

2.2

$$\textcircled{1} \quad y'' + 4y' + 3y = 18$$

$$m^2 + 4m + 3 = (m+1)(m+3) = 0$$

$$m = -1, -3$$

$$y_c = c_1 e^{-x} + c_2 e^{-3x}$$

$$y_p = A \quad y_p' = 0 \quad y_p'' = 0$$

$$3A = 18 \rightarrow A = 6$$

$$c_1 e^{-x} + c_2 e^{-3x} + 6$$

②

$$y''' - 12y'' + 36y' = 12x + 5$$

$$m^3 - 12m^2 + 36m = (m-6)^2 = 0$$

$$y_c = c_1 e^{6x} + c_2 x e^{6x}$$

$$y_p = Ax^2 + B \quad y_p' = 2Ax \quad y_p'' = 2A$$

$$-12A + 36A + 36B = 12x + 5$$

$$36Ax = 12x$$

$$A = \frac{12}{36} = \frac{1}{3}$$

$$36B - 12A = 5$$

$$36B = \frac{29}{3}$$

$$B = \frac{29}{108}$$

$$y = c_1 e^{6x} + c_2 x e^{6x} + \frac{1}{3}x^2 + \frac{29}{108}$$

③

$$y''' - 8y'' + 20y' = 100x^2 - 65xe^x$$

$$m^3 - 8m^2 + 20m = 0 \quad m = \frac{8 \pm \sqrt{64 - 80}}{2}$$

$$m = 4 \pm 2i$$

$$y_c = e^{4x} (c_1 \cos 2x + c_2 \sin 2x)$$

$$y_p = 5x^2 + 4x + 11/10$$

$$y_c =$$

$$y_p = Ax^2 + Bx + C$$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

$$2A - 16Ax = 8B + 20Ax^2 + 20Bx + 20C = 100x^2$$

$$A = 5 - 16A + 20B = 0$$

$$B = \frac{16 \cdot 5}{20} = 4$$

$$20C = 8B - 2A$$

$$C = \frac{22}{10} = 11/10$$

$$y_{p2} = (0x + C) e^x$$

$$y_{p1} = (0x + E + D) e^x$$

$$y_{p2} = (0x + E + 2D) e^x$$

$$0x + E + 2D - 8Dx - 8E - 8D + 2G0x + 20E = 65x$$

$$y = e^{4x} (c_1 \cos(2x) + c_2 \sin(2x)) + 5x^2 + 4x + \frac{11}{8} - 5xe^x - \frac{50}{13} e^x$$

①

$$y'' + 2y' = 2x + 7 - e^{-2x}$$

$$m^2 + 2m = 0 \quad (m + 2) = 0 \quad m \neq 0, -2$$

$$y_c = C_1 e^{-2x} + C_2$$

$$y_{p1} = \frac{1}{2} x^2 + 2x$$

$$y = C_1 e^{-2x} + C_2 + \frac{1}{2} x^2 + 2x + \frac{1}{2} x e^{-2x}$$

$$y_{p1} = Ax^2 + Bx$$

$$y_{p2} = Cx e^{-2x}$$

$$2A + 2 \cdot 2Ax + 2B = 2x + 7$$

$$4A = 2 \quad 2(A+B) = 7$$

$$A = \frac{1}{2} \quad B = \frac{7}{2} - \frac{1}{2} = 3$$

$$y_{p2}' = Cx e^{-2x} + -2Cx e^{-2x}$$

$$y_{p2}'' = -2Cx e^{-2x} + -2Cx e^{-2x} + 2Cx e^{-2x}$$

$$-4Cx e^{-2x} + 6Cx e^{-2x} + 3Cx e^{-2x} - 6Cx e^{-2x} =$$

$$-3Cx e^{-2x} = e^{-2x}$$

③ $y'' + 4y = 6 \sin(2x)$

$$y = y_c + y_p$$

$$y_c: y'' + 4y = 0 \quad y = e^{mx}$$

$$m^2 e^{mx} + 4e^{mx} = 0 \quad e^{mx} (m^2 + 4) = 0$$

$$m^2 + 4 = 0 \rightarrow m^2 = -4 \rightarrow m = \pm \sqrt{-4} = \pm 2i$$

$$m = -2i \quad m = 2i$$

$$e^{2ix} = (\cos(2x) + i \sin(2x)), e^{-2ix} = (\cos(-2x) + i \sin(-2x))$$

$$y_c = C_1 \cos(2x) + C_2 \sin(2x) - \frac{3}{2} x \cos(2x)$$

$$6/4 = 3/2$$