The University of Texas at Arlington Department of Mechanical and Aerospace Engineering

MAE4314

Mechanical Vibrations

Final Project – Fall 2022

Cart and Pendulum System

&

Self-Defined Project:

Three Masses Spring and Damper System

Submit to:

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ABSTRACT:

I: Applying Newton's and Lagrange's methods to analyze a Cart and Pendulum System, including deriving equations of motion, natural frequencies and mode shapes, linearization of the equations, calculating and plotting the free vibration response under various initial conditions, FRFs, developing the MATLAB codes to solve the equations numerically.

II: Using the same approach in part I for a three-spring damper mass system which can model a proposed concept of a quarter car suspension system incorporates the mass of the driver in the seat.

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List of Symbols

Greek Symbols:

ψ	Mode shape

Roman Symbols:

C	Damper coefficient
i	Horizontal direction
j	Vertical direction
k	Spring stiffness
N	Normal force
M	Mass of the cart
m	Mass of the pendulum
g	Gravitational constant
a	Acceleration
v	Velocity
x	Position

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I. Cart and Pendulum System (CPS)

The objective is to apply Newton's and Lagrange's methods to analyze a Cart and Pendulum System, including deriving equations of motion, natural frequencies and mode shapes, linearization of the equations, calculating and plotting the free vibration response under various initial conditions, FRFs, developing the MATLAB codes to solve the equations numerically.

Setup - Cart and Pendulum System (CPS)

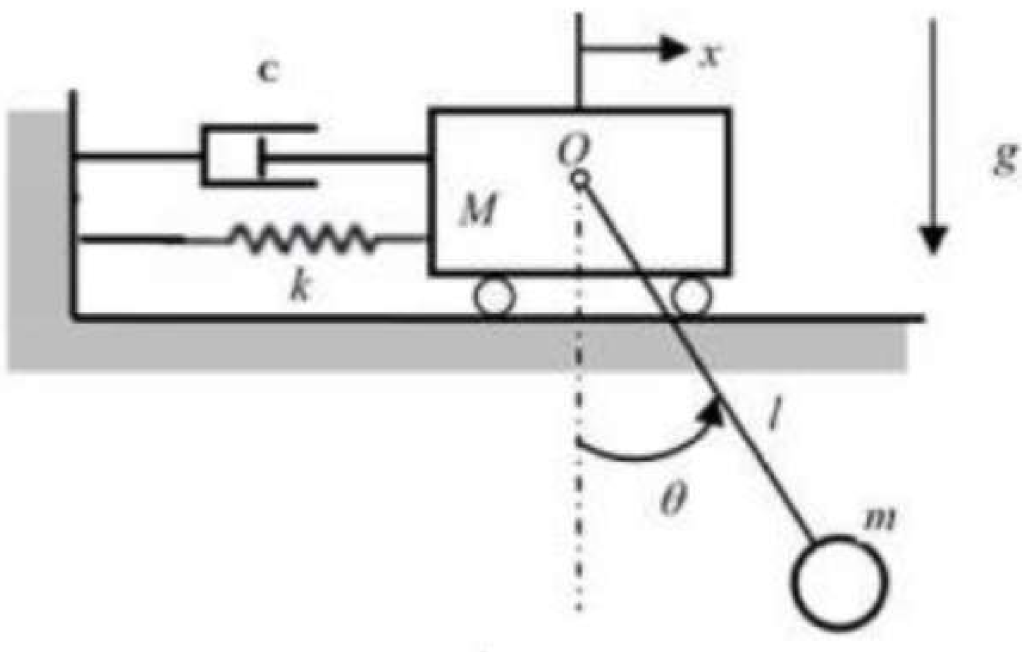

Figure 1. Cart and Pendulum System.

A cart is moving along the horizontal x-axis in this simulation. A pendulum hangs from the cart. The cart and wall are joined by a spring. The cart experiences a damping force.

Procedure and Results - Cart and Pendulum System (CPS)

Equation of motion by Newton's Method

A body's location, speed, or acceleration in relation to a specified frame of reference can be expressed mathematically using an equation of motion. The fundamental equation of motion in classical mechanics is Newton's second law, which says that the force F acting on a body is equal to that body's mass m times its center of mass's acceleration a, or F = ma. It is theoretically possible to deduce the velocity and position of the body as functions of time from Newton's equation using a technique known as integration if the force acting on the body is known to be a function of time.

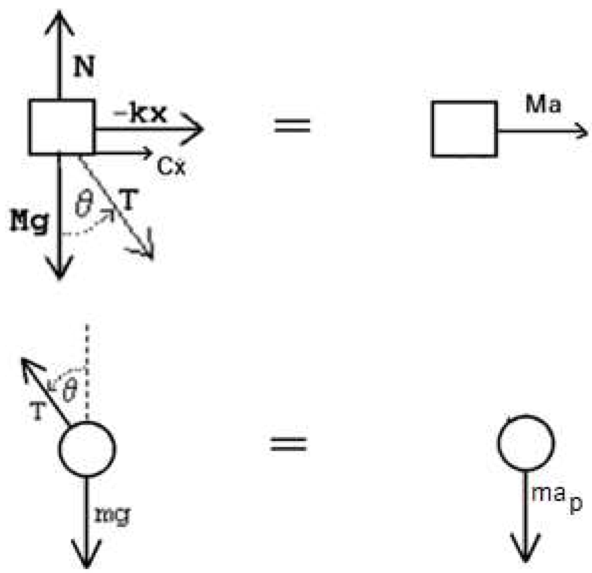

Figure 2. Free Body Diagram of the CPS System.

The kinematics for the pendulum

$$x_p = xi + l\sin\theta i - l\cos\theta j$$

$$v_p = \dot{x}i + l\dot{\theta}\cos\theta i + R\dot{\theta}\sin\theta j$$

$$a_p = \ddot{x}i + l\ddot{\theta}\cos\theta i - l\dot{\theta}2\sin\theta i + l\ddot{\theta}\sin\theta j + l\dot{\theta}2\cos\theta j$$

Apply the equilibrium of forces:

$$\mathrm{Ma} = Nj - Mgj - T\cos\theta j + T\sin\theta i - kxi - C\dot{x}i \tag{1}$$

$$T\cos\theta j - T\sin\theta i - mgj \ = ma_P \tag{2}$$

Rearrange equations (1) and (2) which gives 4 simultaneous equations.

$$T\sin\theta - kx - C\dot{x} = M\ddot{x} \tag{3}$$

$$N - Mg - T\cos\theta = 0 \tag{4}$$

$$-T\sin\theta = m\left(\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}2\sin\theta\right) \tag{5}$$

$$T\cos\theta = m(l\ddot{\theta}\sin\theta + l\dot{\theta}2\cos\theta) \tag{6}$$

Rearrange equations (3), (4), (5), (6) and which gives equations of motion.

$$(M+m)\ddot{x} = ml\dot{\theta}^2\sin\theta - \mathrm{ml}\ddot{\theta}\cos\theta - \mathrm{kx} - C\dot{x} \tag{7}$$

$$m\ddot{x}\cos\theta + ml\ddot{\theta} + mg\sin\theta = 0 \tag{8}$$

Equation of motion by Lagrange's Method

The energy in the system is used in Lagrange's mechanics. The Lagrangian, a function that encapsulates the dynamics of the entire system, is the primary quantity in Lagrangian mechanics. The Lagrangian has energy units overall, but there is no universal formulation for all physical systems.

,

$$\frac{d}{\mathrm{dt}}\left(\frac{dT}{d\dot{q}_i}\right) - \frac{dT}{dq_i} + \frac{dU}{dq_i} + \frac{dD}{dq_i} = F_i$$

$$T = \frac{1}{2}M(\dot{x}i)^2 + \frac{1}{2}m(\dot{x}i + l\dot{\theta}\cos\theta i + l\dot{\theta}\sin\theta j)^2$$

$$V = \frac{1}{2}(kx^2) + mg(l - l\cos\theta)$$

$$D = \frac{1}{2} C\dot{x}$$

For qi = x Lagrange's Equation:
For the qi = θ variable

$$(M+m)\ddot{x} = ml\dot{\theta}^2 \sin\theta - \text{ml}\ddot{\theta}\cos\theta - \text{kx} - C\dot{x}$$

$$m\ddot{x}\cos\theta + ml\ddot{\theta} + mg\sin\theta = 0$$

Linearization of Equations of motion

sin θ =θ
cos θ =1
Let

$$(M+m)\ddot{x} = ml\dot{\theta}^2\theta - \text{ml}\ddot{\theta} - \text{kx} - C\dot{x}$$

$$m\ddot{x} + \text{ml}\ddot{\theta} + mg\theta = 0$$

$$\begin{bmatrix} M+m & ml \\ m & ml \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} C & -ml\dot{\theta} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & mg \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

<u>Data Analysis, Interpretation, and Discussion – Cart and Pendulum System (CPS)</u>

Natural frequencies and mode shapes

For an MDOF system, the natural frequencies and the mode shapes are two of the most crucial characteristics of the system response. The eigenvalue issue is written out in the equations below to be solved for the natural frequencies. The characteristic equation, which has four solutions, is determined by computing the determinant.

$$\begin{bmatrix} (M+m)s^2 + k & \text{ml}s^2 \\ ms^2 & \text{ml}s^2 + \text{mg} \end{bmatrix} = [0]$$

$$((M+m)s^2 + k)(\text{ml}s^2 + \text{mg}) - (\text{ml}s^2)(ms^2) = 0$$

$$\lambda = \pm \frac{\sqrt{-\frac{kl+gm+gM \pm \sqrt{-4gklM + (kl+g(m+M))^2}}{lM}}}{\sqrt{2}} \quad \text{(rad/s)}$$

$$\{\psi\} = \begin{bmatrix} 1 \\ \frac{k+(m+M)\lambda^2}{lm\lambda^2} \end{bmatrix}$$

An eigenvalue problem, also known as a characteristic value problem, can be expressed using the equations above. The numbers are the eigenvalues, or characteristic values, and the accompanying displacement vectors express the eigenvectors, or mode shape which is the distortion that a component would experience when vibrating at its native frequency is known as a mode shape.

Calculate and plot the free vibration response

The free vibration response can be solved by MATLAB ode45 function. The Runge-Kutta 4-5 solving method is used by the MATLAB ode45 function to numerically discover the system's solution. The derivatives for input of ode45 function below are developed by the previous EOMs.

$$\ddot{X} = \frac{ml\dot{\theta}2\sin\theta + mg\sin\theta\cos\theta - kx - C\dot{x}}{M + m\sin^2\theta}$$

$$\ddot{\theta} = \frac{-ml\dot{\theta}2\sin\theta\cos\theta - (m + M)g\sin\theta + kx\cos\theta + C\dot{x}\cos\theta}{l(M + m\sin^2\theta)}$$

$$x_1 = x \Rightarrow \dot{x_1} = x_3$$

$$x_2 = \theta \Rightarrow \dot{x_2} = x_4$$

$$x_3 = \dot{x} \Rightarrow \dot{x_3} = \frac{ml{x_2}^2\sin x_2 + mg\sin x_2\cos x_2 - kx - Cx_3}{M + m\sin^2 x_2}$$

$$x_4 = \dot{\theta} \Rightarrow \dot{x_4} = \frac{-mlx_4 2\sin x_2\cos x_2 - (m + M)g\sin x_2 + kx_1\cos x_2 + C\dot{x}\cos x_2}{l(M + m\sin^2 x_2)}$$

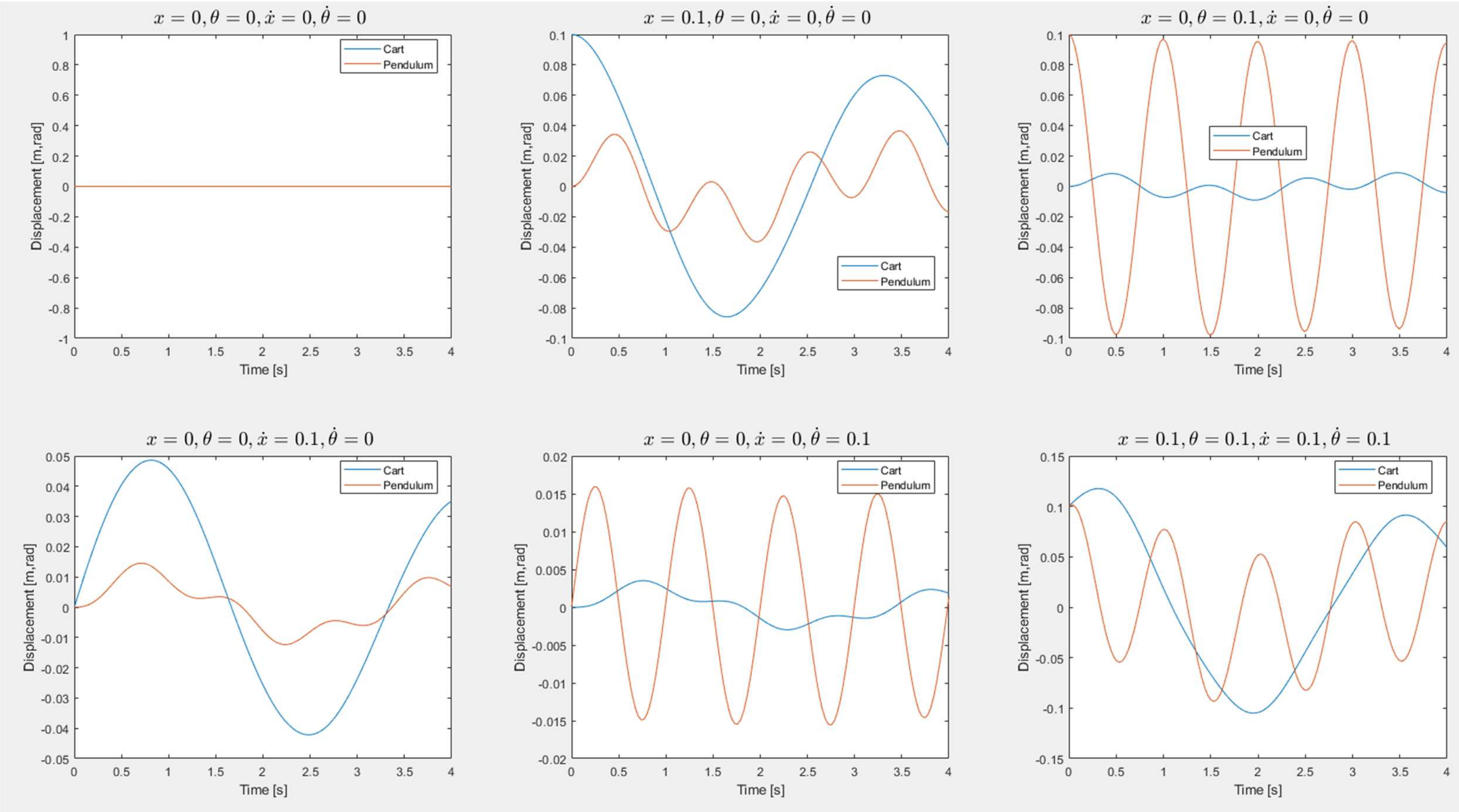

Figure 3. Free Vibration Responses for different Initial Conditions.

Figure 3 shows the free vibration responses for the system for cart mass = 10 kg, pendulum mass = 2 kg, spring constant k = 40 N/m g=9.81 m/s^2, damping factor C = 2 Ns/m and length l=0.5m. The full MATLAB code can be found in Appendix under CPS code.

Calculate and plot FRFs

$$H(\omega) = \frac{1}{-\omega^2 [M] + j\omega[C] + [k]}$$

With previously calculated [M], [C], [k], the free Frequency response (FRFs) for the system can be computed and plot by MATLAB.

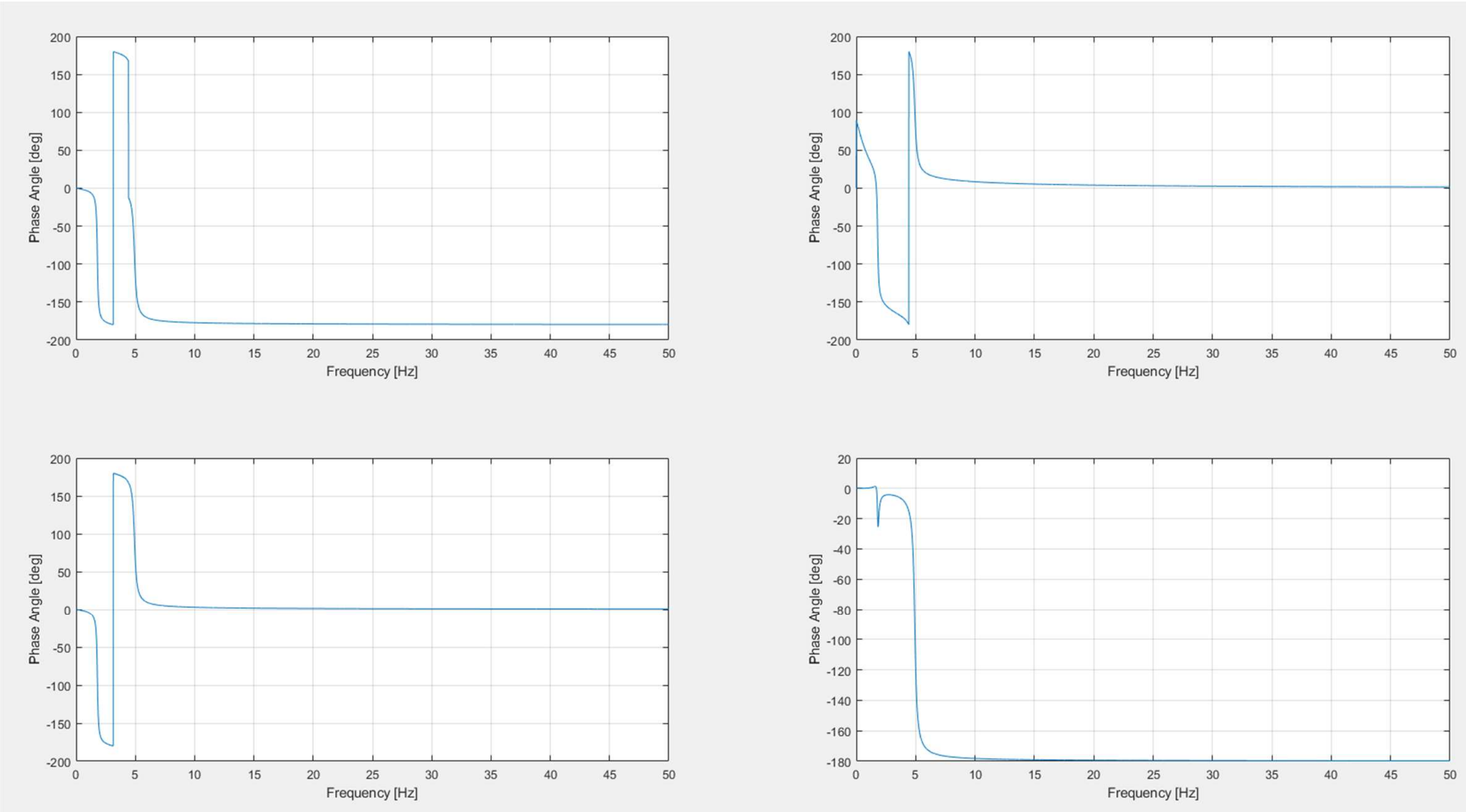

Figure 4. Plot Frequency Response Phase Angle.

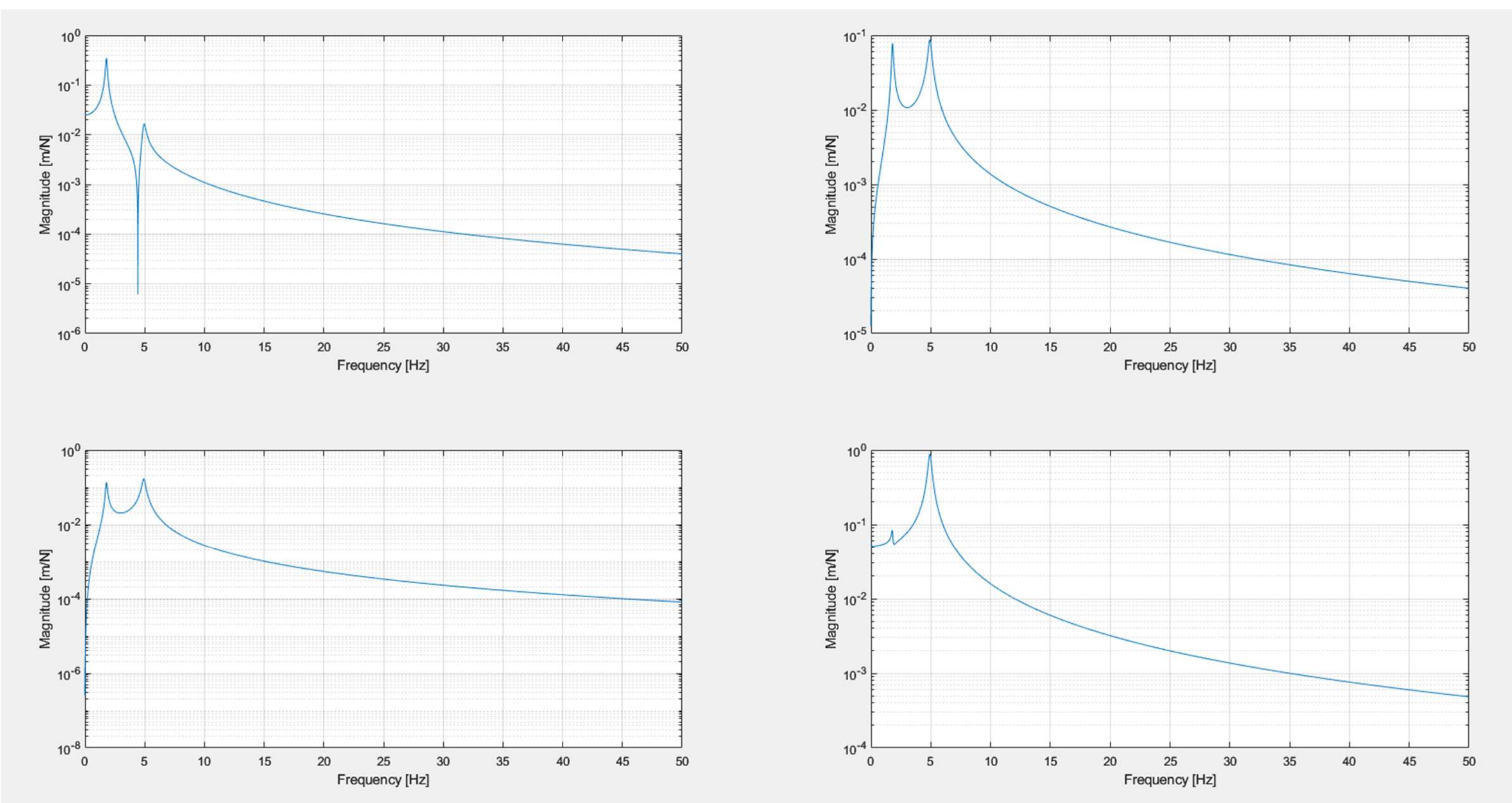

Figure 5. Plot Frequency Response Magnitude.

From figure 5, 2 peaks show the magnitude of the frequency response of 2 degree of freedom system.

Conclusions – Cart and Pendulum System (CPS)

The analysis of cart and pendulum is performed for modeling, simulation, control, and optimization of complicated physical systems is provided by using mathematical analysis and numerical solution. In conclusion, we have solved the CPS using both Newton's and Lagrange's Methods, allowing us to linearize the equations of motion and solve for the natural frequencies and modes of vibration. We have also obtained the Free Vibration Response plots and Frequency Response Function plots for a set of initial conditions.

II. Three Masses Spring and Damper System

The objective for problem 2 is to apply Newton's and Lagrange's method to analyze a Three Mass Spring and Damper System (TMSDS) to find the system solution. We have selected this system because it provides an opportunity for complex analysis and has real word application because it has similarities to real car suspension systems. Our analysis procedure is similar to that in problem 1, such that it includes deriving equations of motion, followed by the most important features of a vibration system, which are the natural frequencies and mode shapes. We also aim to achieve the linearization of the equations, calculating and plotting the free vibration response under various initial conditions, Frequency Response Functions (FRFs), and programming the MATLAB code in order to solve the equations numerically. Provided below is a schematic for the setup of our problem.

Three masses damper system is very common system which can be applied to car suspension or building structural. For example, infrastructure systems frequently show signs of damage. Damage may result from the abrupt failure of a structural element as a result of an over response during a catastrophic natural disaster, such as a powerful earthquake, or it may develop over time owing to material fatigue from repeated stress or chemical corrosion in a dangerous environment. A damper system is used to lower the sudden undesired force on the system.

Setup – Three Masses Spring and Damper System

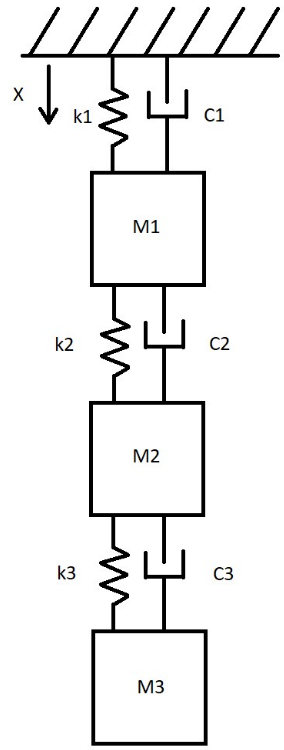

Figure 6. Three Masses Spring and Damper System.

The Three Masses Spring and Damper System consists of 3 masses, 3 springs and damper are placed in a sequence. Three masses are held suspended from the ceiling by 3 springs in series with different spring constants. The masses are allowed to move along the vertical x-axis in this simulation. The three masses each have different masses, and experience three different forms of damping.

<u>Procedure and Results – TMSDS</u>

Equation of motion by Newton's Method

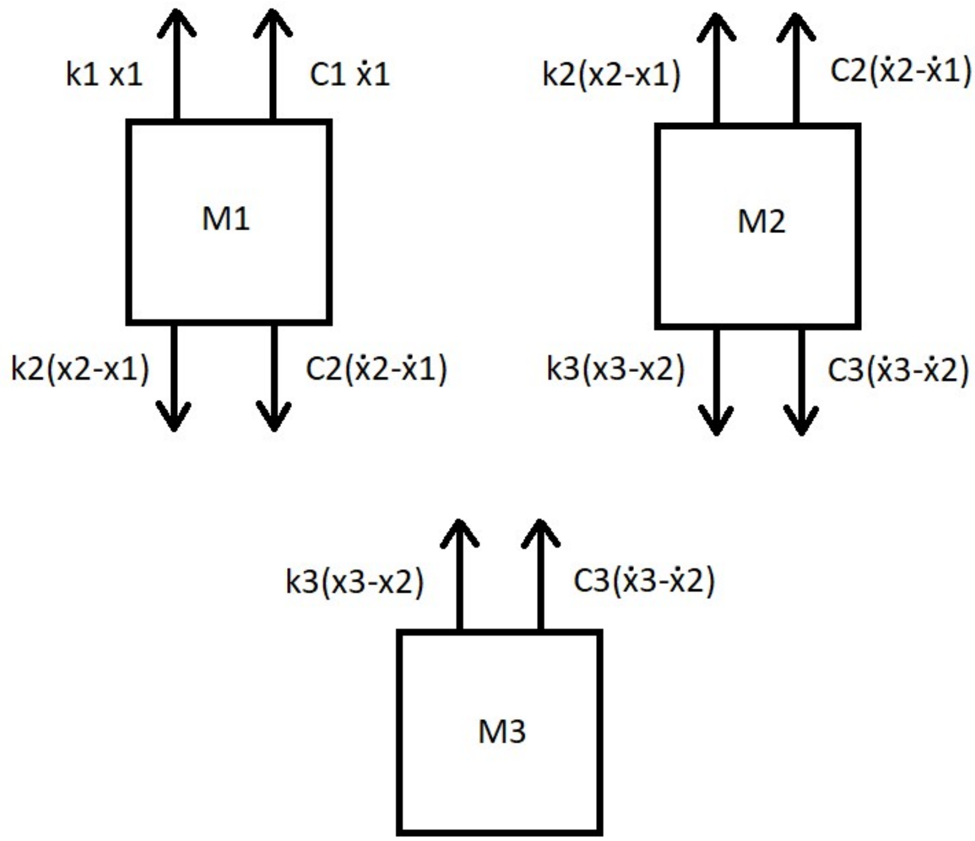

Figure 7. Free Body Diagram of the TMSDS System.

Equation of motion by Newton's Method

$$M_1\ddot{x}_1 = -(C_1 - C_2)\dot{x}_1 - (k_1 + k_2)x_1 + C_2\dot{x}_2 + k_2x_2$$

$$M_2\ddot{x}_2 = C_2\dot{x}_2 + k_2x_2 - (C_2 - C_3)\dot{x}_2 - (k_2 + k_3)x_2 + C_3\dot{x}_3 + k_3x_3$$

$$M_3\ddot{x}_3 = C_2\dot{x}_2 + k_2x_2 + C_3\dot{x}_3 + k_3x_3$$

Equation of motion by Lagrange's Method

$$T = \frac{1}{2}M_1(\dot{x}_1)^2 + \frac{1}{2}M_2(\dot{x}_2)^2 + \frac{1}{2}M_3(\dot{x}_3)^2$$

$$U = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 + \frac{1}{2}k_3x_3^2$$

$$D = \frac{1}{2}C_1\dot{x}_1 + \frac{1}{2}C_2\dot{x}_2 + \frac{1}{2}C_3\dot{x}_3$$

$$\frac{d}{\mathrm{dt}}(\frac{dT}{d\dot{q}_i}) - \frac{dT}{dq_i} + \frac{dU}{dq_i} + \frac{dD}{dq_i} = F_i$$

Applying Lagrange's Method

$$M_1\ddot{x}_1 + M_2\ddot{x}_2 + M_3\ddot{x}_3 + (C_1 + C_2)\dot{x}_1 - (k_1k_2)x_1 - C_2\dot{x}_2 - k_2x_2 - C_2\dot{x}_2 + (C_2C_3) - k_2x_2 + (k_2 + k_3)x_2 - C_3\dot{x}_3 - k_3x_3 - k_3x_2 - C_3\dot{x}_2 + C_3\dot{x}_3 + k_3x_3 - P = 0$$

Lagrange's and Newton's Method give the same result in matrix form.

$$\begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} C_1 + C_2 & -C_2 & 0 \\ -C_2 & C_2 + C_3 & -C_3 \\ 0 & -C_3 & C_3 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}\begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ \mathrm{x}_3 \end{bmatrix} = 0$$

<u>Data Analysis, Interpretation, and Discussion – TMSDS</u>

Using MATLAB to simulate the system with parameters configuration

$$M_1 = 100kg, M_2 = 100kg, M_3 = 100kg$$
$$k_1 = 4000\frac{N}{m}, \quad k_1 = 2000\frac{N}{m}, \quad k_1 = 1000\frac{N}{m}$$
$$C_1 = 5\ \frac{Ns}{m}, C_2 = 10\ \frac{Ns}{m} \quad C_3 = 15\ \frac{Ns}{m}$$

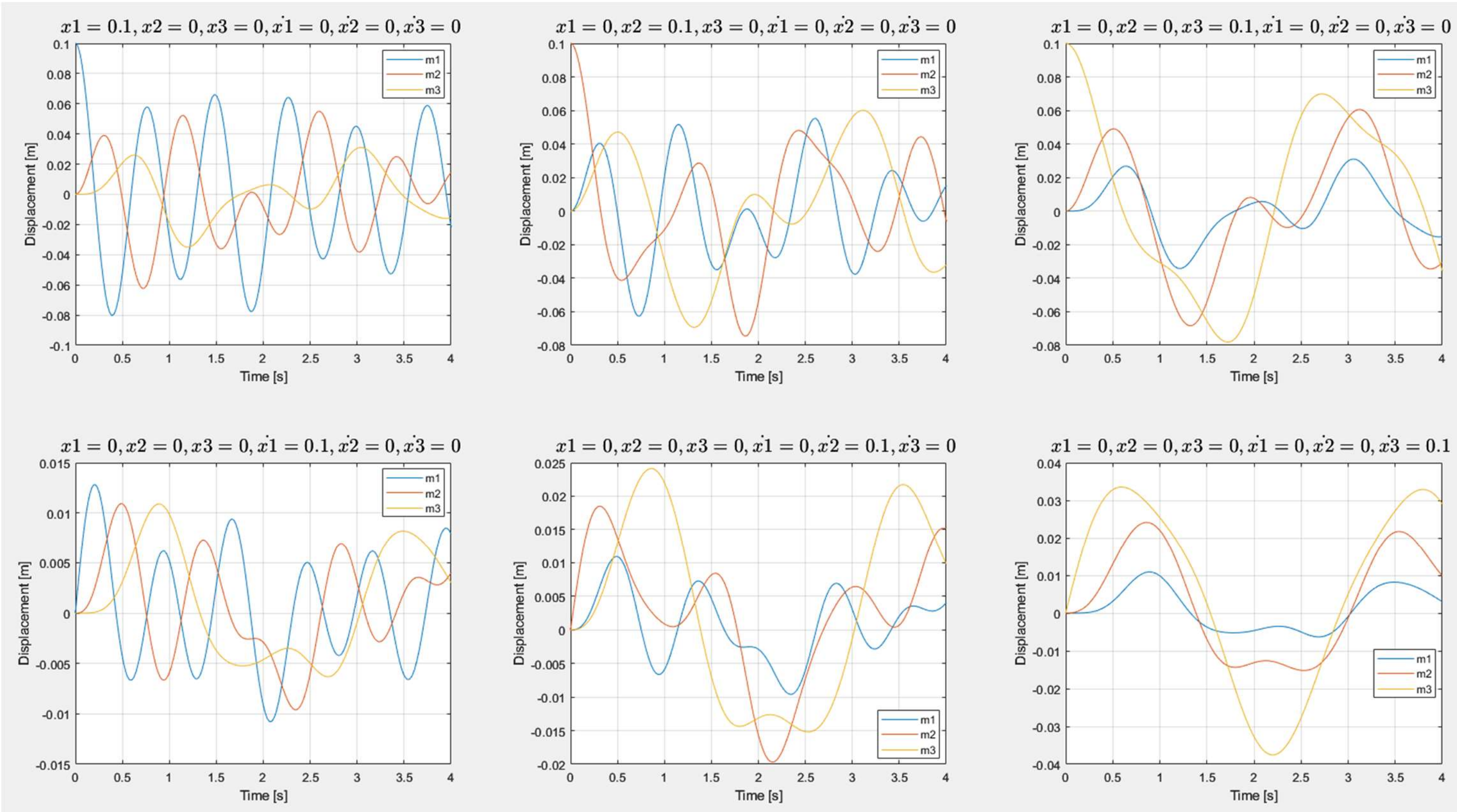

Figure 7. Free Vibration Responses for different Initial Conditions.

From figure 7, responses of the system are observed depend on the initial input. The system may respond by amplifying or canceling the input signal depending on the amplitude and frequency combinations used. One can anticipate the general behavior. No matter what the input amplitude is, the response might be greater if the input force frequency is close to the resonance peaks.

Calculate and plot FRFs

$$\begin{bmatrix} M_1\lambda^2 + k_1 + k_2 & k_2 & 0 \\ -k_2 & M_2\lambda^2 + k_2 + k_3 & -k_3 \\ 0 & -k_3 & M_3\lambda^2 + k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} Det = 2k_2k_3 \; + \; 2k_2^2M_3\lambda^2 \; + \; k_1k_2k_3 \; + \; k_1k_2M_3\lambda^2 \; + \; k_1k_3M_2\,\lambda^2 \\ + \; k_2k_3M_1\lambda^2 \; + \; k_1k_3M_3\lambda^2 \; + \; k_2k_3M_2\lambda^2 \; + \; k_2k_3M_3\lambda^2 \\ + \; k_1M_2M_3\lambda^4 \; + \; k_2M_1M_3\lambda^4 \; + \; k_3M_1M_2\lambda^4 \; + \; k_2M_2M_3\lambda^4 \\ + \; k_3M_1M_3\lambda^4 \; + \; M_1M_2M_3\lambda^6 \; \frac{rad}{s} \end{aligned}$$

$Using\ MATLAB\ symbolic\ function\ to\ solve\ for\ \lambda.$

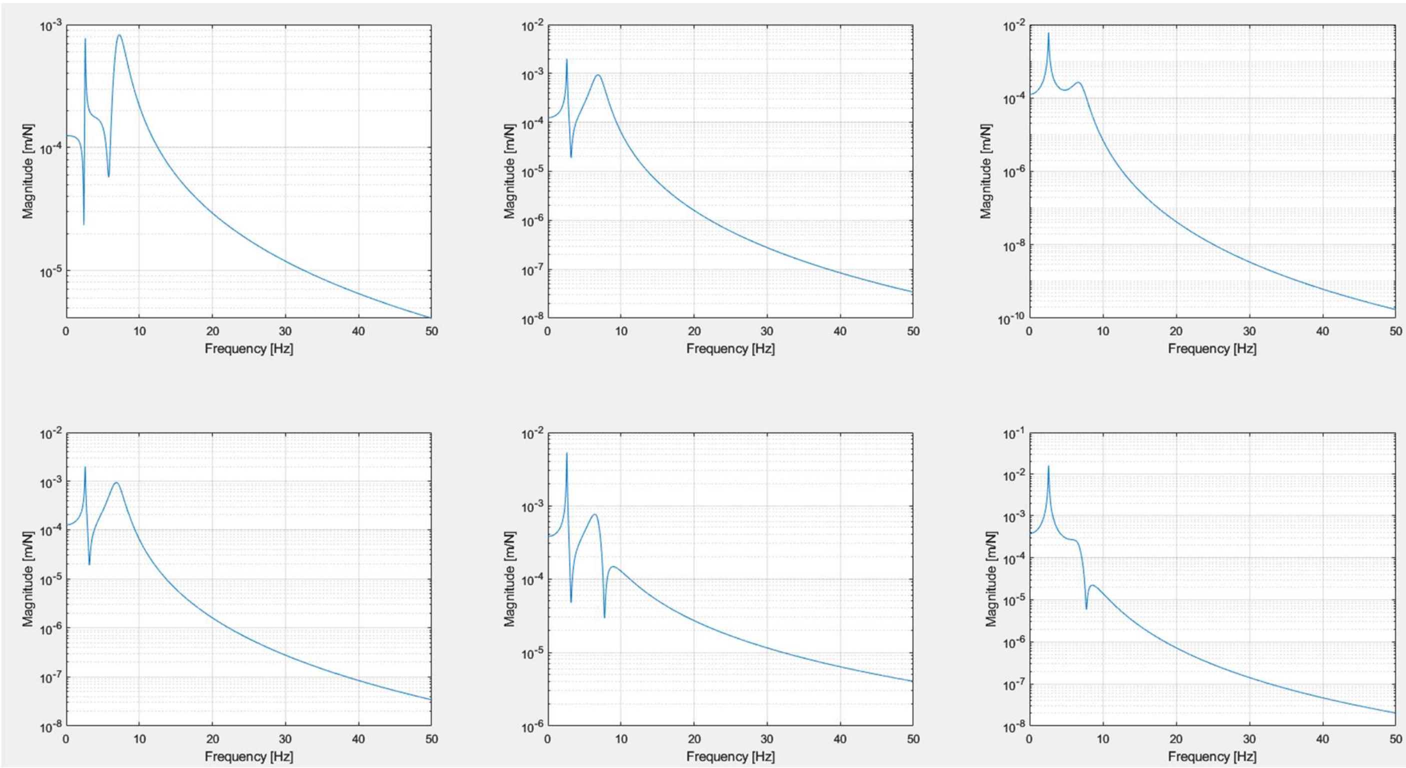

Figure 8. Plot Frequency Response Magnitude.

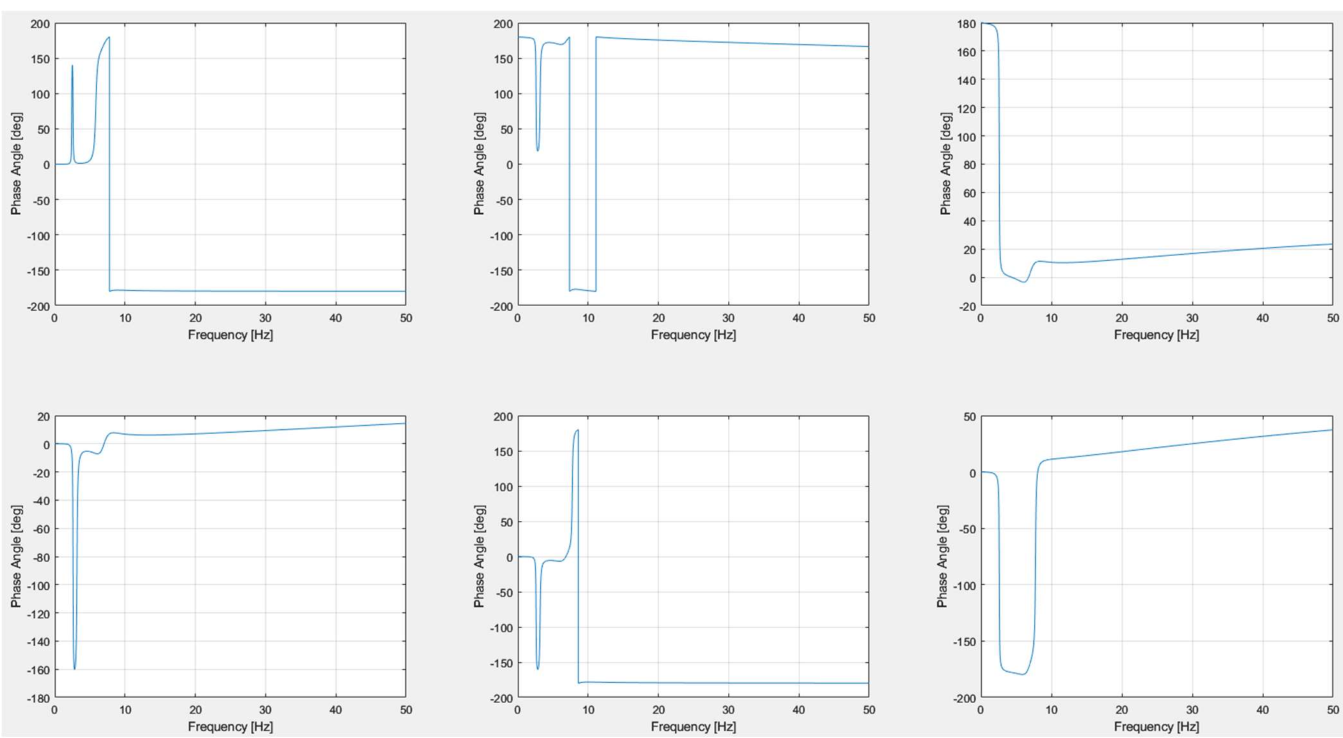

Figure 9. Plot Frequency Response Phase Angle.

The system's mass and rigidity have a direct impact on the resonant frequencies for the system. As a result, various masses or different stiffness of spring will produce various resonance frequencies. From the Frequency Response plots, the 3 peaks are observed which shows 3 mode shape of the 3DOF system. However, some plots only show 2 peaks because frequency was canceled.

Conclusions – TMSDS

In conclusion, we have solved the TMSDS using both Newton's and Lagrange's Methods, allowing us to linearize the equations of motion and solve for the natural frequencies and modes of vibration. We have also obtained the Free Vibration Response plots and Frequency Response Function plots for an initial condition.

Summary

In this project, we attempt to solve two problems. The first problem is finding the solution to a Cart and Pendulum System (CPS) of vibration. The second problem is finding the solution to a Three masses Spring and Damper System (TMSDS) in a vertical orientation against gravity. In our solution, we used both method analysis such as Free Body Diagrams, Newton's Method and Lagrange's Method and software such as MATLAB and Mathematica.

Reference

[1] “Cart & Pendulum.” *MyPhysicsLab Cart + Pendulum*, https://www.myphysicslab.com/pendulum/cart-pendulum-en.html.

[2] Vazquez, J. (n.d.). Honors Contract Report: Multiple Degree of Freedom Analysis for a Quarter Car Suspension Model. essay.

Appendix

CPS Code

```
clear
clc
close all
%Define Model Parameters

%% ADJUST variable

global M m k kt C g l
M = 10; %[kg]
m = 2; %[kg]
k = 40; %[N/m]
g=9.81;%[m/s^2]
kt = m*g; %[N/m]
C = 2;%[Ns/m]
l=0.5;%[m]

%%

omeg = 0:.01:50;
%Define Matrix Coefficients
M_matrix = [M+m m*l; m m*l];
C_matrix = [C -m*l; 0 0];
K_matrix =  [k 0; 0 m*g];
%Calculate Natural Frequencies and Mode Shapes
[V, D] = eig(K_matrix,M_matrix);
f = sqrt(diag(D))/(2*pi);
%Calculate and Plot FRF's
H1 = zeros(2,2,length(omeg));

for n=1:length(omeg)
H1(:,:,n) = inv(-omeg(n)^2.*M_matrix+C_matrix.*1i.*omeg(n)+K_matrix);
end
H11(1:length(omeg))=H1(1,1,:);
H12(1:length(omeg))=H1(1,2,:);
H21(1:length(omeg))=H1(2,1,:);
H22(1:length(omeg))=H1(2,2,:);


%% Plot Frequency Response
figure('Name','Frequency Response Magnitudes')
subplot(2,2,1);
semilogy(omeg,abs(H11));
```

```
xlabel('Frequency [Hz]');
ylabel('Magnitude [m/N]')
grid on
subplot(2,2,2);
semilogy(omeg,abs(H12));
xlabel('Frequency [Hz]');
ylabel('Magnitude [m/N]')
grid on
subplot(2,2,3);
semilogy(omeg,abs(H21));
xlabel('Frequency [Hz]');
ylabel('Magnitude [m/N]')
grid on
subplot(2,2,4);
semilogy(omeg,abs(H22));
xlabel('Frequency [Hz]');
ylabel('Magnitude [m/N]')
grid on

figure('Name','Frequency Response Phase Angle')
subplot(2,2,1);
plot(omeg,angle(H11)*180/pi);
xlabel('Frequency [Hz]');
ylabel('Phase Angle [deg]')
grid on
subplot(2,2,2);
plot(omeg,angle(H12)*180/pi);
xlabel('Frequency [Hz]');
ylabel('Phase Angle [deg]')
grid on
subplot(2,2,3);
plot(omeg,angle(H21)*180/pi);
xlabel('Frequency [Hz]');
ylabel('Phase Angle [deg]')
grid on
subplot(2,2,4);
plot(omeg,angle(H22)*180/pi);
xlabel('Frequency [Hz]');
ylabel('Phase Angle [deg]')
grid on


%% Plot Free Vibration Response
ti = 0; tf = 4;
[t1,X1] = ode45(@FreeResponse,[ti tf],[0,0,0,0]);
[t2,X2] = ode45(@FreeResponse,[ti tf],[0.1,0,0,0]);
```

```
[t3,X3] = ode45(@FreeResponse,[ti tf],[0,0.1,0,0]);
[t4,X4] = ode45(@FreeResponse,[ti tf],[0,0,0.1,0]);
[t5,X5] = ode45(@FreeResponse,[ti tf],[0,0,0,0.1]);
[t6,X6] = ode45(@FreeResponse,[ti tf],[0.1,0.1,0.1,0.1]);
figure('Name','Free Vibration Responses')

subplot(2,3,1);
 plot(t1,X1(:,1),t1,X1(:,2));
 legend('Cart','Pendulum','Location','Best')
 xlabel('Time [s]'); ylabel('Displacement [m,rad]')
title('$x=0, \theta=0, \dot{x}=0, \dot{\theta} =0$ ' , ...
    'FontWeight','bold','FontSize',16, ...
    'FontName','Arial','Interpreter','latex')

subplot(2,3,2);
 plot(t2,X2(:,1),t2,X2(:,2));
 legend('Cart','Pendulum','Location','Best')
 xlabel('Time [s]'); ylabel('Displacement [m,rad]')
title('$x=0.1, \theta=0, \dot{x}=0, \dot{\theta} =0$ ' , ...
    'FontWeight','bold','FontSize',16, ...
    'FontName','Arial','Interpreter','latex')

subplot(2,3,3);
 plot(t3,X3(:,1),t3,X3(:,2))
 legend('Cart','Pendulum','Location','Best')
 xlabel('Time [s]'); ylabel('Displacement [m,rad]')
title('$x=0, \theta=0.1, \dot{x}=0, \dot{\theta} =0$ ' , ...
    'FontWeight','bold','FontSize',16, ...
    'FontName','Arial','Interpreter','latex')

subplot(2,3,4);
 plot(t4,X4(:,1),t4,X4(:,2))
 legend('Cart','Pendulum','Location','Best')
 xlabel('Time [s]'); ylabel('Displacement [m,rad]')
title('$x=0, \theta=0, \dot{x}=0.1, \dot{\theta} =0$ ' , ...
    'FontWeight','bold','FontSize',16, ...
    'FontName','Arial','Interpreter','latex')

subplot(2,3,5);
 plot(t5,X5(:,1),t5,X5(:,2))
 legend('Cart','Pendulum','Location','Best')
 xlabel('Time [s]'); ylabel('Displacement [m,rad]')
title('$x=0, \theta=0, \dot{x}=0, \dot{\theta} =0.1$ ' , ...
    'FontWeight','bold','FontSize',16, ...
    'FontName','Arial','Interpreter','latex')
```

```
subplot(2,3,6);
 plot(t6,X6(:,1),t6,X6(:,2))
 legend('Cart','Pendulum','Location','Best')
 xlabel('Time [s]'); ylabel('Displacement [m,rad]')
title('$x=0.1, \theta=0.1, \dot{x}=0.1, \dot{\theta} =0.1$ ' , ...
    'FontWeight','bold','FontSize',16, ...
    'FontName','Arial','Interpreter','latex')

%global M m k kt C g w
%Function to calculate Free Vibration Response
function dx = FreeResponse(t,x)
global M m k C g l
dx = zeros(4,1);
dx(1) = x(3);
dx(2) = x(4);
dx(3) = (m*l*x(4)^2*x(2)+m*g*x(2)-k*x(1)-C*x(3))/(M+m*x(2)^2);
dx(4) = (-m*l*x(4)^2*x(2) - (m*M)*g*x(2) + k*x(1) + C*x(3))/(l*(M+m*x(2)^2));
end
```

TMSDS Code

```
clear
clc
close all
%Define Model Parameters

%% ADJUST variable
global m1 m2 m3 k1 k2 k3 c1 c2 c3
m1 = 100; %[kg]
m2 = 100; %[kg]
m3= 100;
k1 = 4000; %[N/m]
k2 = 2000;
k3= 1000;
g=9.81;%[m/s^2]
c1 = 5;%[Ns/m]
c2 = 10;
c3 = 15;

%%

omeg = 0:.001:50;
%Define Matrix Coefficients
M_matrix = [m1 0 0; 0 m2 0; 0 0 m3];
C_matrix = [c1+c2 , -c2, 0; -c2, c2+c3, -c3; 0 , -c3, c3];
K_matrix =  [k1+k2, k2, 0; -k2, k2+k3,-k3; 0, -k3 k3];
%Calculate Natural Frequencies and Mode Shapes
[V, D] = eig(K_matrix,M_matrix);
f = sqrt(diag(D))/(2*pi);
%Calculate and Plot FRF's
H1 = zeros(3,3,length(omeg));
```

```
for n=1:length(omeg)
H1(:,:,n) = inv(-omeg(n)^2.*M_matrix+C_matrix.*1i.*omeg(n)+K_matrix);
end
H11(1:length(omeg))=H1(1,1,:);
H12(1:length(omeg))=H1(1,2,:);
H13(1:length(omeg))=H1(1,3,:);

H21(1:length(omeg))=H1(2,1,:);
H22(1:length(omeg))=H1(2,2,:);
H23(1:length(omeg))=H1(2,3,:);

%% Plot Frequency Response
figure('Name','Frequency Response Magnitudes')
subplot(2,3,1);
semilogy(omeg,abs(H11));
xlabel('Frequency [Hz]');
ylabel('Magnitude [m/N]')
grid on
subplot(2,3,2);
semilogy(omeg,abs(H12));
xlabel('Frequency [Hz]');
ylabel('Magnitude [m/N]')
grid on
subplot(2,3,3);
semilogy(omeg,abs(H13));
xlabel('Frequency [Hz]');
ylabel('Magnitude [m/N]')
grid on
subplot(2,3,4);
semilogy(omeg,abs(H21));
xlabel('Frequency [Hz]');
ylabel('Magnitude [m/N]')
grid on
subplot(2,3,5);
semilogy(omeg,abs(H22));
xlabel('Frequency [Hz]');
ylabel('Magnitude [m/N]')
grid on
subplot(2,3,6);
semilogy(omeg,abs(H23));
xlabel('Frequency [Hz]');
ylabel('Magnitude [m/N]')
grid on


figure('Name','Frequency Response Phase Angle')
subplot(2,3,1);
plot(omeg,angle(H11)*180/pi);
xlabel('Frequency [Hz]');
ylabel('Phase Angle [deg]')
grid on
subplot(2,3,2);
plot(omeg,angle(H12)*180/pi);
xlabel('Frequency [Hz]');
```

```
ylabel('Phase Angle [deg]')
grid on
subplot(2,3,3);
plot(omeg,angle(H13)*180/pi);
xlabel('Frequency [Hz]');
ylabel('Phase Angle [deg]')
grid on
subplot(2,3,4);
plot(omeg,angle(H21)*180/pi);
xlabel('Frequency [Hz]');
ylabel('Phase Angle [deg]')
grid on
subplot(2,3,5);
plot(omeg,angle(H22)*180/pi);
xlabel('Frequency [Hz]');
ylabel('Phase Angle [deg]')
grid on
subplot(2,3,6);
plot(omeg,angle(H23)*180/pi);
xlabel('Frequency [Hz]');
ylabel('Phase Angle [deg]')
grid on


%% Plot Free Vibration Response
ti = 0; tf = 4;
[t1,X1] = ode45(@FreeResponse,[ti tf],[0,0,0,0,0,0]);
[t2,X2] = ode45(@FreeResponse,[ti tf],[0.1,0,0,0,0,0]);
[t3,X3] = ode45(@FreeResponse,[ti tf],[0,0.1,0,0,0,0]);
[t4,X4] = ode45(@FreeResponse,[ti tf],[0,0,0.1,0,0,0]);
[t5,X5] = ode45(@FreeResponse,[ti tf],[0,0,0,0.1,0,0]);
[t6,X6] = ode45(@FreeResponse,[ti tf],[0,0,0,0,0.1,0]);
[t7,X7] = ode45(@FreeResponse,[ti tf],[0,0,0,0,0,0.1]);
[t8,X8] = ode45(@FreeResponse,[ti tf],[0.1,0.1,0.1,0,0,0.1]);
figure('Name','Free Vibration Responses')


subplot(2,3,1);
 plot(t2,X2(:,1),t2,X2(:,2),t2,X2(:,3))
 legend('m1','m2','m3','Location','Best')
 xlabel('Time [s]'); ylabel('Displacement [m]')
title('$x1=0.1, x2=0, x3 =0, \dot{x1}=0,\dot{x2}=0,\dot{x3}=0 $ ' , ...
    'FontWeight','bold','FontSize',16, ...
    'FontName','Arial','Interpreter','latex')
grid on

subplot(2,3,2);
 plot(t3,X3(:,1),t3,X3(:,2),t3,X3(:,3))
  legend('m1','m2','m3','Location','Best')
 xlabel('Time [s]'); ylabel('Displacement [m]')
title('$x1=0, x2=0.1, x3 =0, \dot{x1}=0,\dot{x2}=0,\dot{x3}=0 $ ' , ...
    'FontWeight','bold','FontSize',16, ...
    'FontName','Arial','Interpreter','latex')
grid on
```

```matlab
subplot(2,3,3);
 plot(t4,X4(:,1),t4,X4(:,2),t4,X4(:,3))
 legend('m1','m2','m3','Location','Best')
 xlabel('Time [s]'); ylabel('Displacement [m]')
title('$x1=0, x2=0, x3 =0.1, \dot{x1}=0,\dot{x2}=0,\dot{x3}=0 $ ' , ...
    'FontWeight','bold','FontSize',16, ...
    'FontName','Arial','Interpreter','latex')
grid on

subplot(2,3,4);
 plot(t5,X5(:,1),t5,X5(:,2),t5,X5(:,3))
 legend('m1','m2','m3','Location','Best')
 xlabel('Time [s]'); ylabel('Displacement [m]')
title('$x1=0, x2=0, x3 =0, \dot{x1}=0.1,\dot{x2}=0,\dot{x3}=0 $ ' , ...
    'FontWeight','bold','FontSize',16, ...
    'FontName','Arial','Interpreter','latex')
grid on

subplot(2,3,5);
 plot(t6,X6(:,1),t6,X6(:,2),t6,X6(:,3))
 legend('m1','m2','m3','Location','Best')
 xlabel('Time [s]'); ylabel('Displacement [m]')
title('$x1=0, x2=0, x3 =0, \dot{x1}=0,\dot{x2}=0.1,\dot{x3}=0 $ ' , ...
    'FontWeight','bold','FontSize',16, ...
    'FontName','Arial','Interpreter','latex')
grid on

subplot(2,3,6);
 plot(t7,X7(:,1),t7,X7(:,2),t7,X7(:,3));
 legend('m1','m2','m3','Location','Best')
 xlabel('Time [s]'); ylabel('Displacement [m]')
title('$x1=0, x2=0, x3 =0, \dot{x1}=0,\dot{x2}=0,\dot{x3}=0.1 $ ' , ...
    'FontWeight','bold','FontSize',16, ...
    'FontName','Arial','Interpreter','latex')
grid on

%global M m k kt C g w
%Function to calculate Free Vibration Response
function dx = FreeResponse(t,x)
global m1 m2 m3 k1 k2 k3 c1 c2 c3
dx = zeros(6,1);
dx(1) = x(4);
dx(2) = x(5);
dx(3) = x(6);
dx(4) = (-(c1+c2)*x(4)-(k1+k2)*x(1)+c2*x(5)+k2*x(2))/m1;
dx(5) = (c2*x(4)+k2*x(1)-(c2+c3)*x(5)-(k2+k3)*x(2)+c3*x(6)+k3*x(3))/m2;
dx(6) = (c3*x(5)+k3*x(2)-c3*x(6)-k3*x(3))/m3;
end
```