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The Circular Justification in the MM Proposition

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Abstract: The real contributions of the MM Proposition are its central assumptions of perfect capital markets and the associated arbitrage argument. In this text, we review the perfect market assumptions and no-arbitrage principle. Then, to explain the evolution of the understanding of arbitrage from a deterministic world into an uncertain world, we restate and comment on the proofs of the MM Proposition in current perspectives. With the no-arbitrage principle in mind, we clearly read the circular justification in the MM Proposition and the misleading concept of cost of equity. From the perspective of the distribution of a corporation's value creation, we find that shareholders prefer a maximum degree of debt. However, we believe that the capital structures are mainly constrained by industry characteristics and the market timing of equity and debt financing.

Keywords: Perfect Market, No-Arbitrage Principle, MM Proposition, Capital Structure, Cost of Equity

JEL Classification: G12, G32, D24

The most important contribution of MM1958 (Modigliani and Miller, 1958) is the assumption that financial markets are *perfect*, which has now become the standard in teaching finance and in academic literature. Another key contribution of MM1958 is the arbitrage proof of the MM Proposition (capital structure irrelevance), which is an enlightening viewpoint in finance. This idea of arbitrage has affected the development of finance, and the arbitrage argument has been recognized as the no-arbitrage principle.

MM Proposition was a shock at the time of its publication, debates arose soon and were lasting for more than ten years. However, the conclusion whether the capital structure is irrelevant or not is not the most important contribution of MM1958; the capital structure is a toy only. As Stulz (2000, p. 120) points out: "What makes Proposition I memorable and changed finance is not the Proposition itself but its proof." The introduction of the arbitrage argument has a far-reaching impact on finance beyond the imagination of that time. The no-arbitrage analysis marks the unique methodology of finance; compared to the equilibrium analysis of economic theory, the no-arbitrage analysis is a revolutionary idea in finance.

This paper is structured as follows. Section 1 presents a full list of the perfect market assumptions. Section 2 investigates the no-arbitrage principle; we focus on the fundamental theorem of asset pricing and explain the representation and usage of the linear pricing function. Section 3 clarifies the circular justification in the MM Proposition with careful retrospections on its proofs. We review the struggle to understand and refine the concept of arbitrage. In particular, we review the hard journey from the world with certainty into the world with uncertainty. Without unfolding the black box of a firm, we may not have a proper view on capital structure; thus, section 4 discusses securitization, where firms create value through production function. Knowing the source of value creation, we have a new viewpoint on capital structure and the cost of equity. Section 5 concludes.

1 Perfect Market Assumptions

Miller (1988) points out that the then-novel arbitrage proof of the MM Proposition and the key assumption, the assumption of perfect capital markets, have now become the standard course materials taught to students. The perfect market assumptions are similar to the zero net force condition for Newton's first law of motion, which allow us to proceed from the simplest case and to gradually understand the real capital market from the simplified ideal world.

Market frictions are the only concerns of the perfect market assumptions at their early stages, then liquidity and information are added. In a competitive capital market, the perfect market assumptions are:

1. Perfect market trading: frictionless transactions, and the trading price is the equilibrium price.
   - There are no bid-ask spreads, i.e., the selling price is equal to the buying price of any asset.
   - There are no transaction costs of trading.
   - There are no taxes.
   - There are no bankruptcy costs.
2. Perfect market liquidity: buy and/or sell any quantity instantaneously at market price.
   - Short positions, as well as fractional holdings, are allowed.
   - The market is completely liquid, i.e., it is always possible to buy and/or sell unlimited quantities on the market. In particular it is possible to borrow unlimited amounts from the bank (by selling bonds short).
3. Perfect market information: asset prices fully reflect all available information.
   - The information set of future states and probabilities are known.
   - Investors are rational and fully grasp the current and past information.
   - There are no costs to obtain and process information immediately.

The perfect capital market assumptions are clean test tubes for an antiseptic world. By starting from this immaculate laboratory equipment, we can analyze the consequences of financial models, test some hypotheses, and determine how closely our theory accords with the real world. However, due to the initiatives of investors, the financial market is far more complicated than the physical world. We are likely to add new items to the perfect market assumptions to simplify certain conditions for a better understand of the financial market.

In a perfect market, we do not assume that investors are risk-averters. We have to admit that we can not define liquidity unambiguously. Apart from the intuitive feeling of its sudden disappearance, we have not a scintilla of knowledge of liquidity. In our opinion, the efficient markets hypothesis[1] (EMH) is not a core idea of finance. EMH is only part of the perfect

[1]The phrase of efficient markets hypothesis is descriptive; it is designed more to capture an intuition than to state

market assumptions where agents are fully rational and informed, such that all past information available up to current time is impounded in the current price.

The further expansion of financial theory beyond the known territories is the process of relaxing the perfect market assumptions. For instance, behavioral finance is primarily concerned with the rationality of investors. Emotion and psychology influence our decisions. We make decisions based on approximate rules of thumb and not strict logic; thus, we behave in unpredictable or irrational ways, such as herd behavior, overconfidence, and overreaction.

2 No-Arbitrage Principle

The financial market is absent of arbitrage. Any opportunity for arbitrage will be exploited and disappears promptly in a well-developed market. Throughout the years, finance has found it difficult to understand exactly how to measure the risk for securities; fortunately, arbitrage arguments bypass this problem altogether, for they do not rely on attitudes towards risk. The arbitrage argument is a cornerstone of modern finance, and Rubinstein (2003, p. 12) credits MM with pointing others in the direction of arbitrage reasoning as the most fundamental tool to derive results in financial economics.

2.1 Arbitrage Opportunity

What is an arbitrage opportunity? It is a transaction involving no cash outlay that results in a sure profit (Varian, 1987, p. 55). However, to be free of sure win (deterministic arbitrage opportunity) is not enough for a financial market to function properly. For a zero-investment in a finite-state market, if there is a positive payoff in at least one case but non-negative payoff in every state, it is an arbitrage opportunity even if there is not a sure win.

We call the random variable[2] X *weak-positive*, denoted as $X \gneqq 0$, if $X \geqslant 0$ (almost surely) and $\mathrm{P}(X = 0) < 1$. Let (P, X) be a portfolio with a current price P and a future payoff X.

a formal mathematical result. The definition of information efficiency is purposely vague; the exact mechanism by which prices incorporate information is still a mystery.

[2]Information on the financial market is modelled by probability theory; thus, an asset's payoff is depicted by a random variable. We assume that $X \in \mathsf{L}^p(\Omega, \mathcal{F}, \mathrm{P})$, $p > 1$.

Then, the portfolio (P, X) is an *arbitrage opportunity* if

$$\begin{bmatrix} -P \\ X \end{bmatrix} \gneq 0 \tag{2.1}$$

This definition contains three cases:

1. The immediate arbitrage opportunity turns something into nothing: $P < 0, X = 0$. The profit is realized immediately and the investors can go away without any obligations.
2. The zero-investment arbitrage opportunity turns nothing into something:

$$P = 0, X \gneq 0 \tag{2.2}$$

   If $X > 0$, the future payoff is strictly positive. It is called a strong arbitrage opportunity. Furthermore, if $X = a > 0$ for some constant a, it is called a naïve arbitrage opportunity.
3. The mixed arbitrage opportunity, $P < 0, X \gneq 0$, is a portfolio of immediate and zero-investment arbitrage opportunities.

The immediate arbitrage opportunities and naïve arbitrage opportunities are called *deterministic arbitrage opportunities*, or *risk-free arbitrage opportunities*. In general, there exists a risk-free asset[3] in the market. With a risk-free asset, equation (2.1) amounts to equation (2.2); we produce something out of nothing, there is no chance for a loss, and there is a positive probability of gain.

2.2 Positive Linear Pricing Function

Mathematically, the financial market is a closed convex set if the number of assets or states is finite. Additionally, the set of all arbitrage opportunities is a convex set. If the market is free of arbitrage, these two sets are not intersected. Then, they are separated by a hyperplane by the separating hyperplane theorem. The "normal" of the hyperplane happens to be the pricing function, and it points to the positive quadrant of arbitrage opportunities. Therefore, the linear pricing function must be positive. We present the following profound theorem termed by Dybvig and Ross (1987):

Theorem 1 (First Fundamental Theorem of Asset Pricing): *In a perfect capital market, the market is free of arbitrage if and only if there exists a positive linear pricing function.*

[3] We can put the money in the pocket.

By Riesz representation theorem, for portfolio (P, X), the pricing function $\wp(\cdot)$ is expressed by an inner product:

$$P = \wp(X) = \mathrm{E}(\Psi X) \qquad \Psi > 0 \tag{2.3}$$

where the positive random variable Ψ is the *stochastic discount factor* (SDF), for it acts like a discount factor in the deterministic world. Because of the risk-free rate $r \geqslant 0$, we have $0 < \wp(1) = \frac{1}{1+r} \leqslant 1$.

Let the present and future time be represented as 0 and 1, the price of risk-free asset $B_0 = 1$, and the future value $B_1 = 1+r$. Define $G = \Psi B_1/B_0$, then G is the Radon-Nikodým derivative from the P measure to the Q measure, and $\mathrm{E}^Q(Y) = \mathrm{E}(YG)$ for any payoff Y, where $\mathrm{E}^Q(\cdot)$ is the expectation under Q. The probability measure Q is the risk-neutral measure for

$$\frac{P}{B_0} = \frac{\mathrm{E}(\Psi X)}{B_0} = \frac{\mathrm{E}^Q(\Psi X/G)}{B_0} = \mathrm{E}^Q\left(\frac{X}{B_1}\right) \tag{2.4}$$

We see that the market is free of arbitrage if and only if there exists an equivalent martingale measure. The no-arbitrage principle is the core of the option pricing theory, which is now the most successful area of modern finance.

2.3 Pricing Formula

By way of the no-arbitrage analysis, there are the Black-Scholes formula (Black and Scholes, 1973) and the binomial options pricing model (Cox et al., 1979). However, the risk-neutral measure is still somewhat mysterious. It will be illustrated in the following example.

Example 2.1 (Horse Racing): There are three horses with odds

$$o_1 = 5 \quad o_2 = 4 \quad o_3 = 2$$

Let x_i be the stake on horse i. No matter which horse wins, there is a sure win of 9 if

$$x_1 = 5 \qquad x_2 = 6 \qquad x_3 = 10$$

Why is there a betting scheme that results in a sure win? Because equation (2.4) shows that if $\sum_i \frac{1}{1+o_i} \neq 1$, there does not exist a risk-neutral measure, and arbitrage opportunities exist. In more detail, let q_i be the winning probability of horse i in Q world. Since $B_1 = B_0 = 1$ (put the money in the pocket), by equation (2.4) we have $q_i = \frac{1}{1+o_i}$. However,

$$q_1 + q_2 + q_3 = \frac{1}{6} + \frac{1}{5} + \frac{1}{3} = \frac{7}{10} \neq 1$$

Therefore it cannot be a probability measure. Besides, to detect an arbitrage opportunity or adjust the odds, we must admit that the risk-neutral pricing is the best way here, while the equilibrium analysis is an impossible mission.

It is worth noting that the CAPM formula is not a pricing formula. First, it has a logical circularity; the expected return of an asset depends on the market return, but the market return itself is a linear combination of all assets. Second, the CAPM formula does coexist with arbitrage opportunities (Abad, 2020).

3 MM Proposition

Twenty years before MM1958, Williams (1938) had stated the irrelevance of capital structure, to which he gave the name "the Law of the Conservation of Investment Value." A few years before MM1958, Durand (1952) and Morton (1954) also had stated close equivalents of the MM Proposition. However, these writers did not insist and suggest immediately that the results were not applicable to the actual capital markets. Additionally, their reasonings were mostly by intuition or verbal illustration rather than by attempting a mathematical proof.

3.1 Historical Appraisal

Both Modigliani (in 1985) and Miller (in 1990) won Nobel Prizes in Economic Science. In the award ceremony speech of the 1985 Nobel Prizes, Professor Ragnar Bentzel of the Royal Academy of Sciences stated: "It was not till Modigliani and Miller presented their theorems that more stringent theorizing began to appear in this field. By treating financing decisions within the framework of a theory of financial-marketplace equilibrium, Modigliani and Miller provided the general guidelines for continued research in this area." The unprecedented appraisal was offered in the press release of the 1990 Nobel Prizes, "The MM Propositions have therefore become the natural basis, or norm of comparison for theoretical and empirical analysis in corporate finance."

The capital structure irrelevance has been accepted as a new paradigm. Ross (1988) reminds: "the older view that capital structure did matter has about it the flavor of phlogiston." Weston (1989) claims that MM transformed the study of finance from an institutional to an economic orientation, and that the influence of the MM Propositions on financial economics is comparable to the impact of Keynes in macroeconomics. More recently, a number of writers rank the MM

Proposition among the references or benchmarks of economic analysis, such as the Arrow-Debreu model of general equilibrium and the Coase Theorem.

3.2 Proofs

The MM Proposition is the Proposition I in Modigliani and Miller (1958, pp. 268–71), the well-known irrelevancy of capital structure. Which is presented as follows:

Proposition 2 (The MM Proposition): For any firm j, let S_j and D_j be the market value of its common shares and debts, respectively. Denote $V_j = S_j + D_j$ as the market value of all its securities, or firm value for short. Then,

$$V_j = S_j + D_j = \mathrm{E}(X_j)/\rho_k$$

where X_j is the return (the firm's future value), and ρ_k is the discounted rate of risk class k to which firm j belongs. Thus, the value of any firm is independent of its capital structure (D_j/S_j) and is given by discounting its expected return $\mathrm{E}(X_j)$ at the rate ρ_k appropriate to its class.

First, here are some remarks. At that time, the pricing method of risky assets was expectation-pricing, which was adapted from the present value formula of deterministic cash flows, by replacing the deterministic returns with the expected returns and replacing the risk-free discount rate with the risk-adjusted discount rate. MM introduced the concept of risk class to set the discounted rate to price risky cash flows. The risk class notation has been discarded years later. MM1958 proved a slight modification of the MM Proposition:

In a perfect market, supposing that two firms have the same expected returns, but one is financed by equity only, and the other has both debt and equity outstanding, then their firm values are equal. Thus, the value of any firm is independent of its capital structure.

Using today's financial terminology, here is the proof in MM1958:

Proof (*MM1958*): The two firms have the same expected returns, $\mathrm{E}(X_1) = \mathrm{E}(X_2) = \mathrm{E}(X) > 0$. Given $D_1 = 0$ and $D_2 > 0$, then $V_1 = S_1 > 0$, and $V_2 = S_2 + D_2 > 0$. If $V_1 < V_2$, MM form the following zero-investment portfolio:

1. Short $s_2 = \alpha S_2$ dollars' worth of the shares of firm 2, with $0 < \alpha < 1$. The expected

return[a] of this portfolio is

$$\mathrm{E}(Y_2) = -\alpha(\mathrm{E}(X) - D_2(1+r)) \tag{3.1}$$

2. Long $s_1 = \alpha(S_2 + D_2)$ dollars' worth of the shares of firm 1 and short αD_2 dollars of bonds, the expected return of this portfolio is

$$\mathrm{E}(Y_1) = \frac{s_1}{S_1}\,\mathrm{E}(X) - \alpha D_2(1+r) = \frac{\alpha V_2}{V_1}\,\mathrm{E}(X) - \alpha D_2(1+r) \tag{3.2}$$

Clearly, the net investment is

$$P_M = -s_2 + (s_1 - \alpha D_2) = -s_2 + \alpha S_2 = 0$$

and the expected payoff is

$$\mathrm{E}(X_M) = \mathrm{E}(Y_1) + \mathrm{E}(Y_2) = \alpha(V_2 - V_1)\,\mathrm{E}(X)/V_1 > 0$$

That is, $P_M = 0$ and $\mathrm{E}(X_M) > 0$, a positive expected return with null input; thus, there is an arbitrage opportunity (or so MM thought), and $V_1 < V_2$ could not be true.

If $V_1 > V_2$, MM form a new zero-investment portfolio: short $s_1 = \alpha S_1$ dollars' worth of the shares of firm 1, long $s_2 = s_1 S_2/V_2$ dollars' worth of the shares of firm 2 and $d = s_1 D_2/V_2$ dollars of bonds, the net investment is

$$P_M = -s_1 + (s_2 + d) = -s_1 + s_1 = 0$$

and the expected payoff is

$$\begin{aligned}\mathrm{E}(X_M) &= \frac{-s_1}{S_1}\,\mathrm{E}(X) + \left(\frac{s_2}{S_2}(\mathrm{E}(X) - D_2(1+r)) + d\cdot(1+r)\right)\\ &= -\alpha\,\mathrm{E}(X) + \alpha S_1\,\mathrm{E}(X)/V_2 = \alpha(V_1 - V_2)\,\mathrm{E}(X)/V_2 > 0\end{aligned}$$

Similarly, $V_1 > V_2$ could not be true either. MM conclude that in equilibrium there must be $V_1 = V_2$.

[a] Equation (3.1) and (3.2) are corresponding to MM1958's equation (5) and (6), respectively. Although the conclusion is not affected, the principals are omitted in Modigliani and Miller (1958, 1969) and related communications.

The MM1958 proof remains in the deterministic world, and the expected value is used[4],

[4] The explaination can be found in Modigliani and Miller (1969, p. 594): "no generally valid method for discounting uncertain streams was (or is yet) available, ..., by analogy with the perpetuity formula in the certainty case, to define the value of the firm as the capitalized value of the mathematical expectation of that average return." If the MM1958 proof had applied this pricing formula, $V_j = \mathrm{E}(X_j)/\rho_k$ (no need to know the specific value of ρ_k as it takes same value for both firms), the firm value would have nothing to do with the capital structure, and the

not the random payoff itself. Using the knowledge of the deterministic world, the world of uncertainty is simplified by taking expectation. Eleven years later, HS1969 (Heins and Sprenkle, 1969) succeeded in applying the random return itself, allowing both firms to have equal random return rather than the expected value only. Accordingly, the MM Proposition was updated:

Proposition 3 (The MM Proposition, popular version): In a perfect market, supposing that two firms have identical returns, but one is financed by equity only and the other has both debt and equity outstanding, then their firm values are equal. Thus, the value of any firm is independent of its capital structure.

Proof (*HS1969*): If $V_1 < V_2$, short αS_2 dollars' worth of the shares of firm 2, long αS_1 dollars' worth of the shares of firm 1, and short $\alpha(S_1 - S_2)$ dollars of bonds, the net investment is

$$P_H = -\alpha S_2 + (\alpha S_1 - \alpha(S_1 - S_2)) = 0$$

However, the return corresponding to equation (3.2) becomes HS's equation (5a):

$$Y_1^* = \frac{\alpha S_1}{S_1} X - \alpha(S_1 - S_2)(1+r) = \alpha X - \alpha D_2(1+r) - \alpha(1+r)(V_1 - V_2)$$

As for equation (3.1), the return Y_2 is

$$Y_2 = -\alpha(X - D_2(1+r))$$

Thus, the future payoff is

$$X_H = Y_1^* + Y_2 = -\alpha(1+r)(V_1 - V_2) > 0$$

We see that $P_H = 0$ and $X_H > 0$, and there exists a naïve arbitrage opportunity. Similarly, $V_1 > V_2$ results in a naïve arbitrage opportunity.

Inspired by HS1969's method, MM1969 (Modigliani and Miller, 1969) construct a portfolio that has zero return and a negative price, or, an immediate arbitrage opportunity. MM1969's method was ready-to-acceept and gained popularity instantly. In fact, MM did not realize that an immediate arbitrage opportunity is equivalent to a naïve arbitrage opportunity when there is a risk-free asset.

Proof (*MM1969*): If $V_U > V_L$ (the subscripts 1 and 2 in MM1958 are replaced by U and L, respectively), short a fraction α of the unlevered shares, long a fraction α of both the

arbitrage method might have been postponed for many a year.

levered shares and bonds, the cost is

$$P_A = -\alpha S_U + (\alpha S_L + \alpha D_L) = -\alpha V_U + \alpha V_L = \alpha(V_L - V_U) < 0$$

and the return[a] is

$$X_A = -\frac{\alpha S_U}{S_U} X + \left(\frac{\alpha S_L}{S_L}(X - D_L(1+r)) + \alpha D_L(1+r) \right) = -\alpha X + \alpha X = 0$$

For $P_A < 0$, $X_A = 0$, there is an immediate arbitrage opportunity. If $V_U < V_L$, MM did not reverse the above holding but formed a new immediate arbitrage portfolio.

[a]Strictly, consider the possiblilty of default when $0 \leqslant X < D_L(1+r)$

$$X_A = -\frac{\alpha S_U}{S_U} X + \left(\frac{\alpha S_L}{S_L} \max(X - D_L(1+r), 0) + \alpha \min(D_L(1+r), X) \right) = 0$$

Because when insolvency occurs, shareholders bear limited liability.

Due to its simplicity and intuition, this method of proof has become the standard method for current textbooks. For example, Graham et al. (2009, p. 420), Ross et al. (2015, pp. 495–6), and Berk and DeMarzo (2017, p. 527). The latter two textbooks also take into account the uncertainty, such as Ross et al. (2015) considering three cases of return on asset. By now, countless alternative proofs of the MM Proposition have been provided. Among them are:

- Assuming that a firm alters its capital structure without changing its real earns, Hamada (1969) reproduces MM's results by the CAPM formula.
- Smith (1972) introduces a production function to model the future payoff and proves the MM Proposition by maximizing expected utility.
- Ross (1978) applies the linearity of pricing function to prove the irrelevance.
- Ingersoll (1987) proves the MM Proposition in continuous time settings.

In the above various proofs of MM Proposition, besides the basic assumptions such as the perfect market, the core assumption is that the output of a firm is not affected by its capital structure. That is, the capital structure of the two firms is different but the future output is identical. Many documents express this core assumption in similar words, such as

- Assume that they have identical cash flows, and that one is capitalized with equity alone while the other has both debt and equity outstanding. (Ross, 1988, p. 128)
- Use the assumption that cash flows are unaffected by capital structure. (Stulz, 2000, p. 120)
- Suppose the firm changes its capital structure in such a way that leaves its operating

income unchanged state by state. (Rubinstein, 2003, p. 10)

Therefore, in the proof of the MM Proposition, the core task is to prove: If $X_i = X_j$, then $V_i = V_j$. In today's view, the task is quite simple: that the market is free of arbitrage implies that there is a positive linear pricing function $\wp$, such that[5]

$$V_i = \wp(X_i) = \wp(X_j) = V_j$$

It is clear now that the various arbitrage proofs in the MM Proposition are probably nothing more than repetitive arguments of the linear pricing rule (positivity not needed). The capital structure of a levered company may take any value between unlevered and fully indebted, indicating that for any capital structure, the value of a firm's securities is equal to that of a unlevered firm as long as future returns are the same. Thus, *given the same payoff in the future*, the value of any firm is independent of its capital structure.

Eventually we can point out the logical error of the MM Proposition——the circular justification: ***Since equal return in the future must give rise to the same current value, the assumption that the future return of the firm is irrelevant to the capital structure is equivalent to the assumption that the firm value is irrelevant to the capital structure.*** Bound by the idea of expectation-pricing, scholars at that time did not notice that the asset price is completely determined by its payoff; they were accustomed to finding a proper discount rate for each risky asset for valuation. They subconsciously believed that the internal composition or provider of assets would affect the risk-adjusted discount rates. Thus, they paid too much attention to the so-called risk and did not deem that the prices would be equal even when the payoffs were identical.

For a long time, the opponents of the MM Proposition had been at a disadvantage. Because they had not yet realized that equal future payoffs yield the same current value, Durand (1959) and Rose (1959) and other refutations, failed to find the fatal bug in MM's proofs. Confused by the method of arbitrage, the circular argumentation of MM Proposition was covered. At present, the question before us is how to clarify the original problem. With respect to the assumptions of

[5]At time 1, considering limited liability, $S_{1i} + D_{1i} = X \geqslant 0$, where $S_{1i} = \max(X - D_i(1+r), 0)$, and $D_{1i} = \min(X, D_i(1+r))$. Thus,

$$V_i = S_i + D_i = \wp(S_{1i}) + \wp(D_{1i}) = \wp(S_{1i} + D_{1i}) = \wp(X)$$

Similarly, $V_j = \wp(X)$, we have $V_i = V_j$.

the MM Proposition, can firms with different capital structures produce the same future returns? Is there an optimal capital structure? We will return to these issues in §4.2.

3.3 Arbitrage: Determinacy to Uncertainty

Accompanying the argumentation of the MM Proposition, the concept of arbitrage is becoming clearer. MM1958 (p. 269) explain the term arbitrage as an immediate arbitrage opportunity: "an investor could buy and sell stocks and bonds in such a way as to exchange one income stream for another stream, identical in all relevant respects but selling at a lower price." However, in the MM1958 proof, MM considered a zero-investment portfolio with a positive expected return (payoff) as an arbitrage opportunity. This mathematical expression of arbitrage was then-exotic. Rose (1959) argues that MM's arbitrage operation is admissible only if the stocks in both firms represent the same thing. Since the stock of a levered firm is junior to the debt while the stock of an unlevered firm is subject to no similar debt, they are different investments; thus, the arbitrage operation is inadmissible. Furthermore, the stock of a levered firm has a greater risk than does the stock of an unlevered firm. For this reason, HS1969 improve MM1958 proof such that the variance of Y_1^* is the same as the variance of Y_2. Unfortunately, they believe that the arbitrage is legal only when the risks (variances) are equal. It can be seen that the understanding of arbitrage at that time was still entangled with risks. This may be the reason why MM1969's arbitrage argument is so popular; where an immediate arbitrage opportunity offsets the future payoff and has a lower cost, the concept of risk class is not needed. MM1969's arbitrage argument is more in line with our thinking habits formed in a deterministic world even if an immediate arbitrage opportunity is equivalent to a naïve arbitrage opportunity in current settings. This kind of immediate arbitrage method has been widely adopted since then, and the current corporate finance textbooks almost all use this method to illustrate the arbitrage argument.

Influenced by the traditional law of one price in deterministic worlds or commodity markets, the understanding of arbitrage has long been confined to the meaning of deterministic arbitrage. The requirement of homogeneity in the traditional law of one price hampers our understanding of the price of security in an uncertain environment: the difference in capital structures casts doubt on homogeneity, for the current circumstance is not exactly the same, and the future is random. The argumentation process of the MM Proposition introduces uncertainty into the arbitrage thinking. Although only in the region of deterministic arbitrage, the consideration of uncertainty is a key breakthrough of the arbitrage idea toward the no-arbitrage principle. Facing the uncertain nature of the risky asset, rather than retreating back into the deterministic world by calculating the expected value, the precise mathematical definition of arbitrage is then developed.

The discussion of the MM Proposition opens the door to the world of uncertainty. Subsequently, the arbitrage method boosts the field of asset pricing, the most successful of which is the option pricing formula. The dominance (X is dominant over Y) defined in Merton (1973) is equivalent to the weak-positive property ($X - Y \gneq 0$) of payoffs. Through the dominance argument, Merton exploits many properties of option prices. In this vein, Ross (1977, p. 202) gives a rigorous mathematical definition of arbitrage in the finite-state market setting, and assures that in the absence of arbitrage, a positive state price vector exists. Finally, scholars accept that the arbitrage operation is not without risk (uncertainty), and the arbitrage opportunity is then stated unambiguously. An arbitrage opportunity makes something out of nothing, there is at least one situation profitable, and there is always no loss. Based on Ross (1977), Cox and Ross (1976) summarize the properties of pricing function, such as the linearity and nonnegativity. Shortly, Ross (1978) uses the Hahn-Banach Separation Theorem to prove for the first time that a positive linear pricing function exists when the market is free of arbitrage, which paves the way to the first fundamental theorem of asset pricing.

4 Securitization

As an entity organization, an enterprise has initiative and is able to create value. Obviously, an enterprise is not simply a stack of workers, money, and facilities, but an organic integration such that the whole is larger than the sum of all its components. Therefore, there is a specific value that is holistic, which is called a built-in value. The built-in value comes from the innovation ability and decision-making and management levels of the enterprise as a production organization. It reflects the prospects of the company, especially the value creation ability of the enterprise and the economic outlook for the future.

4.1 Value Creation

In a single-period model, suppose that a firm raises fund $I_0 > 0$ at time 0 and operates with the production function $f(\cdot)$. At time 1,

$$O_1 = f(I_0)$$

where the total output (revenue) O_1 is a random variable. Let $J_0 = \wp(O_1) - I_0$ be the value creation, there is

$$V_0 = I_0 + (J_0 + Z_0) = S_0 + D_0 \tag{4.1}$$

where Z_0 is the built-in value, S_0 is the value of stock, and D_0 is the value of bond. Equation (4.1) links the firm's value with the producer (creator). In this process of securitization, the value creation comes from the following two sources:

1. $Z_0 \geqslant 0$ from the creativity of a production organization
2. $J_0 \geqslant 0$ from production function

For investors, the appreciation of wealth is

$$V_0 - I_0 = J_0 + Z_0 = \wp(O_1) - I_0 + Z_0$$

Please note that if the company does not go bankrupt, according to the institutional arrangements, the creditor does NOT have a share of the cake because the creditor receives a reward of $(1+r)D_0$ at time 1, and $\wp((1+r)D_0) - D_0 = D_0 - D_0 = 0$. Thus, the value-added of the company's securitization is exclusive to all shareholders.

4.2 Capital Structure

If a firm's value is independent of its capital structure, what is the essential reason? It is because the value of securities depends on the overall earning power and operating environment of the company. From the value creation of the companies discussed above, we know that whether the funds are obtained through equity financing or through debt financing, as long as the total amount of capital invested is the same, the output will be the same. Once they are used in production, the funds are indistinguishable. What matters is the total amount of capital, and the specific composition of equity and debt has no effect. If the capital input is fixed, the total output will remain unchanged.

Since the capital structure is irrelevant, is the choice of capital structure arbitrary? Obviously not, as there is indeed an optimal capital structure for shareholders. If there is no need to worry about bankruptcy, the shareholders would try to be fully indebted. Why? We know that the total value-added of a company is exclusively owned by shareholders. So, it is a wise move for the shareholders to maximize the value of the unit investment; in other words, to minimize the amount of their funds. Especially in real economic life, debt has a tax shield, so maximizing debt financing is undoubtedly the best capital structure.

If shareholders will maximize debt financing, then why are not companies in the real world 100% indebted? What factors affect the debt ratio? Why are there stylized facts in the capital structure of industries? We think the main factors affecting the leverage are as follows:

- Financial distress: Debt has a maturity date. If the company encounters difficulties and even goes bankrupt, the company's built-in value will be zero, and the shareholders will lose their money. Therefore, shareholders need to choose a suitable capital structure.
- Industry characteristics: Each industry has its characteristics of production and operation, which determine the growth, the demand for working capital, and the stability of cash flows. The debt ratio is higher when the cash flows are more stable. For example, compared to the manufacturing sector, the utility sector is seen to have higher leverage because the revenues of the utility sector are more stable than that of the manufacturing sector.
- Financing convenience: The market is changing. When investors are not optimistic about the economic outlook, abundant funds enter the bond market. It is more convenient for companies to carry out debt financing at this time. In addition, the new debts are issued to pay back the old debts, and the leverage ratio is self-locking by the rigidity of debt financing. Until a new market environment occurs, a large amount of debts are paid off, and then the capital structure is changed accordingly.
- Agency costs (separation of ownership and control): In extreme examples such as the Chinese stock market, companies have been keen to seize money from the market because managers do not need to be accountable to shareholders. Debts have repayment dates, but moneys raised by stock issuances are not required to be returned.

In the real world, these factors are intertwined and need to be considered as a whole. Among the factors affecting the capital structure, industry characteristics and financing convenience are vital. In the study of capital structures, We believe that it is very much needed to arrive at a theoretical unity and usefulness in explaining empirical data.

4.3 Cost of Equity

The cost of equity refers to the rate of return required on an investment in equity. The cost of equity capital is misleading as it is an illusion: debts have a cost, for they need to pay interests, the money paid for the use of someone else's money. However, the equity capitals are owned and used by the shareholders, and there is no additional expense once the fund is raised. There is also a big difference between a cost and a required return. The required return is not binding, and it is not an opportunity cost. It is never a true cost.

The cost of equity is estimated for the computation of the WACC (weighted average cost of capital). In fact, the WACC is misinforming us; it is neither a cost nor a required return but a weighted average of a cost and a required return. The corporate finance community promotes the WACC as a comprehensive cost of capital to guide investment decisions. This practice is wrong. Suppose that the risk-free rate is 5%, and the required rate of return is 10%. If the current $D/S = 1/4$, then the WACC is 9%. Given a project with a constant rate of return at 6%, it is far below the WACC, but the project is profitable because it is an arbitrage opportunity to finance the project by borrowing money. We see that we cannot take the WACC as a hurdle rate; the decision on a project should be based on the value-added ratio of the project (the total value-added divided by the input, where the value-added ratio for an arbitrage opportunity is infinite). If the value-added ratio of the project is higher than the current level of the enterprise, it shows that the production function of the project is more efficient than the current one. Then, undertaking the project will improve the entire production function of the enterprise.

The WACC method is taught to evaluate a firm by discounting the expected free cash flow (FCF, see Jensen, 1986) at its WACC. This practice is an expectation-pricing, which will violate the law of one price since companies have different WACCs, and the same free cash flow will have different values. Furthermore, when there are multiple risky assets in the market, the expected rate of return is not determined by the asset itself because the price of securities is endogenous and is determined by the overall market. In Example 2.1, the setting of each horse's odds is not completely determined by itself. It is jointly set by the three horses and not by any individual; otherwise, there would be arbitrage opportunities.

It is a long-established principle in finance that the evaluation of an investment should not depend on the way it is funded. The WACC, "the most measured number in finance", is an accompaniment of expectation-pricing. Although the WACC valuation method is prevalent, just as the expectation-pricing method has been abandoned by actuaries for centuries, we firmly believe that the WACC will eventually be abandoned. As a legacy of expectation-pricing, with the skin gone, what can the hair adhere to? Using the WACC to guide investment decisions, and minimizing the WACC for the optimal capital structure, both are following faulty ideologies. As brainchildren of deterministic and isolated thinking, just as the scientific community finally abandoned the "phlogiston" and "aether" dogmas, we must discard the hypothetical cost of equity capital and renounce the postulated WACC as soon as possible.

5 Conclusion

The argumentation of the MM Proposition draws attention to the no-arbitrage analysis in finance theory. The debates of the MM Proposition demonstrate the difficulty of understanding the uncertain world. Our thinking and action often unknowingly fall into the familiar and deterministic world. An arbitrage opportunity makes something out of nothing, when we are not obsessed with a constant something, we accept that the arbitrage operation is not without risk (uncertainty), and we only need to make a weak-positive something out of nothing.

Despite the existence of circular argumentation, the whole argument process of the MM Proposition has developed the no-arbitrage principle. Based on the no-arbitrage pricing method, we clearly see the fallacy of expectation-pricing, especially the improper concept of the WACC. Regarding the study of capital structures, we believe that we cannot deviate from the production attributes of an enterprise and the distribution of its value creation. Although shareholders tend to have more debts, they cannot ignore factors such as industry characteristics and financing convenience.

Perfect market assumptions free us from the intricacies and entanglements in the real financial market. This ideal experimental environment is dustless and sterile, which have become the standard assumptions of finance teaching and research. Perfect market assumptions point out the direction of the development of financial disciplines. The no-arbitrage principle gets rid of the heavy cross of the economic equilibrium theory so that finance has its own methodology. Perfect market assumptions and no-arbitrage principles allow us to recognize the logical consistency of financial theory between asset pricing and corporate finance.

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