\pi/4} -5x^2 \sin(2x^3) \cos(2x^3)\, dx$$

-0.0310

Challenge Problems

1. Write a function file that accepts 2 inputs: a function, and a value. Your function file should differentiate the input function and calculate the derivative at the point specified. The output of the function should be a plot of the input function and the tangent line at the point specified. Warn the user if there are any problems with the derivative.

2. Develop an M-file to implement polynomial regression. Pass the M-file two vectors holding the x and y values along with the desired order, m. Test it by solving Question 1 using it.