Phase Geometry Series — Part III
Phase-Field Newtonian Gravity and Phase Clocks

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Abstract

We propose a simple phase-field model that reproduces standard Newtonian gravity while tying the gravitational potential to the gradient energy of a static phase field. The construction is deliberately modest: it is non-relativistic, static, and restricted to the weak-field regime, but it is mathematically consistent and compatible with the usual Newtonian picture. Any static phase configuration $\varphi(\mathbf{x})$ generates an effective mass density $\rho_{\text{eff}}(\mathbf{x}) \propto |\nabla\varphi(\mathbf{x})|^2$, which acts as a source for the Newtonian potential $\Phi(\mathbf{x})$ through the Poisson equation. Localised phase defects then behave as effective gravitating masses with a $1/r$ far-field potential.

We further connect this phase-field gravity to phase-based clocks (Josephson clocks) by embedding the potential $\Phi(\mathbf{x})$ into the weak-field relation for the metric component g_{00} and the corresponding gravitational redshift of clock frequencies. In this way, the same phase field that sources the gravitational potential also controls the local ticking rate of phase clocks. The result is a simple and transparent "Newtonian floor" for phase geometry: standard Newtonian dynamics and weak-field time dilation expressed entirely in terms of a static phase texture.

The phase-field coupling constant κ introduced here is a phenomenological parameter that measures how strongly phase-gradient energy sources an effective Newtonian potential. It is *distinct from* the dimensionless phase–flux coefficient $\alpha(d, \omega)$ used in Parts I and I-B of the series for superconducting phase rotators.

Contents

1 Introduction

The idea that geometry and phase are connected appears in many parts of physics: in superconductivity and superfluidity, in Berry phases, and in analogue gravity. In this series of works on phase geometry we explore this connection systematically. Part I discussed resonant field symmetry in superconductors, and Part II introduced phase-based clocks (Josephson clocks) in weak gravitational fields using a five-dimensional phase-fibre geometry.

This work is Part III of the Phase Geometry Series: *Superconductivity and Weak Gravity*, which develops a unified phase-based view of resonant field symmetry, clocks and gravity. The conceptual foundations are outlined in the Series Overview, while Part I discusses resonant field symmetry and Part I-B develops a calculational framework for phase rotators and the phase–flux coefficient $\alpha(d, \omega)$ in thick SNS weak links. Part II then introduces phase clocks (Josephson clocks) in weak gravitational fields.

Here, in Part III, we focus on a phenomenological phase-field model of Newtonian gravity and its coupling to phase clocks. We deliberately stay away from strong fields and full general relativity. Instead, we construct a phase-field Newtonian gravity in which:

- a static phase field $\varphi(\mathbf{x})$ carries gradient energy;
- this energy density plays the role of an effective mass density $\rho_{\text{eff}}(\mathbf{x})$;
- the Newtonian potential $\Phi(\mathbf{x})$ obeys the usual Poisson equation with ρ_{eff} as a source;
- test particles move in $\Phi(\mathbf{x})$ according to standard Newtonian mechanics;
- phase clocks (such as Josephson clocks) experience gravitational redshift determined by $\Phi(\mathbf{x})$ in the weak-field limit.

The outcome is a minimal, internally consistent model: it does not compete with general relativity, but it provides a clear "playground" where both gravity and clock rates are determined by a single static phase field.

2 Phase field and effective mass density

We consider a real scalar phase field $\varphi(\mathbf{x})$ defined on three-dimensional space. In this work we restrict to static configurations, so φ depends only on position $\mathbf{x}$, not on time.

The basic assumption is that the gradient energy of φ acts as a source of gravity. We introduce an effective mass density

$$\rho_{\text{eff}}(\mathbf{x}) = \frac{\kappa}{c^2}\, |\nabla\varphi(\mathbf{x})|^2, \tag{2.1}$$

where c is the speed of light (used to convert energy density to mass density) and κ is a phenomenological coupling constant. The quantity $|\nabla\varphi|^2 = \nabla\varphi \cdot \nabla\varphi$ is the squared gradient of the phase.

We then postulate that the Newtonian potential $\Phi(\mathbf{x})$ satisfies the standard Poisson equation

$$\nabla^2\Phi(\mathbf{x}) = 4\pi G\, \rho_{\text{eff}}(\mathbf{x}), \tag{2.2}$$

with G the Newtonian gravitational constant and ∇^2 the Laplacian in three-dimensional Euclidean space.

Equation (2.2) is structurally identical to the usual Newtonian field equation for gravity, with ρ_{eff} playing the role of the mass density. The novelty is only in the definition (2.1) of ρ_{eff} in terms of the phase-field gradient. The parameter κ controls how strongly a given phase-texture sources the effective Newtonian potential.

3 General solutions and superposition

Given $\rho_{\text{eff}}(\mathbf{x})$, the solution to the Poisson equation (2.2) with boundary condition $\Phi(\mathbf{x}) \to 0$ as $|\mathbf{x}| \to \infty$ is the standard Newtonian potential

$$\Phi(\mathbf{x}) = -G \int \frac{\rho_{\text{eff}}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}\, d^3x'. \tag{3.1}$$

Using (2.1), we can write this directly as a functional of the phase field:

$$\Phi(\mathbf{x}) = -\frac{G\kappa}{c^2} \int \frac{|\nabla\varphi(\mathbf{x}')|^2}{|\mathbf{x} - \mathbf{x}'|}\, d^3x'. \tag{3.2}$$

This expression states that the gravitational potential at point $\mathbf{x}$ is determined by the full spatial distribution of the phase-gradient energy.

Although ρ_{eff} is quadratic in $\nabla\varphi$ and therefore nonlinear in φ, the Poisson equation is linear in Φ. As a result, superposition holds at the level of the potential. If we decompose the phase field into a sum of contributions,

$$\varphi(\mathbf{x}) \approx \sum_i \varphi_i(\mathbf{x}), \tag{3.3}$$

then

$$\rho_{\text{eff}}(\mathbf{x}) = \frac{\kappa}{c^2}\, |\nabla\varphi(\mathbf{x})|^2 = \frac{\kappa}{c^2} \left|\sum_i \nabla\varphi_i(\mathbf{x})\right|^2, \tag{3.4}$$

and the individual contributions to $\Phi(\mathbf{x})$ can still be superposed via (3.1), even though ρ_{eff} is not simply a sum of separate densities.

4 Effective mass of a localized phase defect

We are particularly interested in localized phase structures, or *phase defects*, whose gradient energy is concentrated in a finite region of space. For such a defect, the effective mass is defined as

$$M_{\text{eff}} = \int \rho_{\text{eff}}(\mathbf{x})\, d^3x = \frac{\kappa}{c^2} \int |\nabla\varphi(\mathbf{x})|^2\, d^3x. \tag{4.1}$$

If the defect is localized such that ρ_{eff} vanishes sufficiently quickly at large $|\mathbf{x}|$, then for distances $r = |\mathbf{x}|$ much larger than the size of the defect, the potential takes the familiar form

$$\Phi(r) \approx -\frac{GM_{\text{eff}}}{r}, \tag{4.2}$$

up to corrections of order R/r, where R is a characteristic size of the defect. Thus, a localized phase defect behaves as an effective gravitating mass, with M_{eff} determined by the integrated phase-gradient energy.

In this sense, the phase-field model provides a concrete realisation of the idea that phase textures can mimic point-like gravitating objects at long distances, while retaining a smooth internal structure.

5 Cylindrical phase defect: qualitative example

To illustrate the construction, consider a simple cylindrical defect centred on the z-axis. In cylindrical coordinates (r, θ, z) we assume translational symmetry along z and rotational symmetry in θ, so that φ depends only on r:

$$\varphi = \varphi(r). \tag{5.1}$$

We further assume that $\varphi(r)$ interpolates between two constant values over a radial scale R, with negligible gradients outside a finite region.

The gradient of the phase is radial, and we have

$$|\nabla\varphi|^2 = \left(\frac{d\varphi}{dr}\right)^2. \tag{5.2}$$

The effective mass per unit length along the z-axis is then

$$\frac{M_{\text{eff}}}{L_z} = \int \rho_{\text{eff}}(\mathbf{x})\, d^2x_\perp = \frac{\kappa}{c^2}\int \left(\frac{d\varphi}{dr}\right)^2 2\pi r\, dr, \tag{5.3}$$

where the integral is over the transverse plane. For a simple profile with characteristic gradient $|\mathrm{d}\varphi/\mathrm{d}r| \sim \Delta\varphi/R$ over a radial shell of width $\sim R$, we obtain the order-of-magnitude estimate

$$\frac{M_{\text{eff}}}{L_z} \sim \frac{\kappa}{c^2}\, 2\pi R \left(\frac{\Delta\varphi}{R}\right)^2 R \sim \frac{2\pi\kappa}{c^2}\,\frac{(\Delta\varphi)^2}{R}. \tag{5.4}$$

Thus, for fixed phase contrast $\Delta\varphi$, narrower defects carry more effective mass per unit length, reflecting the higher concentration of gradient energy.

This cylindrical example is only qualitative, but it shows how specific phase profiles map to effective mass distributions in the present framework.

6 Static Lagrangian formulation

The phase-field model of Newtonian gravity can be encoded in a static Lagrangian (or, more precisely, an energy functional). We consider a functional of the fields $\varphi(\mathbf{x})$ and $\Phi(\mathbf{x})$:

$$\mathcal{E}[\varphi,\Phi] = \int d^3x \left[\frac{\kappa}{2c^2}|\nabla\varphi|^2 + \frac{1}{8\pi G}|\nabla\Phi|^2\right]. \tag{6.1}$$

The first term represents the gradient energy of the phase field, while the second is the usual field-energy of the Newtonian potential.

To impose the Poisson relation between Φ and ρ_{eff}, one can either:

- treat (6.1) as a functional of Φ only and use the standard variational principle $\delta\mathcal{E}/\delta\Phi = 0$ to obtain $\nabla^2\Phi = 0$ with sources encoded in boundary conditions; or
- introduce a Lagrange multiplier field to enforce the constraint (2.2) explicitly.

For the purposes of this phenomenological model, it is usually simpler to regard (2.2) as the defining equation for Φ in terms of φ, with (6.1) providing a useful way to think about the total energy stored in both the phase field and the gravitational potential.

7 Test-particle dynamics

Once the potential $\Phi(\mathbf{x})$ is known, the motion of a classical test particle of mass m is governed by the usual Newtonian dynamics. The Lagrangian for a test particle with position $\mathbf{x}(t)$ is

$$L_{\text{particle}} = \frac{1}{2}m\dot{\mathbf{x}}^2 - m\Phi(\mathbf{x}), \tag{7.1}$$

and the Euler–Lagrange equations yield

$$m\ddot{\mathbf{x}} = -m\nabla\Phi(\mathbf{x}), \tag{7.2}$$

or simply

$$\ddot{\mathbf{x}} = -\nabla\Phi(\mathbf{x}). \tag{7.3}$$

Thus, test particles move as they would in classical Newtonian gravity, with the only twist being that the potential $\Phi(\mathbf{x})$ is itself sourced by the phase-gradient energy according to (2.1) and (2.2).

8 Phase clocks and Josephson-based superconducting oscillators

In Part II of the Phase Geometry Series, phase-based clocks were described within a five-dimensional phase-fibre geometry. There the key result was a phase-rate law of the form

$$\frac{d\phi}{dt} = \omega_0 \sqrt{g_{00}(\mathbf{x})}, \tag{8.1}$$

where g_{00} is the time-time component of the metric, ω_0 is a reference frequency, and ϕ is an internal phase variable. In the weak-field limit, with

$$g_{00}(\mathbf{x}) \simeq 1 + \frac{2\Phi(\mathbf{x})}{c^2}, \tag{8.2}$$

this becomes

$$\frac{d\phi}{dt} \simeq \omega_0 \left(1 + \frac{\Phi(\mathbf{x})}{c^2}\right), \tag{8.3}$$

which reproduces the familiar gravitational redshift: clocks deeper in the potential well tick more slowly.

In the present phase-field Newtonian model, we take $\Phi(\mathbf{x})$ to be the potential generated by the phase-gradient energy of $\varphi(\mathbf{x})$. Phase-based clocks (including Josephson clocks) placed at different positions in this potential therefore experience different ticking rates

$$\omega(\mathbf{x}) \simeq \omega_0 \left(1 + \frac{\Phi(\mathbf{x})}{c^2}\right), \tag{8.4}$$

with the fractional frequency shift

$$\frac{\Delta\omega}{\omega_0} \simeq \frac{\Delta\Phi}{c^2}, \tag{8.5}$$

where $\Delta\Phi$ is the potential difference between two locations.

The superconducting implementations developed in Parts I and I-B provide concrete examples of phase clocks: Josephson oscillators and phase rotators whose frequencies and phases can, in principle, be measured with high precision. In that context, the present Part III can be viewed as a phenomenological model for how engineered phase textures could mimic weak gravitational wells and redshift these clock frequencies.

9 Limits and domain of validity

The phase-field Newtonian model constructed here is deliberately modest in scope. Its domain of validity is restricted by several assumptions:

- **Static phase field.** The phase field $\varphi(\mathbf{x})$ is assumed to be time-independent. Allowing $\varphi(\mathbf{x}, t)$ to vary in time would introduce additional terms in both the effective energy density and the metric, and is left for future work.
- **Weak-field regime.** The use of the Newtonian potential $\Phi(\mathbf{x})$ and the weak-field approximation for g_{00} restrict the model to situations where $|\Phi|/c^2 \ll 1$.
- **Non-relativistic test particles.** Test particles are treated within non-relativistic mechanics. Relativistic corrections can be incorporated, but are not considered here.
- **Phenomenological coupling.** The parameter κ is phenomenological: it is not derived from an underlying microscopic theory, and may in principle depend on the physical system in which the phase field is realised. It is also independent of the superconducting phase–flux coefficient $\alpha(d, \omega)$ used in Parts I and I-B.

Within these limits, the Newtonian sector of the phase-field model is well-defined and internally consistent: any static phase texture $\varphi(\mathbf{x})$ yields an effective mass density $\rho_{\mathrm{eff}}(\mathbf{x})$, a Newtonian potential $\Phi(\mathbf{x})$ with the usual $1/r$ far-field behaviour for localised defects, and standard test-particle dynamics and clock redshift in that potential.

10 Discussion and outlook

We have constructed a simple phase-field model of Newtonian gravity in which:

- the effective mass density is given by the gradient energy of a static phase field, $\rho_{\mathrm{eff}} = (\kappa/c^2)|\nabla\varphi|^2$;
- the Newtonian potential obeys the usual Poisson equation with this source;
- localised phase defects carry a finite effective mass M_{eff} and generate a $1/r$ far-field potential $\Phi(r) = -GM_{\mathrm{eff}}/r$;
- test particles move as they would in classical Newtonian gravity;
- phase-based clocks experience gravitational redshift determined by the same potential $\Phi(\mathbf{x})$, which is itself a functional of the phase texture.

On this basis, one can say that the "Newtonian floor" of phase geometry is complete: the standard physics of slow bodies and weak-field time dilation has been re-expressed in terms of a single static phase field. The coupling constant κ encapsulates how strongly phase-gradient energy is allowed to act as an effective source for Newtonian curvature.

Although deliberately modest in scope, the present phase-field construction has several useful consequences. First, it provides a single phase-based description of effective mass, Newtonian potential, and clock rates: the same static phase texture determines the effective mass density, the gravitational potential $\Phi(\mathbf{x})$, and the local ticking of phase clocks via $g_{00}(\mathbf{x})$. This offers a simple and conceptually transparent way to view weak gravity and time dilation through the lens of phase geometry.

Second, the model can serve as a theoretical basis for analogue gravity systems in superconductors and other phase-ordered media. By engineering phase textures (defects, arrays, patterned structures), one can in principle design effective gravitational potentials and study the motion of quasiparticles and clocks in those backgrounds, including lensing-like effects and potential horizon analogues.

Third, the static Newtonian framework developed here provides a starting point for dynamical generalisations. Allowing the phase field $\varphi(\mathbf{x}, t)$ to become time dependent would induce fluctuations of the effective potential and g_{00}, opening a route towards phase-field analogues of gravitational waves and other time-dependent gravitational phenomena in controllable laboratory settings.

In this sense the phase-field construction is best viewed as a convenient playground for toy and analogue models of gravity, rather than as a competitor to general relativity. Its Newtonian, phase-based formulation makes it particularly suitable for designing and analysing controllable "mini-gravity" scenarios in superconductors and other phase-ordered media where the underlying phase field is directly accessible and, at least in principle, tunable.

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References

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