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① $8y'' + y' = 0$

$8m^2 + m = 0 = m(8m+1)$

$m = 0, -1/8 \qquad y = C_1 + C_2 e^{-x/8}$

②

$y'' - y' - 20y = 0$

$D^2 - D - 20 = 0$

$m^2 - m - 20 = 0$

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y'' = \frac{d^2y}{dx^2}$

$y' = \frac{dy}{dx}$

$m = \frac{1 \pm \sqrt{1+81}}{2}$

$m = \frac{1+9}{2} = 5 \qquad m = \frac{1-9}{2} = -4$

$y = C_1 e^{-4x} + C_2 e^{5x}$

③

$y'' + 6y' + 9y = 0$

$m^2 + 6m + 9 = (m+3)^2 = 0$

$m = -3$

$y = C_1 e^{-3x} + C_2 x e^{-3x}$

④ $y'' + 36y = 0$

$m^2 + 36 = 0$

$m = \pm 6i$

$y = C_1 \cos(6x) + C_2 \sin 6x$

⑤ $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} - 5y = 0 \quad y(1) = 0 \quad y'(1) = 7$

$m^2 - 4m - 5 = (m+1)(m-5) = 0$

$m = -1, 5$

$y = C_1 e^{-t} + C_2 e^{5t} \rightarrow y = C_1 e^{-t} + C_2 e^{5t} \rightarrow y' = -C_1 e^{-t} + 5C_2 e^{5t}$

$y(1) = 0 = C_1 e^{-1} + C_2 e^{5}$

$y'(1) = 7 = -C_1 e^{-1} + 5C_2 e^{5}$

$7 = 6C_2 e^{5} \rightarrow C_2 = \frac{7}{6} e^{-5}$

$C_2 = \frac{7e^{-5}}{6}$

$y = -\frac{7}{6} e^{1-t} + \frac{7}{6} e^{3t-5}$

$C_1 e^{-1} + C_2 e^{3} = 0$

$C_1 e^{-1} + \frac{e^{-5}}{6} e^{3} = 0$

$C_1 e^{-1} = -\frac{7}{6} \rightarrow C_1 = \frac{-7}{6} e$