CLEMSON UNIVERSITY, SCHOOL OF COMPUTING
CPSC 3500 FOUNDATIONS OF COMPUTER SCIENCE

Assignment 5: Regular Expressions

Return by 10/10/2019

Problem 1 10%

In each case below, find a string of minimum length in $\{a, b\}^*$ not in the language corresponding to the given regular expression.

(a) $b^*(ab)^*a^*$

(b) $(a^* + b^*)(a^* + b^*)(a^* + b^*)$

(c) $a^*(baa^*)^*b^*$

(d) $b^*(a + ba)^*b^*$

Problem 2 10%

Consider the two regular expressions

$$r = a^* + b^* \text{ and } s = ab^* + ba^* + b^*a + (a^*b)^*$$

(a) Find a string corresponding to r but not to s.

(b) Find a string corresponding to s but not to r.

(c) Find a string corresponding to both r and s.

(d) Find a string in $\{a, b\}^*$ corresponding to neither r nor s.

Problem 3 10%

Let r and s be arbitrary regular expressions over the alphabet Σ. In each case below, find a simpler equivalent regular expression.

(a) $r(r^*r + r^*) + r^*$

(b) $(r + \Lambda)^*$

(c) $(r + s)^*rs(r + s)^* + s^*r^*$

Problem 4 10%

It is not difficult to show using mathematical induction that for every integer $n \geq 2$, there are non-negative integers i and j such that $n = 2i + 3j$. With this in mind, simplify the regular expression $(aa + aaa)(aa + aaa)^*$.

Problem 5 10%

Suppose w and z are strings in $\{a, b\}^*$. Find regular expressions corresponding to each of the languages defined recursively below.

(a) $\Lambda \in L$; $\forall x \in L$, wx and xz are elements of L.

(b) $a \in L$; $\forall x \in L$; wx, xw, and xz are elements of L.

(c) $\Lambda \in L$; $a \in L$; $\forall x \in L$, wx and zx are in L.

Problem 6 30%

Find a regular expression corresponding to each of the following subsets of $\{a, b\}^*$.

(a) The language of all strings containing exactly two a's.

(b) The language of all strings containing at least two a's.

(c) The language of all strings that do not end with ab.

(d) The language of all strings that begin or end with aa or bb.

(e) The language of all strings not containing the substring aa.

(f) The language of all strings in which the number of a's is even.

(g) The language of all strings containing no more than one occurrence of the string aa. (The string aaa should be viewed as containing two occurrences of aa.)

(h) The language of all strings in which every a is followed immediately by bb.

(i) The language of all strings containing both bb and aba as substrings.

(j) The language of all strings not containing the substring aaa.

Problem 7 10%

(a) The regular expression $(b+ab)^*(a+ab)^*$ describes the set of all strings in $\{a,b\}^*$ not containing the substring ___x___ for any x. (Fill in the blanks appropriately.)

(b) The regular expression $(a+b)^*(aa^*bb^*aa^* + bb^*aa^*bb^*)(a+b)^*$ describes the set of all strings in $\{a,b\}^*$ containing both the substrings ___ and ___. (Fill in the blanks appropriately.)

Problem 8 10%

Prove that every finite language is regular. Hint: use induction.

Problem 9 10%

Given an FA $M = (Q, \Sigma, q_0, A, \delta)$ that accepts language $L(M)$. Describe two algorithms for construction of (not necessarily optimal) FA's that accept $\overline{L(M)}$.

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Assignment 6: NFA

Problem 1 5%

(a) If L is the language corresponding to the regular expression $(aab + bbaba)^*\, baba$, find a regular expression corresponding to the language of the L-reverse strings

$$r(L) = \{r(x) | x \in L\}.$$

(b) Give a recursive definition of the reverse $r(e)$ of a regular expression e.

Problem 2 5%

The star height of a regular expression r over Σ, denoted by $sh(r)$, is defined as follows:

- $sh(\emptyset) = 0$.
- $sh(\Lambda) = 0$.
- $sh(\sigma) = 0\ \forall \sigma \in \Sigma$.
- $sh((rs)) = sh((r + s)) = max(sh(r), sh(s))$.
- $sh((r^*)) = sh(r) + 1$.

Find the star heights of the following regular expressions

$$(a(a + a^*aa) + aaa)^*, \text{ and } (((a + a^*aa)aa)^* + aaaaaa^*)^*.$$

Problem 3 5%

For both the regular expressions in the previous exercise, find an equivalent regular expression of star height 1.

Problem 4 10%

Describe an algorithm that could be used to eliminate the symbol Λ from any regular expression whose corresponding language does not contain the null string. The input of the algorithm is a string with regular expression r. The output is a string with regular expression s, such that $L(r) = L(s)$.

Problem 5 10%

Prove that if $\Lambda \in L^k$, and $k \in \mathbb{N}$, then $L^k \subseteq L^{k+1}$.

Problem 6 30%

The order of a regular language L is the smallest integer k for which $L^k = L^{k+1}$, if there is one, and ∞ otherwise.

(a) Show that if L is a nonempty regular language, the order of L is finite if and only if there is an integer k such that $L^k = L^*$, and that in this case the order of L is the smallest k such that $L^k = L^*$. Hint: does L include Λ?

(b) What is the order of the regular language $\{\Lambda\} \cup \{aa\}\{aaa\}^*$?

(c) What is the order of the regular language $\{a\} \cup \{aa\}\{aaa\}^*$?

(d) What is the order of the language corresponding to the regular expression

$$(\Lambda + b^* a)(b + ab^* ab^* a)^*$$

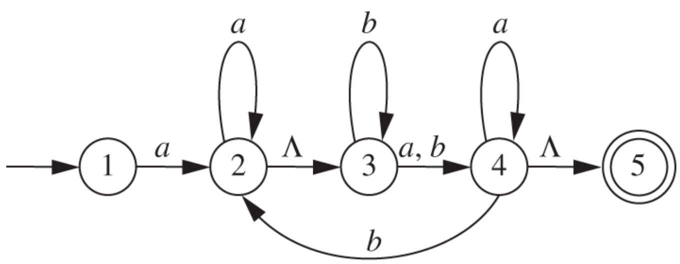
Problem 7 5%

For each string below, say whether the NFA in the diagram accepts it.

(a) aba

(b) abab

(c) aaabbb

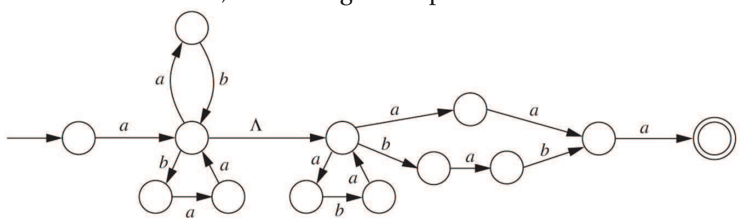


Problem 8 10%

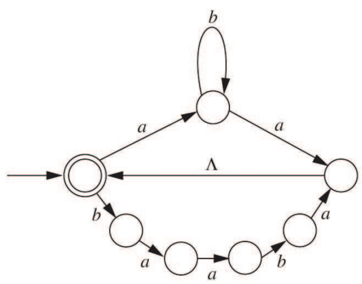
Find a regular expression corresponding to the language accepted by the NFA in problem above. You should be able to do it without applying Kleene's theorem: First find a regular expression describing the most general way of reaching state 4 the first time, and then find a regular expression describing the most general way, starting in state 4, of moving to state 4 the next time.

Problem 9 20%

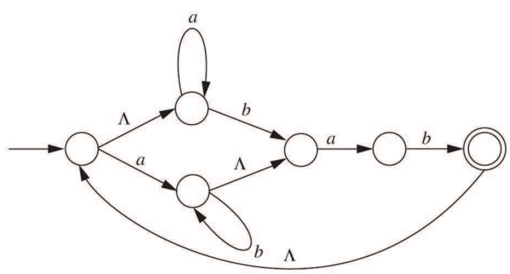
For each of the NFA below, find its regular expression.



(a)



(b)



(c)

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Assignment 7: Nondeterminism, and Kleene's Theorem

Problem 1 5%

Given the transition table for an NFA with states 1-5 and input alphabet $\{a, b\}$. There are no Λ-transitions.

(a) Draw a transition diagram.

(b) Calculate $\delta^*(1, ab)$.

(c) Calculate $\delta^*(1, abaab)$.

q	$\delta(q, a)$	$\delta(q, b)$
1	$\{1, 2\}$	$\{1\}$
2	$\{3\}$	$\{3\}$
3	$\{4\}$	$\{4\}$
4	$\{5\}$	$\emptyset$
5	$\emptyset$	$\{5\}$

Problem 2 10%

A transition table is given for an NFA with seven states. Find

(a) $\Lambda(\{2, 3\})$

(b) $\Lambda(\{1\})$

(c) $\Lambda(\{3, 4\})$

(d) $\delta^*(1, ba)$

(e) $\delta^*(1, ab)$

(f) $\delta^*(1, ababa)$

q	$\delta(q, a)$	$\delta(q, b)$	$\delta(q, \Lambda)$
1	$\emptyset$	$\emptyset$	$\{2\}$
2	$\{3\}$	$\emptyset$	$\{5\}$
3	$\emptyset$	$\{4\}$	$\emptyset$
4	$\{4\}$	$\emptyset$	$\{1\}$
5	$\emptyset$	$\{6, 7\}$	$\emptyset$
6	$\{5\}$	$\emptyset$	$\emptyset$
7	$\emptyset$	$\emptyset$	$\{1\}$

Problem 3 5%

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an NFA with no Λ-transitions. Show that for every $q \in Q$ and every $\sigma \in \Sigma$, $\delta^*(q, \sigma) = \delta(q, \sigma)$.

Problem 4 10%

It is easy to see that if $M = (Q, \Sigma, q_0, A, \delta)$ is an FA accepting L, then the FA $\overline{M} = (Q, \Sigma, q_0, Q - A, \delta)$ accepts $\overline{L}$ (the FA obtained from $\overline{L} = \Sigma^* - L$). Does this still work if M is an NFA? If so, prove it. If not, find a counterexample.

Problem 5 20%

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an NFA. This exercise involves properties of the Λ-closure of a set S. Since $\Lambda(S)$ is defined recursively, structural induction can be used to show that $\Lambda(S)$ is a subset of some other set.

(a) Show that if S and T are subsets of Q for which $S \subseteq T$, then $\Lambda(S) \subseteq \Lambda(T)$.

(b) Show that for any $S \subseteq Q$, $\Lambda(\Lambda(S)) = \Lambda(S)$.

(c) Show that if $S, T \subseteq Q$, then $\Lambda(S \cup T) = \Lambda(S) \cup \Lambda(T)$.

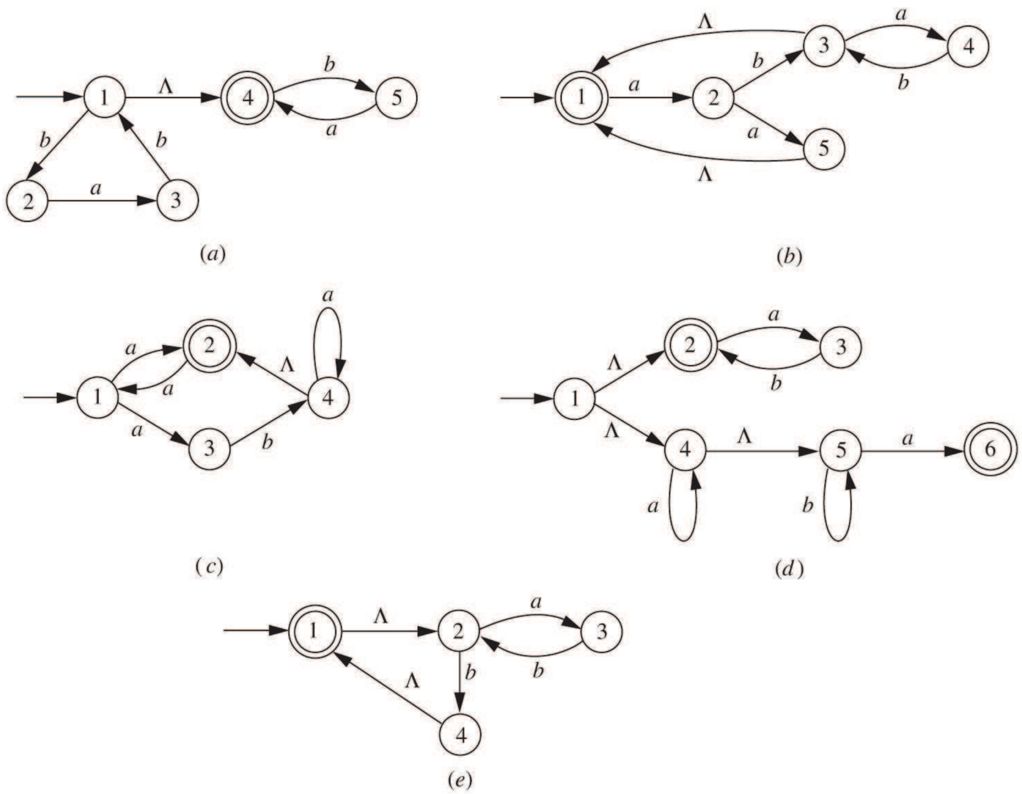
(d) Show that if $S \subseteq Q$, then $\Lambda(S) = \cup\{\Lambda(\{p\}) | p \in S\}$.

(e) Draw a transition diagram to illustrate the fact that $\Lambda(S \cap T)$ and $\Lambda(S) \cap \Lambda(T)$ are not always the same. Which is always a subset of the other?

(f) Draw a transition diagram illustrating the fact that $\Lambda(S')$ and $\Lambda(S)'$ are not always the same. Which is always a subset of the other? Under what circumstances are they equal?

Problem 6 20%

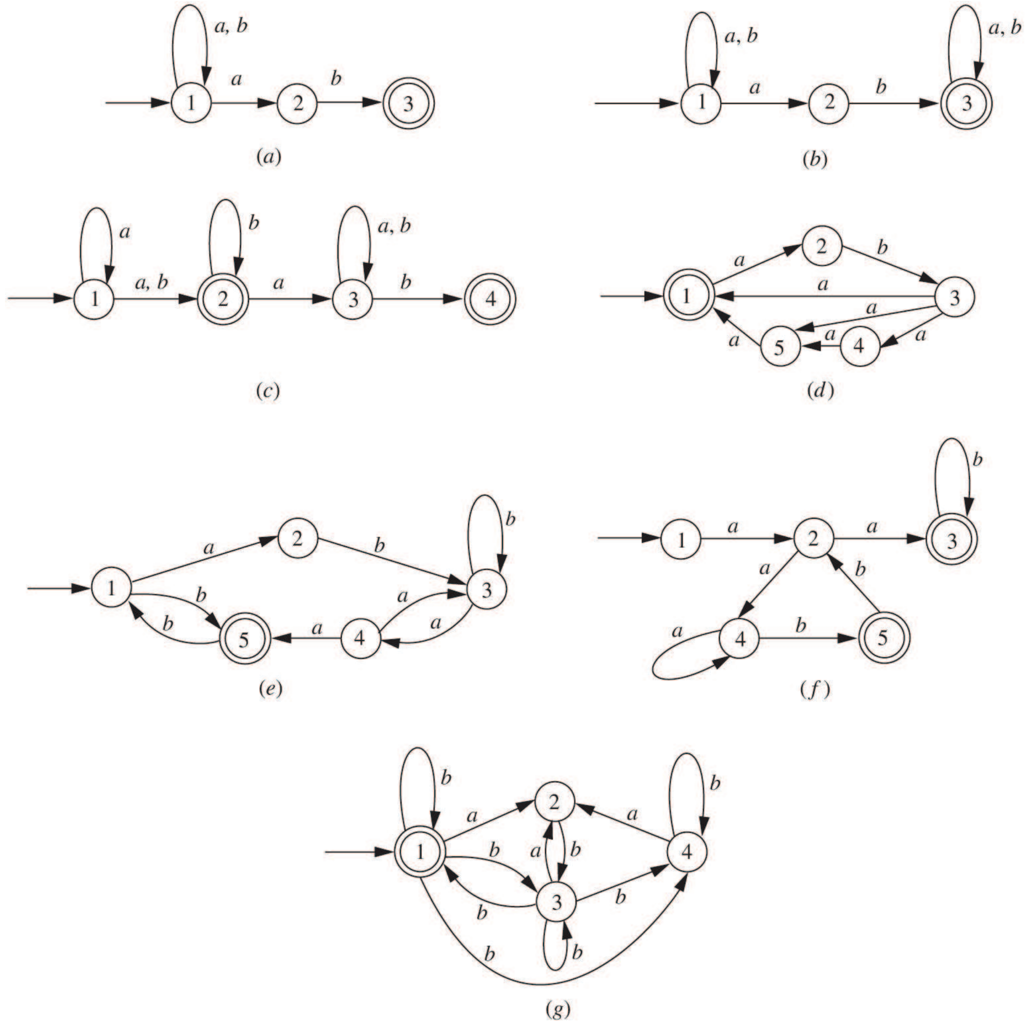
In each part of Figure below is pictured an NFA. Use the algorithm described in the proof of Theorem 3.17 to draw an NFA with no Λ-transitions accepting the same language.



(a) (b) (c) (d) (e)

Problem 7 30%

Each part of the figure below pictures an NFA. Using the subset construction, draw an FA accepting the same language. Label the final picture so as to make it clear how it was obtained from the subset construction.


(a)
(b)
(c)
(d)
(e)
(f)
(g)