Required Textbooks:

1. Quarks and Leptons: An Introductory Course in Modern Particle Physics
   Francis Halzen and Alan D. Martin
   Wiley; 1st edition (January 6, 1984)

2. Introduction to Elementary Particles
   David Griffiths
   Wiley-VCH; 2nd edition (October 21, 2008)

Schedule of Assessment Tasks for Students During the Semester:

Exercises & Homeworks :	20 points
Written Test (1):	20 points
Written Test (2):	20 points
Final Exam:	40 points

The Fundamentals of The Theory of Modern Physics

by

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Chapter 6: Elementary Particle Physics and The Unification of The Forces

> "Three quarks for Muster Mark!
> Sure he hasn't got much of a bark
> and sure any he has it's all beside the mark"
> James Joyce, Finnegan's Wake

6.1 Introduction

Man has always searched for simplicity in nature. Recall that the ancient Greeks tried to describe the entire physical world in terms of the four quantities of earth, air, fire, and water. These, of course, have been replaced with the fundamental quantities of length, mass, charge, and time in order to describe the physical world of space, matter, and time. We have seen that space and time are not independent quantities, but rather are a manifestation of the single quantity — spacetime — and that mass and energy are interchangeable, so that energy could even be treated as one of the fundamental quantities. We also found that energy is quantized and therefore, matter should also be quantized. What is the smallest quantum of matter? That is, what are the fundamental or elementary building blocks of matter? What are the forces that act on these fundamental particles? Is it possible to combine these forces of nature into one unified force that is responsible for all the observed interactions? We shall attempt to answer these questions in this chapter.

6.2 Particles and Antiparticles

As mentioned in chapter 20, the Greek philosophers Leucippus and Democritus suggested that matter is composed of fundamental or elementary particles called atoms. The idea was placed on a scientific foundation with the publication, by John Dalton, of *A New System of Chemical Philosophy* in 1808, in which he listed about 20 chemical elements, each made up of an atom. By 1896 there were about 60 known elements. It became obvious that there must be a way to arrange these different atoms in an orderly way in order to make sense of what was quickly becoming chaos. In 1869 the Russian chemist, Dimitri Mendeleev, developed the periodic table of the elements based on the chemical properties of the elements. Order was brought to the chaos of the large diversity of elements. In fact, new chemical elements were predicted on the basis of the blank spaces found in the periodic table. Later with the discovery of the internal structure of the atom, the atom could no longer be considered as elementary

By 1932, only four elementary particles were known; the electron, the proton, the neutron, and the photon. Things looked simple again. But this simplicity was not to last. Other particles were soon discovered in cosmic rays. Cosmic rays are particles from outer space that impinge on the top of the atmosphere. Some of them make it to the surface of the earth, whereas others decay into still other particles before they reach the surface. Other new particles were found in the large accelerating machines made by man. Today, there are hundreds of such particles. Except for the electron, proton, and neutron, most of these elementary particles decay very quickly. We are again in the position of trying to make order out of the chaos of so many particles.

The first attempt at order is the classification of particles according to the scheme shown in figure 6.1. All the elementary particles can be grouped into particles called hadrons or leptons.

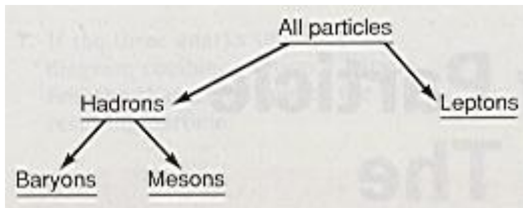


Figure 6.1 First classification of the elementary particles.

Leptons

*The **Leptons** are particles that are not affected by the strong nuclear force.* They are very small in terms of size, in that they are less than 10^{-19} m in diameter. They all have spin ½ in units of $\hbar$. There are a total of six leptons: the electron, e^-, the muon, μ^- and the tauon, τ^-, each with an associated neutrino. They can be grouped in the form

$$\begin{matrix} (\nu_e) & (\nu_\mu) & (\nu_\tau) \\ (e^-) & (\mu^-) & (\tau^-) \end{matrix} \tag{6.1}$$

There are thus three neutrinos: the neutrino associated with the electron, ν_e; the neutrino associated with the muon, ν_μ; and the neutrino associated with the tauon, ν_τ. The muon is very much like an electron but it is much heavier. It has a mass about 200 times greater than the electron. It is not stable like the electron but decays in about 10^{-6} s.

Originally the word lepton, which comes from the Greek word *leptos* meaning small or light in weight, signified that these particles were light. However, in 1975 the τ lepton was discovered and it has twice the mass of the proton. That is, the τ lepton is a heavy lepton, certainly a misnomer.

Leptons are truly elementary in that they apparently have no structure. That is, they are not composed of something still smaller. Leptons participate in the weak nuclear force, while the charged leptons, e^-, μ^-, τ^-, also participate in the electromagnetic interaction.

The muon was originally thought to be Yukawa's meson that mediated the strong nuclear force, and hence it was called a μ^- meson. This is now known to be a misnomer, since the muon is not a meson but a lepton.

Hadrons

***Hadrons** are particles that are affected by the strong nuclear force*. There are hundreds of known hadrons. Hadrons have an internal structure, composed of what appears to be truly elementary particles called quarks. The hadrons can be further broken down into two subgroups, the baryons and the mesons.

1. ***Baryons.*** *Baryons are heavy particles that, when they decay, contain at least one proton or neutron in the decay products.* The baryons have half-integral spin, that is, 1/2 $\hbar$, 3/2 $\hbar$, and so on. We will see in a moment that all *baryons are particles that are composed of three quarks.*

2. ***Mesons.*** Originally, mesons were particles of intermediate-sized mass between the electron and the proton. However many massive mesons have since been found, so the original definition is no longer appropriate. *A meson is now defined as any particle whose decay products do not include a baryon.* We will see that *mesons are particles that are composed of a quark-antiquark pair.* All mesons have integral spin, that is, 0, 1, 2, 3, and so on. The mass of the meson increases with its spin. A list of some of the elementary particles is shown in table 6.1.

Table 6.1 List of Some of the Elementary Particles		
Leptons	electron,	e^-
	muon,	μ^-
	tauon,	τ^-
	neutrinos,	ν_e, ν_μ, ν_τ
Hadrons		
Baryons	proton,	p
	neutron,	n
	delta,	Δ
	lambda,	λ
	Sigma,	Σ
	Hyperon,	Λ
	Omega	Ω
Mesons	pi,	π
	eta,	η
	rho,	ρ
	omega,	Ω
	delta,	δ
	phi	ϕ

In 1928, Paul Dirac merged special relativity with the quantum theory to give a relativistic theory of the electron. A surprising result of that merger was that his equations predicted two energy states for each electron. One is associated with the electron, whereas the other is associated with a particle, like the electron in every way, except that it carries a positive charge. This new particle was called the *antielectron* or the *positron*. This was the first prediction of the existence of antimatter. The positron was found in 1932.

For every particle in nature there is associated an ***antiparticle****. The antiparticle of the proton is the antiproton.* It has all the characteristics of the proton except that it carries a negative charge. Some purely neutral particles such as the photon and the π^0 meson are their own antiparticles. Antiparticles are written with a bar over the symbol for the particle. Hence, $\bar{p}$ is an antiproton and $\bar{n}$ is an antineutron.

Matter consists of electrons, protons and neutrons, whereas ***antimatter*** *consists of antielectrons (positrons), antiprotons, and antineutrons.* Figure 6.2 shows atoms of matter and antimatter. The same electric forces that hold matter

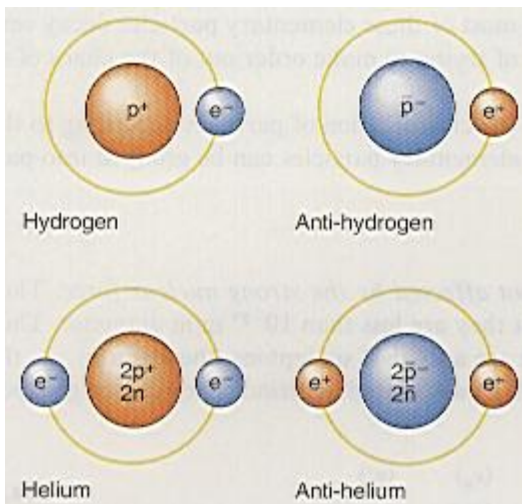


Figure 6.2 Matter and antimatter.

together, hold antimatter together. (Note that the positive and negative signs are changed in antimatter.) The antihelium nucleus has already been made in high-energy accelerators.

Whenever particles and antiparticles come together they annihilate each other and only energy is left. For example, when an electron comes in contact with a positron they annihilate according to the reaction

$$e^- + e^+ \rightarrow 2\gamma \tag{6.2}$$

where the 2γ's are photons of electromagnetic energy. (Two gamma rays are necessary in order to conserve energy and momentum.) This energy can also be used to create other particles. Conversely, particles can be created by converting the energy in the photon to a particle-antiparticle pair such as

$$\gamma \rightarrow e^- + e^+ \tag{6.3}$$

Creation or annihilation can be shown on a spacetime diagram, called a *Feynman diagram*, after the American physicist Richard Feynman (1918-1988), such as in figure 6.3. Figure 6.3(a) shows the creation of an electron-positron pair. A

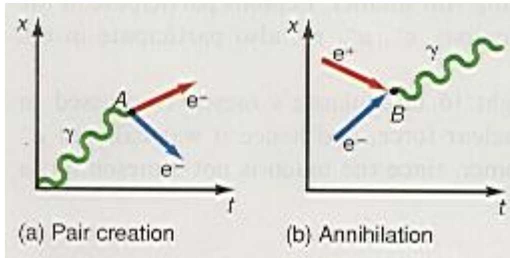


Figure 6.3 Creation and annihilation of particles.

photon γ moves through spacetime until it reaches the spacetime point *A,* where the energy of the photon is converted into the electron-positron pair. Figure 6.3(b) shows an electron and positron colliding at the spacetime point *B* where they annihilate each other and only the photon γ now moves through spacetime. (In order to conserve momentum and energy in the creation process, the presence of a relatively heavy nucleus is required.)

6.3 The Four Forces of Nature

In the study of nature, four forces that act on the particles of matter are known. They are:

1. *The Gravitational Force.* The gravitational force is the oldest known force. It holds us to the surface of the earth and holds the entire universe together. It is a long-range force, varying as $1/r^2$. Compared to the other forces of nature it is by far the weakest force of all.
2. *The Electromagnetic Force.* The electromagnetic force was the second force known. In fact, it was originally two forces, the electric force and the magnetic force, until the first unification of the forces tied them together as a single electromagnetic force. The electromagnetic force holds atoms, molecules, solids, and liquids together. Like gravity, it is a long-range force varying as $1/r^2$.
3. *The Weak Nuclear Force.* The weak nuclear force manifests itself not so much in holding matter together, but in allowing it to disintegrate, such as in the decay of the neutron and the proton. The weak force is responsible for the fusion process occurring in the sun by allowing a proton to decay into a neutron such as given in equation 5.21. The proton-proton cycle then continues until helium is formed and large quantities of energy are given off. The nucleosynthesis of the chemical elements also occurred because of the weak force. Unlike the gravitational and electromagnetic force, the weak nuclear force is a very short range force.
4. *The Strong Nuclear Force.* The strong nuclear force is responsible for holding the nucleus together. It is the strongest of all the forces but is a very short range force.

That is, its effects occur within a distance of about 10^{-15} m, the diameter of the nucleus. At distances greater than this, there is no evidence whatsoever for its very existence. The strong nuclear force acts only on the hadrons.

Why should there be four forces in nature? Einstein, after unifying space and time into spacetime, tried to unify the gravitational force and the electromagnetic force into a single force. Although he spent a lifetime trying, he did not succeed. The hope of a unification of the forces has not died, however. In fact, we will see shortly that the electromagnetic force and the weak nuclear force have already been unified theoretically into the electroweak force by Glashow, Weinberg, and Salam, and experimentally confirmed by Rubbia. A grand unification between the electroweak and the strong force has been proposed. Finally an attempt to unify all the four forces into one superforce is presently underway.

6.4 Quarks

In the attempt to make order out of the very large number of elementary particles, Murray Gell-Mann and George Zweig in 1964, independently proposed that the hadrons were not elementary particles but rather were made of still more elementary particles. Gell-Mann called these particles, **quarks.** He initially assumed there were only three such quarks, but with time the number has increased to six. The six quarks are shown in table 6.2. *The names of the quarks are: up, down, strange, charmed, bottom, and top.* One of the characteristics of these

Table 6.2
The Quarks

Name (Flavor)	Symbol	Charge	Spin
up	u	2/3	1/2
down	d	−1/3	1/2
strange	s	−1/3	1/2
charmed	c	2/3	1/2
bottom	b	−1/3	1/2
top	t	2/3	1/2

quarks is that they have fractional electric charges. That is, the up, charmed, and top quark has 2/3 of the charge found on the proton, whereas the down, strange, and bottom quark has 1/3 of the charge found on the electron. They all have spin 1/2, in units of $\hbar$. Each quark has an antiquark, which is the same as the original quark except it has an opposite charge. The antiquark is written with a bar over the letter, that is $\bar{q}$.

We will now see that all of the hadrons are made up of quarks. The baryons are made up of three quarks:

$$\text{Baryon} = qqq \tag{6.4}$$

While the mesons are made up of a quark-antiquark pair:

$$\text{Meson} = q\bar{q} \tag{6.5}$$

As an example of the formation of a baryon from quarks, consider the proton. The proton consists of two up quarks and one down quark, as shown in figure 6.4(a).

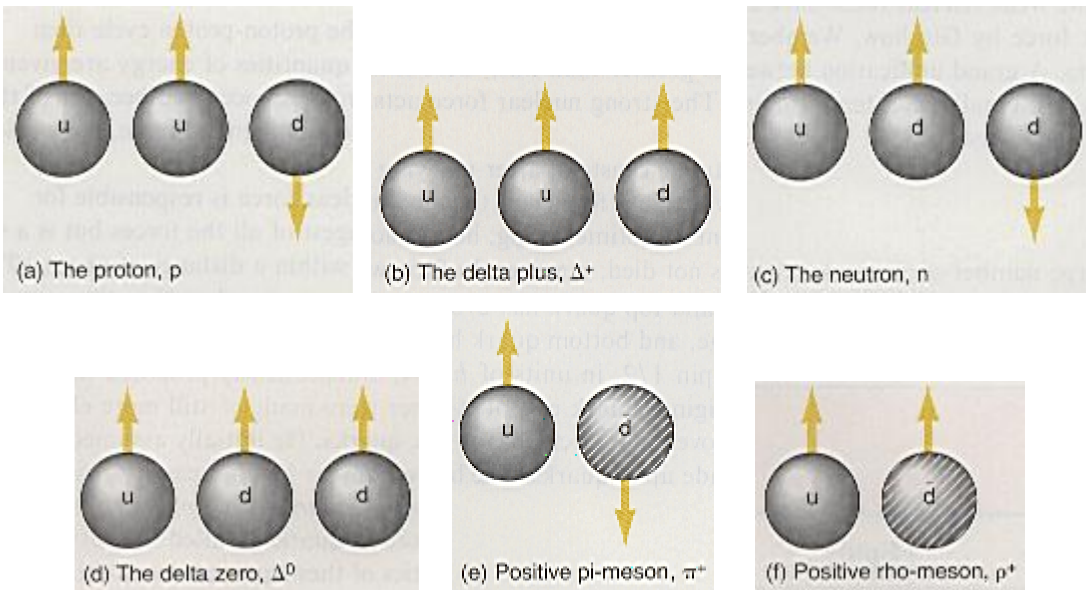


Figure 6.4 Some quark configurations of baryons and mesons.

The electric charge of the proton is found by adding the charges of the constitutive quarks. That is, since the u quark has a charge of 2/3, and the d quark has a charge of −1/3, the charge of the proton is

$$2/3 + 2/3 - 1/3 = 1$$

which is exactly as expected. Now the proton should have a spin of 1/2 in units of $\hbar$. In figure 6.4(a), we see the two up quarks as having their spin up by the direction of the arrow on the quark. The down quark has its arrow pointing down to signify that its spin is down. Because each quark has spin 1/2, the spin of the proton is found by adding the spins of the quarks as

$$1/2 + 1/2 - 1/2 = 1/2$$

We should note that the names up and down for the quarks are just that, a name, and have nothing to do with the direction of the spin of the quark. For example, the delta plus Δ^+ baryon is made from the same three quarks as the proton, but their spins are all aligned in the same direction, as shown in figure 6.4(b). Thus, the spin of the Δ^+ particle is

$$1/2 + 1/2 + 1/2 = 3/2$$

That is, the Δ^+ particle has a spin of 3/2. Since it takes more energy to align the spins in the same direction, when quark spins are aligned, they have more energy. This manifests itself as an increased mass by Einstein's equivalence of mass and energy ($E = mc^2$). Thus, we see that the mass of the Δ^+ particle has a larger mass than the proton. Hence, in the formation of particles from quarks, we not only have to know the types of quarks making up the particle but we must also know the direction of their spin.

Figure 6.4(c) shows that a neutron is made up of one up quark and two down quarks. The total electric charge is

$$2/3 - 1/3 - 1/3 = 0$$

While its spin is

$$1/2 + 1/2 - 1/2 = 1/2$$

Again note that the delta zero Δ^0 particle is made up of the same three quarks, figure 6.4(d), but their spins are all aligned.

As an example of the formation of a meson from quarks, consider the pi plus π^+ meson in figure 6.4(e). It consists of an up quark and an antidown quark. Its charge is found as

$$2/3 + [-(-1/3)] = 2/3 + 1/3 = 1$$

That is, the d quark has a charge of $-1/3$, so its antiquark $\bar{d}$ has the same charge but of opposite sign $+1/3$. The spin of the π^+ is

$$1/2 - 1/2 = 0$$

Thus, the π^+ meson has a charge of +1 and a spin of zero.

If the spins of these same two quarks are aligned, as in figure 6.4(f), the meson is the positive rho-meson ρ^+, with electric charge of +1 and spin of 1.

The quark structure of some of the baryons is shown in table 6.3, whereas table 6.4 shows the quark structure for some mesons.

Particles that contain the strange quark are called strange particles. The reason for this name is because these particles took so much longer to decay than the other elementary particles, that it was considered strange.

If a proton or neutron consists of quarks, we would like to "see" them. Just as Rutherford "saw" inside the atom by bombarding it with alpha particles, we can "see" inside a proton by bombarding it with electrons or neutrinos. In 1969, at the Stanford Linear Accelerator Center (SLAC), protons were bombarded by high-energy electrons. *It was found that some of these electrons were scattered at very large angles, just as in Rutherford scattering, indicating that there are small constituents within the proton.* Figure 6.5 shows the picture of a proton as

Table 6.3
Quark Structure of Some of the Baryons

Name	Symbol	Structure	Charge (units of e)	Spin (units of $\hbar$)	Mass (GeV)
Proton	p	u u d	1	1/2	0.938
Neutron	n	u d d	0	1/2	0.94
Delta plus plus	Δ^{++}	u u u	2	3/2	1.232
Delta plus	Δ^{+}	u u d	1	3/2	
Delta zero	Δ^{0}	u d d	0	3/2	
Delta minus	Δ^{-}	d d d	–1	3/2	
Lambda zero	Λ^{0}	u d s	0	1/2	1.116
Positive sigma	Σ^{*+}	u u s	1	3/2	1.385
Positive sigma	Σ^{+}	u u s	1	1/2	1.189
Neutral sigma	Σ^{*0}	u d s	0	3/2	1.385
Neutral sigma	Σ^{0}	u d s	0	1/2	1.192
Negative sigma	Σ^{*-}	d d s	–1	3/2	1.385
Negative sigma	Σ^{-}	d d s	–1	1/2	1.197
Negative xi	Ξ^{-}	s d s	–1	1/2	1.321
Neutral xi	Ξ^{o}	s u s	0	1/2	1.315
Omega minus	Ω^{-}	s s s	–1	3/2	1.672
Charmed lambda	Λ^{+}_{c}	u d c	1	1/2	2.281

Table 6.4
Quark Structure of Some Mesons

Name	Symbol	Structure	Charge (units of e)	Spin (units of $\hbar$)	Mass (GeV)
Positive pion	π^{+}	$\underline{d}$ u	1	0	0.14
Positive rho	ρ^{+}	$\underline{d}$ u	1	1	0.77
Negative pion	π^{-}	$\underline{u}$ d	–1	0	0.14
Negative rho	ρ^{-}	$\underline{u}$ d	–1	1	0.77
Pi zero	π^{0}	50%($\underline{u}$ u) + 50%$\underline{d}$ d)	0	0	0.135
Positive kaon	K^{+}	u $\underline{s}$	1	0	0.494
Neutral kaon	K^{0}	$\underline{s}$ d	0	0	0.498
Negative kaon	K^{-}	$\underline{u}$ s	–1	0	0.494
J/Psi (charmonium)	J/Ψ	c $\underline{d}$	0	1	3.097
Charmed eta	η_{c}	c $\underline{c}$	0	0	2.98
Neutral D	D^{0}	$\underline{u}$ c	0	0	1.863
Neutral D	D^{*0}	$\underline{u}$ c	0	1	
Positive D	D^{+}	$\underline{d}$ c	1	0	1.868
Zero B-meson	B^{0}	$\underline{d}$ b	0		5.26
Negative B-meson	B^{-}	$\underline{u}$ b	–1		5.26
Upsilon	υ	$\underline{b}$ b	0	1	9.46
Phi-meson	Φ	s $\underline{s}$	0	1	1.02
F-meson	F^{+}	c $\underline{s}$	0	1	2.04

observed by scattering experiments. The scattering appears to come from particles with charges of +2/3 and −1/3 of the electronic charge. (Recall that the up quark has a charge of +2/3, whereas the down quark has a charge of −1/3.) *There is thus, experimental evidence for the quark structure of the proton.* Similar experiments have also been performed on neutrons with the same success. The scattering also confirmed the existence of some quark-antiquark pairs within the proton. Recall that quark-antiquark pairs are the constituents of mesons. The experiments also showed the existence of other particles within the nucleons, called *gluons*. The gluons are the exchange particles between the quarks that act to hold the quarks together. They are the nuclear glue.

The one difficulty with the quark model at this point is that there seems to be a violation of the Pauli exclusion principle. Recall that the Pauli exclusion principle stated that no two electrons can have the same quantum numbers at the same time. The Pauli exclusion principle is actually more general than that, in that it applies not only to electrons, but to any particles that have half-integral spin, such as 1/2, 3/2, 5/2, and so on. *Particles that have half-integral spin are called fermions.* Because quarks have spin 1/2, they also must obey the Pauli exclusion principle. But the Δ^{++} particle is composed of three up quarks all with the same spin, and the Ω^- particle has three strange quarks all with the same spin. *Thus, there must be an additional characteristic of each quark, that is different for each quark, so that the Pauli exclusion principle will not be violated. This new attribute of the quark is called "color."*

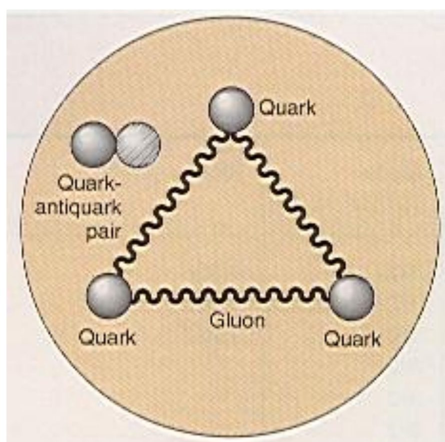


Figure 6.5 Structure of the proton. (From D. H. Perkins, "Inside the Proton" in The Nature of Matter, Clarendon Press, Oxford. 1981)

Quarks come in three colors: red, green, and blue. We should note that these colors are just names and have no relation to the real colors that we see everyday with our eyes. The words are arbitrary. As an example, they could just as easily have been called A, B, and C. We can think of color in the same way as electric charges. Electric charges come in two varieties, positive and negative. Color charges come in three varieties: red, green, and blue. *Thus, there are three types of up* *quarks; a red-up quark* u_R, *a green-up quark* u_G, *and a blue-up quark* u_B. Hence the delta plus-plus particle Δ^{++} can be represented as in figure 6.6(a). In this way there is no violation of the Pauli exclusion principle since each up quark is different.

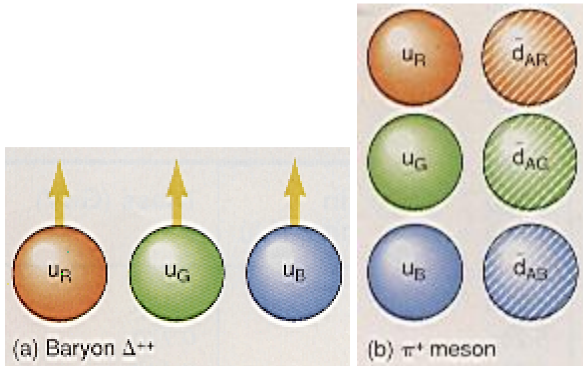
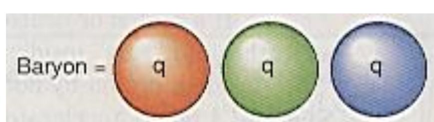


Figure 6.6 Colored quarks.

All baryons are composed of red, green, and blue quarks. Just as the primary colors red, green, and blue add up to white, the combination of a red, green, and blue quark is said to make up the color white. All baryons are, therefore, said to be white, or colorless. Just as a quark has an antiquark, each color of quark has an anticolor. Hence, a red-up quark has an up antiquark that carries the color antired, and is called an antired-up quark. The varieties of quarks are called flavors, such as up, down, strange, and so on. Hence, each flavor of quark comes in three colors to give a total of six flavors times three colors equals 18 quarks. Associated with the 18 quarks are 18 antiquarks. *Mesons, like baryons, must also be white or colorless. Hence, one colored quark of a meson must always be associated with an anticolor, since a color plus its anticolor gives white.* Thus, possible formations of a π^+ meson are shown in figure 6.6(b). That is, a red-up quark u_R combines with an antidown quark that carries the color antired $\bar{d}_{AR}$ to form the white π^+ meson. (The anticolor quark is shown with the hatched lines in figure 6.6.) Similarly the π^+ meson can be made out of green and antigreen $u_G\bar{d}_{AG}$ and blue and antiblue quarks $u_B\bar{d}_{AB}$ and a linear combination of them, such as $u_R\bar{d}_{AR} + u_G\bar{d}_{AG} + u_B\bar{d}_{AB}$ We can rewrite equations 6.4 and 6.5 as



(6.6)

(6.7)

The force between a quark carrying a color and its antiquark carrying anticolor is always attractive. Similarly the force between three quarks each of a different color is also attractive. All other combinations of colors gives a repulsive force. We will say more about colored quarks when we discuss the strong nuclear force in section 6.8.

6.5 The Electromagnetic Force

The electromagnetic force has been discussed in some detail in your previous general physics course. To summarize the results from there, Coulomb's law gave the electric force between charged particles, and the electric field was the mediator of that force. The relation between electricity and magnetism was first discovered by Ampère when he found that a current flowing in a wire produced a magnetic field. Faraday found that a changing magnetic field caused an electric current. James Clerk Maxwell synthesized all of electricity with all of magnetism into his famous equations of electromagnetism. That is, the separate force of electricity and the force of magnetism were unified into one electromagnetic force.

The merger of electromagnetic theory with quantum mechanics has led to what is now called ***quantum electrodynamics,*** *which is abbreviated* ***QED****. In QED the electric force is transmitted by the exchange of a virtual photon.* That is, the force between two electrons can be visualized as in figure 6.7. Recall from chapter 3 that the Heisenberg uncertainty relation allows for the creation of a virtual particle as long as the energy associated with the mass of the virtual particle is repaid in a time interval Δt that satisfies equation 3.56. In figure 6.7, two electrons approach each other. The first electron emits a virtual photon and recoils as shown.

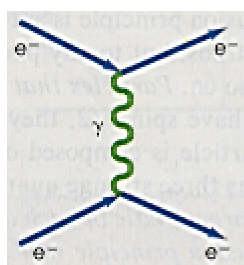


Figure 6.7 The electric force as an exchange of a virtual photon.

When the second electron absorbs that photon it also recoils as shown, leading to the result that the exchange of the photon caused a force of repulsion between the two electrons. As pointed out in chapter 3, this exchange force is strictly a quantum mechanical phenomenon with no real classical analogue. So it is perhaps a little more difficult to visualize that the exchange of a photon between an electron and a proton produces an attractive force between them. *The exchanged photon is the mediator or transmitter of the force. All of the forces of nature can be represented by an exchanged particle.*

Because the rest mass of a photon is equal to zero, the range of the electric force is infinite. This can be shown with the help of a few equations from chapter 3. The payback time for the uncertainty principle was

$$\Delta t = \frac{\hbar}{\Delta E} \tag{3.56}$$

While the energy ΔE was related to the mass Δm of the virtual particle by

$$\Delta E = (\Delta m)c^2$$

Substituting this into equation 3.56, gave for the payback time

$$\Delta t = \frac{\hbar}{(\Delta m)c^2} \tag{3.57}$$

The distance a virtual particle could move and still return during that time Δt, was given as

$$d = c\,\frac{\Delta t}{2} \tag{3.58}$$

This distance is called the *range* of the virtual particle. Substituting equation 3.57 into 3.58 gives for the range

$$d = \frac{c\,\hbar}{2(\Delta m)c^2}$$

$$d = \frac{\hbar}{2c}\,\frac{1}{\Delta m} \tag{6.8}$$

For a photon, the rest mass Δm is equal to zero. So as the denominator of a fraction approaches zero, the fraction approaches infinity. Hence, the range d of the particle goes to infinity. Thus, the electric force should extend to infinity, which, of course, it does.

6.6 The Weak Nuclear Force

The weak nuclear force is best known for the part it plays in radioactive decay. Recall from chapter 5 on nuclear physics that the initial step in beta β^- decay is for a neutron in the nucleus to decay according to the relation

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e} \tag{6.9}$$

Whereas the proton inside the nucleus decays as

$$p \rightarrow n + e^{+} + \nu_e \tag{6.10}$$

and is the initial step in the beta β^{+} decay. Finally, the radioactive disintegration caused by the capture of an electron by the nucleus (electron capture), is initiated by the reaction

$$e^{-} + p \rightarrow n + \nu_{\mathbf{e}} \tag{6.11}$$

These three reactions are just some of the reactions that are mediated by the weak nuclear force.

*The **weak nuclear force** does not exert the traditional push or pull type of force known in classical physics. Rather, it is responsible for the transmutation of the subatomic particles. The weak nuclear force is independent of electric charge and acts between leptons and hadrons and also between hadrons and hadrons.* The range of the weak nuclear force is very small, only about 10^{-17} m. The decay time is relatively large in that the weak decay occurs in about 10^{-10} seconds, whereas decays associated with the strong interaction occur in approximately 10^{-23} seconds.

The weak nuclear force is the weakest force after gravity. A product of weak interactions is the neutrino. The neutrinos are very light particles. Some say they have zero rest mass while others consider them to be very small, with an upper limit of about 10^{-30} eV for the ν_e neutrino. The neutrino is not affected by the strong or electromagnetic forces, only by the weak force. Its interaction is so weak that it can pass through the earth or the sun without ever interacting with anything.

6.7 The Electroweak Force

Steven Weinberg, Abdus Salam, and Sheldon Glashow proposed a unification of the electromagnetic force with the weak nuclear force and received the Nobel Prize for their work in 1979. This force is called the **electroweak force.** Just as a virtual photon mediates the electromagnetic force between charged particles, it became obvious that there should also be some particle to mediate the weak nuclear force. The new electroweak force is mediated by four particles: the photon and three intermediate *vector bosons* called W^{+}, W^{-}, and Z^{0}. The photon mediates the electromagnetic force, whereas the vector bosons mediate the weak nuclear force. In terms of the exchange particles, the decay of a neutron, equation 6.9, is shown in figure 6.8(a). *A neutron decays by emitting a W^{-} particle, thereby converting the neutron into a proton. The W^{-} particle subsequently decays within 10^{-26} s into an electron and an antineutrino.* The decay of the proton in a radioactive nucleus, equation 6.10, is shown in figure 6.8(b). *The proton emits the positive intermediate vector boson, W^{+}, and is converted into a neutron. The W^{+} subsequently decays into a positron and a neutrino.* An electron capture, equation 6.11, is shown in figure 6.8(c) as a collision between a proton and an electron. The proton emits a W^{+} and is converted into a neutron. The W^{+} then combines with the electron forming a neutrino. The Z^{0} particle is observed in electron-neutrino scattering, as shown in figure 6.8(d).

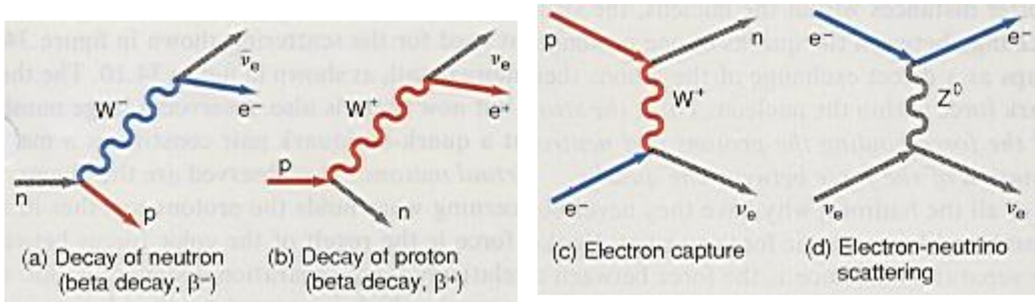


Figure 6.8 Examples of the electroweak force.

The vector bosons, W^+ and W^-, were found experimentally in proton-antiproton collisions at high energies, at the European Center for Nuclear Research (CERN), in January 1983, by a team headed by Carlo Rubbia of Harvard University. The Z^0 was found a little later in May 1983. The mass of the $W^\pm$ was around 80 GeV, while the mass of the Z^0 was about 90 GeV. Referring to equation 6.8, we see that for such a large mass, Δm in that equation gives a very short range d for the weak force, as found experimentally.

At very high energies, around 100 GeV, the electromagnetic force and the weak nuclear force merge into one electroweak force that acts equally between all particles: hadrons and leptons, charged and uncharged.

6.8 The Strong Nuclear Force

As mentioned previously, the **strong nuclear force** is responsible for holding the protons and neutrons together in the nucleus. The strong nuclear force must indeed be very strong to overcome the enormous electrical force of repulsion between the protons. Yukawa proposed that an exchange of mesons between the nucleons was the source of the nuclear force. But the nucleons are themselves made up of quarks. What holds these quarks together?

In quantum electrodynamics (QED), the electric force was caused by the exchange of virtual photons. One of the latest theories in elementary particle physics is called ***quantum chromodynamics (QCD)*** *and the force holding quarks together is caused by the exchange of a new particle, called a "gluon."* That is, a gluon is the nuclear glue that holds quarks together in a nucleon. Figure 6.9(a) shows the force between quarks as the exchange of a virtual gluon. Gluons, like quarks, come in colors and anticolors. *A gluon interacting with a quark changes the color of a quark.* As an example, figure 6.9(b) shows a red-up quark u_R emitting a red-antiblue gluon $(R\bar{B})$ The up quark loses its red color and becomes blue. That is, *in taking away an anticolor, the color itself must remain.* Hence, taking away an antiblue from the up quark, the color blue must remain. When the first blue- up quark

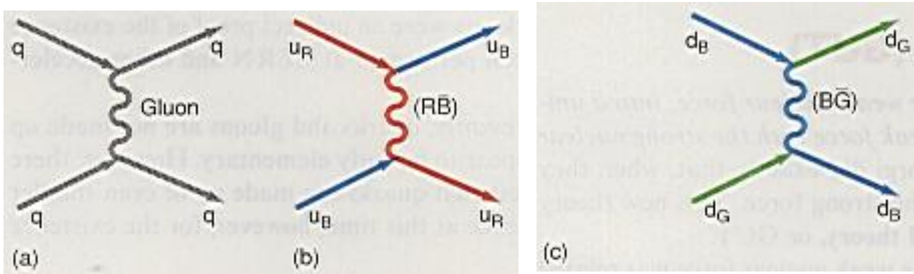


Figure 6.9 Exchange of gluons between quarks.

receives the red-antiblue gluon $(R\overline{B})$, the blue of the up quark combines with the antiblue of the gluon canceling out the color blue. (A color and its anticolor always gives white.) The red color of the gluon is now absorbed by the up quark turning it into a red-up quark. *Thus, in the process of exchanging the gluon, the quarks changed color*. Figure 6.9(c) shows a blue-down quark emitting a blue-antigreen gluon $(B\overline{G})$, changing the blue-down quark into a green-down quark. When the first green-down quark absorbs the $(B\overline{G})$ gluon, the color green cancels and the down quark becomes a blue-down quark.

All told, there are eight different gluons and each gluon has a mass. Each gluon always carries one color and one anticolor. Occasionally a gluon can transform to a quark-antiquark pair.

At energies greater than that used for the scattering shown in figure 6.5, scattering from protons reveals even more detail, as shown in figure 6.10. The three valence quarks are shown as before, but now there is also observed a large

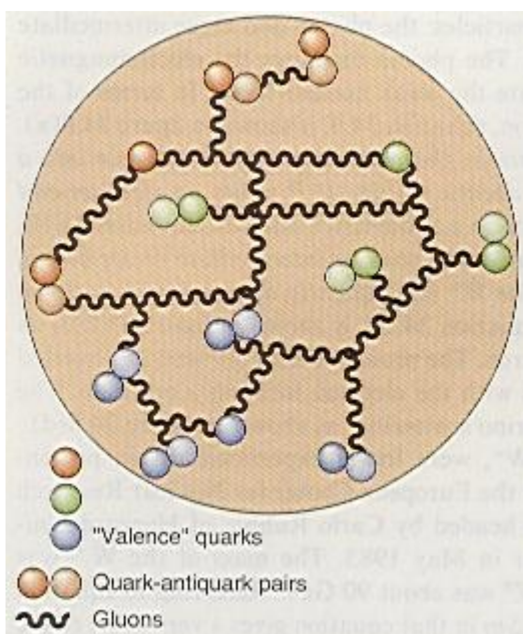


Figure 6.10 More detailed structure of the proton. (After D. H. Perkins, "The Nature of Matter", Oxford University Press)

number of quark-antiquark pairs. Recall that a quark-antiquark pair constitutes a meson. *Hence, the proton is seething with virtual mesons.* Also observed are the gluons. To answer the traditional questions concerning what holds the protons together in the nucleus, we can say that the strong force is the result of the color forces between the quarks within the nucleons. At relatively large separation distances within the nucleus, the quark-antiquark pair (meson), which is created by the gluons, is exchanged between the nucleons. At shorter distances within the nucleus, the strong force can be explained either as an exchange between the quarks of one proton and the quarks of another proton, or perhaps as a direct exchange of the gluons themselves, which give rise to the quark-quark force within the nucleon. *Thus, the strong force originates with the quarks, and the force binding the protons and neutrons together in the nucleus is the manifestation of the force between the quarks.*

If quarks are the constituents of all the hadrons, why have they never been isolated? The quark-quark force is something like an elastic force given by Hooke's law, $F = kx$. For small values of the separation distance x, the force between the quarks is small and the quarks are relatively free to move around within the particle. However, if we try to separate the quarks through a large separation distance x, then the force becomes very large, so large, in fact, that the quarks cannot be separated at all. *This condition is called the confinement of quarks*. *Thus, quarks are never seen in an isolated state because they cannot escape from the particle in which they are constituents.*

But is there any evidence for the existence of quarks? The answer is yes. Experiments were performed in the new PETRA storage ring at DESY (Deutsches Electronen-Synchronton) in Hamburg, Germany, in 1978. Electrons and positrons, each at an energy of 20 GeV, were fired at each other in a head-on collision. The annihilation of the electron and its antiparticle, the positron, produce a large amount of energy; it is from this energy that the quarks are produced. The experimenters found a series of "quark jets," which were the decay products of the quarks, exactly as predicted. (A quark jet is a number of hadrons flying off from the interaction in roughly the same direction.) These quark jets were an indirect proof of the existence of quarks. Similar experiments have been performed at CERN and other accelerators.

As far as can be determined presently, quarks and gluons are not made up of still smaller particles; that is, they appear to be truly elementary. However, there are some speculative theories that suggest that quarks are made up of even smaller particles called preons. There is no evidence at this time, however, for the existence of preons.

6.9 Grand Unified Theories (GUT)

If it is possible to merge the electric force with the weak nuclear force, into a unified electroweak force, why not merge the electroweak force with the strong nuclear force?

In 1973 Sheldon Glashow and Howard Georgi did exactly that, when they published a theory merging the electroweak with the strong force. This new theory was the first of many to be called the **grand unified theory,** or GUT.

The first part of this merger showed how the weak nuclear force was related to the strong nuclear force. Let us consider the decay of the neutron shown in equation 6.9:

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

We can now visualize this decay according to the diagram in figure 6.11. According

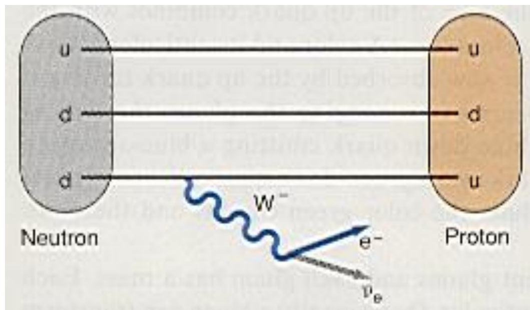


Figure 6.11 The decay of the neutron.

to the quark theory, a neutron is composed of one up quark and two down quarks. One of the down quarks of the neutron emits the W^{-} boson and is changed into an up quark, transforming the neutron into a proton. (Recall, that the proton consists of two up quarks and one down quark.) The W^{-} boson then decays into an electron and an antineutrino. Thus, *the weak force changes the flavor of a quark, whereas the strong force changes only the color of a quark.*

Above 10^{15} GeV of energy, called the *grand unification energy,* we can no longer tell the difference between the strong, weak, and electromagnetic forces. Above this energy there is only one unified interaction or force that occurs. Of course, this energy is so large that it is greater than anything we could ever hope to create experimentally. As we shall see, however, it could have been attained in the early stages of the creation of the universe — the so-called "Big Bang."

The strong nuclear force operates between quarks, whereas the weak nuclear force operates between quarks and leptons. If the strong and weak forces are to be combined, then the quarks and leptons should be aspects of one more fundamental quantity. That is, the grand unified force should be able to transform quarks into leptons and vice versa. In the grand unified theories, there are 24 particles that mediate the unified force and they are listed in table 6.5. In grand unified theories, the forces are unified because the forces arise through the exchange of the same family of particles. As seen before, the photon mediates the electromagnetic force; the vector bosons mediate the weak force; the gluons mediate the strong force; and there are now 12 new particles called X particles (sometimes progenitor and/or lepto-quark particles) that mediate the unified force. It is these X particles that are capable of converting hadrons into leptons by changing quarks to leptons.

Table 6.5 Family of Particles that Mediate the Unified Force		
Particle	Number	Force Mediated
Photon	1	Electromagnetic
Vector bosons (W^+, W^-, Z^0)	3	Weak
Gluons	8	Strong
X particles	12	Strong-electroweak

The X particles come in four different electrical charges, ±1/3 and ±4/3. Thus, the X particles can be written as $X^{1/3}$, $X^{-1/3}$, $X^{4/3}$, and $X^{-4/3}$. Each of these X particles also comes in the three colors red, blue, and green, thereby giving the total of 12 X particles. The X particles can change a quark into a lepton, as shown in figure 6.12. An X particle carrying an electrical charge of −4/3, and a color charge

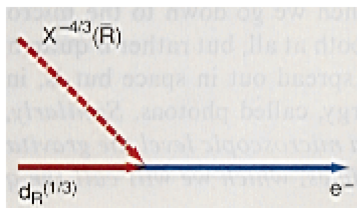


Figure 6.12 Changing a quark to an electron.

of antired combines with a red-down quark, which carries an electrical charge of 1/3. The colors red and antired cancel to give white, while the electrical charge becomes 1/3 − 4/3 = −3/3 = −1 and an electron is created out of a quark. This type of process is not readily seen in our everyday life because the mass of the virtual X particle must be of the order of 10^{15} GeV, which is an extremely large energy. A similar analysis shows that an isolated proton should also decay. The lifetime, however, is predicted to be 10^{32} yr. Experiments are being performed to look for the predicted decays. However, at the present time no such decay of an isolated proton has been found. An isolated proton seems to be a very stable particle, indicating that either more experiments are needed, or the GUT model needs some modifications.

6.10 The Gravitational Force and Quantum Gravity

As has been seen throughout this book, physics is a science of successive approximations to the truth hidden in nature. Newton found that celestial gravity was of the same form as terrestrial gravity and unified them into his law of universal gravitation. However, it turned out that it was not quite so universal. Einstein started the change in his special theory of relativity, which governed systems moving with respect to each other at constant velocity. As he generalized this theory to systems that were accelerated with respect to each other, he found the equivalence between accelerated systems and gravity. The next step of course was to show that matter warped spacetime and gravitation was a manifestation of that warped spacetime. Thus, general relativity became a law of gravitation, and it was found that Newton's law of gravitation was only a special case of Einstein's theory of general relativity.

We have also seen that the quantum theory is one of the great new theories of modern physics, which seems to say that nature is quantized. There are quanta of energy, mass, angular momentum, charge, and the like. But general relativity, in its present format, is essentially independent of the quantum theory. It is, in this sense, still classical physics. It, too, must be only an approximation to the truth hidden in nature. *A more general theory should fuse quantum mechanics with general relativity — that is, we need a quantum theory of gravity*.

In order to combine quantum theory with general relativity (hereafter called Einstein's theory of gravitation), we have to determine where these two theories merge. Remember the quantum theory deals with very small quantities, because of the smallness of Planck's constant h, whereas Einstein's theory of gravitation deals with very large scale phenomena, or at least with very large masses that can significantly warp spacetime.

One of the important characteristics of the quantum theory is the wave-particle duality; waves can act as particles and particles can act as waves. And as has also been seen, waves can exist in the electromagnetic field. Let us, for the moment, compare electromagnetic fields with gravitational fields. On a large scale the electric field appears smooth. It is only when we go down to the microscopic level that we see that the electric field is not smooth at all, but rather is quite bumpy, because the energy of the electric field is not spread out in space but is, instead, stored in little bundles of electromagnetic energy, called photons. *Similarly, from the quantum theory we should expect that on a microscopic level the gravitational field should also be quantized into little particles, which we will call the quanta of the gravitational field — the* ***gravitons.***

But what is a gravitational field but the warping of spacetime? Hence, a quantum of gravitation must be a quantum of spacetime itself. Thus, the graviton would appear to be a quantum of spacetime. Therefore, on a microscopic level, spacetime itself is probably not smooth but probably has a graininess or bumpiness to it. At this time, no one knows for sure what happens to spacetime on this microscopic level, but it has been conjectured that spacetime may look something like a foam that contains "wormholes."

At what point do the quantum theory and Einstein's theory of gravitation merge? The answer is to be found in Heisenberg's uncertainty principle.

$$\Delta E \Delta t \geq \hbar \tag{31.55}$$

For the electric field, small quantities of energy ΔE of the electric field are turned into small quanta of energy, the photons. In a similar manner, small quantities of energy ΔE of the gravitational field should be turned into little bundles or quantums of gravity, the gravitons. Since the range of a force is determined by the mass of the exchanged particle, and the range of the gravitational force is known to be infinite, it follows that the rest mass of the graviton must be zero. Hence, a quantum fluctuation should appear as a gravitational wave moving at the speed of light *c*. Therefore, if we consider a fluctuation of the gravitational field that spreads out spherically, the small time for it to move a distance *r* is

$$\Delta t = \frac{r}{c} \tag{6.12}$$

To obtain an order of magnitude for the energy, we drop the greater than sign in the uncertainty principle and on substituting equation 6.12 into equation 3.55 we get, for the energy of the fluctuation,

$$\Delta E \Delta t = \Delta E \frac{r}{c} = \hbar$$

and

$$\Delta E = \frac{\hbar c}{r} \tag{6.13}$$

The value of *r* in equation 6.13, wherein the quantum effects become important, is unknown at this point; in fact, it is one of the things that we wish to find. So further information is needed. Let us consider the amount of energy required to pull this little graviton or bundle of energy apart against its own gravity. The work to pull the graviton apart is equal to the energy necessary to assemble that mass by bringing small portions of it together from infinity. Let us first consider the problem for the electric field, and then use the analogy for the gravitational field. Recall that the electric potential for a small spherical charge is

$$V = \frac{kq}{r}$$

But the electric potential *V* was defined as the potential energy per unit charge, that is,

$$V = \frac{\text{PE}}{q}$$

So if a second charge *q* is brought from infinity to the position *r*, the potential energy of the system of two charges is

$$\text{PE} = qV = \frac{kq^2}{r}$$

In a similar vein, a gravitational potential Φ could have been derived using the same general technique used to derive the electric potential. The result for the gravitational potential would be

$$\Phi = \frac{GM}{r} \tag{6.14}$$

where G, of course, is the gravitational constant, M is the mass, and r is the distance from the mass to the point where we wish to determine the gravitational potential. The gravitational potential of a spherical mass is defined, similar to the electric potential, as the gravitational potential energy per unit mass. That is,

$$\Phi = \frac{\text{PE}}{M} \tag{6.15}$$

Hence, if another mass M is brought from infinity to the position r, the potential energy of the system of two equal masses is

$$\text{PE} = M\Phi = \frac{GM^2}{r} \tag{6.16}$$

This value of the potential energy, PE to assemble the two masses, is the same energy that would be necessary to pull the two masses apart. Applying the same reasoning to the assembly of the masses that constitutes the graviton, the potential energy given by equation 6.16 is equal to the energy that would be necessary to pull the graviton apart. This energy can be equated to the energy of the graviton found from the uncertainty principle. Thus,

$$\text{PE} = \Delta E$$

Substituting for the PE from equation 6.16 and the energy ΔE from the uncertainty principle, equation 6.13, we get

$$\frac{GM^2}{r} = \frac{\hbar c}{r} \tag{6.17}$$

But the mass of the graviton M can be related to the energy of the graviton by Einstein's mass-energy relation as

$$\Delta E = Mc^2$$

or

$$M = \frac{\Delta E}{c^2} \tag{6.18}$$

Substituting equation 6.18 into equation 6.17 gives

$$\frac{G(\Delta E)^2}{r(c^2)^2} = \frac{\hbar c}{r}$$

Solving for ΔE, we get

$$\Delta E = \sqrt{\frac{\hbar c^5}{G}} \tag{6.19}$$

Equation 6.19 represents the energy of the graviton.

Example 6.1

The energy of the graviton. Find the energy of the graviton.

Solution

The energy of the graviton, found from equation 6.19, is

$$\Delta E = \sqrt{\frac{\hbar c^5}{G}}$$

$$\Delta E = \sqrt{\frac{(1.05\times10^{-34}\ \text{J s})(3.00\times10^{8}\ \text{m/s})^5}{6.67\times10^{-11}\ (\text{N m}^2)/\text{kg}^2}}$$

$$\Delta E = 1.96\times10^{9}\ \text{J}$$

This can also be expressed in terms of electron volts as

$$\Delta E = (1.96\times10^{9}\ \text{J})\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\text{J}}\right)\left(\frac{1\ \text{GeV}}{10^{9}\ \text{eV}}\right)$$

$$= 1.20\times10^{19}\ \text{GeV}$$

This is the energy of a graviton; it is called the *Planck energy*.

From the point of view of particle physics, then, the graviton looks like a particle of mass 10^{19} GeV/c^2. This is an enormous mass and energy when compared to the masses and energies of all the other elementary particles. However, for any elementary particles of this size or larger, both quantum theory and gravitation must be taken into account. Recall that in all the other interactions of the elementary particles, gravity was ignored. From the point of view of ordinary gravity, this energy is associated with a mass of 2×10^{-5} g, a very small mass.

The distance in which this quantum fluctuation occurs can now be found by equating ΔE from equation 6.13 to ΔE from equation 6.19, that is,

$$\Delta E = \frac{\hbar c}{r} = \Delta E = \sqrt{\frac{\hbar c^5}{G}}$$

Solving for r we get

$$r = \frac{\hbar c}{\sqrt{\hbar c^5/G}}$$

$$r = \sqrt{\frac{\hbar G}{c^3}} \tag{6.20}$$

Equation 6.20 is the distance or length where quantum gravity becomes significant. This distance turns out to be the same distance that Max Planck found when he was trying to establish some fundamental units from the fundamental constants of nature, and is called the *Planck length* L_P. Hence, the Planck length is

$$L_P = \sqrt{\frac{\hbar G}{c^3}} \tag{6.21}$$

Example 6.2

The Planck length. Determine the size of the Planck length.

Solution

The Planck length, determined from equation 6.21, is

$$L_P = \sqrt{\frac{\hbar G}{c^3}}$$

$$L_P = \sqrt{\frac{(1.05\times 10^{-34}\text{J s})(6.67\times 10^{-11}(\text{N m}^2)/\text{kg}^2)}{(3.00\times 10^8 \text{m/s})^3}}$$

$$= 1.61\times 10^{-35}\text{ m} = 1.61\times 10^{-33}\text{cm}$$

Thus, quantum fluctuations of spacetime start to occur at distances of the order of 1.61×10^{-33} cm. We can now find the interval of time, within which this quantum fluctuation of spacetime occurs, from equation 6.12 as

$$\Delta t = \frac{r}{c} = \frac{L_P}{c}$$

This time unit is called the *Planck time* T_P and is

$$T_P = \frac{L_P}{c} \tag{6.22}$$

$$= \frac{1.61\times 10^{-35}\text{m}}{3.00\times 10^8 \text{m/s}}$$

$$= 5.37\times 10^{-44}\text{s}$$

Thus, intervals of space and time given by the Planck length and the Planck time are the regions in which quantum gravity must be considered. This distance and time are extremely small. Recall that the size of the electron is about 10^{-19} m. Thus, quantum gravity occurs on a scale much smaller than that of an atom, a nucleus, or even an electron. There is relatively little known about quantum gravity at this time, but research is underway to find more answers dealing with the ultimate structure of spacetime itself.

6.11 The Superforce — Unification of All the Forces

An attempt to unify all the forces into one single force — a kind of **superforce** — continues today. One of the techniques followed is called *supersymmetry,* where the main symmetry element is spin. (Recall that all particles have spin.) Those particles that obey the Pauli exclusion principle have half-integral spin, that is, spin $\hbar/2$, $3\hbar/2$, and so on. Those particles that obey the Pauli exclusion principle are called *fermions.* All the quarks and leptons are fermions. Particles that have integral spin, $\hbar$, $2\hbar$, and so on, do not obey the Pauli exclusion principle. These particles are called *bosons.* All the mediating particles, such as the photon, $W^{\pm}$, Z^0, gluons, and the like, are bosons. *Hence, fermions are associated with particles of matter, whereas bosons are associated with the forces of nature, through an exchange of bosons*. *The new theories of supersymmetry attempt to unite bosons and fermions.*

A further addition to supersymmetry unites gravity with the electroweakstrong or GUT force into the superforce that is also called super gravity. Super gravity requires not only the existence of the graviton but also a new particle, the "gravitino," which has spin 3/2. However, this unification exists only at the extremely high energy of 10^{19} GeV, an energy that cannot be produced in a laboratory. However, in the initial formation or creation of the universe, a theory referred to as the Big Bang, such energies did exist.

The latest attempt to unify all the forces is found in the *superstring theory*. The superstring theory assumes that the ultimate building blocks of nature consist of very small vibrating strings. As we saw in our study of wave motion, a string is capable of vibrating in several different modes. The superstring theory assumes that each mode of vibration of a superstring can represent a particle or a force. Because there are an infinite number of possible modes of vibration, the superstring can represent an infinite number of possible particles. The graviton, which is responsible for the gravitational interaction, is caused by the lowest vibratory mode of a circular string. (Superstrings come in two types: open strings, which have ends, and closed strings, which are circular.) The photon corresponds to the lowest mode of vibration of the open string. Higher modes of vibrations represent different particles, such as quarks, gluons, protons, neutrons, and the like. In fact, the gluon is considered to be a string that is connected to a quark at each end. In this theory, no particle is more fundamental than any other, each is just a different mode of vibration of the superstrings. The superstrings interact with other superstrings by breaking and reforming. The four forces are considered just different manifestations of the one unifying force of the superstring. The superstring theory assumes that the universe originally existed in ten dimensions, but broke into two pieces — one of the pieces being our four-dimensional universe. Like the theories of supersymmetry and super gravity, the energies needed to test this theory experimentally are too large to be produced in any laboratory.

A simple picture of the unifications is shown in table 6.6. A great deal more work is necessary to complete this final unification.

Table 6.6
The Forces and Their Unification

Electricity				
	Electromagnetism			
Magnetism		Electroweak force		
Weak force			Grand unified theories (GUT)	
Strong force				Superforce
Gravity				

Have you ever wondered ... ?
An Essay on the Application of Physics
The Big Bang Theory and the Creation of the Universe

Have you ever wondered how the world was created? In every civilization throughout time and throughout the world, there has always been an account of the creation of the world. Such discussions have always belonged to religion and philosophy. It might seem strange that astronomers, astrophysicists, and physicists have now become involved in the discussion of the creation of the universe. Of course, if we think about it, it is not strange at all. *Since physics is a study of the entire physical world; it is only natural that physics should try to say something about the world's birth.*

The story starts in 1923 when the American astronomer, Edwin Hubble, using the Doppler effect for light, observed that all the galactic clusters, outside our own, in the sky were receding away from the earth. When we studied the Doppler effect for sound, we saw that when a train recedes from us its frequency decreases. A decrease in the frequency means that there is an increase in the wavelength. Similarly, a Doppler effect for light waves can be derived. The equations are different than those derived for sound because, in the special theory of relativity, the velocity of light is independent of the source. However, the effect is the same. That is, a receding source that emits light at a frequency ν, is observed by the stationary observer to have a frequency ν', where ν' is less than ν. Thus, since the frequency decreases, the wavelength increases. Because long waves are associated with the red end of the visible spectrum, all the observed wavelengths are shifted toward the red end of the spectrum. The effect is called the *cosmological red shift,* to distinguish it from the gravitational red shift discussed in chapter 2. *Hubble found that the light from the distant galaxies were all red shifted indicating that the distant galaxies were receding from us.*

It can, therefore, be concluded that if all the galaxies are receding from us, the universe itself must be expanding. Hubble was able to determine the rate at which the universe is expanding. If the universe is expanding now, then in some time in the past it must have been closer together. If we look far enough back in time, we should be able to find when the expansion began. (Imagine taking a movie picture of an explosion showing all the fragments flying out from the position of the explosion. If the movie is run backward, all the fragments would be seen moving backward toward the source of the explosion.)

The best estimate for the creation of the universe, is that the universe began as a great bundle of energy that exploded outward about 15 billion years ago. This great explosion has been called the ***Big Bang.*** *It was not an explosion of matter into an already existing space and time, rather it was the very creation of space and time, or spacetime, and matter themselves.*

As the universe expanded from this explosion, all objects became farther and farther apart. A good analogy to the expansion of spacetime is the expansion of a toy balloon. A rectangular coordinate system is drawn on an unstretched balloon, as shown in figure 1(a), locating three arbitrary points, *A, B,* and *C.* The balloon is

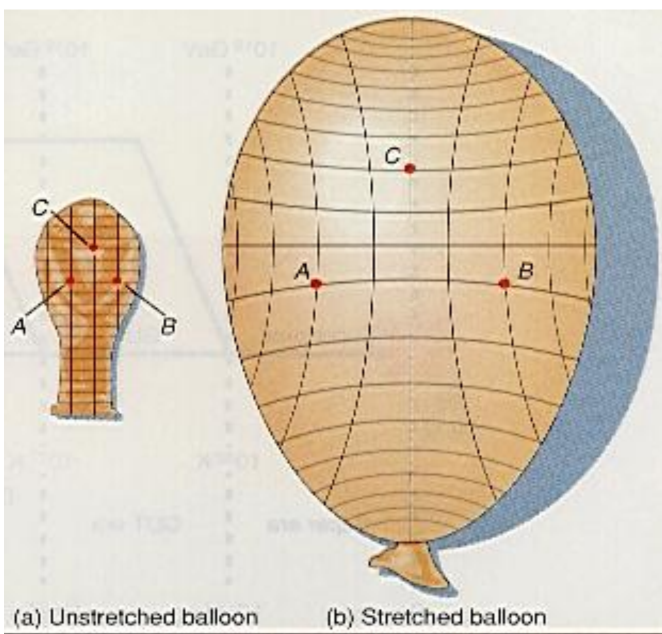


Figure 1 An Analogy to the expanding universe.

then blown up. As the balloon expands the distance between points *A* and *B, A* and *C,* and *B* and *C* increases. So no matter where you were on the surface of the balloon you would find all other points moving away from you. This is similar to the distant galaxies moving away from the earth. To complete the analogy to the expanding universe, we note that the simple flat rectangular grid in which Euclidean geometry holds now become a curved surface in which Euclidean geometry no longer holds.

If everything in the universe is spread out and expanding, the early stages of the universe must have been very compressed. To get all these masses of stars of the present universe back into a small compressed state, that compressed state must have been a state of tremendous energy and exceedingly high density and temperature. Matter and energy would be transforming back and forth through Einstein's mass-energy formula, $E = mc^2$. Work done by particle physicists at very high energies allows us to speculate what the universe must have looked like at these very high energies at the beginning of the universe.

The early history of the universe is sketched in figure 2. The Big Bang is shown occurring at time $t = 0$, which is approximately 15 billion years ago.

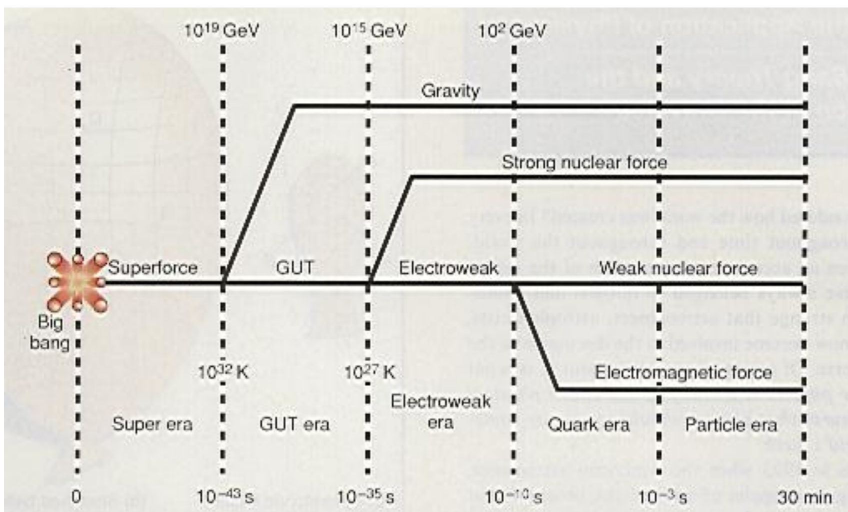


Figure 2 Creation of the four forces from the superforce.

1. *From the Big Bang to 10^{-43} s*

   Between the creation and the Planck time, 0 to 10^{-43} s, the energy of the universe was enormous, dropping to about 10^{19} GeV at the Planck time. The temperature was greater than 10^{33} K. Relatively little is known about this era, but the extremely high energy would cause all the forces to merge into one superforce. That is, gravity, the strong force, the weak force, and the electromagnetic force would all be replaced by one single superforce. This is the era being researched by present physicists in the supersymmetry and super gravity theories. There is only one particle, a super particle, that decays into bosons and fermions, and continually converts fermions to bosons and vice versa, so that there is no real distinction between them.

2. *From 10^{-43} s to 10^{-35} s*
As the universe expands, the temperature drops and the universe cools to about 10^{32} K. The energy drops below 10^{19} GeV and the gravitational force breaks away from the superforce as a separate force, leaving the grand unified force of GUT as a separate force. Now two forces exist in nature. We are now in the GUT era, that era governed by the grand unified theories. The X particle and its antiparticle $\overline{X}$ are in abundance. The X particles decay into quarks and leptons, whereas the $\overline{X}$ particles decay into antiquarks and antileptons. However, the decay rate of X and $\overline{X}$ are not the same and more particles than antiparticles are formed. This will eventually lead to the existence of more matter than antimatter in the universe. The X particles continually convert quarks into leptons and vice versa. There are plenty of quarks, electrons, neutrinos, photons, gluons, X particles, and their antiparticles present, but they have effectively lost their individuality.

3. *From 10^{-35} s to 10^{-10}* s
As further expansion of the universe continues, the temperature drops to 10^{27} K and the energy drops to 10^{15} GeV. At this low energy all the X particles disappear, and quarks and leptons start to have an individual identity of their own. No longer can they be converted into each other. The lower energy causes the strong nuclear force to break away leaving the electroweak force as the only unified force left. There are now three forces of nature: gravity, strong nuclear, and the electroweak. There are quarks, leptons, photons, neutrinos, $W^{\pm}$ and Z^0, and gluon particles present. It is still too hot for the quarks to combine.

4. *From 10^{-10} s to 10^{-3} s*
As the universe continues to expand, it cools down to an energy of 10^2 GeV. The $W^{\pm}$ and Z^0 particles disappear because there is not enough energy to form them anymore. The weak nuclear force breaks away from the electroweak force, leaving the electromagnetic force. There are now present the four familiar forces of nature: gravity, strong nuclear, weak nuclear, and electromagnetic. Quarks now combine to form baryons, qqq, and mesons, $q\overline{q}$ The familiar protons and neutrons are now formed. Because of the abundance of quarks over antiquarks, there will also be an excess of protons and neutrons over antiprotons and antineutrons.

5. *From 10^{-3} s to 30 mi*n
The universe has now expanded and cooled to the point where protons and neutrons can combine to form the nucleus of deuterium. The deuterium nuclei combine to form helium as described in section 5.9 on fusion. There are about 77% hydrogen nuclei, and 23% helium nuclei present at this time and this ratio will continue about the same to the present day. There are no atoms formed yet because the temperature is still too high. What is present is called a *plasma.*

6. *From 30 min to 1 Billion Years*
   Further expansion and cooling now allows the hydrogen and helium nuclei to capture electrons and the first chemical elements are born. Large clouds of hydrogen and helium are formed.

7. *From 1 Billion Years to 10 Billion Years*
   The large rotating clouds of hydrogen and helium matter begin to concentrate due to the gravitational force. As the radius of the cloud decreases, the angular velocity of the cloud increases in order to conserve angular momentum. (Similar to the spinning ice skater) These condensing, rotating masses are the beginning of galaxies.

   Within the galaxies, gravitation causes more and more matter to be compressed into spherical objects, the beginning of stars. More and more matter gets compressed until the increased pressure of that matter causes a high enough temperature to initiate the fusion process of converting hydrogen to helium and the first stars are formed. Through the fusion process, more and more chemical elements are formed. The higher chemical elements are formed by neutron absorption until all the chemical elements are formed.

   These first massive stars did not live very long and died in an explosion — a supernova — spewing the matter of all these heavier elements out into space. The fragments of these early stars would become the nuclei of new stars and planets.

8. *From 10 Billion Years to the Present*
   The remnants of dead stars along with hydrogen and helium gases again formed new clouds, which were again compressed by gravity until our own star, the sun, and the planets were formed. All the matter on earth is the left over ashes of those early stars. Thus, even we ourselves are made up of the ashes of these early stars. As somebody once said, there is a little bit of star dust in each of us.

The Language of Physics

Leptons
Particles that are not affected by the strong nuclear force (p.).

Hadrons
Particles that are affected by the strong nuclear force (p.).

Baryons
A group of hadrons that have half-integral spin and are composed of three quarks (p.).

Mesons
A group of hadrons that have integral spin, that are composed of quark-antiquark pairs (p.).

Antiparticles
To each elementary particle in nature there corresponds another particle that has the characteristics of the original particle but opposite charge. Some neutral particles have antiparticles that have opposite spin, whereas the photon is its own antiparticle. The antiparticle of the proton is the antiproton. The antiparticle of the electron is the antielectron or positron. If a particle collides with its antiparticle both are annihilated with the emission of radiation or other particles. Conversely, photons can be converted to particles and antiparticles (p.).

Antimatter
Matter consists of protons, neutrons, and electrons, whereas antimatter consists of antiprotons, antineutrons, and antielectrons (p.).

Quarks
Elementary particles that are the building blocks of matter. There are six quarks and six antiquarks. The six quarks are: up, down, strange, charmed, bottom, and top. Each quark and antiquark also comes in three colors, red, green, and blue. Each color quark also has an anticolor quark. Baryons are composed of red, green, and blue quarks and mesons are made up of a linear combination of colored quark-antiquark pairs (p.).

Quantum electrodynamics (QED)
The merger of electromagnetic theory with quantum mechanics. In QED, the electric force is transmitted by the exchange of a virtual photon (p.).

Weak nuclear force
The weak nuclear force does not exert the traditional push or pull type of force known in classical physics. Rather, it is responsible for the transmutation of the subatomic particles. The weak force is independent of electric charge and acts between leptons and hadrons and also between hadrons and hadrons. The weak force is the weakest force after gravity (p.).

Electroweak force
A unification of the electromagnetic force with the weak nuclear force. The force is mediated by four particles: the photon and three intermediate vector bosons called W^+, W^-, and Z^0 (p.).

The strong nuclear force
The force that holds the nucleons together in the nucleus. The force is the result of the color forces between the quarks within the nucleons. At relatively large

separation distances within the nucleus, the quark-antiquark pair (meson), which is created by the gluons, is exchanged between the nucleons. At shorter distances within the nucleus, the strong force can be explained either as an exchange between the quarks of one proton and the quarks of another proton, or perhaps as a direct exchange of the gluons themselves, which give rise to the quark-quark force within the nucleon (p.).

Quantum chromodynamics (QCD)
In QCD, the force holding quarks together is caused by the exchange of a new particle, called a gluon. A gluon interacting with a quark changes the color of a quark (p.).

Grand unified theory
A theory that merges the electroweak force with the strong nuclear force. This force should be able to transform quarks into leptons and vice versa. The theory predicts the existence of 12 new particles, called X particles that are capable of converting hadrons into leptons by changing quarks to leptons. This theory also predicts that an isolated proton should decay. However, no such decays have ever been found, so the theory may have to be modified (p.).

Gravitons
The quanta of the gravitational field. Since gravitation is a warping of spacetime, the graviton must be a quantum of spacetime (p.).

Superforce
An attempt to unify all the forces under a single force. The theories go under the names of supersymmetry, super gravity, and superstrings (p.).

The Big Bang theory
The theory of the creation of the universe that says that the universe began as a great bundle of energy that exploded outward about 15 billion years ago. It was not an explosion of matter into an already existing space and time, rather it was the very creation of spacetime and matter (p.).

Questions for Chapter 6

*1. Discuss the statement, “A graviton is a quantum of gravity. But gravity is a result of the warping of spacetime. Therefore, the graviton should be a quantum of spacetime. But just as a quantum of the electromagnetic field, the photon, has energy, the graviton should also have energy. In fact, we can estimate the energy of a graviton. Therefore, is spacetime another aspect of energy? Is there only one fundamental quantity, energy?”

*2. Does antimatter occur naturally in the universe? How could you detect it? Where might it be located?

3. When an electron and positron annihilate, why are there two photons formed instead of just one?

4. Murray Gell-Mann first introduced three quarks to simplify the number of truly elementary particles present in nature. Now there are six quarks and six antiquarks, and each can come in three colors and three anticolors. Are we losing some of the simplicity? Discuss.

5. Discuss the experimental evidence for the existence of structure within the proton and the neutron.

6. How did the Pauli exclusion principle necessitate the introduction of colors into the quark model?

*7. If the universe is expanding from the Big Bang, will the gravitational force of attraction of all the masses in the universe eventually cause a slowing of the expansion, a complete stop to the expansion, and finally a contraction of the entire universe?

*8. Just as there are electromagnetic waves associated with a disturbance in the electromagnetic field, should there be gravitational waves associated with a disturbance in a gravitational field? How might such gravitational waves be detected?

*9. Einstein's picture of gravitational attraction is a warping of spacetime by matter. This has been pictured as the rubber sheet analogy. What might antimatter do to spacetime? Would it warp spacetime in the same way or might it warp spacetime to cause a gravitational repulsion? Would this be antigravity? Would the antiparticle of the graviton then be an antigraviton? Instead of a black hole, would there be a white hill?

10. Discuss the similarities and differences between the photon and the neutrino.

Problems for Chapter 6

Section 6.2 Particles and Antiparticles

1. How much energy is released when an electron and a positron annihilate? What is the frequency and wavelength of the two photons that are created?

2. How much energy is released when a proton and antiproton annihilate?

3. How much energy is released if 1.00 kg of matter annihilates with 1.00 kg of antimatter? Find the wavelength and frequency of the resulting two photons.

4. A photon "disintegrates," creating an electron-positron pair. If the frequency of the photon is 5.00×10^{24} Hz, determine the linear momentum and the energy of each product particle.

Section 6.4 Quarks

5. If the three quarks shown in the diagram combine to form a baryon, find the charge and spin of the resulting particle.

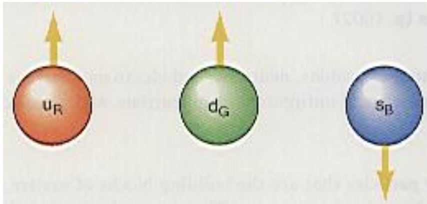


Diagram for problem 5.

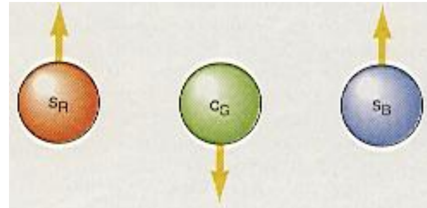


Diagram for problem 6.

6. If the three quarks shown in the diagram combine to form a baryon, find the charge and spin of the resulting particle.

7. If the three quarks shown in the diagram combine to form a baryon, find the charge and spin of the resulting particle.

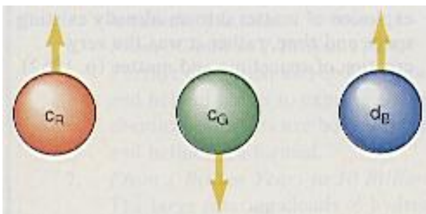


Diagram for problem 7.

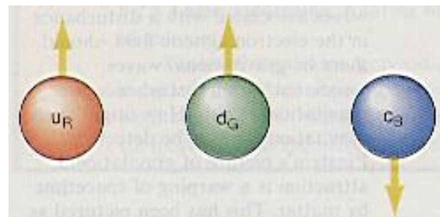


Diagram for problem 8.

8. Find the charge and spin of the baryon that consists of the three quarks shown in the diagram.

9. If the two quarks shown in the diagram combine to form a meson, find the charge and spin of the resulting particle.

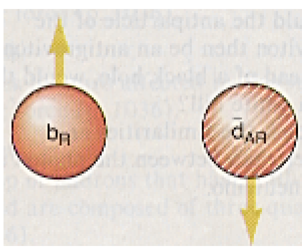


Diagram for problem 9.

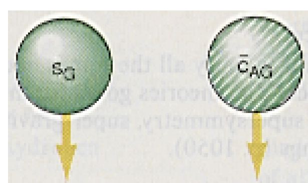


Diagram for problem 10.

10. If the two quarks shown in the diagram combine to form a meson, find the charge and spin of the resulting particle.

11. Find the charge and spin of the meson that consists of the two quarks shown in the diagram.

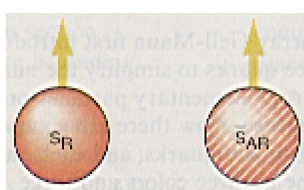


Diagram for problem 11.

12. Which of the combinations of particles in the diagram are possible and which are not. If the combination is not possible, state the reason.

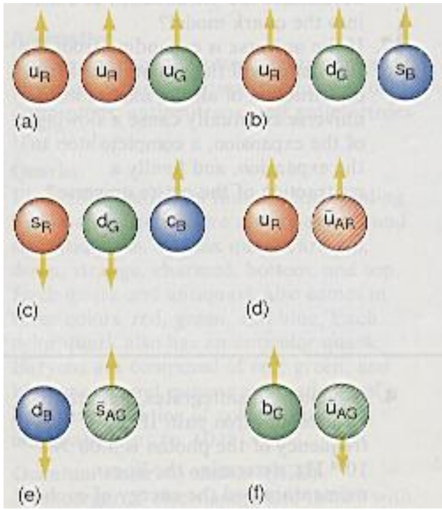


Diagram for problem 12.

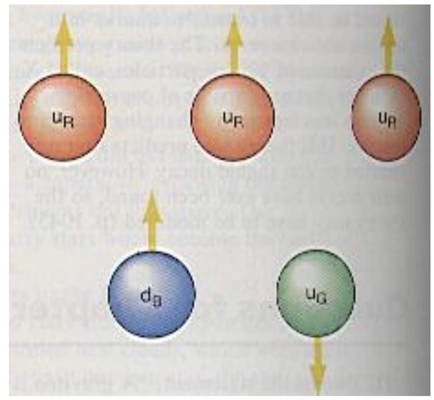


Diagram for problem 13.

13. Why are the two particles in the diagram impossible?

14. A baryon is composed of three quarks. It can be made from a total of six possible quarks, each in three possible colors, and each with either a spin-up or spin-down. From this information, how many possible baryons can be made?

15. A meson is composed of a quark-antiquark pair. It can be made from a total of six possible quarks, each in three possible colors, and each with either a spin-up or spin-down, and six possible antiquarks each in three possible colors, and each with either a spin-up or spin-down. Neglecting linear combinations of these quarks, how many possible mesons can be made?

16. From problems 14 and 15 determine the total number of possible hadrons, ignoring possible mesons made from linear combinations of quarks and antiquarks. Could you make a “periodic table” from this number? Discuss the attempt to attain simplicity in nature.

17. Determine all possible quark combinations that could form a baryon of charge +1 and spin ½.

To go to another chapter, return to the table of contents by clicking on this sentence.

Special Relativity

Kinematics

- Lorentz Transformations
- Four-Vectors
- Energy, Momentum, and Mass
- Collisions
- Examples

Lorentz Transformations

- Relate coordinates in: $S \xrightarrow{v} S'$
- Derived from the postulates of relativity
- For motion along the x-axis:

$$\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{v}{c^2}x\right) \end{aligned}$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

- Length Contraction: moving objects are shortened: $L = L'/\gamma$
- Time Dilation: moving clocks run slow: $T = \gamma T'$

Application: Cosmic Ray Muons

- With $\tau_\mu = 2.2\ \mu s$, a muon produced in the upper atmosphere could nominally travel (at $v \sim c$) 660 m before decaying
- The muon lifetime is enhanced through *time dilation* by a factor of γ. Supposing $\gamma \sim 10$, this allows a typical muon to travel 6.6 km before decaying.
- Recall $\gamma = E/m$ and $\beta = \sqrt{1 - 1/\gamma^2}$
- Decay length in laboratory frame is $\gamma c \tau$

Four-Vectors and Tensors

- Write $x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$ so that

$$(x')^\mu = \Lambda^\mu_\nu x^\nu \qquad \text{with} \qquad \Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- A *four-vector* is a four-component object which behaves like x^μ under Lorentz transformations.
- x^μ is the contravariant and x_μ is the covariant four-vector
- The invariant $I = x^\mu x_\mu$

Relativistic Invariant

- It can be shown that

$$(x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 = (x'^0)^2 - (x'^1)^2 - (x'^2)^2 - (x'^3)^2$$

  for any two frames related by a Lorentz transformation.

- If we define the *metric* tensor $g_{\mu\nu} = diag(1, -1, -1, -1)$, we have $x_\mu = g_{\mu\nu}x^\nu$ and

$$x^2 = x \cdot x = x_\mu x^\mu = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

  is a *Lorentz scalar*.

Energy, Momentum, and Mass

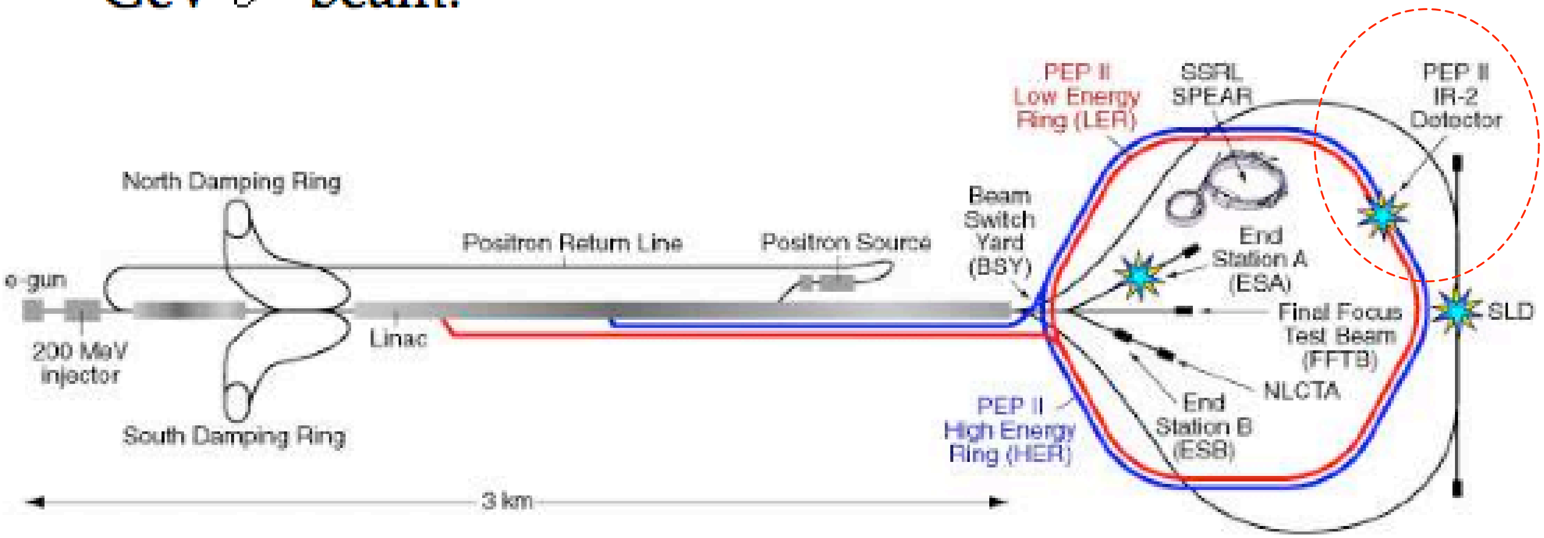
- Relativistic energy is $E = \gamma m$
- Relativistic momentum is $\mathbf{p} = \gamma m \mathbf{v}$
- Define the *four-momentum* by $p^\mu = (E, \mathbf{p})$.
  Then, $p^2 = m^2$ is a relativistic invariant.
- For massless particles, $E = |\mathbf{p}| = h\nu$
- Classically, we always conserve 3-momentum ($\mathbf{p}$), always conserve mass, sometimes conserve kinetic energy, and always conserve total energy even if we don't keep track of it all.
- Relativistically, we always conserve 3-momentum ($\mathbf{p}$), sometimes conserve mass, sometimes conserve kinetic energy, and always conserve total energy. More succinctly, four-momentum is conserved.

Conserved vs. Invariant quantities

- A conserved quantity remains the same, *in a particular frame*, before and after an event.
- An invariant quantity is the same *in all inertial reference frames*.
  - Energy is conserved, but not invariant.
  - Mass is invariant, but not conserved.

Examples

- BaBar experiment : Here, a 9 GeV e^- beam collides with a 3.1 GeV e^+ beam.



- What are the speeds of the colliding particles?
- What are the energies of the particles in the center of momentum (CM) frame?

Speeds

- Use $E = \gamma m$ and $m = 0.511$ MeV to determine that $\gamma_- = 17600$ for the electrons and $\gamma_+ = 6070$ for the positrons.
- Then, with $\gamma = 1/\sqrt{1-\beta^2}$, we solve for $\beta = v/c$

$$\Rightarrow \beta = \sqrt{1 - \frac{1}{\gamma^2}} \simeq 1 - \frac{1}{2\gamma^2}$$

- Using γ_- and γ_+, we have

$$\begin{aligned} v_{e^-} &= (1 - 10^{-9})c \\ v_{e^+} &= (1 - 10^{-8})c \end{aligned}$$

- In the CM frame, $p_{e^-} = (E_{CM}, \mathbf{p}_{CM})$ and $p_{e^+} = (E_{CM}, -\mathbf{p}_{CM})$ so that the (invariant) square of the total four-momentum is:

$$\begin{aligned}(p_{e^-} + p_{e^+})^2 &= (2E_{CM}, \mathbf{0})^2 \\ &= 4E_{CM}^2\end{aligned}$$

- In the lab frame, $p_{e^-} = (E_-, \mathbf{p}_-)$ and $p_{e^+} = (E_+, \mathbf{p}_+)$ so that

$$\begin{aligned}(p_{e^-} + p_{e^-})^2 &= p_{e^-}^2 + p_{e^+}^2 + 2p_{e^-} \cdot p_{e^+} \\ &= m^2 + m^2 + 2(E_- E_+ - \mathbf{p}_- \cdot \mathbf{p}_+) \\ &\simeq 2(E_- E_+ + |\mathbf{p}_-||\mathbf{p}_+|) \\ &\simeq 4E_- E_+\end{aligned}$$

- Equating the CM and lab expressions for the invariant, we have

$$E_{CM} = \sqrt{E_- E_+} = \sqrt{(9\ \mathrm{GeV})(3.1\ \mathrm{GeV})} = 5.3\ \mathrm{GeV}$$

Fixed Targets vs. Colliding Beams

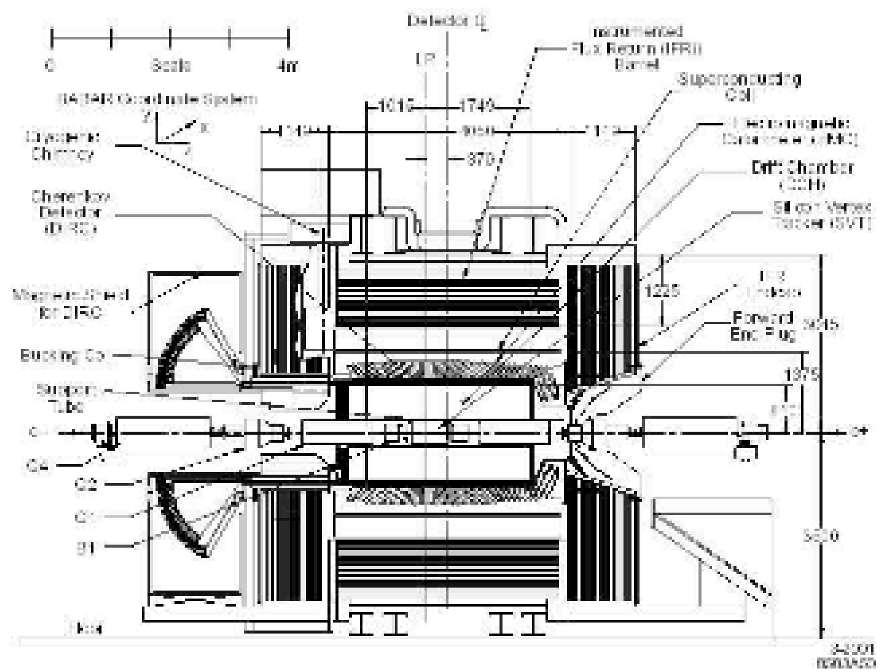
- In BaBar, $(9+3.1) = 12.1$ GeV of beam energy leads to $(2\times 5.3) = 10.6$ GeV of CM energy that can be used to make new particles (in this case, the $\Upsilon(4S)$).
- How much beam energy would it take to produce this CM energy if the target were fixed?
- Use the total four-momentum as an invariant. The individual four-momenta will be $(m, 0)$ and $(E, \mathbf{p})$, and therefore

$$\begin{aligned}(p_{e^-} + p_{e^-})^2 &= m^2 + m^2 + 2\left[(m, 0)\cdot(E, \mathbf{p})\right] \\ &\simeq 2Em\end{aligned}$$

With $2Em = 4E_{CM}^2$ we find that $E = 10^5$ GeV!

Fixed Targets vs. Colliding Beams III

- Why is the energy of the electrons and positrons at SLAC different?
- What does this mean for design of the BaBar detector?



Two-Body Decays

- Consider the decay $\pi \to \mu + \nu$
- In the CM frame, the final-state energies are unique, since the two particles must emerge back to back (to conserve momentum).
- How can we calculate these energies? Use an invariant, of course.

- Four-vectors: $p_\pi = (m_\pi, \mathbf{0})$, $p_\mu = (E_\mu, \mathbf{p})$, and $p_\nu = (E_\nu, -\mathbf{p})$.
- Conservation of four-momentum: $p_\pi = p_\mu + p_\nu$
- $p_\mu = p_\pi - p_\nu$ leads to the invariant

$$
\begin{aligned}
p_\mu^2 &= (p_\pi - p_\nu)^2 \\
m_\mu^2 &= p_\pi^2 + p_\nu^2 - 2p_\pi \cdot p_\nu \\
m_\mu^2 &= m_\pi^2 - 2m_\pi E_\nu \\
\Rightarrow E_\nu &= \frac{m_\pi^2 - m_\mu^2}{2m_\pi}
\end{aligned}
$$

- Similarly, $p_\nu = p_\pi - p_\mu$ leads to the invariant

$$
\begin{aligned}
p_\nu^2 &= (p_\pi - p_\mu)^2 \\
0 &= p_\pi^2 + p_\mu^2 - 2p_\pi \cdot p_\mu \\
0 &= m_\pi^2 + m_\mu^2 - 2m_\pi E_\mu \\
\Rightarrow E_\mu &= \frac{m_\pi^2 + m_\mu^2}{2m_\pi}
\end{aligned}
$$

- Notice that

$$
\begin{aligned}
E_\nu + E_\mu &= \frac{m_\pi^2 - m_\mu^2}{2m_\pi} + \frac{m_\pi^2 + m_\mu^2}{2m_\pi} \\
&= m_\pi
\end{aligned}
$$

as we require for energy conservation.

Three-Body Decays

- Consider decays such as $n \to p + e + \overline{\nu}_e$ and $\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$
- In the CM frame, the final-state energies are *not* unique.
- The observation that there was a range of electron energies in the two decays above played a large role in the postulate of the existence of neutrinos.

Mandelstam Invariants

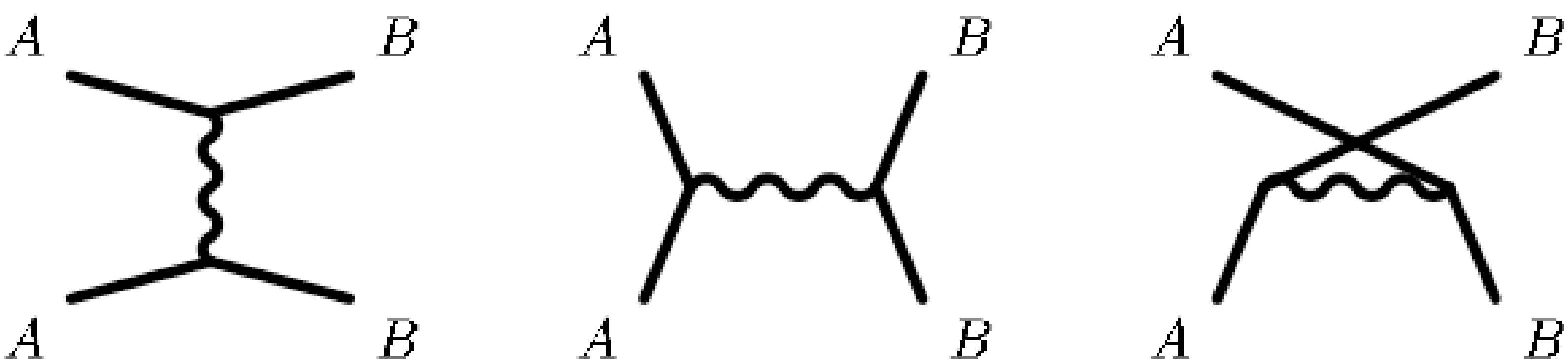
- For a scattering process like $A + B \longrightarrow C + D$, define the Mandelstam invariants by red

$$
\begin{aligned}
s &= (p_A + p_B)^2 \\
t &= (p_A - p_C)^2 \\
u &= (p_A - p_D)^2
\end{aligned}
$$

- We typically define a scattering angle θ in terms of the direction of C with respect to A.

Channels

- For $A + B \longrightarrow A + B$ scattering in some unspecified theory, the Mandelstam invariants s, t, and u are related to 3 distinct topological *channels* with which Feynman diagrams might be drawn to represent the interaction:



s-channel t-channel u-channel

Summary

- Special Relativity is an essential foundation of particle physics.
- The CM frame vs laboratory frame
- 2-body decays are much simpler than 3-body decays.
- Whenever possible, work with invariants formed from the contraction of four-vectors.
- The Mandelstam invariants are so common and useful that we give them their own symbols: s, t, and u.

5 The Dirac Equation and Spinors

In this section we develop the appropriate wavefunctions for fundamental fermions and bosons.

5.1 Notation Review

The three dimension differential operator is $\vec{\nabla}$:

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \tag{5.1}$$

We can generalise this to four dimensions ∂_μ:

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \tag{5.2}$$

5.2 The Schrödinger Equation

First consider a classical *non-relativistic* particle of mass m in a potential U. The energy-momentum relationship is:

$$E = \frac{p^2}{2m} + U \tag{5.3}$$

we can substitute the differential operators:

$$\hat{E} \rightarrow i\hbar\frac{\partial}{\partial t} \qquad \hat{p} \rightarrow -i\hbar\vec{\nabla} \tag{5.4}$$

to obtain the non-relativistic **Schrödinger Equation** (with $\hbar = 1$):

$$i\frac{\partial \psi}{\partial t} = \left(-\frac{1}{2m}\vec{\nabla}^2 + U \right)\psi \tag{5.5}$$

For $U = 0$, the free particle solutions are:

$$\psi(\vec{x}, t) \propto e^{-iEt}\psi(\vec{x}) \tag{5.6}$$

and the probability density ρ and current $\vec{j}$ are given by:

$$\rho = |\psi(x)|^2 \qquad \vec{j} = -\frac{i}{2m}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^* \right) \tag{5.7}$$

with conservation of probability giving the continuity equation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{j} = 0, \tag{5.8}$$

Or in Covariant notation:

$$\partial_\mu j^\mu = 0 \ \text{ with } \ j^\mu = (\rho, \vec{j}) \tag{5.9}$$

The Schrödinger equation is 1st order in $\partial/\partial t$ but second order in $\partial/\partial x$. However, as we are going to be dealing with relativistic particles, space and time should be treated equally.

5.3 The Klein-Gordon Equation

For a *relativistic particle* the energy-momentum relationship is:

$$p \cdot p = p_\mu p^\mu = E^2 - |\vec{p}|^2 = m^2 \tag{5.10}$$

Substituting the equation (5.4), leads to the relativistic **Klein-Gordon equation**:

$$\left(-\frac{\partial^2}{\partial t^2} + \vec{\nabla}^2\right)\psi = m^2\psi \tag{5.11}$$

The free particle solutions are plane waves:

$$\psi \propto e^{-ip\cdot x} = e^{-i(Et - \vec{p}\cdot\vec{x})} \tag{5.12}$$

The Klein-Gordon equation successfully describes **spin 0 particles** in relativistic quantum field theory.

There are problems with the interpretation of the positive and negative energy solutions of the Klein-Gordon equation, $E = \pm\sqrt{p^2 + m^2}$, since the negative energy solutions have negative probability densities ρ.

5.4 The Dirac Equation

The problems with the Klein-Gordon equation led Dirac to search for an alternative relativistic wave equation in 1928, in which the time and space derivatives are first order. The **Dirac equation** can be thought of in terms of a "square root" of the Klein-Gordon equation. In covariant form it is written:

$$\left(i\gamma^0\frac{\partial}{\partial t} + i\vec{\gamma}\cdot\vec{\nabla} - m\right)\psi = 0 \qquad \left(i\gamma^\mu\partial_\mu - m\right)\psi = 0 \tag{5.13}$$

where we have introduced the coefficients $\gamma^\mu = (\gamma^0, \vec{\gamma}) = (\gamma^0, \gamma^1, \gamma^2, \gamma^3)$, which have to be determined.

As we will see in equation (5.20), the Dirac equation is simply four coupled differential equations, describing a wavefunction ψ with four components.

5.5 The Gamma Matrices

To find what the γ^μ, $\mu = 0, 1, 2, 3$ objects are, we first multiply the Dirac equation by its conjugate equation:

$$\psi^\dagger\left(-i\gamma^0\frac{\partial}{\partial t} - i\vec{\gamma}\cdot\vec{\nabla} - m\right)\left(i\gamma^0\frac{\partial}{\partial t} + i\vec{\gamma}\cdot\vec{\nabla} - m\right)\psi = 0 \tag{5.14}$$

and demand that this be consistent with the Klein-Gordon equation, (5.11). This leads to the following conditions on the γ^μ:

$$\begin{gathered}(\gamma^0)^2 = 1, \quad (\gamma^i)^2 = -1 \quad \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 0 \text{ for } \mu \neq \nu \\ \text{with } i = 1, 2, 3,\ \mu, \nu = 0, 1, 2, 3\end{gathered} \tag{5.15}$$

Equivalently in terms of anticommutation relations and the metric tensor (equation (3.3)):

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu, \gamma^\nu + \gamma^\nu, \gamma^\mu = 2g^{\mu\nu} \qquad \mu, \nu = 0, 1, 2, 3 \tag{5.16}$$

The simplest solution for the γ^μ, that satisfies these anticommutation relations, are 4×4 **unitary matrices**. We will use the following representation for the γ matrices:

$$\gamma^0 = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix} \tag{5.17}$$

where $\mathbf{I}$ denotes a 2×2 identity matrix, $\mathbf{0}$ denotes a 2×2 null matrix, and the σ^i are the **Pauli spin matrices**:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5.18}$$

Let's write out the gamma matrices in full:

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{5.19}$$

Please note, despite the μ superscript, the γ^μ are not four vectors. However they do remain constant under Lorentz transforms.

Finally let's write out the Dirac Equation in full:

$$\begin{pmatrix} i\frac{\partial}{\partial t} - m & 0 & i\frac{\partial}{\partial z} & i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \\ 0 & i\frac{\partial}{\partial t} - m & i\frac{\partial}{\partial x} - \frac{\partial}{\partial y} & -i\frac{\partial}{\partial z} \\ -i\frac{\partial}{\partial z} & -i\frac{\partial}{\partial x} - \frac{\partial}{\partial y} & -i\frac{\partial}{\partial t} - m & 0 \\ -i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} & i\frac{\partial}{\partial z} & 0 & -i\frac{\partial}{\partial t} - m \end{pmatrix} \begin{pmatrix} \psi^1 \\ \psi^2 \\ \psi^3 \\ \psi^4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{5.20}$$

5.6 Spinors

The Dirac equation describes the behaviour of spin-1/2 fermions in relativistic quantum field theory. For a free fermion the wavefunction is the product of a plane wave and a **Dirac spinor**, $u(p^\mu)$:

$$\psi(x^\mu) = u(p^\mu)e^{-ip\cdot x} \tag{5.21}$$

Substituting the fermion wavefunction, ψ, into the Dirac equation:

$$(\gamma^\mu p_\mu - m)u(p) = 0 \tag{5.22}$$

For a particle at rest, $\vec{p} = 0$, we find the following equations:

$$\left(i\gamma^0 \frac{\partial}{\partial t} - m\right)\psi = \left(\gamma^0 E - m\right)\psi = 0 \qquad \hat{E}\, u = \begin{pmatrix} m\mathbf{I} & 0 \\ 0 & -m\mathbf{I} \end{pmatrix} u \tag{5.23}$$

The solutions are four eigenspinors:

$$u^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad u^3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad u^4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{5.24}$$

and the associated wavefunctions of the fermion is:

$$\psi^1 = e^{-imt} u^1 \qquad \psi^2 = e^{-imt} u^2 \qquad \psi^3 = e^{+imt} u^3 \qquad \psi^4 = e^{+imt} u^4 \tag{5.25}$$

Note that the spinors are 1×4 column matrices, and that there are four possible states. The spinors are, however, not four-vectors: the four components do not represent t, x, y, z.

The four components are a suprise: we would expect only two spin states for a spin-1/2 fermion! Note also the change of sign in the exponents of the plane waves in the states ψ^3 and ψ^4. The four solutions in equations (5.24) and (5.25) describe two different spin states ($\uparrow$ and $\downarrow$) with $E = m$, and two spin states with $E = -m$.

5.7 Negative Energy Solutions & Antimatter

To describe the negative energy states, Dirac postulated that an electron in a positive energy state is produced from the vacuum accompanied by a *hole* with negative energy. The hole corresponds to a physical **antiparticle**, the positron, with charge $+e$.

Another interpretation (Feynman-Stückelberg) is that the $E = -m$ solutions can either describe a negative energy particle which propagates backwards in time, or a positive energy antiparticle propagating forward in time:

$$e^{-i[(-E)(-t)-(-\vec{p})\cdot(-\vec{x})]} = e^{-i[Et-\vec{p}\cdot\vec{x}]} \tag{5.26}$$

5.8 Spinors for Moving Particles

For a moving particle, $\vec{p} \neq 0$ the Dirac equation becomes (using (5.13) and (5.17)):

$$(\gamma^\mu p_\mu - m)\left(\begin{array}{c} u_A \; u_B \end{array}\right) = \begin{pmatrix} E - m & -\vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -E - m \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix} = 0 \tag{5.27}$$

where u_A and u_B denote the 1×2 upper and lower components of u respectively. The equations for u_A and u_B are coupled:

$$u_A = \frac{\vec{\sigma}\cdot\vec{p}}{E - m} u_B \qquad u_B = \frac{\vec{\sigma}\cdot\vec{p}}{E + m} u_A \tag{5.28}$$

The solutions are obtained by successively setting: $u_A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $u_A = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $u_B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and $u_B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, to give:

$$u^1 = \begin{pmatrix} 1 \\ 0 \\ p_z/(E+m) \\ (p_x+ip_y)/(E+m) \end{pmatrix} \quad u^2 = \begin{pmatrix} 0 \\ 1 \\ (p_x-ip_y)/(E+m) \\ -p_z/(E+m) \end{pmatrix}$$
$$u^3 = \begin{pmatrix} -p_z/(-E+m) \\ (-p_x-ip_y)/(-E+m) \\ 1 \\ 0 \end{pmatrix} \quad u^4 = \begin{pmatrix} (-p_x+ip_y)/(-E+m) \\ p_z/(-E+m) \\ 0 \\ 1 \end{pmatrix} \tag{5.29}$$

The u^1 and u^2 solutions describe an electron of energy $E = +\sqrt{m^2+\vec{p}^2}$, and momentum $\vec{p}$:

$$\psi = u^1(p^\mu)e^{-ip\cdot x} \qquad \psi = u^2(p^\mu)e^{-ip\cdot x} \tag{5.30}$$

The u^3 and u^4 of equation (5.29) describe a positron of energy $E = -\sqrt{m^2+\vec{p}^2}$, and momentum $\vec{p}$.

It is usual to change to the spinors $v^2(p) \equiv u^3(-p)$ and $v^1(p) \equiv u^4(-p)$ to describe these *positive* energy antiparticle states, $E = +\sqrt{m^2+\vec{p}^2}$

$$v^2(p^\mu) \equiv u^3(-p^\mu) = \begin{pmatrix} p_z/(E+m) \\ (p_x+ip_y)/(E+m) \\ 1 \\ 0 \end{pmatrix} \qquad \psi = v^2(p^\mu)e^{-ip\cdot x} = u^3(-p^\mu)e^{i(-p)\cdot x}$$

$$v^1(p^\mu) \equiv u^4(-p^\mu) = \begin{pmatrix} (p_x-ip_y)/(E+m) \\ -p_z/(E+m) \\ 0 \\ 1 \end{pmatrix} \qquad \psi = v^1(p^\mu)e^{-ip\cdot x} = u^4(-p^\mu)e^{i(-p)\cdot x} \tag{5.31}$$

The u and v are the solutions of:

$$(i\gamma^\mu p_\mu - m)u = 0 \qquad (i\gamma^\mu p_\mu + m)v = 0 \tag{5.32}$$

5.9 Spin and Helicity

The two different solutions for each of the fermions and antifermions corresponds to two possible spin states. For a fermion with momentum $\vec{p}$ along the z-axis, $\psi = u^1(p^\mu)e^{-ip\cdot x}$ describes a spin-up fermion and $\psi = u^2(p^\mu)e^{-ip\cdot x}$ describes a spin-down fermion. For an antifermion with momentum $\vec{p}$ along the z-axis, $\psi = v^1(p^\mu)e^{-ip\cdot x}$ describes a spin-up antifermion and $\psi = v^2(p^\mu)e^{-ip\cdot x}$ describes a spin-down antifermion.

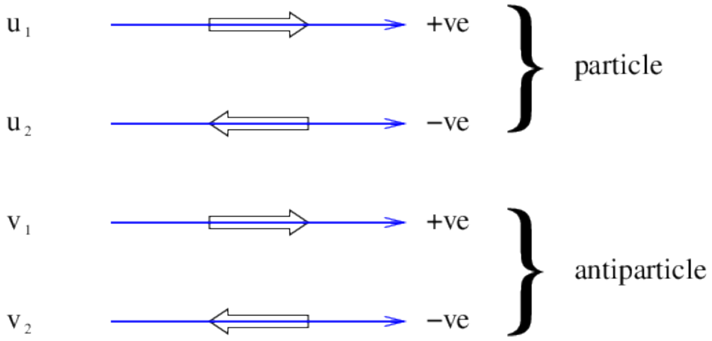


Figure 5.1: Helicity eigenstates for a particle or antiparticle travelling along the $+z$ axis.

The u^1, u^2, v^1, v^2 spinors are only eigenstates of $\hat{S}_z$ for momentum $\vec{p}$ along the z-axis. There's nothing special about projecting out the component of spin along the z-axis, that's just the conventional choice. For our purposes it makes more sense to project the spin along the particle's direction of flight, this defines the **helicity**, h of the particle.

$$\hat{h} = \frac{\vec{S} \cdot \vec{p}}{|\vec{S}||\vec{p}|} = \frac{2\vec{S} \cdot \vec{p}}{|\vec{p}|} \tag{5.33}$$

For a spin-1/2 fermion, the two possible values of h are $h = +1$ or $h = -1$. We call $h = +1$ **right-handed** and $h = -1$ **left-handed**.

The possible states of particles and antiparticles are shown in Figure 5.1. As we will see, the concept of left- and right-handedness plays an important role in calculating matrix elements and in the weak force.

If it also worth noting here, massless fermions, are purely left-handed (only u^2); massless antifermions are purely right handed (only v^1).

5.10 Projection Operators

Helicity is not a Lorentz invariant quantity, therefore we also define a related Lorentz invariant quantity: **chirality**.

Chirality can be defined in terms for the **chiral projection operators**, P_L and P_R:

$$P_L = \frac{1}{2}(1 - \gamma_5) \qquad P_R = \frac{1}{2}(1 + \gamma_5) \tag{5.34}$$

where γ_5 is another 4×4 matrix:

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{pmatrix} \qquad (\gamma^5)^2 = 1 \qquad \{\gamma^5, \gamma^\mu\} = 0 \tag{5.35}$$

P_L and P_R project out the left-handed and right-handed chiral components of a spinor:

$$u_L = P_L u \quad u_R = P_R u \tag{5.36}$$

For the two antifermions states remember that the direction of the momentum was reversed in going from the u to the v spinors. Hence the projections of the v spinors are:

$$v_R = P_L v \quad v_L = P_R v \tag{5.37}$$

In the highly relativistic limit, $E \gg m$, $\beta \to 1$ the left-handed chiral states and the same as the left-handed helicity states, and similarly for the right-handed states.

Setting $m = 0$ and $p = p_z = E$ in the highly relativistic limit $\beta \to 1$:

$$u^1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad u^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \quad v^2 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad v^1 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \tag{5.38}$$

In a future lecture we'll see that the weak $W^{\pm}$ couplings contain $P_L = (1 - \gamma^5)/2$, and hence only couple to left-handed particles or right-handed antiparticles.

6 Quantum Electrodynamics

6.1 Notation: the Metric Tensor

In this section we can't avoid using the metric tensor, equation (3.3):

$$g_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{6.1}$$

6.2 Fermion currents

We need to define a Lorentz invariant quantity to describe fermion currents for QED. We define the **adjoint spinor** $\bar{\psi} \equiv \psi^\dagger \gamma^0$, where $\psi^\dagger$ is the hermitian conjugate (complex conjugate transpose) of ψ:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \qquad \psi^\dagger = (\psi^*)^T = (\psi_1^*, \psi_2^*, \psi_3^*, \psi_4^*) \qquad \bar{\psi} \equiv \psi^\dagger \gamma^0 = (\psi_1^*, \psi_2^*, -\psi_3^*, -\psi_4^*) \tag{6.2}$$

The adjoint Dirac equation can be formed by taking the hermitian conjugate of the Dirac equation (5.13), and multiplying it from the right by γ^0:

$$i\partial_\mu \bar{\psi} \gamma^\mu + m\bar{\psi} = 0 \tag{6.3}$$

Multiplying the adjoint Dirac equation (6.3) by ψ from the right, (or the original Dirac equation by $\bar{\psi}$ from the left) gives the continuity equation (c.f. equation **??**equ:continuity)):

$$\partial_\mu(\bar{\psi}\gamma^\mu\psi) = \bar{\psi}\gamma^\mu(\partial_\mu\psi) + (\partial_\mu\bar{\psi})\gamma^\mu\psi = 0 \quad \text{or} \quad \partial_\mu j^\mu = 0 \tag{6.4}$$

Where j^μ is the four-vector fermion current:

$$j^\mu = \bar{\psi}\gamma^\mu\psi = (\bar{\psi}\gamma^0\psi, \bar{\psi}\vec{\gamma}\psi) = (\rho, \vec{j}) \tag{6.5}$$

and ρ is the probability density:

$$\rho = j^0 = \bar{\psi}\gamma^0\psi = \psi^\dagger\psi \tag{6.6}$$

The fermion current $j^\mu = \bar{\psi}\gamma^\mu\psi$ and is has the properties of a Lorentz four-vector, which is what we required. Additionally, the probability density ρ is positive definite for all four possible spinor states. This is only true if we use the *adjoint* form with $\bar{\psi}$.

6.3 Maxwell's Equations

The photon is described by Maxwell's equations.

For electromagnetic interactions, Maxwell's equations can be written in a Lorentz covariant form:

$$\partial^2 A^\mu = 4\pi j^\mu \qquad \partial_\mu A^\mu = 0 \qquad \partial_\mu j^\mu = 0 \tag{6.7}$$

where $\partial^2 = \partial_\nu \partial^\nu = 1/c^2 dt^2 - \nabla^2$ $A^\mu = (\phi, \vec{A})$ is the electric and magnetic potential four-vector and $j^\mu = (\rho, \vec{j})$ is the charge/current density four-vector.

Plane wave solutions can be written as $A^\mu = \epsilon^\mu(s)e^{-ip\cdot x}$. $\epsilon^\mu(s)$ is the polarisation vector, which depends on the spin, s, of the photon.

6.4 Polarisation vectors

For spin-one bosons, there are three spin projections corresponding to three possible helicity states $s = +1, 0, -1$. $s = 0$ is known as *longitudinal* polarisation, and the $s = \pm 1$ are *transverse* polarisations (actually left and right-handed circular polarisations). For massless particles, the $s = 0$ state does not exist.

The equivalent of a fermion spinor is a **polarisation vector**, ϵ^μ, and the boson wavefunction is written:

$$\psi = \epsilon^\mu(p; s)e^{-ip\cdot x} \tag{6.8}$$

The polarisation vector is Lorentz gauge invariant:

$$p_\mu \epsilon^\mu = 0 \tag{6.9}$$

Virtual photons have $q^2 \neq 0$, and thus can have both longitudinal and transverse polarisations. This is also true for the massive W and Z bosons.

6.5 Feynman Rules for QED

Now we know how to describe photons and fermion currents, we can write down the Feynman rules for QED. These are shown in figure 6.1.

We use the Feynman Rules to write down the unique matrix element, $\mathcal{M}$, for the process. The matrix element represents the transition probability per unit time for that process to happen.

Write down the term for each piece of the Feynman diagram and multiply them all together to obtain $\mathcal{M}$:

- Start with a fermion line, and follow the arrows backwards. First write down the spinor for the (anti)fermion. Trace the arrow back to the vertex; write down the vertex term. Finally write down the spinor for the last part of the fermion line.

(If there is more than one vertex on the line, you will have to write down all the vertex term, and use the term for the internal fermion lines.) Do this for all the fermion lines.

- Write down the propagator term for the internal photons.

6.5.1 Summing Diagrams

If there is more than one Feynman diagram for a process, then you have to sum all the matrix elements:

$$\mathcal{M}_{\text{total}} = \mathcal{M}_1 + \mathcal{M}_2 + \cdots \tag{6.10}$$

In Fermi's Golden Rule (equation 4.2),we use the matrix element squared $|\mathcal{M}|^2$. When we sum diagrams:

$$|\mathcal{M}_{\text{total}}|^2 = (\mathcal{M}_1 + \mathcal{M}_2 + \cdots)(\mathcal{M}_1^* + \mathcal{M}_2^* + \cdots) \tag{6.11}$$

where $\mathcal{M}_1^*$ is the complex conjugate.

If we want to calculate the unpolarised cross section we need to **average over initial state spins and sum over final state spins**.

Remember we are using perturbation theory: each Feynman diagram represents only part of the process. Ideally we should sum all possible diagrams. However for each vertex the diagram is suppressed by $\sim 1/\sqrt{\alpha}$, therefore we often only need to consider only the lowest order diagrams (the ones with the least number of possible vertices). The most precise calculations in QED use diagrams with up to ten vertices, $\mathcal{O}(\alpha^5)$.

6.6 High energy $e^-\mu^- \rightarrow e^-\mu^-$ scattering

The process $e^-\mu^- \rightarrow e^-\mu^-$ is similar to spinless electromagnetic scattering in section 4.7, apart from the addition of the fermion spinors. The Feynman diagram, showing the spinor, vertex and propagator terms, is shown in figure 6.2.

The matrix element is written in terms of the two fermion currents:

$$\mathcal{M} = e^2 \frac{g_{\mu\nu}}{q^2} (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma^\nu u_2) \tag{6.12}$$

Where u_i are the spinors for the incoming and outgoing particles, $i = 1, 2, 3, 4$.

The $g_{\mu\nu}$ allows us to change (contract) the γ^ν in the second bracket into γ^μ:

$$\mathcal{M} = \frac{e^2}{q^2} (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_4 \gamma^\mu u_2) \tag{6.13}$$

For the unpolarised matrix element squared we must *average* over the initial spin states,

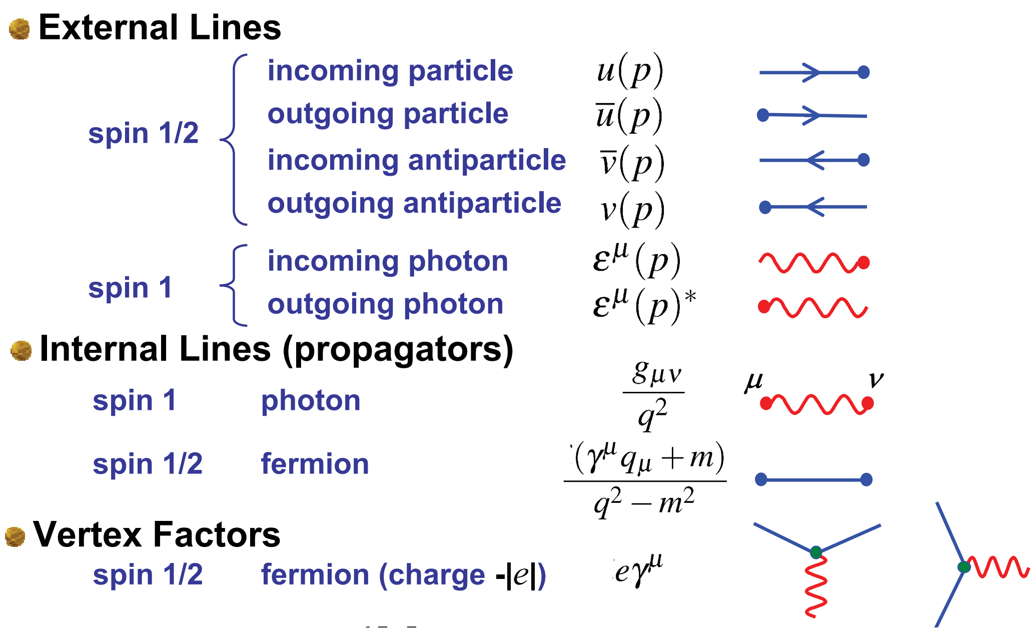


Figure 6.1: Feynman rules for QED.

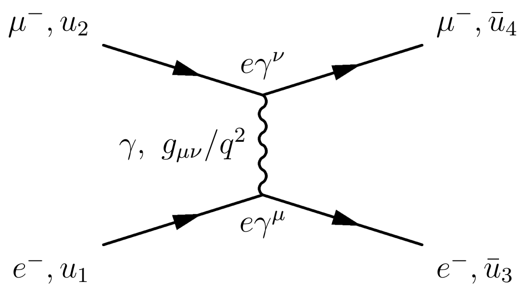


Figure 6.2: Feynman diagram for $e^-\mu^- \to e^-\mu^-$. The spinor, vertex and propagator terms used to calculate $\mathcal{M}$ are given.

and *sum* over the final spin states. We separate out the electron and muon parts:

$$\begin{aligned}
|\mathcal{M}|^2 &= \frac{e^4}{q^4}\frac{1}{(2S_1+1)(2S_2+1)}\sum_{S_3,S_4}(\bar{u}_3\gamma^{\mu}u_1)\,(\bar{u}_3\gamma^{\nu}u_1)^*(\bar{u}_4\gamma^{\mu}u_2)(\bar{u}_4\gamma^{\nu}u_2)^* \\
&= \frac{e^4}{q^4}\left(\frac{1}{(2S_1+1)}\sum_{S_3}(\bar{u}_3\gamma^{\mu}u_1)\,(\bar{u}_3\gamma^{\nu}u_1)^*\right)\left(\frac{1}{(2S_2+2)}\sum_{S_4}(\bar{u}_4\gamma^{\mu}u_2)(\bar{u}_4\gamma^{\nu}u_2)^*\right) \\
&= \frac{e^4}{q^4}\,L_e\,L_m \qquad (6.14)
\end{aligned}$$

where:

$$L_e = \frac{1}{(2S_1+1)}\sum_{S_3}(\bar{u}_3\gamma^{\mu}u_1)\,(\bar{u}_3\gamma^{\nu}u_1)^* \qquad (6.15)$$

where the initial electron spin is $S_1 = 1/2$, and the final spin states are S_3. For L_m replace $1 \to 2, 3 \to 4$. The calculation of the sum over the products of the spinors and gamma matrices looks horrible, but there is a trick using *trace theorems* which is discussed in detail on P.254/255 of Griffiths. The most relevant ones are:

$$Tr(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu} \qquad Tr(\gamma_\mu \not{p}_1 \not{p}_3 \gamma^\mu) = 4(p_1 \cdot p_3) \tag{6.16}$$

with the conventional abbreviation $\not{p} = \gamma \cdot p = \gamma^\mu p_\mu = \sum_{\mu=0}^{3} \gamma^\mu p_\mu$. The electron sum gives:

$$L_e^{\mu\nu} = 2[p_3^\mu p_1^\nu + p_3^\nu p_1^\mu - (p_3 \cdot p_1 - m_e^2) g^{\mu\nu}] \tag{6.17}$$

and similarly for the muon part. Notice that there are no spinors left: the calculation is now just four vectors and matrices multiplied together. For high energy scattering the lepton masses (m_e, m_μ) can be neglected, and the matrix element squared is:

$$|\mathcal{M}|^2 = 8\frac{e^4}{q^4}[(p_3 \cdot p_4)(p_1 \cdot p_2) + (p_3 \cdot p_2)(p_1 \cdot p_4)] = 2e^4 \frac{s^2 + u^2}{t^2} \tag{6.18}$$

This differs from the spinless result which has $(s-u)^2$ rather than $s^2 + u^2$.

For polarised cross sections there are $2^4 = 16$ possible spin configurations, but only 4 of these are allowed at high energy, because helicity change at the vertices is suppressed. The four polarised matrix elements are:

$$\mathcal{M}(\uparrow\uparrow\uparrow\uparrow) = \mathcal{M}(\downarrow\downarrow\downarrow\downarrow) = e^2 \frac{u}{t} \qquad \mathcal{M}(\uparrow\downarrow\uparrow\downarrow) = \mathcal{M}(\downarrow\uparrow\downarrow\uparrow) = e^2 \frac{s}{t} \tag{6.19}$$

which gives the unpolarised result when the squares are summed.

6.6.1 $e^-\mu^- \to e^-\mu^-$ Cross Section

To calculate the cross section, we can use equation (4.16):

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{\mathcal{S}|\mathcal{M}|^2}{(E_1 + E_2)^2} \frac{|\vec{p}_f^{\,*}|}{|\vec{p}_i^{\,*}|}$$

Making the following substitutions:

- centre of mass energy, $(E_1 + E_2)^2 = s$
- $|\vec{p}_f^{\,*}| = |\vec{p}_i^{\,*}|$ for elastic scattering
- $\mathcal{S} = 1$ as no identical particles in the final state
- $\alpha = e^2/(4\pi)$
- $|\mathcal{M}|^2$ from equation (6.18)

gives,

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\pi s} \left(\frac{s^2 + u^2}{t^2}\right) \tag{6.20}$$

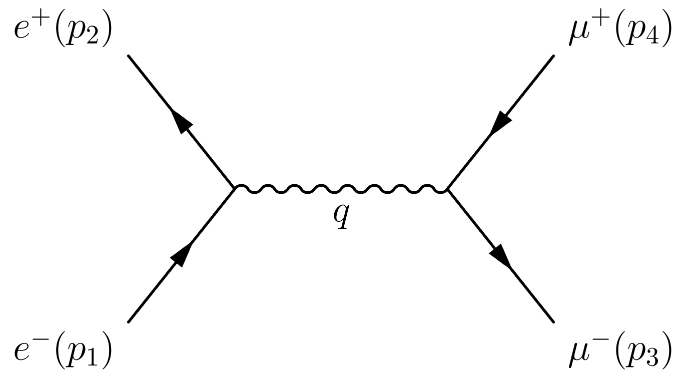


Figure 6.3: Lowest order Feynman diagram for $e^+e^- \to \mu^+\mu^-$

6.7 $e^+e^- \to \mu^+\mu^-$ Annihilation

The process $e^+e^- \to \mu^+\mu^-$ proceeds through the annihilation of the electron and positron into a virtual photon. The lowest order Feynman diagram is shown in figure 6.3.

The matrix element is:

$$\mathcal{M} = \frac{e^2}{q^2} \left(\bar{v}_2 \gamma^\mu u_1\right) \left(\bar{u}_3 \gamma_\mu v_4\right) \tag{6.21}$$

As above there are 16 possible helicity configurations but only 4 remain in the high energy limit, corresponding to annihilation/creation of fermion/antifermion pairs with opposite helicity states:

$$\mathcal{M}(\uparrow\downarrow\uparrow\downarrow) = \mathcal{M}(\downarrow\uparrow\downarrow\uparrow) = e^2 \frac{u}{s} = \frac{e^2}{2}(1 + \cos\theta^*) \tag{6.22}$$

$$\mathcal{M}(\uparrow\downarrow\downarrow\uparrow) = \mathcal{M}(\downarrow\uparrow\uparrow\downarrow) = e^2 \frac{t}{s} = \frac{e^2}{2}(1 - \cos\theta^*) \tag{6.23}$$

where θ is the scattering angle in the centre-of-mass system. It might be useful to think of this using the spin of the photon, $S = 1$. In the relativistic limit, helicity (or spin) is conserved, therefore the spin of the two incoming particles must add up to one: we must have one electron with $h = +1$ and one with $h = -1$. Likewise the two muons will be produced such that one has $h = +1$ and one has $h = -1$.

Note that $e^-\mu^- \to e^-\mu^-$ and $e^+e^- \to \mu^+\mu^-$ are related by a *crossing symmetry*. The matrix elements are related by the exchange of $s \leftrightarrow t$.

The total unpolarised matrix element squared for $e^+e^- \to \mu^+\mu^-$ is:

$$|\mathcal{M}|^2 = 2e^4 \frac{t^2 + u^2}{s^2} = e^4 \left(1 + \cos^2\theta^*\right) \tag{6.24}$$

Using equation (4.16), or by $t \leftrightarrow s$ in equation (6.20), the differential cross section in the centre-of-mass system is:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \left(1 + \cos^2\theta\right) \tag{6.25}$$

To get the total cross section we integrate over the solid angle, $d\Omega = d\cos\theta d\phi$:

$$\begin{aligned}\sigma &= \int \frac{d\sigma}{d\Omega}\, d\Omega = \frac{\alpha^2}{4\pi s} \int (1+\cos^2\theta) d\cos\theta d\phi \\ &= \frac{\alpha^2}{4\pi s} [\phi]_{-\pi}^{\pi} \left[\cos\theta + \frac{1}{3}\cos^3\theta\right]_{\cos\theta=-1}^{\cos\theta=+1} = \frac{4\alpha^2}{3s}\end{aligned} \quad (6.26)$$

The total cross section is proportional to $1/s$. Figure 6.4 shows measurements of this cross section. The agreement is pretty good, given that this we only used 1st order in pertubation theory!

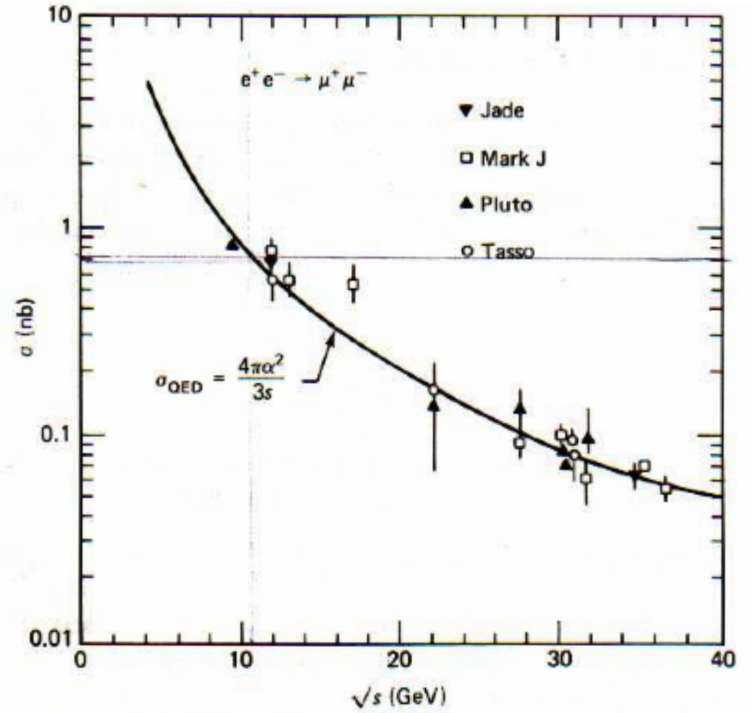


Fig. 6.6 The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

Figure 6.4: Measured total cross section for $e^+e^- \to \mu^+\mu^-$ as a function of centre of mass energy, s, compared to our prediction.

6.8 Matrix elements for QED processes

A Table of QED matrix elements in the **high energy** limit from P.129 of Halzen & Martin are given in figure 6.5.

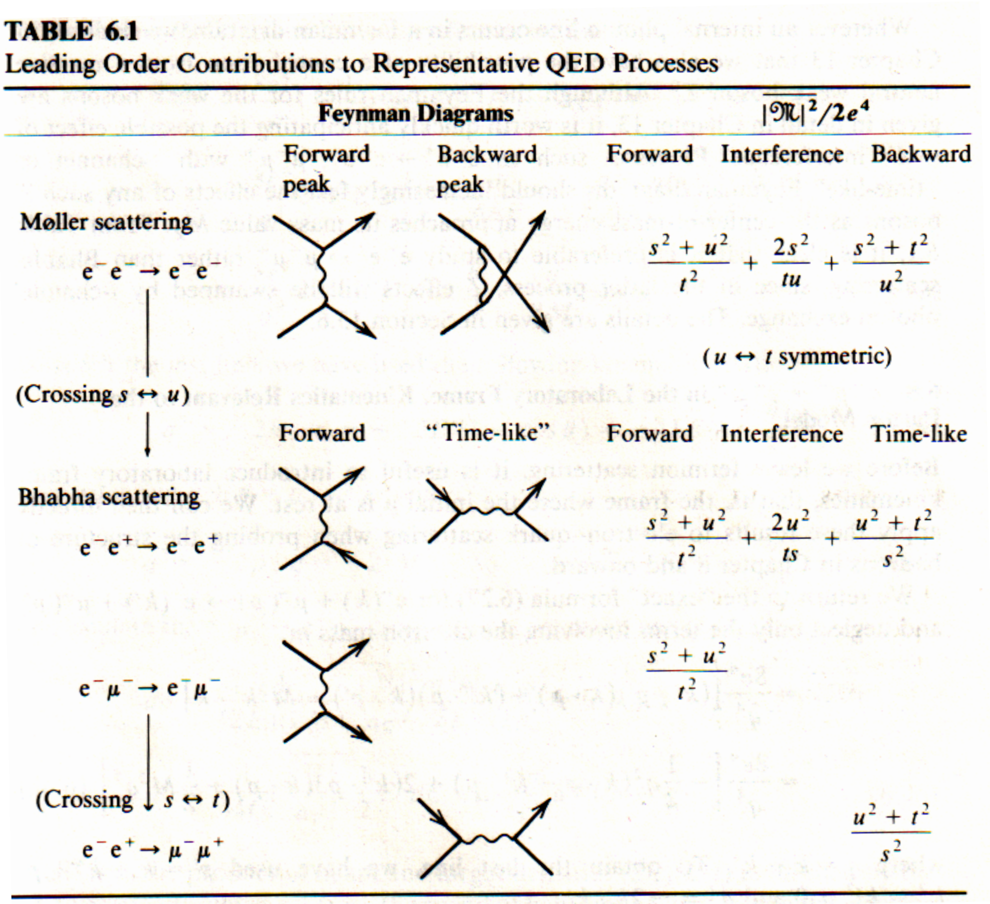
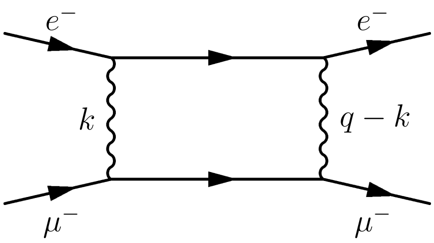


TABLE 6.1
Leading Order Contributions to Representative QED Processes

	Feynman Diagrams		$\overline{\lvert\mathcal{M}\rvert^2}/2e^4$		
	Forward peak	Backward peak	Forward	Interference	Backward
Møller scattering $e^-e^- \to e^-e^-$			$\dfrac{s^2+u^2}{t^2}$	$+\ \dfrac{2s^2}{tu}$	$+\ \dfrac{s^2+t^2}{u^2}$
			($u \leftrightarrow t$ symmetric)		
(Crossing $s \leftrightarrow u$)					
	Forward	"Time-like"	Forward	Interference	Time-like
Bhabha scattering $e^-e^+ \to e^-e^+$			$\dfrac{s^2+u^2}{t^2}$	$+\ \dfrac{2u^2}{ts}$	$+\ \dfrac{u^2+t^2}{s^2}$
$e^-\mu^- \to e^-\mu^-$			$\dfrac{s^2+u^2}{t^2}$		
(Crossing $s \leftrightarrow t$)					
$e^-e^+ \to \mu^-\mu^+$					$\dfrac{u^2+t^2}{s^2}$

Figure 6.5: QED matrix elements for lowest order processes in terms of the Mandelstam variables.



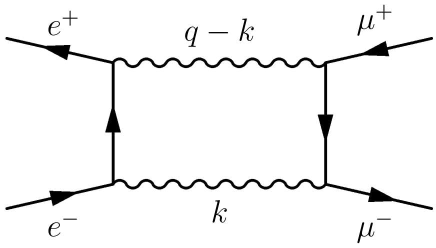


Figure 7.1: Higher order box diagrams with two photon exchange. Left: $e^-\mu^- \to e^-\mu^-$ scattering. Right: $e^+e^- \to \mu^+\mu^-$.

7 Renormalisation and the Weak Force

7.1 Higher Order Diagrams

As shown in figure 7.1 second photon can be exchanged we have considered above $e^-\mu^- \to e^-\mu^-$ scattering or in $e^+e^- \to \mu^+\mu^-$. These are examples of a higher order diagram known as a "box" diagram. There are two additional vertices, so the amplitude is reduced by a factor $\alpha = 1/137$ compared to the lowest order diagram with one photon exchange.

The overall four-momentum transfer is still q, but it has to be divided between the two photons with the first photon taking k. In fact k does not have to be less than q. It can have any value, including much larger than q, or opposite in sign. The fermion propagators between the two photons are also modified by k. It is simplest to think of the four momentum k as something that flows round the box in a clockwise or anticlockwise direction. Since it is only present for the virtual particles it is unobservable.

When calculating the amplitude for these diagrams it is necessary to perform an integral over the unknown four momentum k over a range which goes from zero to infinity. This integral diverges logarithmically. It took two decades of the twentieth century for Feynman and others to work out how to deal with these divergences, and obtain physically meaningful results for higher order diagrams.

> "All the infinities are miraculously swept up into formal expressions for quantities like physical mass and charge of the particles. These formally infinite expressions are then replaced by their finite physical values." (Aitchison & Hey P.51)

This process is known as **renormalization**.

These other higher order diagrams are also smaller than the lowest order diagram by a factor $\alpha = 1/137$. They all contain a loop with an arbitrary four momentum k that modifies all the propagators around the loop. In each case the integral over k leads to logarithmic infinities in the amplitudes which have to be removed by renormalization.

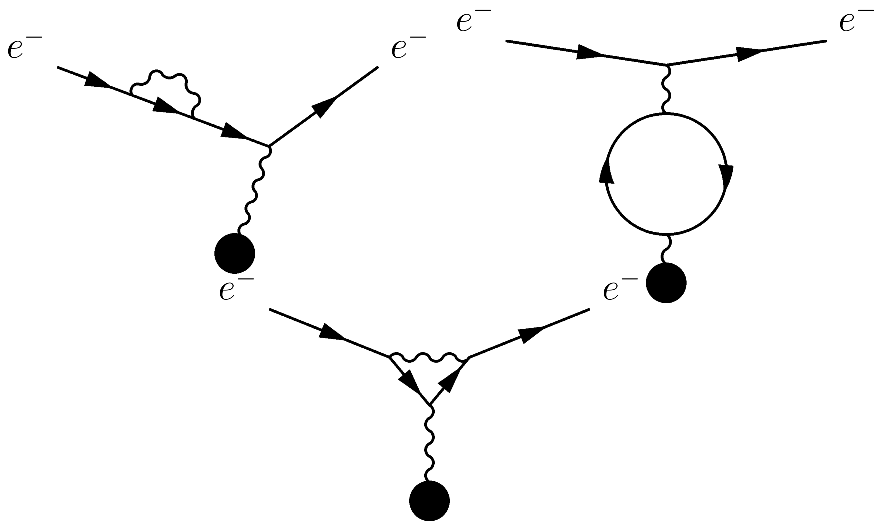


Figure 7.2: Some other higher order diagrams. Top left: a "dressed fermion" emits and reabsorbs a virtual photon. Top right: a "bubble" diagram where a virtual photon creates and annihilates a fermion-antifermion pair. Bottom: a "vertex correction" where a virtual photon connects two fermion lines across a vertex. The black blob represents an electromagnetic vertex which is not described in full.

7.2 Renormalization

In QED the divergent terms from higher order contributions to the amplitude are absorbed into a redefinition of the **charge** $e = \sqrt{4\pi\alpha}$ responsible for the vertex coupling, and a redefinition of the **fermion mass** m which enters through fermion propagators:

$$e_R = e_0 + \delta e \qquad m_R = m_0 + \delta m \tag{7.1}$$

The quantities e_0 and m_0 are known as the "bare" charge and mass. These would be the physical values if there were no higher order diagrams. Since there are always higher order diagrams, the bare values are unmeasurable. The remormalised physical values e_R and m_R are the ones we measure: they include δe and δm corrections from higher order diagrams, even though these are infinite! Rather bizarrely there must be cancellations between the infinities in δe, δm and the bare quantities e_0, m_0.

The renormalization process can be explicitly shown by imposing a "cutoff" mass M on the integral over the loop four momentum k. The integral then gives a finite part independent of M, and a part dependent on M which goes to infinity when the cutoff is removed (see Griffiths P. 219, P.264 and Halzen & Martin P. 157). The renormalized charge absorbs the infinite M dependent part (here we have just included electron-positron loops):

$$e_R = e_0 \sqrt{1 - \frac{e_0^2}{12\pi^2} ln \left(\frac{M^2}{q^2} \right)} \tag{7.2}$$

Taking a finite scale for M leads to a variation of the physical charge e as a function of the four momentum transfer q^2. This "running coupling constant" is discussed in the next section.

A way to understand renormalization is to say that the higher order contributions are absorbed into re-definitions of the Feynman diagrams:

- A dressed fermion contribution can be absorbed into a re-definition of the spinor, modifying the fermion mass.
- A bubble diagram can be absorbed into a re-definition of the photon propagator. This introduces lepton-antilepton and quark-antiquark components into the photon!
- A vertex correction can be absorbed into a re-definition of the coupling charge at the vertex. This can be thought of as being a result of charge screening due to fermion/antifermion pairs.

7.3 Running Coupling Constants

The finite parts of the higher order corrections lead to a q^2 dependence of the vertex couplings. Starting from the renormalized physical charge at $q^2 = 0$ the "running" coupling constant is:

$$\alpha(q^2) = \alpha(0)\left(1 + \alpha(0)\frac{z_f}{3\pi} ln(\frac{q^2}{M^2})\right) \tag{7.3}$$

where $z_f = \sum_f Q_f^2$ is the sum over the charges of all the active fermion/antifermion pairs (in units of e). z_f has a dependence on q^2 as more fermions become active. At low q^2 we can use $z_f = 1$ (only e^+e^- pairs), but at 100 GeV we need $z_f = 38/9 \approx 4$ (only $t\bar{t}$ pairs not active).

Since M is an arbitrary large cut-off mass, the above form is not particularly useful. What is usually done is to select a **renormalization scale**, $q^2 = \mu^2$, relative to which α at any other value of q^2 can be defined:

$$\alpha(q^2) = \alpha(\mu^2)\left(1 - \alpha(\mu^2)\frac{z_f}{3\pi} ln(\frac{q^2}{\mu^2})\right)^{-1} \tag{7.4}$$

We can choose any value of μ where make a initial measurement of α, but once we do the evolution of the values of α are determined by equation (7.4). It is usual to take either a low energy reference from atomic physics ($\mu = 1$ MeV) or a high energy reference from LEP ($\mu = m_Z = 91\ GeV$)data:

$$\alpha(\mu = 0) = \frac{1}{137} \qquad \alpha(\mu^2 = m_Z^2) = \frac{1}{128} \tag{7.5}$$

Note that the running of the electromagnetic coupling α is quite small. We will see later that this is not the case for the strong coupling α_s.

7.4 Measurements of $g - 2$

Non-Examinable section - just included for your interest

The gyromagnetic ratio, g, for an electron, defines its magnetic moment, i.e. the coupling of its spin to a magnetic field:

$$\vec{\mu} = g\mu_B\vec{S} \qquad \mu_B = \frac{e\hbar}{2m_e c} \tag{7.6}$$

where μ_B is the Bohr magneton. One of the successes of the Dirac equation is that it predicts that $g = 2$ for "bare" fermions.

The lowest order Feynman diagram describing the magnetic moment couples the fermion to the electromagnetic field via a single virtual photon. The inclusion of higher order diagrams lead to an *anomalous* value for g slightly different from 2. QED has been used to calculate diagrams up to $\mathcal{O}(\alpha^5)$, of which there are a very large number! The theoretical predictions for the electron and muon are:

$$\left[\frac{g-2}{2}\right]_e = 0.001159652183(8) \qquad \left[\frac{g-2}{2}\right]_\mu = 0.0011659183(5) \tag{7.7}$$

where the numbers in brackets are the errors on the last digits. *Note that the main theoretical uncertainties are no longer associated with QED but with the introduction of strong interaction effects when a bubble diagram involves a quark-antiquark pair.*

The experimental measurements of $g-2$ use spin precession in a magnetic field. The electron is stable and can be held in a Penning trap to make the measurements. In the case of the unstable muon, there has been a heroic series of experiments over the last 50 years using storage rings. The current results are:

$$\left[\frac{g-2}{2}\right]_e = 0.0011596521807(3) \qquad \left[\frac{g-2}{2}\right]_\mu = 0.0011659209(6) \tag{7.8}$$

These tests of QED are among the most accurate tests of a theory in the whole of physics. There is quite a lot of current interest in the $(26 \pm 8) \times 10^{-10}$ difference between experiment and theory for the muon which has about 3σ significance. This may be evidence for physics beyond the Standard Model giving rise to additional Feynman diagrams. *End of non-examinable section*

7.5 Charged Currents

The $W^\pm$ bosons with mass $M_W = 80.4$ GeV mediate weak charged current interactions:

- The heavy virtual W boson propagator is $g^{\mu\nu}/(M_W^2 - q^2)$.
- The current operator is purely left-handed $\gamma^\mu\frac{1}{2}(1-\gamma^5)$.
  This is known as the **V–A theory**.
- The dimensionless coupling constant at each vertex, g_W, is often written in terms of the dimensioned **Fermi constant**, $G_F = 1.16 \times 10^{-5}\text{GeV}^{-2}$:

  $$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} \tag{7.9}$$

  This comes from the low energy limit on the propagator $q^2 \ll M_W^2$.

Figure 7.3: Quark level β decay Feynman diagrams.

- The $W^\pm$ couplings to leptons conserve lepton family number:
  $W^+ \to e^+\nu_e,\ \mu^+\nu_\mu,\ \tau^+\nu_\tau$ and $W^- \to e^-\bar{\nu}_e,\ \mu^-\bar{\nu}_\mu,\ \tau^-\bar{\nu}_\tau$.

- The $W^\pm$ couplings to quarks *change quark flavour.* All possible $\mathrm{u}_i\bar{\mathrm{d}}_j$ or $\bar{\mathrm{u}}_i d_j$ pairs are allowed, where $\mathrm{u}_i = (u, c, t)$, $\mathrm{d}_j = (d, s, b)$.

- The coupling constants for quarks are gV_{ij} where V_{ij} is the 3×3 Cabibbo-Kobayashi-Maskawa matrix.

$$V_{CKM} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \tag{7.10}$$

7.6 Beta decay

Inside a nucleus weak charged current interactions transform a proton into a neutron (β^+ decays), or a neutron into a proton (β^- decays):

$$^{14}\mathrm{O} \to ^{14}\mathrm{N}^* + e^+ + \nu_e \qquad ^{60}\mathrm{Co} \to ^{60}\mathrm{Ni}^* + e^- + \bar{\nu}_e \tag{7.11}$$

A free neutron decays to a proton, electron and antineutrino, with a lifetime of 886 s, but a free proton does not decay to a neutron. This is because the proton mass, 938.3 MeV, is lower than the neutron mass, 939.6 MeV. At the quark level β decay is described by:

$$u \to d + e^+ + \nu_e \qquad d \to u + e^- + \bar{\nu}_e \tag{7.12}$$

If the electron (positron) is transferred from the final to the initial state we have **electron capture**:

$$e^- + u \to d + \nu_e \qquad e^+ + d \to u + \bar{\nu}_e \tag{7.13}$$

If the neutrino (antineutrino) is transferred from the final to the initial state we have **inverse β decay**:

$$\bar{\nu}_e + u \to d + e^+ \qquad \nu_e + d \to u + e^- \tag{7.14}$$

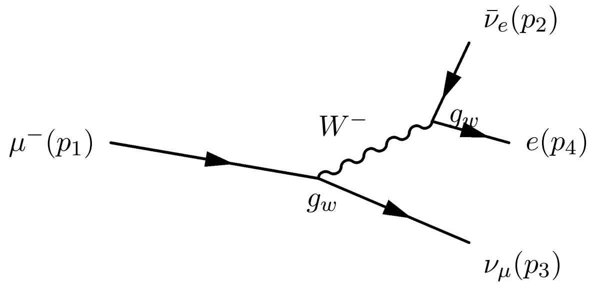
7.6.1 Matrix element for β decay

The matrix element for β decay can be written in terms of quark and lepton currents containing the fermion spinors:

$$\mathcal{M} = \left(\frac{g}{\sqrt{2}}\bar{u}_d\gamma^\mu\frac{1}{2}(1-\gamma^5)u_u\right)\frac{1}{M_W^2-q^2}\left(\frac{g}{\sqrt{2}}\bar{u}_{\nu_e}\gamma^\mu\frac{1}{2}(1-\gamma^5)u_e\right) \tag{7.15}$$

Hadronic interactions (form factors) play a role in decay rate and lifetimes, and the above formula need to be modifed for these effects.

7.7 Muon decay



The muon is a charged lepton that decays to an electron, a muon neutrino and an electron antineutrino:

$$\mu^- \to e^-\nu_\mu\bar{\nu}_e \qquad \mu^+ \to e^+\nu_e\bar{\nu}_\mu \tag{7.16}$$

The matrix element can be written in terms of a muon-type current between the muon and the muon neutrino, and an electron-type current between the electron and the electron antineutrino:

$$|\mathcal{M}|^2 = \left(\frac{g_W^2}{8m_W^2}\right)^2 [\bar{u}(\nu_\mu)\gamma^\mu(1-\gamma^5)u(\mu)]^2[\bar{u}(e)\gamma^\mu(1-\gamma^5)v(\bar{\nu}_e)]^2 \tag{7.17}$$

Neglecting the masses of the electron and neutrinos the transition rate can be calculated (see Griffiths P.311-313):

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{4\pi^3}m_\mu^2E_e^2\left(1-\frac{4E_e}{3m_\mu}\right) \tag{7.18}$$

This is known as the Michel spectrum. Recent measurement of this are shown in figure 7.4. Near the endpoint $E_0 = m_\mu/2$ the rate drops abruptly from a maximum to zero.

We can obtain the decay rate by integrating equation (7.18):

$$\Gamma = \int_0^{m_\mu/2}\frac{d\Gamma}{dE_e}dE_e = \frac{G_F^2m_\mu^2}{4\pi^3}\int_0^{m_\mu/2}E_e^2\left(1-\frac{4E_e}{3m_\mu^2}\right)dE_e \tag{7.19}$$

As $\mu^- \to e^-\overline{\nu_e}\nu_\mu$ is the only possible decay, the lifetime of the muon is:

$$\tau \equiv \frac{1}{\Gamma} = \frac{192\pi^3}{G_F^2m_\mu^5} = \frac{192\pi^3\hbar^7}{G_F^2m_\mu^5c^4} \tag{7.20}$$

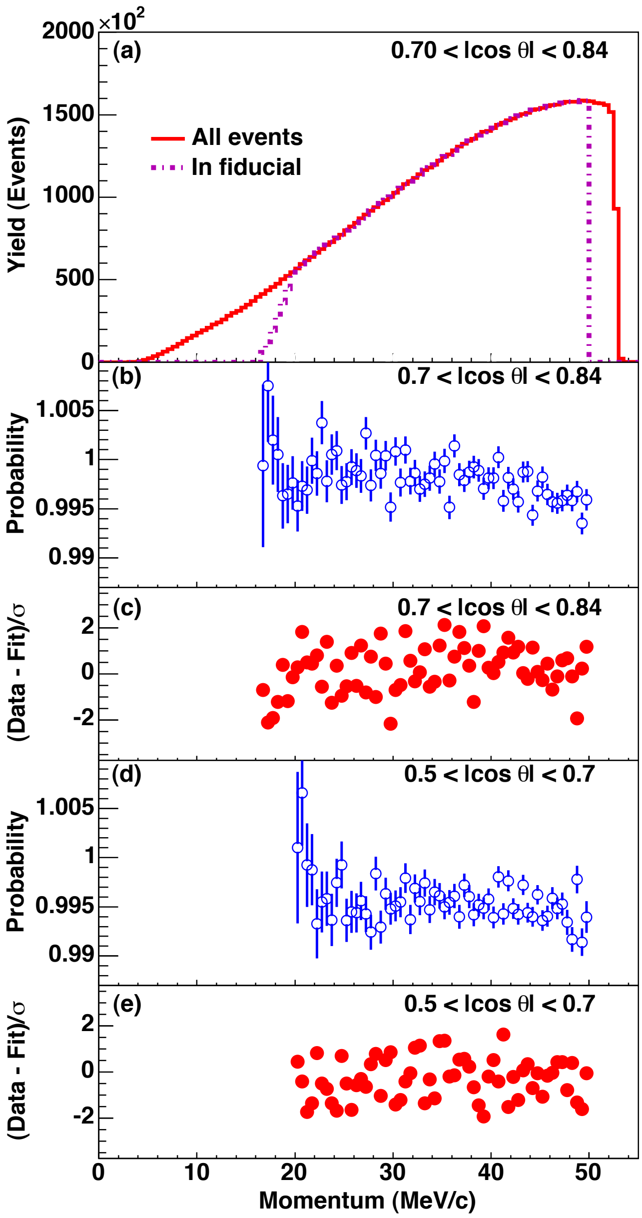


Figure 7.4: TWIST experiment at TRIMF in Canada measures $\mu^+ \to e^+\nu_e\overline{\nu_\mu}$ decay spectrum. Excellent agreement between data and prediction!

Measurements of muon lifetime and mass used to define a value for G_F (values from PDG 2010)

$$\tau = (2.19703 \pm 0.00002) \times 10^6 \text{ s} \qquad m = 105.658367 \pm 0.000004 MeV \tag{7.21}$$

Applying small corrections for finite electron mass and second order effects $G_F = 1.166364(5) \times 10^{-5}$ GeV^{-2}

We can compare the couplings in muon decay and superallowed β decay:

$$G_F(\mu) = 1.166 \times 10^{-5} \text{ GeV}^{-2} \qquad G_F(\beta)|V_{ud}| = (1.136 \pm 0.003) \times 10^{-5} \text{ GeV}^{-2}$$

These give the same value for G_F with $|V_{\rm ud}| \approx 0.97$.

7.8 Neutral Currents

The Z^0 boson with mass $M_Z = 91.2$ GeV mediates weak neutral current interactions:

- The heavy virtual Z boson propagator is $1/(M_Z^2 - q^2)$.
- The current operator is no longer purely left-handed except for neutrinos. It is written as $\gamma^\mu(c_V - c_A\gamma^5)$, where $g_L = (c_V + c_A)/2$ and $g_R = (c_V - c_A)/2$, and $c_V(c_A)$ are the vector (axial-vector) couplings.
- The Z^0 couplings to leptons conserve lepton flavour:
  $Z^0 \to e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$ and $Z^0 \to \nu_e\bar{\nu}_e$, $\nu_\mu\bar{\nu}_\mu$, $\nu_\tau\bar{\nu}_\tau$.
- The Z^0 couplings to quarks conserve quark flavour:
  $Z^0 \to u\bar{u}$, $c\bar{c}$, $d\bar{d}$, $s\bar{s}$, $b\bar{b}$, $(t\bar{t})$
- The coupling constant at each vertex is g, but modified by the values of c_V and c_A which depend on fermion type.

Lepton	$2c_V$	$2c_A$	Quark	$2c_V$	$2c_A$
ν_e, ν_μ, ν_τ	1	1	u, c, t	0.38	1
e, μ, τ	-0.06	-1	d, s, b	-0.68	-1

At low centre-of-mass energies the couplings of the Z^0 to quarks and charged leptons are a small addition to the electromagnetic couplings, which can only be observed in precision experiments. At higher energies, the interference between Z^0 and γ exchange is important. It is explained in terms of the full electroweak theory.

There is one place where the neutral current couplings to Z^0 can be observed unambiguously. This is in neutrino scattering, since neutrinos have no charge and do not experience the electromagnetic interaction. The first observation of these couplings was at CERN in 1973.

The following three chapters taken from the lecture notes given by Prof. Alexander Belyaev (PHYS 3002) in Southampton University

Chapter 13

Accelerators

Particles physics, also known as 'high energy physics' is the study of the fundamental forces of nature and the particles that can be found at very high energies.

The most massive particles that has been discovered so far are W-boson with a mass of 80.4 GeV/c^2, Z-boson with a mass of 91.2 GeV/c^2, and *top*-quark with a mass of 172.0 GeV/c^2. All these particles are 100 times heavier than the proton. So we need a really high energy to produce these particles.

Another way of seeing that we need high energies is to note that we wish to probe very short distances. At the very least we want to probe distances which are small compared with a typical nuclear radius, i.e.

$$x \ll 1\,\mathrm{fm} = 10^{-15}\,\mathrm{m}$$

In order to do this the uncertainty in the position, Δx must be much smaller than 1 fm, and by Heisenberg's uncertainty principle

$$\Delta x \Delta p \geq \hbar/2$$

the uncertainty in momentum Δp must obey the inequality

$$\Delta p \gg \frac{\hbar}{1\,\mathrm{fm}} = 197\,\mathrm{MeV/c}.$$

This in turn means that the momenta of the particle used as a probe must have a momentum much larger than this, and hence an energy large compare with ≈ 200 MeV.

In fact, the weak interactions have a range which is more than two orders of magnitude shorter than this and so particles used to investigate the mechanism of weak interactions have to have energies of at least 100 GeV.

In order to achieve these very high energies particles are accelerated in "accelerators". Incident particles are accelerated to these high energies and scattered against another particle. There is enough energy to smash the initial particles up and produce many other particles in the final state, some of them with considerably higher masses than the incident particles. Such scattering is called "inelastic scattering" (conversely a scattering event in which the final state particles are the same as the initial particles is called "elastic scattering". Rutherford scattering or Mott scattering are examples of elastic scattering.) The word 'elastic' here means that none of the incoming energy is used up in the production of other particles.

In elastic scattering we talk about a differential cross-section (with respect to solid angle), which is the number of particles per incident flux in a given element of solid angle. For inelastic events we can talk about the total cross-section for a particular process. For example, at the LEP accelerator (electron-positron scattering) at CERN one possible process was

$$e^+ + e^- \rightarrow W^+ + W^-,$$

in which the electron and positron annihilate each other and produce two W-bosons instead. The W-boson has a mass of 80.4 GeV/c^2, so that total centre-of-mass energies of over 160 GeV are required for this process to take place. The cross-section $\sigma_{(e^+e^- \rightarrow W^+W^-)}$ is the total number of events in which two W-bosons are produced per unit incident flux (i.e. the number of W-boson pairs produced divided by the number of particle scatterings per unit area)

It is now believed that there exist particles with masses which are an order of magnitude larger than this and modern accelerators can achieve energies of up to 1 TeV (10^{12} eV). This new energy frontier and respectively new small distances can be probed by presently the most powerful accelerator in the world – the Large Hadron Collider (LHC) – at CERN which has resumed running in November 2009 with energy 7.5 TeV.

13.1 Fixed Target Experiments vs. Colliding Beams

The total energy of a projectile particle plus the target particle depends on the reference frame. The frame that is relevant for the production of high mass particles is the centre-of-mass frame for which the projectile and target have equal and opposite momentum p. For simplicity let us suppose that the projectile and target particle are the same, or possibly particle antiparticle (e.g. proton-proton, proton-antiproton, or electron-positron) so that their masses, m are the same. This means that in this frame both the particle have the same energy, E_{CM} (since we are usually dealing with relativistic particles, this means kinetic *plus* rest energy.)

Let us construct the quantity

$$s = \left(\sum_{i=1,2} E_i\right)^2 - \left(\sum_{i=1,2} \mathbf{p}_i\right)^2 c^2$$

In the centre-of-mass frame, where the momenta are equal and opposite the second term vanishes and we have

$$s = 4E_{CM}^2,$$

i.e. s is the square of the total incoming energy in the centre of mass frame - this is a quantity that is often used in particle physics and the notation s is always used. For one particle we know that $E^2 - p^2c^2$ is equal to m^2c^4 and is therefore the same in any frame of reference even though the quantities E and $\mathbf{p}$ will be different in the two frames. Likewise the above quantity, s, is the same in any frame of reference (we say that 'it invariant under Lorentz transformations.')

In the frame in which the target particle is at rest, its energy is mc^2 and its momentum is zero, whereas the projectile has energy E_{LAB} and momentum $\mathbf{p}_{LAB}$ so that we have

$$s \;=\; \left(E_{LAB} + mc^2\right)^2 - \mathbf{p}^2_{LAB}c^2 \;=\; E^2_{LAB} + m^2c^4 + 2mc^2E_{LAB} - \mathbf{p}^2_{LAB}c^2 \;=\; 2m^2c^4 + 2mc^2E_{LAB},$$

where in the last step we have used the relativity relation

$$E^2_{LAB} - \mathbf{p}^2_{LAB}c^2 \;=\; m^2c^4.$$

Equating the two expressions for s (and taking a square root we obtain the relation

$$\sqrt{s} \;=\; 2E_{CM} \;=\; \sqrt{2m^2c^4 + 2mc^2E_{LAB}}.$$

For non-relativistic incident particles with kinetic energy $T \ll mc^2$ for which $E_{LAB} = mc^2 + T$, this gives

$$\sqrt{s} \;=\; 2E_{CM} \;=\; 2mc^2 + T,$$

as expected, but for relativistic particles the centre-of-mass energy is considerably reduced. For example, taking the proton mass be be approximately 1 GeV/c^2, the if we have an accelerator that can accelerate protons up to an energy of 100 GeV, the total centre-of-mass energy achieved is only about 15 GeV - far less than the energy required to produce a particle of mass 100 GeV/c^2.

The solution to this problem is to use colliding beams of particles. In these experiments both the initial particles involved in the scattering emerge from the accelerator and are then stored in storage rings, in which the particles move in opposite directions around the ring, with their high energies maintained by means of a magnetic field. At various point around the rings the beams intersect and scattering takes place. In this way the laboratory frame *is* the centre-of-mass frame and the full energy delivered by the accelerator can be used to produce high mass particles.

13.2 Luminosity

The luminosity $\mathcal{L}$ is the number of particle collisions per unit area (usually quoted in cm^2) per second. The number of events of a particular type which occur per second is the cross-section multiplied by the luminosity. In the example of two W-boson production at LEP the cross-section, $\sigma_{(e^+e^- \to W^+W^-)}$ is 15 pb (p=pico means 10^{-12}) and the luminosity of LEP was 10^{32} per cm^2 per second. The number of these pairs of W-bosons produced per second is given by

$$\frac{dN_{W^+W^-}}{dt} \;=\; (15 \times 10^{-12} \times 10^{-28}) \times (10^{32} \times 10^4) \;=\; 1.5 \times 10^{-3},$$

where the first term in parenthesis is the cross-section converted to m^2 and the second is the luminosity converted to $m^{-2}\,sec^{-1}$. So, the general formula for reaction rate, $R = dN/dt$ is

$$R = \sigma \times \mathcal{L}$$

while for integrated luminosity over the time $L = \int \mathcal{L} dt$ the number of events, N, we will observe is given by

$$N = \sigma \times L$$

.

$\mathcal{L}$ is proportional to the number 'bunches' of particles in each beam, n (typically 5-100), the revolution frequency, f(kHz-MHz), N_1, N_2 – the number of particles in each bunch ($\simeq 10^{10} - 10^{11}$) and inversely proportional to the bean cross section, A (μm^2):

$$\mathcal{L} = \frac{nfN_1N_2}{A}$$

As in the case of radioactivity the cross-section is a probability for a particular event and the actual number of events observed is a random distribution with that probability. If a cross-section predicts N events over a given time-period, the error on that number is $\sqrt{N}$ (this means that there is a 68% probability that the number of events observed will lie in the region $N - \sqrt{N}$ to $N = \sqrt{N}$). To be able to measure the above cross-section at LEP to an accuracy of 1% it was necessary to collect 10000 such W-pairs, which, at a rate of 1.5×10^{-3} per sec., took about three months.

We pay a price for colliding beam experiments in terms of luminosity. For a fixed target experiment we can make an estimate of the luminosity in the case of proton-proton scattering from the fact that the incident particles are travelling almost with the speed of light. The luminosity is given by the number of protons in a column of the target of unit area and length c. For a solid whose density is $10^4\,kg\,m^{-3}$, and assuming that about one half of the target material consists of protons of mass 1.67×10^{-27} kg, this comes out to about 10^{35} per cm^2 per sec. In colliding beams it is necessary to focus the incident beams as tightly as possible using magnetic fields, in order to maximize the luminosity. So far, luminosities of 10^{32} per cm^2 per sec. have been achieved, which means the reaction rate is down by three orders of magnitude compared with a fixed target experiment. However, the LHC is designed to reach a luminosity of 10^{34} per cm^2 per sec. - i.e luminosities within an order of magnitude of that obtained in fixed target experiments.

13.3 Types of accelerators

As we have discussed, the general aim of accelerators is to collide two particles at high(est) energy and create new particles from combined energy and quantum numbers or to probe inside one of the particles to see what it is made of.

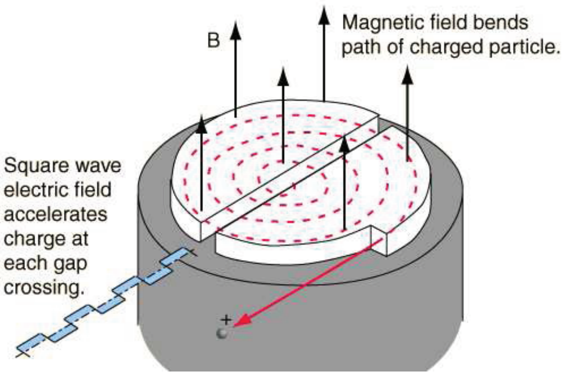
Only stable charged particles can be accelerated: such as electrons, positrons, protons, anti-proton and some ions. Potentially, the long-lived particles such as muon ($\tau \simeq 2 \times 10^{-6}$s) were discussed to be used in the future muon collider.

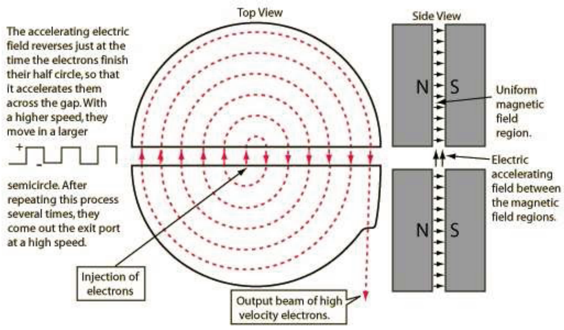
Single DC stage accelerators such as the Van de Graaff Generator can accelerate electrons and protons upto about 20 MeV.

There are two general types of modern accelerators – Circular (Cyclic) and Linear.

13.3.1 Cyclotrons

The prototype design for all circular accelerators is the cyclotron.





This is a device in which the (charged) particles to be accelerated move in two hollow metallic semi-disks (D's) with a large magnetic field B applied normal to the plane of the D's. The particles move in a spiral from the center and an alternating electric field is applied between the D's whose frequency is equal to the frequency of rotation of the charged particles, such that when the particles crosses from one of the D's to the other the electric field always acts in the direction which accelerates the particles.

A charged particle with charge e moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ experiences a force $\mathbf{F}$, where

$$\mathbf{F} \;=\; q\mathbf{v} \times \mathbf{B}.$$

When the magnetic field is perpendicular to the plane of motion of the charged particle, this force is always towards the centre and gives rise to centripetal acceleration, so that at the moment when the particles are moving in a circle of radius r

$$F \;=\; B\,e\,v \;=\; m\frac{v^2}{r}.$$

We see immediately that the angular velocity $\omega = v/r$ is constant, so that the frequency of the alternating electric field remains constant. The maximum energy that the particles can acquire depends on the radius, R, for which the velocity has its maximum value v_{max},

$$v_{max} \;=\; \frac{BeR}{m},$$

leading to a maximum kinetic energy

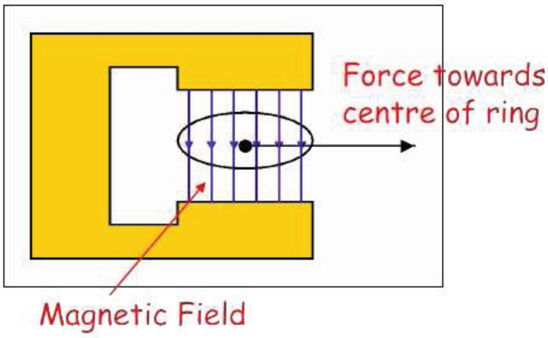
$$T_{max} \;=\; \frac{1}{2}mv_{max}^2 \;=\; \frac{B^2e^2R^2}{2m}.$$

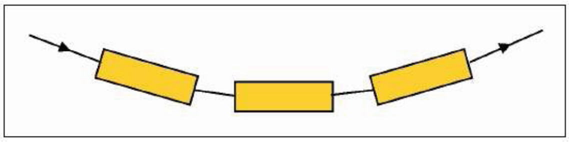
This works fine if the energy of the particle remains non-relativistic. However, in high energy accelerators the particles are accelerated to energies which are extremely relativistic - the particles are travelling very nearly with the velocity of light (at the LHC v/c will be $1 - 10^{-15}$!). Taking relativistic effects into account The angular velocity is now

$$\omega \; = \; \sqrt{1 - v^2/c^2}\frac{Be}{m}.$$

This means that as the particles accelerate, either the frequency of the applied electric field must vary - such machines are called "synchrocyclotrons" - or the applied magnetic field must be varied (or both) - such machines are called "synchrotrons".

At Synchrotron dipole magnets keep particles in circular orbit using $p = 0.3 \times B \times R$ (p in GeV/c, B in Tesla, R in meters), while quadrupole magnets used to focus the beam.





Since the bending field B is limited then the maximum energy is limited by the size of the ring. The CERN SPS (Super Proton Synchrotron) has a radius R = 1.1Km and a momentum of 450 GeV/c. Particles are accelerated by resonators (RF Cavities). The bending field B is increased with time as the energy (momentum p) increases so as to keep R constant [$p = 0.3BR$]. Electron synchrotrons are similar to proton synchrotrons except that the energy losses are greater.

One of the main limiting factors of synchrotron accelerators is the Synchrotron Radiation. A charged particle moving in a circular orbit is accelerating (even if the speed is constant) and therefore radiates. The energy radiated per turn per particle is:

$$\Delta E = \frac{4\pi e^2 \beta^2 \gamma^4}{3R}$$

where e is the charge, $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2} = E/m$, from which follows that

$$\Delta E \propto 1/m^4$$

For relativistic electrons and protons of the same momentum the ratio of energy losses are very large for electrons versus protons:

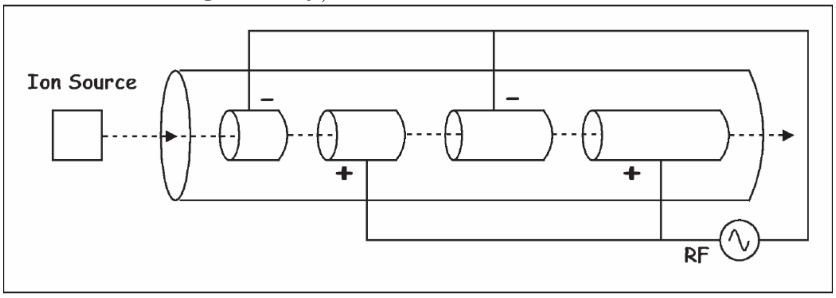
$$\frac{\Delta E_e}{\Delta E_p} = \left(\frac{m_p}{m_e}\right)^4 \simeq 10^{13}$$

.

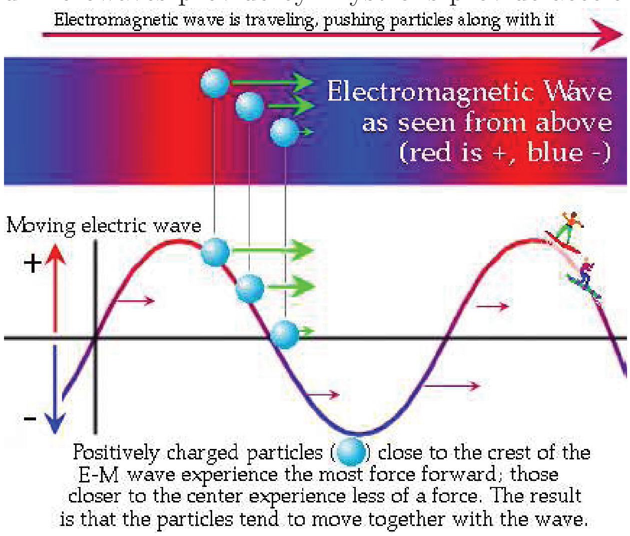
13.3.2 Linear Accelerators

The energy loss due to synchrotron radiation, can be avoided in a linear accelerator. In such a machine the particles are accelerated by means of an applied electric field along a long a tube.

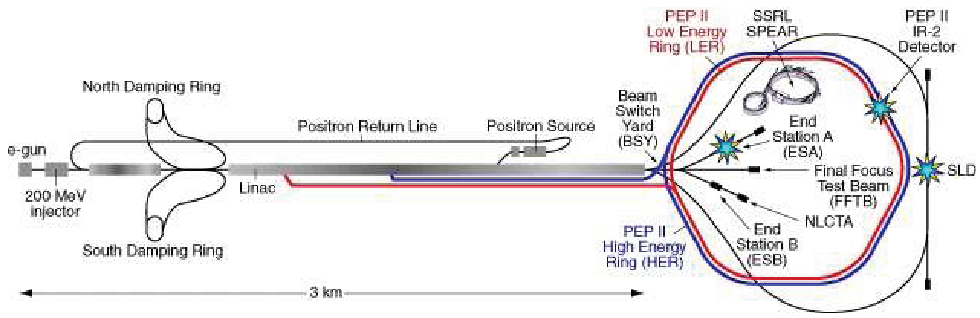
Proton Linear Accelerators (Linacs) use a succession of drift tubes of increasing length (to compensate for increasing velocity).



Particles always travel in vacuum. There is no field inside the drift tubes. External field between ends of tubes changes sign so proton always sees $-ve$ in front and $+ve$ behind. Proton linacs of 10-70m give energies of 30 to 200 MeV. Usually used as injectors for higher energy machines. Above a few MeV, electrons travel at speed of light. The 'tubes' become uniform in length and microwaves provide by Klystrons provide accelerating potential.



The largest linear collider in existence is SLAC (Stanford Linear Collider Center) in California. This is 3 km. long and accelerates both electrons and positrons up to energies of 50 GeV. It is able to accelerate both electrons and positrons simultaneously by sending an electromagnetic wave in the microwave band along the beam pipe and injecting bunches of electrons and positrons which are precisely one half wavelength apart, so that the electric field acting on the positrons is in the forward direction and so accelerates the positrons in the forward direction, whereas the electric field acting on the electrons is in the backwards direction, but because the electrons have negative charge they are also accelerated in the forward direction.



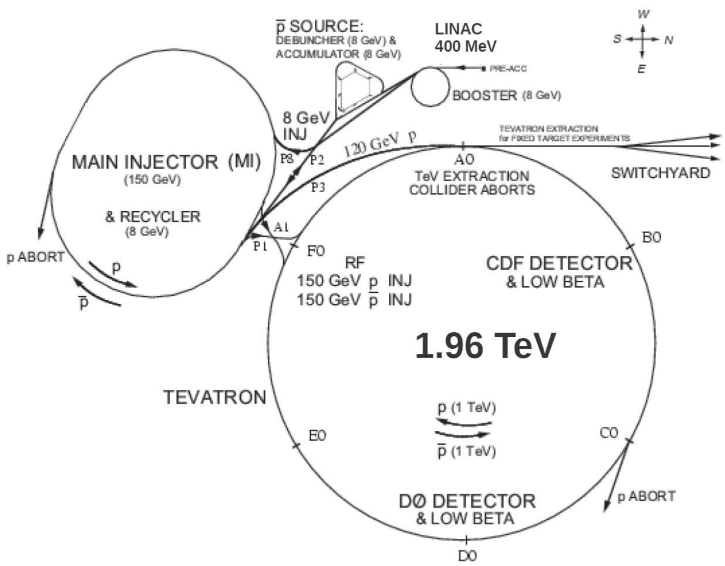
At the end of the tube the electrons and positrons are stored in a storage ring (they go around the storage ring in opposite directions under the influence of the same magnetic field) and there are intersection points where electron-positron scatterings occur.

There are plans (awaiting international approval) to build a much larger linear collider (known as ILC - the International Linear Collider) which will have a total centre-of-mass energy of 500 GeV (or perhaps even 1 TeV).

13.4 Main Recent and Present Particle Accelerators

Here are some of today's main accelerator laboratories.

- **FermiLab:**

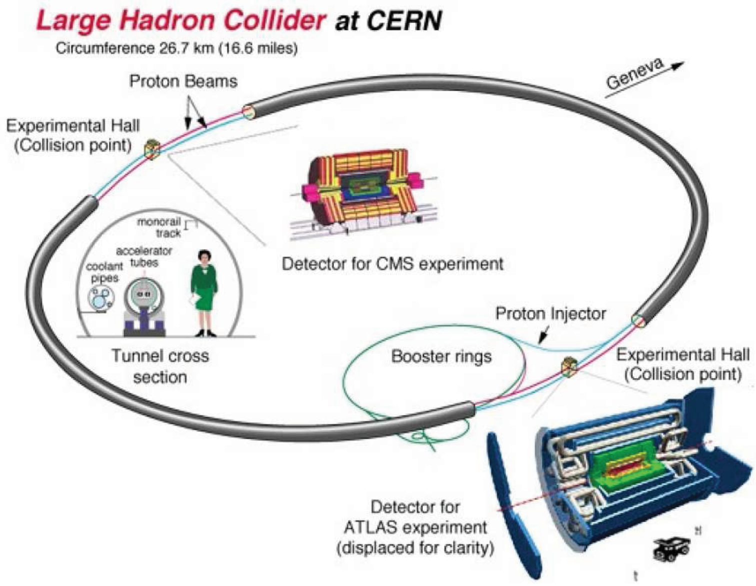


Situated just outside Chicago this is now running the Tevatron in which protons and antiprotons are each accelerated to an energy of 1.96 TeV and then move around a ring of circumference 6 km. This is a synchrotron in which very high magnetic fields are achieved using superconducting (electro-)magnets, which are capable of maintaining very large currents thereby producing large magnetic fields. The luminosity is

$10^{32}\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$

- **CERN:**
  Situated just outside Geneva, until 2001 the main experiment was LEP in which electrons and positrons were each accelerated to an energy of about 100 GeV, and had a luminosity of $10^{32}\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$. This was the largest electron synchrotron in the world with a circumference of 27 km.

  The next project at CERN is the LHC has started in September 2008. After an accident in October 2008, LHC has resumed its operation in November 2009 and now it is colliding protons against protons with energies 3.5TeV × 3.5TeV resulting to a total cms energy of 7 TeV. Using a specially designed magnetic field configuration, two beams of protons moving in opposite direction around the same ring is possible. In the future, protons will each be accelerated to 7 TeV and the design luminosity is $10^{34}\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$.



- **DESY:**
  Situated just outside Hamburg, this laboratory is running the HERA accelerator which accelerated protons to an energy of 820 GeV and electrons (or positrons) to 27 GeV. It is the only accelerator in which the initial particles are not the same - or particle-antiparticle pairs.

Table below presents summary on present and recent colliders as well as comparison of electron and proton accelerators.

Name	Particles	Energies	Where	Status
SLC	e^+e^-	**50+50GeV**	Stanford USA	**Ended**
LEP	e^+e^-	**100+100GeV**	CERN Geneva	**Ended**
Tevatron	$p\bar{p}$	**980+980GeV**	Fermilab USA	**Ended**
HERA	$e^{\pm}p$	**30+820GeV**	DESY Hamburg	**Ended**
PEP II	e^+e^-	**9+3.1GeV**	Stanford USA	**Current**
KEKB	e^+e^-	**8+3.5GeV**	Tsukuba Japan	**Current**
LHC	pp	**4.0+4.0TeV**	CERN Geneva	**Current**

Electron Machines	Proton Machines
Clean - no other particles involved than e^+e^-.	Messy - qq or $q\bar{q}$ interact and rest of p or $\bar{p}$ is junk.
Lower energy for same radius (synchrotron radiation). LEP e^+e^- ~ 200 GeV.	Higher energy for same radius. LHC (pp) in LEP tunnel ~14 TeV.
Energy of e^+e^- known.	Energy of qq or $q\bar{q}$ not known.
Fixed energy (for a given set of operating conditions).	Range of qq or $q\bar{q}$ energies for fixed pp or $p\bar{p}$ energy.
Best for detailed study.	Best for discovering new things.

Chapter 16

Constituent Quark Model

Quarks are fundamental spin-$\frac{1}{2}$ particles from which all hadrons are made up. Baryons consist of three quarks, whereas mesons consist of a quark and an anti-quark. There are six types of quarks called "flavours". The electric charges of the quarks take the value $+\frac{2}{3}$ or $-\frac{1}{3}$ (in units of the magnitude of the electron charge).

Symbol	**Flavour**	**Electric charge (e)**	**Isospin**	**I_3**	**Mass Gev/c^2**
u	up	$+\frac{2}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$	≈ 0.33
d	down	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	≈ 0.33
c	charm	$+\frac{2}{3}$	0	0	≈ 1.5
s	strange	$-\frac{1}{3}$	0	0	≈ 0.5
t	top	$+\frac{2}{3}$	0	0	≈ 172
b	bottom	$-\frac{1}{3}$	0	0	≈ 4.5

These quarks all have antiparticles which have the same mass but opposite I_3, electric charge and flavour (e.g. anti-strange, anti-charm, etc.)

16.1 Hadrons from u,d quarks and anti-quarks

Baryons:

Baryon	**Quark content**	**Spin**	**Isospin**	**I_3**	**Mass Mev/c^2**
p	uud	$\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	938
n	udd	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	940
Δ^{++}	uuu	$\frac{3}{2}$	$\frac{3}{2}$	$+\frac{3}{2}$	1232
Δ^{+}	uud	$\frac{3}{2}$	$\frac{3}{2}$	$+\frac{1}{2}$	1232
Δ^{0}	udd	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	1232
Δ^{-}	ddd	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	1232

- Three spin-$\frac{1}{2}$ quarks can give a total spin of either $\frac{1}{2}$ or $\frac{3}{2}$ and these are the spins of the baryons (for these 'low-mass' particles the orbital angular momentum of the quarks is zero - excited states of quarks with non-zero orbital angular momenta are also possible and in these cases the determination of the spins of the baryons is more complicated).

- The masses of particles with the same isospin (but different I_3) are almost the same - the differences being due to the electromagnetic interactions which distinguish members of the isospin multiplet with different electric charge. If it were possible to 'switch off' the electromagnetic interactions these masses would be exactly equal.

- The baryons which consist of three u-quarks or three d-quarks only occur for spin $\frac{3}{2}$ (we return to this later)

Mesons:

Meson	Quark content	Spin	Isospin	I_3	Mass Mev/c^2
π^+	$u\bar{d}$	0	1	$+1$	140
π^0	$\frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$	0	1	0	135
π^-	$d\bar{u}$	0	1	-1	140
ρ^+	$u\bar{d}$	1	1	$+1$	770
ρ^0	$\frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$	1	1	0	770
ρ^-	$d\bar{u}$	1	1	-1	770
ω	$\frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$	1	0	0	782

- A spin-$\frac{1}{2}$ quark and an anti-quark with the same spin can combine (when the orbital angular momentum is zero) to give mesons of spin-0 or spin-1.

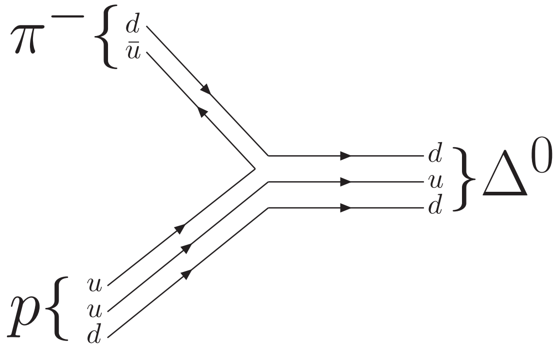
- The neutral mesons are not pure $u\bar{u}$, or $d\bar{d}$ states, but quantum superpositions of these.

- The neutral mesons have $I_3 = 0$. They could be in an isospin $I = 1$ state, (π^0, ρ^0), in which case their masses are similar to those of their charged counterparts, or $I = 0$ (ω) in which case their masses are somewhat different.

The strong interactions conserve flavour. There a d-quark cannot be converted into an s-quark (or vice versa), even though the electric charge is the same.

However, in a scattering process a quark *can* annihilate against an anti-quark of the same flavour, giving some energy which can be converted into mass and used to create a more massive particle. An example of this is

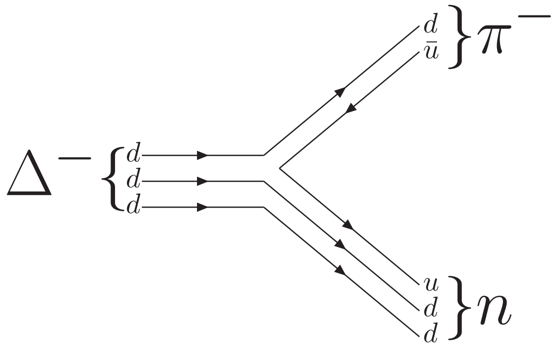
$$\begin{array}{ccccc} \pi^- & + & p & \rightarrow & \Delta^0 \\ (d\bar{u}) & & (uud) & & (udd) \end{array}$$



A u-quark from the proton and a $\bar{u}$ anti-quark from the pion have annihilated and the extra energy goes into the extra mass of the Δ^0, which is very short-lived and appears as a resonance in the $\pi^- \, p$ scattering cross-section.

Likewise in a decay process it is possible for some of the mass of the decaying particle to create a quark and anti-quark pair of the same flavour which go to forming the decay products, e.g.

$$\begin{array}{ccccc} \Delta^- & \rightarrow & \pi^- & + & n \\ (ddd) & & (ddu) & & (d\bar{u}) \end{array}$$



Here a u-quark and $\bar{u}$ anti-quark pair are created when the Δ^- decays and the $\bar{u}$ anti-quark binds with one of the d-quarks in the decaying Δ^- to make a π^-, whereas the u-quark binds with the other two d-quarks in the decaying Δ^- in order to make a neutron.

Quark and anti-quark pair creation is possible in any particle particle scattering process provided there is sufficient energy to create the final state particles. Thus for example it is possible to have the inelastic process

$$\begin{array}{ccccccccccc} p & + & p & \rightarrow & n & + & n & + & \pi^+ & + & \pi^+ \\ (uud) & & (uud) & & (udd) & & (udd) & & (u\bar{d}) & & (u\bar{d}) \end{array}$$

In this process two pairs of d-quarks and $\bar{d}$ anti-quarks are created. The d-quarks bind with the u and d quarks from the incoming protons to form neutrons, whereas the $\bar{d}$ anti-quarks bind with the remaining u-quarks in the incoming protons to form pions. In the centre-of-mass frame in which the total momentum is zero, so that the outgoing particles can be at rest - this is the lowest energy that they can have and is equal to sum of the masses of two neutrons and two pions (times by c^2), which is therefore the lowest total centre-of-mass energy of the incoming protons.

16.2 Hadrons with s-quarks (or $\bar{s}$ anti-quarks)

Baryons:

Baryon	Quark content	Spin	Isospin	I_3	Mass Mev/c^2
Σ^+	uus	$\frac{1}{2}$	1	$+1$	1189
Σ^0	uds	$\frac{1}{2}$	1	0	1193
Σ^-	dds	$\frac{1}{2}$	1	-1	1189
Ξ^0	uss	$\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	1314
Ξ^-	dss	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1321
Λ	uds	$\frac{1}{2}$	0	0	1115
Σ^{*+}	uus	$\frac{3}{2}$	1	$+1$	1385
Σ^{*0}	uds	$\frac{3}{2}$	1	0	1385
Σ^{*-}	dds	$\frac{3}{2}$	1	-1	1385
Ξ^{*0}	uss	$\frac{3}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	1530
Ξ^{*-}	dss	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1530
Ω^-	sss	$\frac{3}{2}$	0	0	1672

- For historical reasons the s-quark was assigned strangeness equal to -1, so these baryons have strangeness -1, -2 or -3 for one, two, or three strange quarks respectively. (likewise the b-quark has bottom flavour -1, whereas the c-quark has flavour charm=+1, and the t-quark has flavour top=+1)
- As in the case of Δ^- and Δ^{++}, the Ω^- which has three s-quarks (strangeness=-3) has spin-$\frac{3}{2}$.

The Ω^- had not discovered when the Quark Model was invented - its existence was a prediction of the Model. Furthermore its mass was predicted from the observation

$$M_{\Sigma^*} - M_\Delta \approx M_{\Xi^*} - M_{\Sigma^*} \approx 150\,\mathrm{MeV/c^2}$$

giving a predicted value for M_Ω of

$$M_\Omega = M_{\Xi^*} + 150 = 1680\,\mathrm{MeV/c^2}.$$

Mesons:

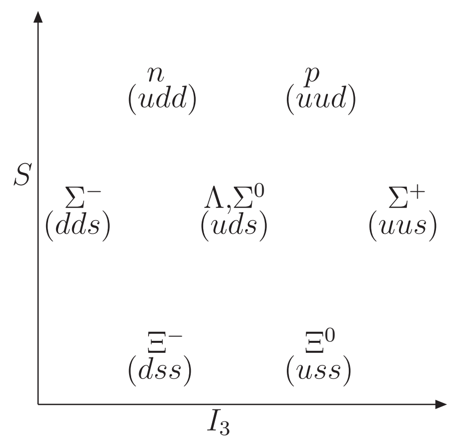
Meson	Quark content	Spin	Isospin	I_3	Mass Mev/c^2
K^+	$u\bar{s}$	0	$\frac{1}{2}$	$+\frac{1}{2}$	495
K^0	$d\bar{s}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	495
$\overline{K^0}$	$s\bar{d}$	0	$\frac{1}{2}$	$+\frac{1}{2}$	495
K^-	$s\bar{u}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	495
η	$(u\bar{u},\, d\bar{d},\, s\bar{s})$	0	0	0	547
K^{*+}	$u\bar{s}$	1	$\frac{1}{2}$	$+\frac{1}{2}$	892
K^{*0}	$d\bar{s}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	896
$\overline{K^{*0}}$	$s\bar{d}$	1	$\frac{1}{2}$	$+\frac{1}{2}$	896
K^{*-}	$s\bar{u}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	892
ϕ	$s\bar{s}$	1	0	0	1020

16.3 Eightfold Way:

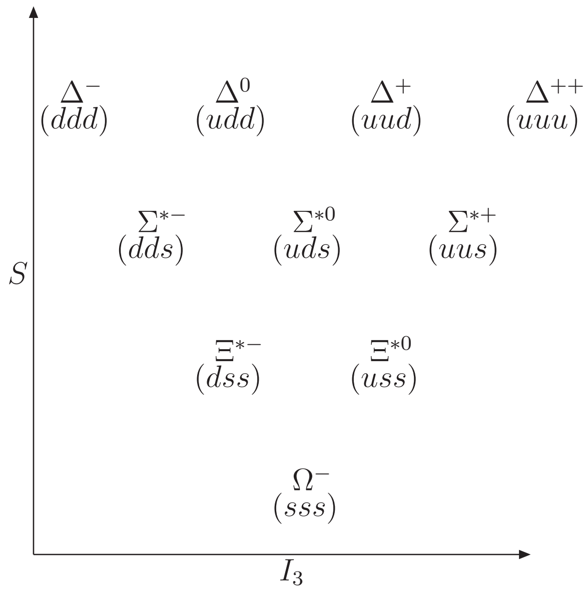
There is a method of classifying hadrons made up from u, d and s quarks and their anti-quarks by plotting particles with the same spin on a plot of strangeness against I_3.

For the lightest mesons and baryons there are eight particles on each plot. For this reason this classification method is known as the "Eightfold Way".

Spin-$\frac{1}{2}$ Baryons:

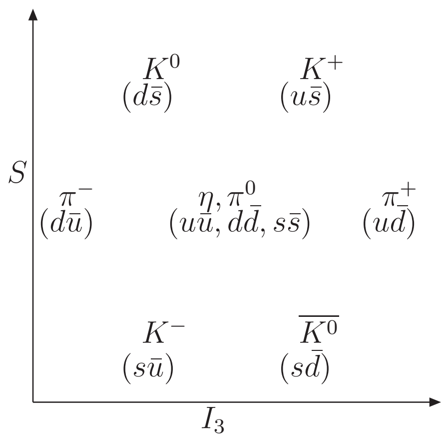


Spin-$\frac{3}{2}$ Baryons:

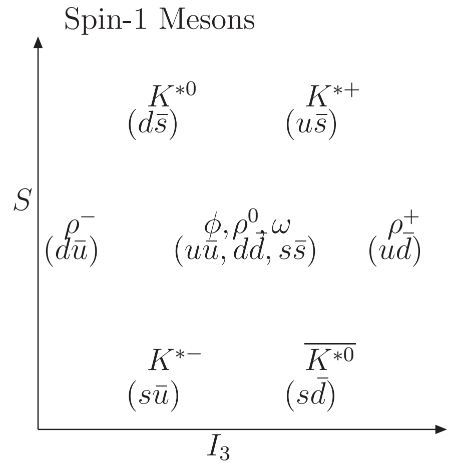


The rows contain the isospin multiplets. However, in the case of the row for $I = 1$, there can also be states with $I = 0$, $I_3 = 0$, so that the point in the middle can have two (or more) entries.

<u>Spin-0 Mesons:</u>



<u>Spin-1 Mesons:</u>



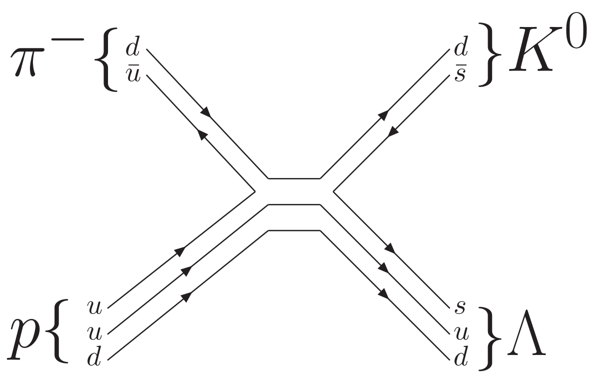
These meson multiplets contain the mesons and their antiparticles (obtained by replacing each quark by its anti-quark and vice versa), whereas the baryon multiplets have separate antiparticle multiplets which are bound states of three anti-quarks.

Some mesons, such as π^0, ρ^0, η are their own antiparticle, because they are bound states of a quark and an anti-quark of the same flavour so that replacing a quark by its anti-quark with the same flavour (and vice versa) produces the same particle. Other charged or neutral particles have separate antiparticles which have opposite electric charge and/or strangeness.

16.4 Associated Production and Decay

In strong interaction processes, quark flavour is conserved. s-quarks cannot be created or destroyed by the strong interactions (they can be created or destroyed by the weak interactions). This means that in a scattering experiment (e.g proton-proton or pion-proton scattering) one can only create a particle containing a strange quark if at the same time there is a particle containing an $\bar{s}$ anti-quark, so that the total strangeness is conserved. An example of such a process is

$$\begin{array}{ccccccc} \pi^- & + & p & \rightarrow & \Lambda & + & K^0 \\ (d\bar{u}) & & (duu) & & (dus) & & (\bar{s}d) \end{array}$$



What happens is that a u-quark annihilates against a $\bar{u}$ anti-quark and an s-quark $\bar{s}$ anti-quark pair has been created. This reaction is only possible above a threshold energy. In the centre-of-mass frame, the lowest total energy of the incoming particles is the sum of the masses of the Λ and the K^0, i.e.

$$\sqrt{s} \;=\; E_{CM}^{TOT} \;=\; (M_\Lambda \,+\, M_{K^0})\,c^2,$$

(here E_{CM}^{TOT} means to the *total* energy of the incoming (or outgoing) particles in the centre-of-mass frame - as the particles are not of the same mass, the individual energies of the particles will be different). In a (proton) fixed target experiment the pions must be accelerated to sufficient energy such that the centre-of-mass energy is greater than this value.

On the other hand the process

$$\begin{array}{ccccccc} \pi^- & + & p & \rightarrow & \Lambda & + & \pi^0 \\ (d\bar{u}) & & (duu) & & (dus) & & (\bar{d}d, \bar{u}u) \end{array}$$

is forbidden because the number of strange quarks in the initial and final states is not the same.

It is possible to scatter charged kaons ($K^\pm$) against nucleons. The K^- contains an s-quark. it is therefore possible to produce strange baryons in this process, such as

$$\begin{array}{ccccccc} K^- & + & n & \rightarrow & \Lambda & + & \pi^- \\ (s\bar{u}) & & (udd) & & (dus) & & (\bar{u}d) \end{array}$$

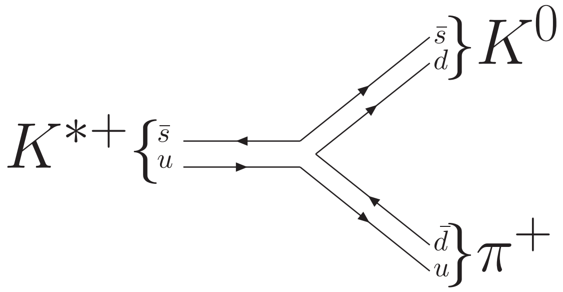
All flavours have been conserved in this reaction. However, $K^+ - n$ scattering will *not* produce a strange baryon because a strange baryon contains s-quarks but no $\bar{s}$ anti-quarks, whereas the K^+ contains a $\bar{s}$ anti-quark, but no s-quark.

Recent evidence has suggested that there is a resonance in $K^+ - n$ scattering at a centre-of-mass energy of 1.5 GeV. This suggests that there is an (unstable) particle with mass $1.5\,\mathrm{GeV/c^2}$. If confirmed this would be a new type of particle called a "pentaquark" since it must be a bound state of four quarks and an anti-quark ($\bar{s}uudd$). Such a particle does *not* fit in with the usual quark model picture of hadrons.

Similarly, in the decays of particles containing s-quarks (or $\bar{s}$ anti-quarks), the decay can proceed via the strong interactions and will be very rapid - leading to a large width - only if the decay products have a total strangeness which is equal to the strangeness of the decaying particle. For such a process to occur, the mass of the decaying particle must be larger than the combined mass of the decay products.

An example is

$$\begin{array}{ccccc} K^{*+} & \rightarrow & K^0 & + & \pi^+ \\ (u\bar{s}) & & (d\bar{s}) & & (u\bar{d}) \end{array}$$



A d-quark and $\bar{d}$ anti-quark pair have been created but the initial and final states both contain an $\bar{s}$ anti-quark - so flavour is conserved.

$$m_{K^*} = 842, \quad m_{K^*} = 495, \quad m_\pi = 135 \ (\mathrm{MeV/c^2})$$

so there is enough energy in the decaying K^* to produce a kaon (K) and a pion, since the mass of the K^* exceeds the sum of the masses of the kaon and pion. This decay therefore can proceed via the strong interactions which means that the K^* has a very short lifetime. It is only seen as a resonance in the centre-of-mass frame of the $K - \pi$ system width a width of 50 MeV.

Likewise the Ξ^* has enough mass to decay into a Ξ plus a pion - the inital and final states both having strangeness -2, and similarly the Σ^* can decay into a Σ plus a pion, or into a Λ plus a pion - conserving strangeness. These decays are therefore very rapid as they proceed though strong interactions.

Most of the lighter strange particles do not have enough energy to decay into other strange particles. They therefore decay through the weak interactions - and therefore have a much longer lifetime. The usually can leave a track in a detector.

Combining associated production and decay one can have an event such as

$$\pi^+ + n \ \rightarrow \ K^{*+} \ + \ \Lambda \ \rightarrow \ K^0 + \pi^+ + \Lambda.$$

The observed particles are the K^0, π^+, Λ but if we look at the energies and momenta of the K^0 and π^+ and construct the quantity

$$P^2_{K\pi} \ = \left(E_{K^0} + E_{\pi^+}\right)^2 / c^2 - \left(\mathbf{p}_{K^0} + \mathbf{p}_{\pi^+}\right)^2,$$

we would get a resonance peak at

$$P_{K\pi} \ = \ 842\,\mathrm{MeV/c},$$

indicating that at such momenta a K^* particle is produced for a very short time.

16.5 Heavy Flavours

When the quark model was invented only $u-$, $d-$ and s-quarks were postulated and all known hadrons could be built out of these three quarks and their anti-quarks. Since then three new quarks, c, b and t gave been discovered. They are much more massive than the $u-$, $d-$ and s-quarks, so they were not discovered until sufficiently large accelerators had been built and were in use. In the same way that there are hadrons containing one or more $s-$quarks (or $\bar{s}$ anti-quarks), there are hadrons which contain these heavy quarks. So far, only hadrons containing one c-quark or one b-quark (or their antiparticles) have been observed. It is believed that a hadron which contained a t-quark would be too unstable to form a bound state.

There are also bound states of $c\,\bar{c}$ and $b\bar{b}$. These are neutral - like the ϕ meson which is a bound state of $s\,\bar{s}$. These heavy quarks were first observed by observing these neutral bound states.

16.6 Quark Colour

There is a difficulty within the quark model when applied to baryons. This can be seen if we look at the Δ^{++} or Δ^{-} or Ω^{-}, which are bound states of three quarks of the same flavour. For these low-mass states the orbital angular momentum is zero and so the spatial parts of the wavefunctions for these baryons is symmetric under interchange of the position of two of these (identical flavour) quarks.

We know that the total wavefunction for a baryon must be antisymmetric as baryons have half-odd-integer spin, so the spin part of the wave function should be antisymmetric. On the other hand these baryons have spin-$\frac{3}{2}$ which means that the spin part of the wavefunction is symmetric (for example the $S_z = +\frac{3}{2}$ state is the state in which all three quarks have $s_z = +\frac{1}{2}$ and this is clearly symmetric under the interchange of two spins).

This is solved by assuming that quarks come in three possible "colour" states - R, G or B. The antisymmetry of the baryon wavefunction is restored by the assumption that the baryon wavefunction is antisymmetric under the interchange of two colours. If a baryon is composed of three quarks with flavours f_1, f_2 and f_3 the these should also have a colour index, e.g. f_1^R, f_1^G or f_1^B etc. The colour antisymmetric wavefunction is written

$$\frac{1}{\sqrt{6}}\left(|f_1^R f_2^G f_3^B\rangle + |f_1^B f_2^R f_3^G\rangle + |f_1^G f_2^B f_3^R\rangle - |f_1^B f_2^G f_3^R\rangle - |f_1^R f_2^B f_3^G\rangle - |f_1^G f_2^R f_3^B\rangle\right)$$

We can see that this changes sign if we interchange any two colours. This means that in order to have a totally antisymmetric wavefunction (including the colour part), the spin and spatial part must be *symmetric* so that a particle in which all three quarks have the same flavour (and zero orbital angular momentum) must be symmetric under the interchange of any two of the spins - and this is the spin-$\frac{3}{2}$ state.

A state of three different colours which is antisymmetric under the interchange of any two of the colours is called a "colour singlet" state - we can think of it as a colourless state.

The quarks themselves are a colour triplet - meaning that they can be in any one of three colour states.

It is assumed that all physically observable particles (i.e all hadrons) are colour singlets (colourless particles). This means that it is not possible to isolate individual quarks and observe them. indeed no individual quark has ever been observed. This is called “quark confinement” and it is the explanation of why the strong interactions are short-range, despite the fact that the gluons, which are the strong-interaction carriers, are massless - you can’t pull a quark too far away from the other quarks or antiquarks in the hadron to which it is bound.

For mesons we also require that the quarks and anti-quarks bind in such a way that the meson is a colour singlet. in the case of a quark and ant-quark bound state this means that the wavefunction is a superposition of R with $\bar{R}$, G with $\bar{G}$, and B with $\bar{B}$. Thus, for example, the wavefunction for the π^+ is written

$$|\pi^+\rangle \quad = \quad \frac{1}{\sqrt{3}}\left(|u^R\overline{d^R}\rangle + |u^G\overline{d^G}\rangle + |u^B\overline{d^B}\rangle\right)$$

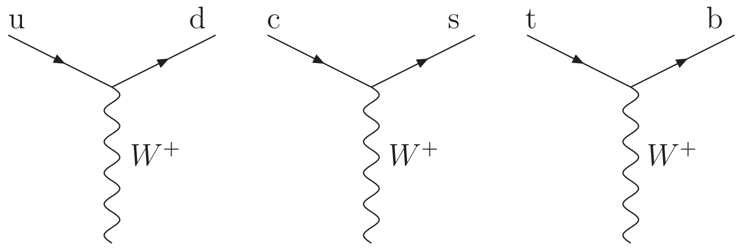
The colourless property is achieved by requiring that a quark of a given colour binds with an antiquark which is the antiparticle of a quark of the same colour - then we have to ‘average’ over all the colours by taking a superposition of all three possible colour pairs.

Chapter 17

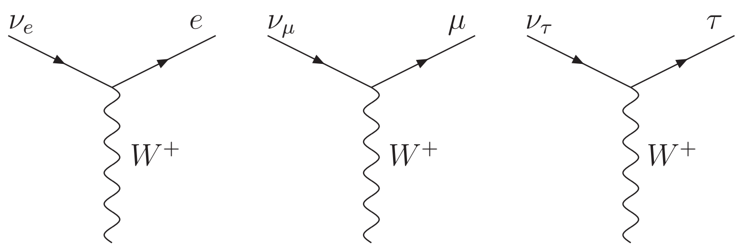
Weak Interactions

The weak interactions are mediated by $W^{\pm}$ or (neutral) Z exchange. In the case of $W^{\pm}$, this means that the flavours of the quarks interacting with the gauge boson can change.

$W^{\pm}$ couples to quark pairs (u, d). (c, s), (t, b) with vertices

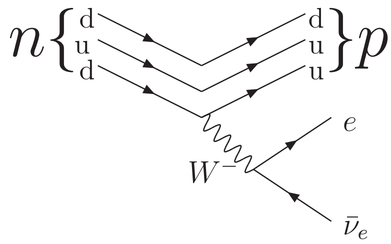


as well as to leptons (ν_e, e). (ν_μ, μ), (ν_τ, τ) with vertices



Note that in these interactions both quark number (baryon number) and lepton number are conserved.

It is this process that is responsible for β-decay. Neutron decays into a proton because a d-quark in the neutron converts into a u-quark emitting a W^- which then decays into an electron and anti-neutrino.



The amplitude for such a decay is proportional to

$$\frac{g_W^2}{(q^2 - M_W^2 c^2)},$$

where g_W is the strength of the coupling of the W^- to the quarks or leptons and $q^2 = E_q^2/c^2 - |\mathbf{q}|^2$, where $\mathbf{q}$ is the momentum transferred between the neutron and proton and E_q is the energy transferred. This momentum is of order 1 MeV/c and so we can neglect it in comparison with $M_W c$ which is 80.4 GeV/c. Thus the amplitude is proportional to

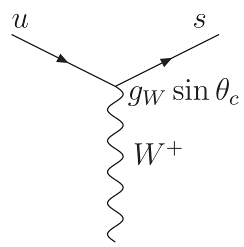
$$\frac{g_W^2}{M_W^2 c^2}.$$

The coupling g_W is not so small. In fact it is twice as large as the electron charge e. Weak interactions are weak because of the large mass term in the denominator.

At modern high energy accelerators, it is possible to produce weak interaction processes in which $|\mathbf{q}| \sim M_W c$ or even $|\mathbf{q}| \gg M_W c$. In such cases weak interactions are larger than electromagnetic interactions and almost comparable with strong interactions.

17.1 Cabibbo Theory

Particles containing strange quarks, e.g. $K^\pm$, K^0, Λ etc. cannot decay into non-strange hadrons via the strong interactions, which have to conserve flavour, but they can decay via the weak interactions. This is possible because $W^\pm$ not only couples a u-quark to a d-quark but can also (with a weaker coupling) couple a u-quark to an s-quark so we have a vertex



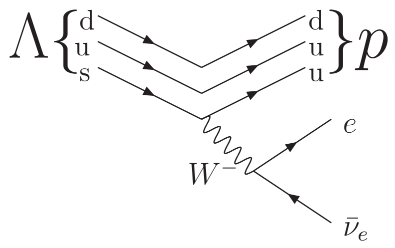
with coupling $g_W \sin\theta_C$, whereas the $u - d - W$ coupling is actually $g_w \cos\theta_C$. θ_C is called the "Cabibbo angle" and its numerical value is $\sin\theta_C \approx 0.22$.

This coupling allows a strange hadron to decay into non-strange hadrons and (sometimes) leptons.

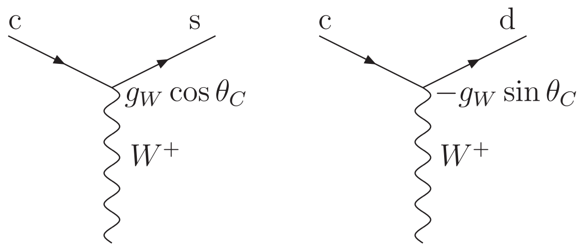
Thus, for example the decay

$$\Lambda \to p + e^- + \bar{\nu}_e$$

occurs when an s-quark converts into a u quark and emits a W^- which then decays into an electron and anti-neutrino. The Feynman graph is



Likewise, the c-quark has a coupling to the s-quark with coupling $g_W \cos\theta_C$ and a coupling to a d-quark with coupling $-g_W \sin\theta_C$.



This implies that charm hadrons are more likely to decay into hadrons with strangeness, because the coupling between a c-quark and a s-quark is larger than between a c-quark and a d-quark.

We can piece this together in a matrix form as follows

$$g_W \begin{pmatrix} d & s \end{pmatrix} \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}$$

This 2×2 matrix is called the "Cabbibo matrix". It is described in terms of a single parameter, the Cabibbo angle.

Since we know that there are, in fact, three generations of quarks this matrix is extended to a general 3×3 matrix as follows

$$g_W \begin{pmatrix} d & s & b \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

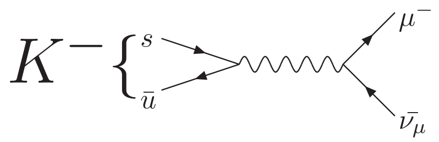
The 3×3 matrix is called the "CKM" (Cabibbo, Kobayashi, Maskawa) matrix. Quantum-mechanical constraints lead to the conclusion that of the nine elements there are only four independent parameters. Comparing the CKM matrix with the Cabibbo matrix we see that to a very good approximation, $V_{ud} \approx V_{cs} \approx \cos\theta_C$ and $V_{us} \approx -V_{cd} \approx \sin\theta_C$.

17.2 Leptonic, Semi-leptonic and Non-Leptonic Weak Decays

Because the $W^{\pm}$ couples either to quarks or to leptons, decays of strange mesons can either be leptonic, meaning that the final state consists only of leptons, semi-leptonic, meaning that the final state consists of both hadrons and leptons, or non-leptonic, meaning that the final state consists only of hadrons. For strange baryons only semi-leptonic and non-leptonic decays are possible because baryon number is strictly conserved - so there must be a baryon in the final state. Lepton number is also strictly conserved which means that a charged lepton is always accompanied by its anti-neutrino (or vice versa) in the final state.

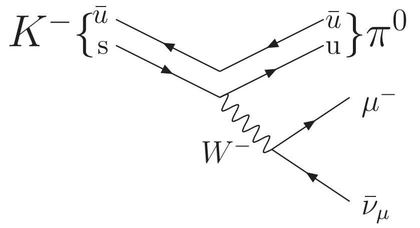
For mesons, examples are:

Leptonic decay $K^- \rightarrow \mu^- + \bar{\nu}_\mu$

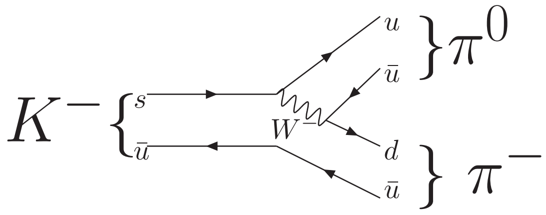


As well as converting an s-quark into a u-quark to emit a W^-, it is also possible to create a W^- from the annihilation of an s-quark with a $\bar{u}$ anti-quark.

Semi-leptonic decay $K^- \rightarrow \mu^- + \bar{\nu}_\mu + \pi^0$



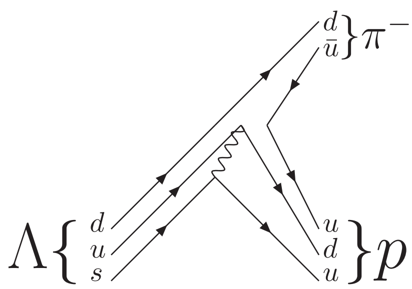
Non-leptonic decay $K^- \rightarrow \pi^0 + \pi^-$



Note that $m_K > 2m_\pi$ which is why this non-leptonic decay mode is energetically allowed.

In the case of baryons, we have already seen an example of a semi-leptonic decay, $\Lambda \to p\,e^-\ \bar{\nu}_e$. An example of a non-leptonic decay is

$$\Lambda \ \to \ p\,\pi^-$$



A W^- is exchanged between the s-quark and the u-quark in the Λ, converting them into a u-quark and a d-quark respectively. A $u - \bar{u}$ quark-antiquark pair is created in the process in order to make up the final state hadrons of a proton and a negative pion.

17.3 Flavour Selection Rules in Weak Interactions

Since in the exchange of a single $W^\pm$ an s-quark can be converted into a non-strange quark, it is highly unlikely that two strange quarks would be converted into non-strange quarks in the same decay process. We therefore have a selection rule for weak decay processes

$$\Delta S \ = \ \pm 1$$

Therefore, hadrons with strangeness -2 which decay weakly must first decay into a hadron with strangeness -1 (which in turn decays into non-strange hadrons). Thus, for example, we have

$$\Xi^0 \ \to \ \Lambda + \pi^0$$

The same selection rules apply for changes in other flavours (charm, bottom).

17.4 Parity Violation

The parity violation observed in β-decay arises because the $W^{\pm}$ tends to couple to quarks or leptons, which are left-handed (negative helicity), i.e. states in which the component of spin in their direction of motion is $-\frac{1}{2}\hbar$.

$W^{\pm}$ always couple to left-handed neutrinos. For quarks and massive leptons the $W^{\pm}$ *can* couple to positive helicity (right-handed) states, but the coupling is suppressed by a factor

$$\frac{mc^2}{E},$$

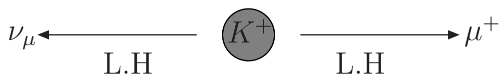
where m is the particle mass and E is its energy. The suppression is much larger for relativistically moving particles

In the case of nuclear β-decay, the nucleus is moving non-relativistically, but the electron typically has energy of a few MeV (and a mass of 0.511 MeV/c^2), so there is a significant suppression of the coupling to right-handed electrons. This is what was observed in the experiment by C.S. Wu on ^{60}Co.

For the coupling of $W^{\pm}$ to anti-quarks or anti-leptons, the helicity is reversed -i.e. the $W^{\pm}$ always couples to positive helicity anti-neutrinos and usually to positive helicity e^+, μ^+, τ^+ or to antiquarks, with a suppressed coupling to left-handed antileptons or anti-quarks.

A striking example of the consequence of this preferred helicity coupling can be seen in the leptonic decay of K^+.

$$K^+ \rightarrow \mu^+ + \nu_\mu$$



In the rest frame of the K^+ the momentum is zero, so the μ^+ and the ν_μ must move in opposite directions. The K^+ has zero spin, so by conservation of angular momentum, the two decay particles must have opposite spin component in any one chosen direction (e.g. the direction of the μ^+. This means that they have the *same* helicity. This means that the $W^{\pm}$ couples to the left-helicity anti-muon, μ^+ and such a coupling is suppressed by

$$\frac{m_\mu c^2}{E_\mu}$$

If we look at the decay mode

$$K^+ \rightarrow e^+ + \nu_e,$$

the same argument would lead to a suppression (of the decay amplitude) of

$$\frac{m_e c^2}{E_e}.$$

Since $m_e \ll m_\mu$ we expect the decay into a positron to be heavily suppresses. In fact we expect the ratio of the partial widths

$$\frac{\Gamma(K^+ \to \mu^+ \nu_\mu)}{\Gamma(K^+ \to e^+ \nu_e)} \;=\; \frac{m_\mu^2}{m_e^2} \;\approx\; 4 \times 10^4$$

This coincides very closely to the experimentally observed ratio.

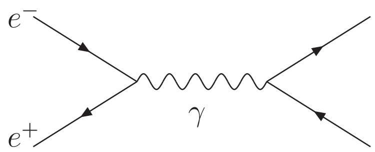
17.5 Z-boson interactions

As well as exchange of $W^\pm$ in which flavour is changed, the weak interactions are also mediated by a neutral gauge-boson, Z. This couples to both quarks and leptons but does not change flavour.

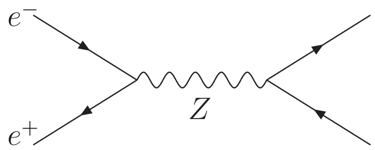
In that sense the interactions of the Z are similar to that of the photon, but there are some important differences.

- The Z couples to neutrinos whereas the photon does not (neutrinos have zero electric charge).
- The Z has a mass of 91.1 Gev/c^2, so the interactions are short range - like the interactions of the $W^\pm$.
- The Z also has a coupling of different strength to left-handed (negative helicity) and right-handed (positive helicity) quarks and leptons and so these interactions also violate parity.

Nevertheless, in any process where there can be photon exchange, there can also be Z exchange. In terms of Feynman diagrams for $e^+ e^-$ scattering into any pair of final state particles, we have



but also



The first diagram (photon exchange) has a propagator

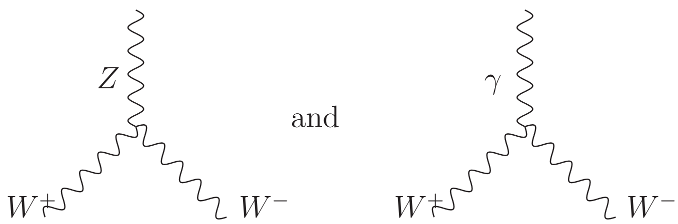
$$1/s,$$

where $\sqrt{s}$ is the centre-of-mass energy, whereas the second diagram (Z exchange) has a propagator

$$\frac{1}{s - M_Z^2 c^4}.$$

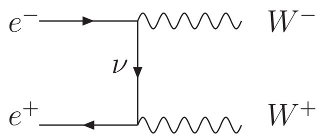
For relatively low centre-of-mass energies for which $\sqrt{s} \ll M_Z c^2$, the second diagram may be neglected and the second diagram gives a negligible contribution. But as $\sqrt{s}$ grows to become comparable (or greater than) $M_Z c^2$ both of these diagrams are equally important.

The Z and photon can both couple to $W^{\pm}$, so we get interaction vertices

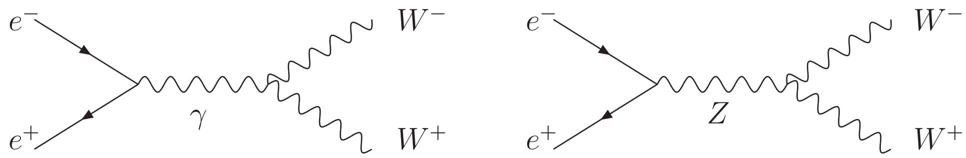


The interaction between the photon and $W^{\pm}$ is not surprising since the $W^{\pm}$ are charged and we would expect them to interact with photons, with coupling e. The interaction of $W^{\pm}$ with the Z is similar but has a different coupling.

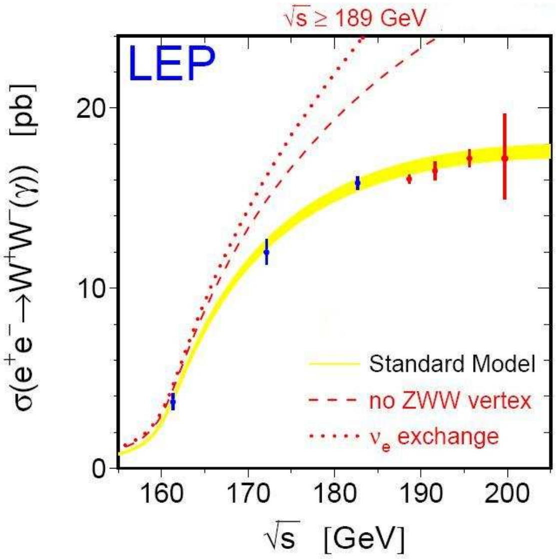
The coupling of the Z and photon to the $W^{\pm}$ was confirmed at the LEPII experiment at CERN where it was possible to accelerate electrons and positrons to sufficient energies to produce a W^+ and a W^- in the final state. From the coupling of the W to electron and neutrino the Feynman diagram for this process is



but because of the coupling of the Z and photon to $W^{\pm}$ we also have diagrams



The data from LEPII clearly show that these graphs have to be taken into account



It turns out that the Standard Model of weak and electromagnetic ("electroweak") interactions, developed in the 1960's by Glashow, Weinberg, and Salam, gives a relation between the weak coupling g_W, the (magnitude of the) electron charge, e and the masses of the Z and $W^{\pm}$

$$\frac{M_W}{M_Z} \;=\; \cos\theta_W$$

where θ_W is known as the weak mixing angle.

$$e \;=\; g_W \sin\theta_W \;=\; g_W \sqrt{1-\frac{M_W^2}{M_Z^2}}$$

This enables us to make an order of magnitude estimate of the rates for weak processes at low energies.

At energies $\ll M_W c^2$, the amplitude for a $W^{\pm}$ exchange process is proportional to

$$\frac{g_W^2}{4\pi\epsilon_0 M_W^2 c^4},$$

so that the rate is proportional to

$$\left(\frac{g_W^2}{4\pi\epsilon_0 M_W^2 c^4}\right)^2.$$

Now for a weak decay rate we want dimensions of inverse time, so we need to multiply this by something with dimensions of the fourth power of energy divided by time. The only quantity proportional to the energy is the Q value of the decay, Q_β and to get inverse time we can divide by $\hbar$ so we get an estimate

$$\text{Rate} \;\sim\; \left(\frac{g_W^2}{4\pi\epsilon_0 \hbar c M_W^2 c^4}\right)^2 . \frac{Q_\beta^5}{\hbar}$$

The pre-factor is actually quite small. For example, for muon decay $Q_\beta \approx m_\mu c^2$, and the muon decay rate is actually

$$\frac{1}{\tau_\mu} = \frac{1}{768\pi^3}\left(\frac{g_W^2}{4\pi\epsilon_0\hbar c}\right)^2 \frac{m_\mu^4}{M_W^4}\frac{m_\mu c^2}{\hbar}.$$

We know

$$\frac{g_W^2}{4\pi\epsilon_0\hbar c} = \frac{e^2}{4\pi\epsilon_0\hbar c\sin^2\theta_W} = \frac{\alpha}{\sin^2\theta_W}$$

and

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}.$$

Therefore from the measured masses of the W and Z we can determine the muon lifetime.

17.6 The Higgs mechanism

There is one further particle predicted by the Standard Model of electroweak interactions which has not yet been discovered.

This arises from the mechanism, discovered by P.Higgs, by which particles acquire their mass. The basic idea is that there exists a field, ϕ called the "Higgs field" which has a constant non-zero value everywhere in space. This constant value is called the "vacuum expectation value", $\langle\phi\rangle$.

In the absence of this field it is assumed that all particles would be massless and would travel with velocity c. But because of their interaction with the background Higgs field they are slowed down - thereby acquiring a mass, M

$$M = \frac{1}{2}\frac{g_H}{\sqrt{\epsilon_0\hbar c}}\langle\phi\rangle,$$

where g_H is the coupling of the particle to the Higgs field (the denominator factor $\sqrt{\epsilon_0\hbar c}$ gives it the correct dimensions.) This mechanism is part of the Standard Model.

The Higgs field couples to $W^\pm$ with coupling g_W so that

$$M_W = \frac{1}{2}\frac{g_W}{\sqrt{\epsilon_0\hbar c}}\langle\phi\rangle.$$

Inserting $g_W = e/\sin\theta_W$ with $\cos\theta_W = M_W/M_Z$ and $M_W = 80.4\,\mathrm{GeV/c^2}$, and $M_Z = 91.2\,\mathrm{GeV/c^2}$, we get the value of the vacuum expectation value

$$\langle\phi\rangle = 250\,\mathrm{GeV/c^2}$$

Other particles couple to the Higgs field with couplings that are proportional to their mass.

In the same way that there are quanta of the electromagnetic field which are particles (photons), so there must be quanta of the Higgs field. These are called "Higgs particles". They must necessarily exist if the Higgs mechanism for generating masses for particles is to be consistent with quantum physics.

As it was mentioned in the introduction, the Higgs boson was discovered on the 4th of July 2012 by ATLAS and CMS collaborations at the LHC, completing the set of particles of the Standard Model. With a high confidence level this particle is confirmed to have the following properties:

1. It has a spin zero. This is consistent with the theoretical predictions since the vacuum expectation value has to be invariant under Lorentz transformations - so that it is the same in all frames of reference.

2. Higgs boson couples to $W^{\pm}$ and Z (which are consequently massive).

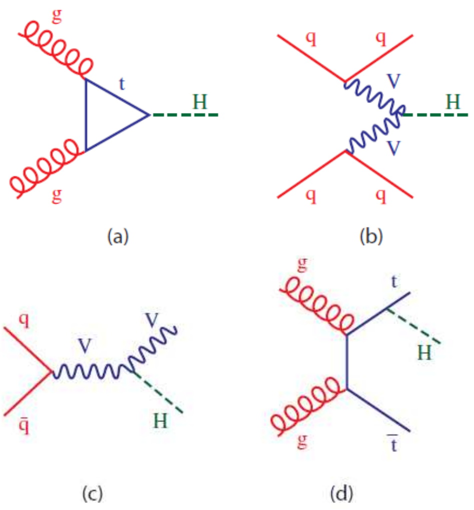
3. It does *not* directly couple to photons (which are massless) so it is uncharged.

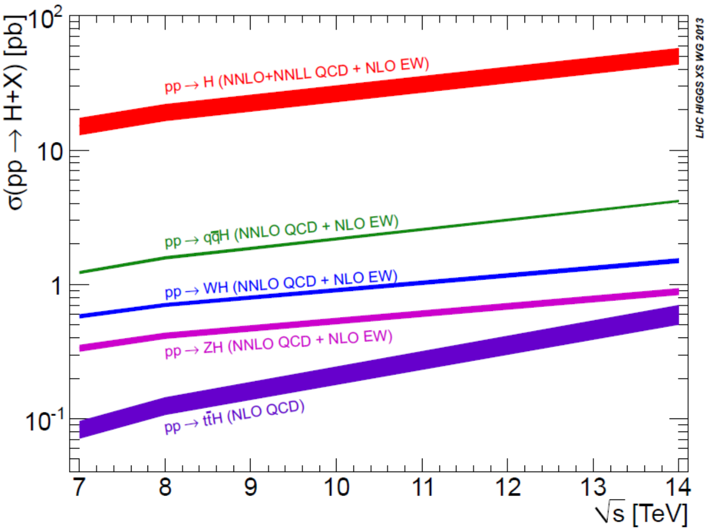
4. it does not not couple directly to gluons (which are massless) and so it does not take part in the strong interactions.

5. Its coupling to massive particles is proportional to the particle mass.

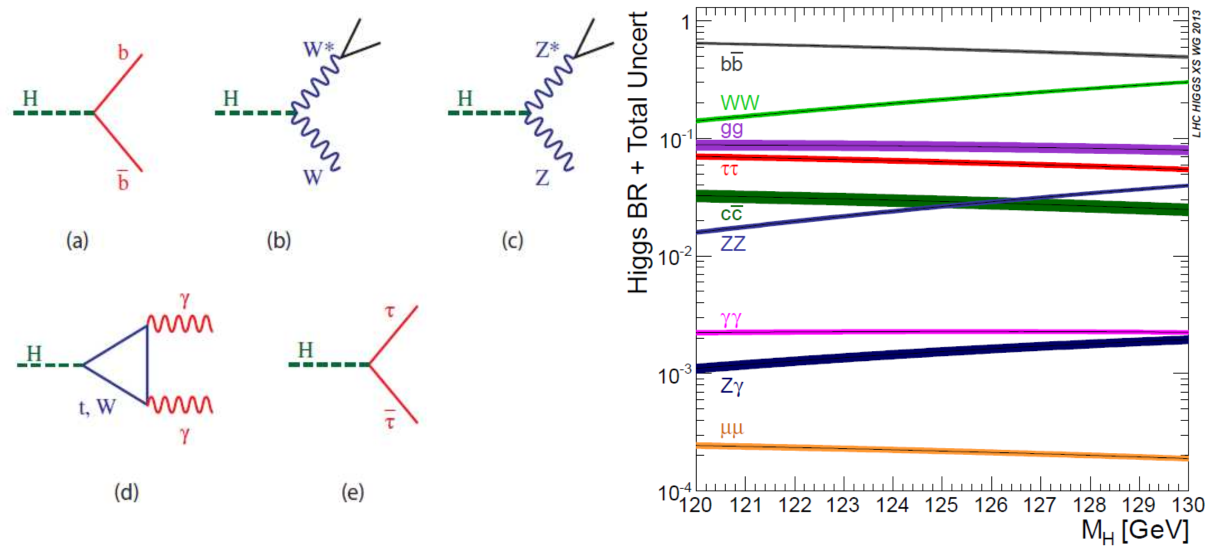
6. Its mass is measured to be about 125 GeV/c^2.

Diagrams for production mechanisms of the Higgs boson at the LHC are shown below (left) together with the respective cross sections (right). They include: (a) gluon fusion, (b) weak-boson fusion, (c) Higgs-strahlung (or associated production with a gauge boson) and (d) associated production with top quarks processes. Note, that the first process is the loop-induce one: while Higgs boson does not interact directly with massless gluons, it actually can interact with gluons via virtual massive quarks (e.g. top-quarks which the strongest coupling to the Higgs boson) in the triangle loop diagram. Actually gluon fusion is the main production process of the Higgs boson, while the weak-boson fusion plays the next to leading role. Theoretical uncertainties are represented by the widths of the cross section bands.





Higgs boson decay is dominated by the most massive particles allowed by its mass because its coupling to particles is proportional to the mass. The t-quark mass is 175 GeV/c^2 so it cannot decay into a $t - \bar{t}$ pair. The next most massive quark is the b-quark so Higgs boson predominantly decays into a $b - \bar{b}$ pair, shown in diagram (a) below. Higgs boson is not sufficiently massive to decay into real W^+W^- or two real Z particles, however it can decay to one real and another virtual W or Z boson (W* and Z*) followed by their decay into fermion-antifermion pair as shown in diagrams (b) and (c). As in case of gluon fusion production process, Higgs boson can also decay into photon pair via its interactions with virtual top quark and W-boson as shown in diagram (d). Higgs boson also decays to $\tau^+\tau^-$ pair, the dominant leptonic decay channel since τ-lepton is the most massive amongst the leptons. The respective branching ratios for Higgs boson decay channels are shown in the right frame of the figure below as a function of the Higgs boson mass.



Higgs boson discovery was based not on the process with the highest production and decay rates, which would be naively the $gg \to H \to b\bar{b}$ process. Actually it was was based on the processes with optimal signal-to-background ratio and the highest signal significance. In particular, one of the most significant and cleanest signatures comes from $H \to \gamma\gamma$ decay for which Standard Model background is relatively low. Another very important and significant signature is based on $H \to ZZ^* \to$ 4leptons decay which also provide clean 4-lepton signature.