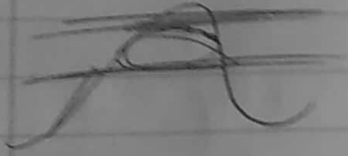
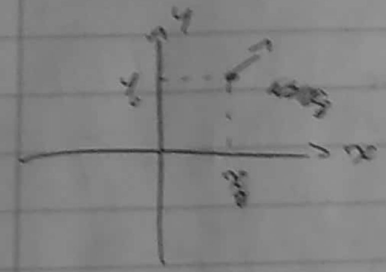
Metodos matematicos para fisica

* $\emptyset(x, y, z) = k \Rightarrow$ Curvas de nivel
Superficie



$$\vec{\nabla}\emptyset = \left(\frac{\partial \emptyset}{\partial x}, \frac{\partial \emptyset}{\partial y}, \frac{\partial \emptyset}{\partial z}\right) \leftarrow$$ Perpendicular a la trayectoria en cada punto



Para cada punto en el espacio, va a existir una curva de nivel

$\vec{E} = -\vec{\nabla}_\emptyset$
↑
Campo electrico

Derivada direccional
$\hat{n} \cdot \vec{\nabla}\emptyset$

Derivada direccional: Como cambia el campo en cierta dirección
Cuando $\hat{n}$ esta en la dirección del gradiente $|\hat{n}||\vec{\nabla}_\emptyset|\cos\theta$
$= |\vec{\nabla}_\emptyset|\cos\theta$

Tarea: Calcula la derivada direccional
de $\emptyset = xy - x^3y^3$

$y - 2x^2y^3$

en el punto $(1, 3, 1)$ y en la dirección $(-1, 2, 2)$

$\emptyset$ = Campo escalar

$$\vec{\nabla}_\emptyset = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\emptyset$$

$$\vec{\nabla} = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \leftarrow$$ Operador nabla.

Operador

$$\nabla_o \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{F}(x, y, z) = \left(i\frac{\partial}{\partial x}, j\frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$
$$= (F_x i + F_y j + F_z k)$$
$$\boxed{= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}}$$

∇_o: del dot operator means to take the divergence
$\vec{E}$: Electric field vector
ρ: Electric charge volume density (coloumb/m^3)
ε_0: Electric permittivity of the free space. Also called dialectric constant

$\nabla \cdot \vec{B} = 0$

Magnetic field flux density vector.

Rotacional para una coordenada cartesiana

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$ o (determinante)

$$\nabla \times B = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$\vec{B}$ = Induced magnetic flux density vector

μ_0 = Magnetic permeability of free space

$\vec{J}$ = Electric current density vector

ε_0 = Electric permitivity of free space

$\frac{\partial E}{\partial t}$ = Rate of change of the electric field

$\phi = xy - x^3 y^3$

$\nabla_\phi = (y - 3x^2y^3, x - 3x^3y^2, 0)$

$a = (1,3,1) \quad b = (-1,2,2)$

$c = b - a \quad c = (-1-1, 2-3, 2-1) \quad c = (-2,-1,1)$

$M = \sqrt{-2^2 + (-1^2) + 1^2} = \sqrt{6}$

$C_{norm} = \left(\frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right) = (-0.81, -0.408, 0.408)$

$C_{norm} \cdot \nabla_\phi = (-0.81, -0.408, 0.408) \cdot (y - 3x^2y^3, \cancel{x} - 3x^3y^2, 0)$

$\nabla_\phi(a) = (3 - 3(1)^2(3)^3, 1 - 3(1)^3(3)^2, 0) = (-78, -26, 0)$

$C_{norm} \cdot \nabla_\phi(a) = (-0.81)(-78) + (-0.408)(-26) + (0.408)(0)$

$= 63.18 + 10.608 + 0 = \boxed{73.788}$

$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$

$\vec{\nabla}\cdot\vec{F}$ $\vec{\nabla}\times\vec{F}$

La placiano $\nabla^2 = \vec{\nabla}\cdot\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$\frac{\partial^2}{\partial x^2}\phi = \frac{1}{v^2}\frac{\partial^2\phi}{\partial t^2}$ ⇐ Para 1D

velocidad de la onda

$\nabla^2\phi = \frac{1}{v}\frac{\partial\phi}{\partial t^2}$ ⇐ Para 3D (laplaciano)

Tarea: $\phi = x^2 + y^2 + z^2$ Calcular $\vec{\nabla}\phi$, $\nabla^2\phi$

$\vec{v} = \vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ $\vec{\nabla}\cdot\vec{v}$, $\vec{\nabla}\times\vec{v}$

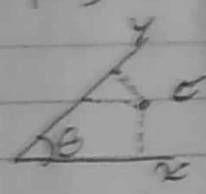
Coordenadas curvilineas ortogonales

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$z = Cte \Rightarrow$ Plano (area)

$z, x = Cte \Rightarrow$ linea

Ortogonales porque los 3 ejes son perpendiculares



Como se escribiria el vector de posición?

$\hat{\imath} = 1, \emptyset, \emptyset$

$\hat{\jmath} = \cos\theta, \sin\theta, \emptyset$

$\hat{k} = \emptyset, \emptyset, 1$

Coordenadas cilindricas
Polares pero le agregamos el eje z

$x \leftrightarrow r$ $r = \sqrt{x^2+y^2}$

$y \leftrightarrow \theta \Rightarrow \theta = \arctan\left(\frac{y}{x}\right)$

$z \leftrightarrow z$ $z = z$

Del vector de posición va a salir todo.

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$= r\cos\theta\hat{\imath} + r\sin\theta\hat{\jmath} + z\hat{k}$

$\frac{\partial\vec{r}}{\partial r} = \cos\theta\hat{\imath} + \sin\theta\hat{\jmath} + 0 =: \hat{r}$ ⇐ Vector radial Unitario

Salio unitario de pura suerte

$\frac{\partial\vec{r}}{\partial z} = \hat{k}$ $\frac{\partial\vec{r}}{\partial\theta} = -r\sin\theta\hat{\imath} + r\cos\theta\hat{\jmath} + 0\hat{k} = \vec{\theta}$

$\hat{\theta} = norm(\theta)$ ⇐ $\vec{\theta} = r(-\sin\theta\hat{\imath} + \cos\theta\hat{\jmath})$