$$\frac{d}{dt}\left(\vec{u}(t)+\vec{v}(t)\right)=\frac{d\vec{u}}{dt}(t)+\frac{d\vec{v}}{dt}(t)$$

$$\frac{d}{dt}\left(\vec{u}(t)\cdot\vec{v}(t)\right)=\frac{d}{dt}\left(\sum_{i=1}^{3}u_i^{(t)}v_i^{(t)}\right)$$

$$\sum_{i=1}^{3}\frac{d}{dt}u_iv_i=\sum_{i=1}^{3}\left(u_i'v_i+u_iv_i'\right)=\sum_{i=1}^{3}u_i'v_i+\sum_{i=1}^{3}u_iv_i'=$$

$\hat{e}_r=\cos\theta\,\hat{\imath}+\operatorname{Sen}\theta\,\hat{\jmath}$ ← Tambien conocido como $\hat{r}$

$$\frac{d\hat{e}_r}{d\theta}=\frac{d\hat{r}}{d\theta}=-\operatorname{Sen}\theta\,\hat{\imath}+\cos\theta\,\hat{\jmath}$$

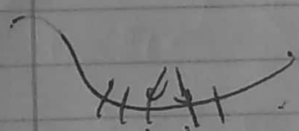
$$\hat{r}\cdot\frac{d\hat{r}}{d\theta}=(\cos\theta\,\hat{\imath}+\operatorname{Sen}\theta\,\hat{\jmath})\cdot(-\operatorname{Sen}\theta\,\hat{\imath}+\cos\theta\,\hat{\jmath})$$

$$=-\cos\theta\operatorname{Sen}\theta+\cos\theta\operatorname{Sen}\theta=0$$

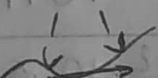
Integral de una función vectorial

$$\int_a^b\hat{r}(t)\,dt=\int_a^b r_1(t)\,dt\,\hat{\imath}+\int_a^b r_2(t)\,dt\,\hat{\jmath}+\int_a^b r_3(t)\,dt\,\hat{k}$$

En este caso tambien tenemos una función vectorial



ΔS_j



$$\Delta S_j=|\vec{r}(t_j)-\vec{r}(t_j+1)|$$

$$\Delta S_j=|\Delta\vec{r}_j|\qquad\lim_{n\to\infty}\sum_{i=1}^{n}\lim_{\Delta t\to 0}\frac{|\vec{r}_j|}{\Delta t}\Delta t$$

$\int_a^b\left|\frac{dr}{dt}\right|dt$ = Longitud del arco

Función longitud de arco

$$S(t)=\int_a^t|\vec{r}\,'(w)|\,dw\qquad\frac{ds}{dt}=|\vec{r}\,'(t)|$$

variable de integración (Random)

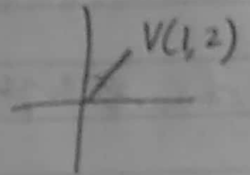
Tarea

$\vec{r}(t) = \cos t\, \hat{i} + \operatorname{Sen} t\, \hat{j} + t\hat{k}$

a) Encuentra la longitud de arco ~~[illegible]~~ desde $(1, 0, 0)$ hasta un punto dado por $\vec{r}(t)$ $(S(t))$

b) Obtén una relación entre S y t

U



Soh Cah Toa

$\operatorname{Tan}\theta_v = \frac{2}{1}$ $\operatorname{Tan}\theta = 2$ $\theta_v\ 63.4349$

$\theta_u = 63.4349 + 60 = 123.4349$ $\vec{U} = \vec{V}$ $\vec{U} = \sqrt{5}$

$\operatorname{Sin}\theta_u = \frac{o}{h}$ $\operatorname{Sin}\theta_u = \frac{y}{\vec{U}}$ $\operatorname{Sin}\theta_u \vec{U} = y = \boxed{1.866}$

$\operatorname{Cos}\theta_u = \frac{a}{h}$ $\operatorname{Cos}\theta_u = \frac{x}{\vec{U}}$ $\operatorname{Cos}\theta_u \vec{U} = x = \boxed{-1.232}$

$\sqrt{1.866^2 + (-1.232)^2} = \sqrt{5}$

$|\vec{U}| = 4$ $\frac{|\vec{U}|}{|\vec{V}|} = \frac{3}{2}$ $U \cdot V?$ $\frac{\vec{U}}{\vec{V}} = \frac{-\vec{U}}{-\vec{V}} = \frac{3}{2}$

$|\vec{U}| = \sqrt{\vec{U} \cdot \vec{U}}$

$4 = \sqrt{\vec{U} \cdot \vec{U}}$

$4^2 = \vec{U} \cdot \vec{U}$

$16 = |\vec{U}||\vec{U}| \operatorname{Cos}(\theta)$

$\vec{U}^2 = 16$

$\vec{U} = 4$

$3 \to 4$ $|\vec{U}||\vec{V}|\operatorname{Cos}\theta = \vec{U} \cdot \vec{V}$

$2 \to 2.6$ $|\vec{U}||\vec{V}|\operatorname{Cos}\theta =$

$U \cdot V = \frac{32}{3} = 10.6$

$|\vec{U}| = 4$ $\vec{U} = V(\frac{3}{2})$

$|UV|$ $\frac{|U|}{|V|} = |U||V|\operatorname{Cos}\theta$

$\frac{\vec{U}}{\vec{V}} = \frac{3}{2}$ $\frac{2}{3} = \frac{V}{U}$ $4\,|U|(\frac{2}{3})\frac{|U|}{(\frac{2}{3})4} = |U||V|\operatorname{Cos}\theta$

$V = (\frac{2}{3})U$

$(4)[(\frac{2}{3})(4)]\left(\frac{4}{(\frac{2}{3})(4)}\right)\ 16$

$|\vec{U}| = 4$ $\quad 3 - 4$ $\quad \frac{\vec{U}}{\vec{V}} = \frac{4}{2.6}$ $\quad \frac{\vec{V}}{\vec{U}} = \frac{2.6}{4}$ $\quad \vec{V} = U\left(\frac{2.6}{4}\right)$ $\quad \vec{V} = 4\left(\frac{2.6}{4}\right)$

$\frac{\vec{U}}{\vec{V}} = \frac{3}{2}$ $\quad 2 - 2.6$

$\vec{V} = 2.6$

$$|U||V| \frac{|U|}{|V|} = |U||V| \cos\theta = \cancel{|\theta|} \, U \cdot V$$

$$(4)(2.6)\frac{4}{2.6} = 16$$

$\vec{i} = 4$ $\quad \cancel{\frac{U}{V} = \frac{2}{3}}$ $\quad \frac{U}{V} = \frac{3}{2}$ $\quad \frac{V}{U} = \frac{2}{3}$ $\quad V = \frac{20}{3}$ $\quad V = \frac{8}{3}$

$$U \times V = (4)\frac{8}{3} = \frac{32}{3}$$

$B = (0, -3, 4)$ $\quad C = (8, 5, 3)$

$$\theta = \cos^{-1}\left(\frac{(0)(8) + (-3)(5) + (4)(3)}{(5)(7\sqrt{2})}\right) = 93.47$$

$$\frac{1}{6} \left| \{ \vec{AB} \times \vec{AC}) \cdot \vec{AD} \} \right|$$

0

$A = (1, 1, 0)$ $\quad B = (1, 5, 0)$ $\quad C = (2, 3, 4)$ $\quad D = (-2, 2, 5)$

$$AB = (1-1)\hat{i} + (5-1)\hat{j} + (0-0)\hat{k} = (0, 4, 0)$$

$$AC = (2-1)\hat{i} + (3-1)\hat{j} + (4-0)\hat{k} = (1, 2, 4)$$

$$AD = (-2-1)\hat{i} + (2-1)\hat{j} + (5-0)\hat{k} = (-3, 1, 5)$$

$$(AB \times AC) \cdot AD = \begin{pmatrix} 0 & 4 & 4 \\ 1 & 2 & 4 \\ -3 & 1 & 5 \end{pmatrix}$$

$$0 \begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix} - 4 \begin{vmatrix} 1 & 4 \\ -3 & 5 \end{vmatrix} + 4 \begin{vmatrix} 1 & 2 \\ -3 & 1 \end{vmatrix}$$

$$= 0 - 4[(5)(-12)] + 4[(1)(-6)] = \cancel{0} + 240 \cancel{4} - 24 = \cancel{264} \; 216$$

$\cancel{(1/6)(-264) = -44}$ $\quad (1/6)(216) = 36$