Signed Number Base Conversions

Signed integers are formatted such that the leftmost bit is a sign bit (a 1 representing a negative number, and a 0 representing a positive number). The most important thing to know when working with signed numbers is how large the number is (how many bits is it). For example:

11011011_2: is this a positive or negative number?

If the above number is an 8 bit number, then it is negative. But, if the number above is a 16 bit number (without the preceding 0s shown) then the number is a positive number.

To convert signed binary numbers to/from octal or hexadecimal numbers, you use the same procedure outlined previously, and as shown below. To convert between decimal and binary, octal, or hexadecimal, the procedure is a little more complex. Using a conversion between binary and decimal as an intermediary step between octal-decimal and hexadecimal-decimal conversions, simplifies the process and will be show below.

Convert 16 Bit Signed Binary Numbers to Octal or Hexadecimal:

There are two important points to keep in mind when converting Signed Binary to Hex or Octal:

#1. If the binary number is a signed number, then so is the octal or hex number
#2. Hex and octal numbers are just short-hand notations for binary

Convert Signed Binary 1100100110111101_2 to Octal

Solution: As done with unsigned numbers, group the binary digits into groups of 3 and replace with the appropriate octal character (starting at the binary (decimal) point).

1 100 100 110 111 101 = 144675_8

Convert Signed Binary 1100100110111101_2 to Hexadecimal

Solution: As done with unsigned numbers, group the binary digits into groups of 4 and replace with the appropriate hex character (starting at the binary (decimal) point).

1100 1001 1011 1101 = $C9BD_x$

Convert Signed Octal 1372_8 to a 16 Bit Binary Number

Solution: As done with unsigned numbers, replace with the octal characters with the appropriate group the binary digits. Then fill in any preceding 0s. You do not always have to fill in any preceding 0s, but without them (or knowing about them) you may not be able to tell if the number is positive or negative

1372_8 = 1 011 111 010 = 0000001011111010_2 which is a positive number

Convert Signed Hexadecimal $3FBD_x$ to a 16 Bit Binary Number

Solution: As done with unsigned numbers, replace with the hex characters with the appropriate group the binary digits. Then fill in any preceding 0s. You do not always have to fill in any preceding 0s, but without them (or knowing about them) you may not be able to tell if the number is positive or negative

$3FBD_x$ = 0011 1111 1011 1101 = 0011111110111101_2 which is a positive number

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Convert an 8 Bit Signed Binary Number to Decimal

Convert Signed Binary 10111101_2 to Decimal

Solution:

If the sign bit is a 0, the conversion is the same as an unsigned integer.

If the sign is a 1, we must first find the magnitude of the number. To do so, the number must be negated by executing the two's complement operation. Note that the sign bit is included in the operations performed on the number:

```
1 0111101      Original Value (sign bit separated for clarification)
```

```
0 1000010      One's Complement
       +1      Add 1
0 1000011      Result.
```

$01000011_2 = 67_{10}$ So, because the original number was a negative number, we have to remember that as a part of the result. Therefore, $10111101_2 = -67_{10}$.

Note: A simple way to perform the 2's complement operation is to scan the bits from right to left, keeping all of the digits up to and including the first logical 1. Then, change each bit.

```
1 0111101      Original Value
        1      Keep the first 1 that is found
0 100001       Toggle the remaining bits
0 1000011      Result.
```

Convert Signed Decimal -102_{10} to Binary

Solution:

If the sign bit is a 0, the conversion is the same as an unsigned integer.

If the sign bit is a 1, the number is first converted as an unsigned integer, and then a 2's complement operation is performed on the result in order to generate a negative answer.

-102 Original Value in decimal, so we will convert +102 to binary using the methods we have already discussed

$$\frac{102}{2} = 51 \quad \text{remainder} = 0$$

$$\frac{51}{2} = 25 \quad \text{remainder} = 1$$

$$\frac{25}{2} = 12 \quad \text{remainder} = 1$$

$$\frac{12}{2} = 6 \quad \text{remainder} = 0$$

$$\frac{6}{2} = 3 \quad \text{remainder} = 0$$

$$\frac{3}{2} = 1 \quad \text{remainder} = 1$$

$$\frac{1}{2} = 0 \quad \text{remainder} = 1$$

1 1 0 0 1 1 0

01100110_2 Binary Equivalent of +102. A preceding 0 had to be added to make it an 8 bit number. We must now take the two's compliment of this number because the original number (-102) was negative.

10011010_2 Two's Complement, and final answer

Signed Decimal-to-Octal, Decimal-to-Hex, Octal-to-Decimal, or Hexadecimal-to-Decimal

To convert positive numbers between number systems, we would perform the same procedures already outlined in the unsigned conversion procedures. To convert negative numbers between binary, octal, or hex to/from decimal is another story. Above, you saw that converting positive or negative numbers from binary to either hex or octal was the same procedure as converting positive numbers. But, converting negative numbers from decimal to binary, or binary to decimal, you must first find the magnitude of the number (convert it to a positive number), and then convert it to the target number system. keeping in mind that the original number is negative, and make the appropriate changes at the end.

Because conversions between binary and hex or octal are so easy, I recommend a two step process when converting negative numbers to/from decimal and octal or hex. I convert the original negative number to binary first, and then convert it to the target number system. For example:

Convert a 16 bit Signed Octal 144675_8 to Decimal

Solution: Convert 144675_8 to binary first:

144675_8 = 1 100 100 110 111 101_2 This number is negative, because the MSb (Most Significant bit) is 1. We must now take the two's compliment to find the magnitude of the number:

1100100110111101	original
0011011001000010	1s compliment
+1	
0011011001000011	2s compliment

We can now convert to decimal:

decimal = $0*2^{15} + 0*2^{14} + 1*2^{13} + 1*2^{12} + 0*2^{11} + 1*2^{10} + 1*2^{9} + 0*2^{8} + 0*2^{7} + 1*2^{6} + 0*2^{5} + 0*2^{4} + 0*2^{3} + 0*2^{2} + 1*2^{1} + 1*2^{0}$

decimal = 0 + 0 + 8192 + 4096 + 0 + 1024 + 512 + 0 + 0 + 64 + 0 + 0 + 0 + 0 + 2 + 1 = 13891 But, the original number was negative, so don't forget the negative sign

decimal = -13891_{10}

Convert a 16 bit Signed Hex $C9BD_x$ to Decimal

Solution: Convert $C9BD_x$ to binary first:

$C9BD_x$ = 1100 1001 1011 1101_2 This number is negative, because the MSb (Most Significant bit) is 1. We must now take the two's compliment to find the magnitude of the number: You will note that this is the same number as above, so our results are same (Wow! that doesn't happen very often!)

1100100110111101	original
0011011001000010	1s compliment
+1	
0011011001000011	2s compliment

We can now convert to decimal:

decimal = $0*2^{15} + 0*2^{14} + 1*2^{13} + 1*2^{12} + 0*2^{11} + 1*2^{10} + 1*2^{9} + 0*2^{8} + 0*2^{7} + 1*2^{6} + 0*2^{5} + 0*2^{4} + 0*2^{3} + 0*2^{2} + 1*2^{1} + 1*2^{0}$

decimal = 0 + 0 + 8192 + 4096 + 0 + 1024 + 512 + 0 + 0 + 64 + 0 + 0 + 0 + 0 + 2 + 1 = 13891 But, the original number was negative, so don't forget the negative sign

decimal = -13891_{10}

The procedure to convert negative decimal numbers to binary, octal, or hex is the same, but in reverse.

#1. Remove the negative sign (but don't forget about it)
#2. Convert the positive decimal number to binary
#3. Take the two's compliment of the binary number (to change it to negative
#4. Convert the negative binary number to octal or hex.