

# Maté

Ipp (integración por partes)

$$\int u dv = uv - \int v du$$

$$\int x^2 \text{Sen}(3x) dx$$

Inversa

Logarítmica

Algebraicas

Trigonométricas

Exponencial

Para decidir  
cual debería  
ser la U

Tabulación

U	dv
$x^2$	$\text{Sen}(3x)$
$2x$	$-\frac{1}{3}\text{Cos}(3x)$
$2$	$-\frac{1}{9}\text{Sen}(3x)$
$0$	$\frac{1}{27}\text{Cos}(3x)$

$$-\frac{x^2}{3}\text{Cos}(3x) + \frac{2x}{9}\text{Sen}(3x) + \frac{1}{27}\text{Cos}(3x)$$

$$\int e^{2x} \text{Cos}(3x) dx \rightarrow \frac{1}{2}e^{2x} \text{Cos}(3x) - \int \frac{3}{2}e^{2x} \text{Sen}(3x) dx$$

Tabulación

U	dv
$\text{Cos}(3x)$	$e^{2x}$
$-3\text{Sen}(3x)$	$\frac{1}{2}e^{2x}$
$\text{Cos}(3x)$	$-\frac{1}{4}e^{2x} \text{Cos}(3x)$

$$\rightarrow \frac{1}{2}e^{2x} \text{Cos}(3x) + \frac{3}{4} \int e^{2x} \text{Sen}(3x) dx$$

$$u = \text{Sen}(3x) \quad dv = e^{2x} dx$$

$$du = 3\text{Cos}(3x) \quad v = \frac{1}{2}e^{2x}$$

$$\frac{13}{4} \int e^{2x} \text{Cos}(3x) dx = \frac{1}{2}e^{2x} \text{Cos}(3x) + \frac{3}{4} \text{Sen}(3x)e^{2x}$$

$$\int \text{Cos} e^{2x} \text{Cos}(3x) dx = \frac{2}{13}e^{2x} \text{Cos}(3x) + \frac{3}{13}e^{2x} \text{Sen}(3x)$$

$$\int \text{arcsen}(x) dx$$

$$u = \text{arcsen}(x) \quad dv = dx$$

$$du = \frac{1}{\sqrt{1-x^2}} dx \quad v = x$$

$$= x \text{arcsen}(x) - \int \frac{x}{\sqrt{1-x^2}} dx \rightarrow \int (1-x^2)^{-1/2} x dx$$

$$= x \text{arcsen}(x) + \frac{1}{2} \sqrt{1-x^2}$$

$$= x \text{arcsen} + \frac{1}{2} (1-x^2)^{1/2}$$

$$u = 1-x^2$$

$$du = -2x dx$$

$$\int \frac{-1/2 du}{2}$$

$$-\frac{1}{2} \int u^{-1/2} du = -2u^{1/2}$$

$$a) \int x^4 e^{-2x} dx$$

$$a) \int x^2 e^{-2x} dx$$

$$b) \int x^2 \cos(2x) dx$$

$$c) \int x^4 \ln(3x) dx$$

Tabulación	
u	dv
+ $x^2$	$e^{-2x}$
- $2x$	$1/2 e^{-2x}$
+ $2$	$1/4 e^{-2x}$
- $0$	$1/8 e^{-2x}$

$$\int x^2 e^{-2x} - \frac{1}{2} x \frac{1}{2} e^{-2x} + \frac{1}{4} e^{-2x} + \frac{1}{8} e^{-2x}$$

$$b) \int x^2 \cos(2x) dx$$

Tabulación	
u	dv
+ $x^2$	$\cos(2x)$
- $2x$	$-2 \sin(2x)$
+ $2$	$2 \cos(2x)$
- $0$	$2 \sin(2x)$

$$= 2x^2 \sin(2x) + 2x^2 \sin(2x) - 2 \cos(2x) - 4 \sin(2x)$$

$$1) \int \frac{2x+1}{x(x^2+1)} dx = \int \frac{A}{x} + \frac{Bx+C}{x^2+1} dx$$

$A \ln(x) \rightarrow \frac{B}{2} \ln(x^2+1) \rightarrow C \ln\left(\frac{x}{x^2+1}\right) + C$

$\int \frac{dx}{a^2+u^2} du = \arctan\left(\frac{u}{a}\right) + C$

$\frac{A(x^2+1) + (Bx+C)x}{x(x^2+1)}$

$$Ax^2 + A + Bx^2 + Cx = 2x + 1$$

$$(A+B)x^2 + Cx + A = 2x + 1 + (0)x^2$$

$$A+B=0 \quad C=2 \quad A=1 \quad B=-1$$

$$b) \int \frac{1}{x^2(2x-1)} dx = \int \left( \frac{A}{x^2} + \frac{B}{x} + \frac{C}{2x-1} \right) dx$$

$$g) \frac{2x-1}{x^2-1} dx \rightarrow \int \frac{2x-1}{(x^2-1)} dx = \int \frac{2x-1}{(x+1)(x-1)} dx$$

$$= \int \frac{A}{x+1} + \frac{B}{x-1} = A \ln(x+1) + B \ln(x-1) + C$$

$$k) \int \frac{dx}{(x^2+4)^{3/2}} = \int \frac{dx}{((x^2+4)^{1/2})^3} = \int \frac{2 \sec^2 \theta d\theta}{((x^2+4)^{1/2})^3}$$

$$\begin{array}{l} \text{Diagram: A right triangle with hypotenuse } \sqrt{x^2+4}, \text{ vertical side } x, \text{ and horizontal side } 2. \text{ Angle } \theta \text{ is at the bottom-left vertex.} \\ \tan \theta = \frac{x}{2} \\ x = 2 \tan \theta \\ dx = 2 \sec^2 \theta d\theta \end{array}$$

$$\frac{\sqrt{x^2+4}}{2} = \sec \theta$$

$$= \int \frac{2 \sec^2 \theta d\theta}{(2 \sec \theta)^3}$$

$$\int \frac{1}{4} \sec^{-2} \theta d\theta$$

$$= \int \frac{2 \sec^2 \theta}{8 \sec^3 \theta} = \frac{1}{4} \int \frac{1}{\sec \theta} d\theta = \frac{1}{4} \int \cos \theta d\theta$$

$$= \frac{1}{4} \left( \frac{x}{\sqrt{x^2+4}} + C \right)$$

$$c) \int x^4 \ln(3x) dx = \left( \frac{x^5}{5} \right) \ln(3x) - \int \left( \frac{x^5}{5} \right) \frac{1}{x} dx$$

$$= \left( \frac{x^5}{5} \right) \ln(3x) - \frac{1}{5} \int x^4 dx$$

$$= \left( \frac{x^5}{5} \right) \ln(3x) - \frac{x^5}{25}$$

Integration by parts:  $\int u dv = uv - \int v du$   
 $u = \ln(3x)$   
 $du = \frac{1}{x}$   
 $dv = x^4$   
 $v = \frac{x^5}{5}$

$$R = \frac{kA(T_h - T_c)}{L}$$

$$L = \frac{kA(T_h - T_c)}{R}$$

$$R = \frac{kAT_h - kAT_c}{kA}$$

$$(238)(250)(65-20)$$

$$1200$$

Ave linear expansion coeff is 1200

$$R = \frac{kAT_h - kAT_c}{kA}$$

$$(238) \left( \frac{5}{100} \right)$$

$$\Delta L = \alpha L_0 \Delta T$$

$$\Delta A = 2\alpha A_0 \Delta T$$

$$\Delta V = 3\alpha V_0 \Delta T$$

$$100$$

$$10 = (\alpha)(100)(\Delta T)$$

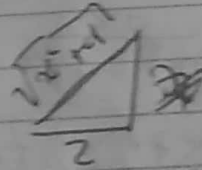
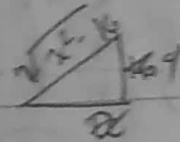
$$\frac{\Delta L}{L_0} = \Delta T$$

$$\Delta T = \frac{10}{(12 \times 10^{-6})(100)} = 91.666$$



$$\Delta A = 2A \Delta T \Rightarrow \frac{\Delta A}{2A} = \Delta T = 2083.33$$

$$\Delta V = 3V \Delta T \Rightarrow \frac{\Delta V}{3V} = \Delta T = 1388.88$$



$$x = 2 \tan(\theta)$$

$$dx = 2 \sec^2 \theta d\theta$$

$$\sqrt{x^2+4} = 2 \sec \theta$$

$$\cos^2 \theta = \frac{1}{\sec^2 \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\int \frac{dx}{(x^2+4)^2} = \int \frac{dx}{(2 \sec \theta)^4} = \int \frac{2 \sec^2 \theta d\theta}{(2 \sec \theta)^4}$$

$$= \int \frac{2 \sec^2 \theta d\theta}{16 \sec^4 \theta} = \frac{1}{8} \int \sec^{-2} \theta d\theta$$

$$= \frac{1}{8} \int \cos^2 \theta d\theta = \frac{1}{8} \int \frac{1 + \cos(2\theta)}{2} d\theta$$

$$\frac{1}{16} \left( \int d\theta + \int \cos 2\theta d\theta \right) = \frac{1}{16} \left( \theta + \frac{1}{2} \sin 2\theta \right) + C$$

0.083

$$= \frac{1}{16} \theta + \frac{1}{32} \sin 2\theta \cos \theta + C$$

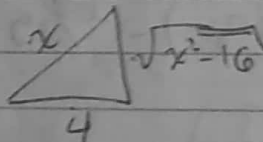
Sigue expresarlo en x

$$= \frac{1}{16} \arctan\left(\frac{x}{2}\right) + \frac{1}{16} \left( \frac{x}{\sqrt{x^2+4}} \right) \left( \frac{2}{\sqrt{x^2+4}} \right)$$

$$= \frac{1}{16} \arctan\left(\frac{x}{2}\right) + \frac{1}{8} \frac{x}{x^2+4}$$

$$= \frac{1}{16} \arctan\left(\frac{x}{2}\right) + \frac{1}{16} \left( \frac{x}{\sqrt{x^2+4}} \right) \left( \frac{2}{\sqrt{x^2+4}} \right)$$

$$\int \frac{\sqrt{x^2-16}}{x^4} = \int \frac{4 \tan \theta}{(4 \sec \theta)^4} 4 \sec \theta \tan \theta d\theta$$



$$\frac{1}{16} \int \frac{\sec^2 \theta}{\cos^3 \theta} d\theta = \frac{1}{16} \int \frac{\cos^3 \theta \sec^2 \theta}{\cos^3 \theta} d\theta$$

$$\sec = \frac{x}{4}$$

$$x = 4 \sec \theta$$

$$dx = 4 \sec \theta \tan \theta d\theta$$

$$\sqrt{x^2-16} = 4 \tan \theta$$

$$= \frac{1}{16} \int \sec^2 \theta \cos \theta d\theta$$

$$= \frac{1}{16} \int u^4 du = \frac{1}{16} \frac{\sec^5 \theta}{5} + C$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

$$u = \sec \theta$$

$$du = \sec \theta \tan \theta d\theta$$