Midterm Exam, Spring 2019

Time: 1 hour 15 minutes. Exam is closed book, notes and any other aids including electronic.

Name: **ID:**

Problem 1 20%

Find a regular expression corresponding to each of the following languages over $\Sigma = \{a, b\}$

1. (6%) The language consisting of all strings in which every third symbol is an a if they are longer than 3 symbols.
2. (7%) Given two strings $w, z \in \{a, b\}^*$. The recursive definition of L is

$$\begin{array}{l} \Lambda, a \in L \\ \forall x \in L \quad wx, \text{ and } zx \text{ are in } L \end{array}$$

   You can use w, and z in the regular expression. There is no need to rewrite them as concatenations of their letters.

3. (7%) The language of all strings that do not end with ab.

Answer:

Problem 2 20%

For each statement below, decide whether it is true or false. If it is true, prove it. If it is not true, give a counterexample. If you prove it, you can use any theorem we saw in class. All items refer to languages over the alphabet $\{a, b\}$.

(a) (5%) If L_1 can be accepted by an FA and neither L_2 nor $L_1 \cap L_2$ can, then $L_1 \cup L_2$ cannot.

(b) (5%) If each of the languages $L_1, L_2, \ldots$ can be accepted by an FA, then $\cup_{n=1}^{\infty} L_n$ can.

(c) (10%) If none of the languages $L_1, L_2, \ldots$ can be accepted by an FA, and $L_i \subseteq L_{i+1}$ for each i, then $\cup_{n=1}^{\infty} L_n$ cannot be accepted by an FA.

Answer:

Problem 3 20%

1. (5%) Formulate the Pumping Lemma. (If you cannot formulate it rigorously, write its main ideas.)

2. (15%) Rigorously establish the regularity or nonregularity of the following language over alphabet $\{a\}$

$$L = \{a^n \mid n \text{ is a power of } 2\}$$

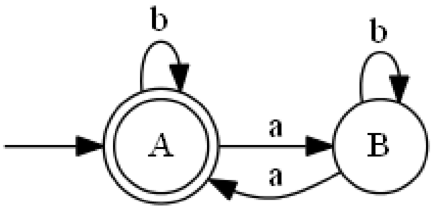
Answer:

Problem 4 15%

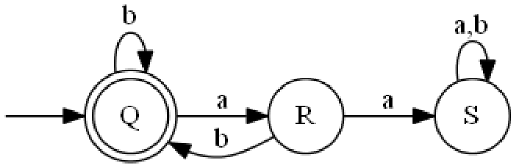
Construct FAs that accept

1. union
2. intersection
3. difference

of languages $L(M_1)$, and $\overline{L(M_2)}$.



M_1

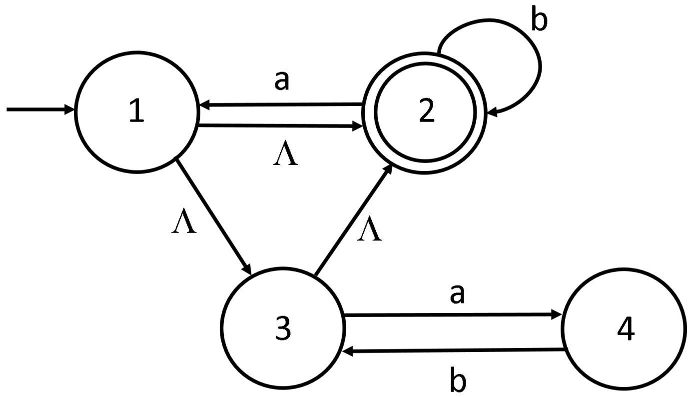


M_2

Answer:

Problem 5 25%

1. (15%) Eliminate nondeterminism in an NFA (with $\Sigma = \{a, b\}$) in two steps
   a) (10%) Eliminate Λ-transitions.
   b) (5%) Convert NFA with no Λ-transitions to FA.



2. (10%) For a string $x \in \Sigma^*$, define $\mathsf{sort}(x)$ to be the string obtained by rearranging the symbols of x such that all a's appear before b's, i.e., we sort the symbols of x in the lexicographic order. For a language L we define a language $\mathsf{sort}(L) = \{\mathsf{sort}(x) \mid x \in L\}$. Prove or disprove the following proposition "If L is regular then $\mathsf{sort}(L)$ is also regular".

Answer:

Answer:

Solution to Problem 1

1. $((a+b)(a+b)a)^*(\Lambda+a+b+(a+b)(a+b))$
2. $(w+z)^*(a+\Lambda)$
3. $\Lambda+b+(a+b)^*a+(a+b)^*bb$

Solution to Problem 2

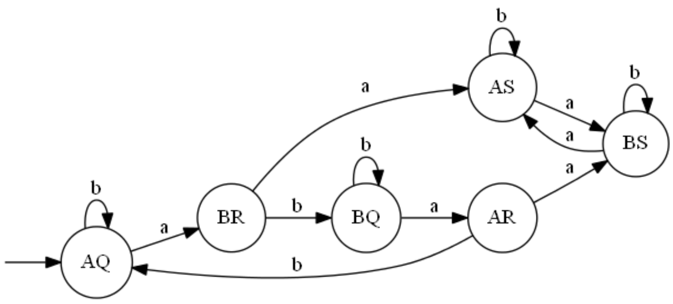
1. False. Example: $L_1 = \Sigma^*$, and $L_2 = \text{PAL}$
2. False. Every language is a one-element palindrome language. Their union is not regular.
3. False. Each L_i is PAL$\cup\{i$ first lexicographically ordered words from $\Sigma^*\}$

Solution to Problem 3

1. See lecture notes for the formulation.
2. Expecting a contradiction, assume L is regular and let $n \geq 1$ be the PL integer. Define $s = a^{2^n}$ and let $s = uvw$, where $|uv| \leq n$, and $|v| \geq 1$. Then, $uv^2w = a^{2^n+k}$ for some k s.t. $|v| = k$. Next string in L that is longer than s is $a^{2^{n+1}} = a^{2^n+2^n}$.

   Since $|v| \geq 1$ but $k \leq 2^n$, string $uv^2w \notin L$, contradicting the PL, i.e., L is not regular.

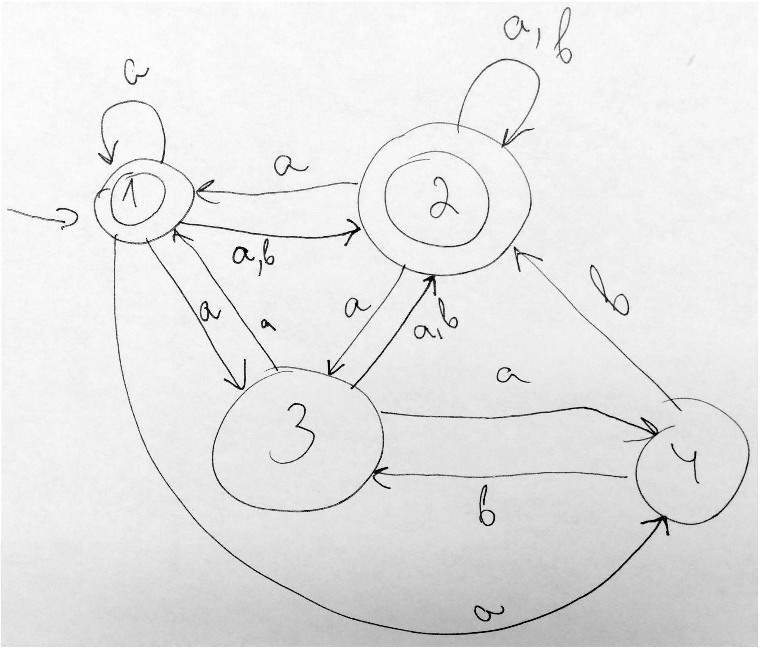
Solution to Problem 4



Accepting states:

1. Union: AQ,AR, AS, BR, BS
2. Intersection: AR, AS
3. Difference: AQ

Solution to Problem 5

1. 

2. $(a+b)^*$

3. False. $L = (ab)^*$ is regular, and $sort(L) = \{a^n b^n\}$ is not.