

1. Prime Generation (Sieve of Eratosthenes)

We wish to generate the first N_p primes. Choose an upper bound

$$B \geq N_p (\ln N_p + \ln \ln N_p).$$

Initialize a boolean array $\text{isPrime}[0 \dots B]$ with $\text{isPrime}[0] = \text{isPrime}[1] = \text{false}$ and $\text{isPrime}[i] = \text{true}$ for $i \geq 2$. Then:

For $i = 2, \dots, \lfloor \sqrt{B} \rfloor$,

if $\text{isPrime}[i]$ then mark $\text{isPrime}[i^2]$, $\text{isPrime}[i^2 + i]$, $\text{isPrime}[i^2 + 2i], \dots = \text{false}$.

The first N_p remaining true indices are $\{p_1, p_2, \dots, p_{N_p}\}$.

2. Extract Outermost Triplets

Let

$$T = \left\lfloor \frac{N_p}{3} \right\rfloor$$

be the number of non-overlapping triplets. For $k = 0, 1, \dots, T - 1$ define

$$X_k = p_{3k+1}, \quad Y_k = p_{3k+2}, \quad Z_k = p_{3k+3}.$$

3. Gap Sequences

For each string $S \in \{X, Y, Z\}$, form

$$\Delta_k^S = S_{k+1} - S_k, \quad k = 0, \dots, T - 2.$$

4. Normalize Gaps to Instantaneous Frequencies

Set $\Delta_{\min} = \min_k \Delta_k^S$, $\Delta_{\max} = \max_k \Delta_k^S$, and

$$\tilde{\Delta}_k = \frac{\Delta_k^S - \Delta_{\min}}{\Delta_{\max} - \Delta_{\min}} \in [0, 1].$$

Choose bounds f_{\min}, f_{\max} , then

$$f_k = f_{\min} + \tilde{\Delta}_k (f_{\max} - f_{\min}), \quad k = 0, \dots, N - 1.$$

5. Upsample and Interpolate

Pick integer u . Define

$$t_i = \frac{i}{u}, \quad i = 0, 1, \dots, uN - 1,$$

and interpolate $\{(k, f_k)\}$ to obtain $f(t_i)$ by linear interpolation.

6. Phase Accumulation and Waveform

Initialize $\Phi_0 = 0$. Then

$$\Phi_i = \Phi_{i-1} + 2\pi \frac{f(t_i)}{u}, \quad s_i = \sin(\Phi_i), \quad i = 0, \dots, uN - 1.$$

7. Two-Dimensional Interference Fields

Given two waves $\{s_i^X\}$ and $\{s_j^Y\}$, define

$$A_{ij} = s_i^X + s_j^Y, \quad M_{ij} = s_i^X \times s_j^Y, \quad i, j = 0, \dots, uN - 1.$$

Plot A and M as heatmaps over the grid $[0, uN - 1]^2$.

Parameter Summary

- N_p : number of primes
- $T = \lfloor N_p/3 \rfloor$: number of triplets
- $N \leq T - 1$: gaps used
- u : upsampling factor
- f_{\min}, f_{\max} : frequency bounds