CLEMSON UNIVERSITY, SCHOOL OF COMPUTING
CPSC 3500 FOUNDATIONS OF COMPUTER SCIENCE

Assignment 6: NFA

Return by 10/9/18

Problem 1 5%

(a) If L is the language corresponding to the regular expression $(aab + bbaba)^* baba$, find a regular expression corresponding to the language of the L-reverse strings

$$r(L) = \{r(x) | x \in L\}.$$

(b) Give a recursive definition of the reverse $r(e)$ of a regular expression e.

Problem 2 5%

The star height of a regular expression r over Σ, denoted by $sh(r)$, is defined as follows:

- $sh(\emptyset) = 0$.
- $sh(\Lambda) = 0$.
- $sh(\sigma) = 0 \; \forall \sigma \in \Sigma$.
- $sh((rs)) = sh((r + s)) = max(sh(r), sh(s))$.
- $sh((r^*)) = sh(r) + 1$.

Find the star heights of the following regular expressions

$$(a(a + a^* aa) + aaa)^*, \text{ and } (((a + a^* aa)aa)^* + aaaaaa^*)^*.$$

Problem 3 5%

For both the regular expressions in the previous exercise, find an equivalent regular expression of star height 1.

Problem 4 10%

Describe an algorithm that could be used to eliminate the symbol Λ from any regular expression whose corresponding language does not contain the null string. The input of the algorithm is a string with regular expression r. The output is a string with regular expression s, such that $L(r) = L(s)$.

Problem 5 10%

Prove that if $\Lambda \in L^k$, and $k \in \mathbb{N}$, then $L^k \subseteq L^{k+1}$.

Problem 6 30%

The order of a regular language L is the smallest integer k for which $L^k = L^{k+1}$, if there is one, and ∞ otherwise.

(a) Show that if L is a nonempty regular language, the order of L is finite if and only if there is an integer k such that $L^k = L^*$, and that in this case the order of L is the smallest k such that $L^k = L^*$. Hint: does L include Λ?

(b) What is the order of the regular language $\{\Lambda\} \cup \{aa\}\{aaa\}^*$?

(c) What is the order of the regular language $\{a\} \cup \{aa\}\{aaa\}^*$?

(d) What is the order of the language corresponding to the regular expression

$$(\Lambda + b^* a)(b + ab^* ab^* a)^*$$

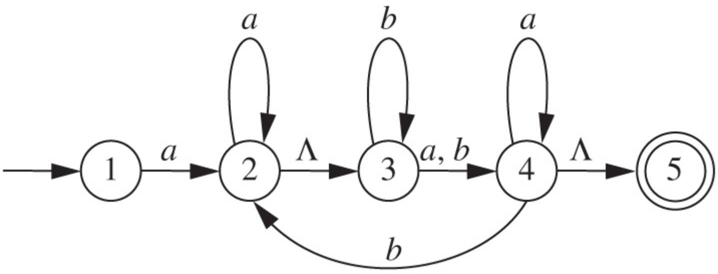
Problem 7 5%

For each string below, say whether the NFA in the diagram accepts it.

(a) aba

(b) abab

(c) aaabbb

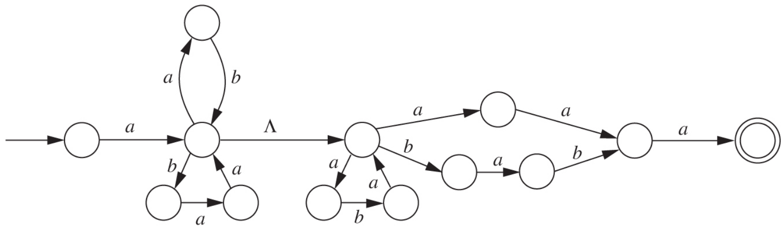


Problem 8 10%

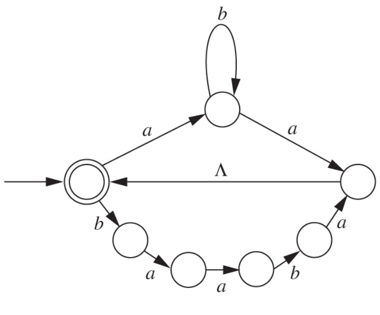
Find a regular expression corresponding to the language accepted by the NFA in problem above. You should be able to do it without applying Kleene's theorem: First find a regular expression describing the most general way of reaching state 4 the first time, and then find a regular expression describing the most general way, starting in state 4, of moving to state 4 the next time.

Problem 9 20%

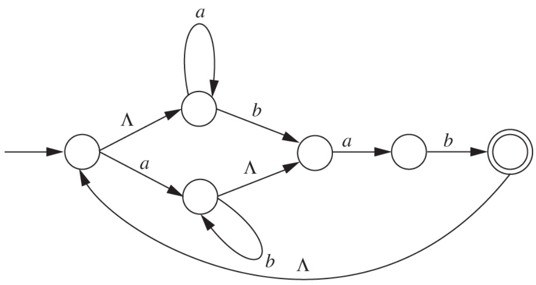
For each of the NFA below, find its regular expression.



(a)



(b)



(c)

Solutions to Assignment 6

Solution to Problem 1

(a) $abab(baa+ababb)^*$

(b) We may define a reverse function recursively on the set of regular expressions as follows: for $e = 0$, $e = \Lambda$, and $e = \sigma$ (where $\sigma \in \Sigma$), $r(e) = e$; and for arbitrary regular expressions e_1 and e_2, $r(e_1+e_2) = r(e_1)+r(e_2)$; $r(e_1e_2) = r(e_2)r(e_1)$; and $r(e_1^*) = (r(e_1))^*$.

Solution to Problem 2

2, and 3.

Solution to Problem 3

$\Lambda + aaa^*$, and $\Lambda + aaaa^*$

Solution to Problem 4

Make a sequence of passes through the expression.In each pass, replace any regular expression of the form (Λr) or $(r\Lambda)$ by r (where r is any regular expression); replace any occurrence of Λ^* by Λ; replace any regular expression of the form $(r+\Lambda)^*$ by (r^*) (where r is any regular expression); and replace any regular expression of the form $(\Lambda + r)s$ by $s + rs$ (where r and s are any regular expressions). Stop after any pass in which no changes are made. If Λ remains in the expression, then Λ is one of the strings corresponding to the expression.

Solution to Problem 5

If $\Lambda \in L^k$ then $\Lambda \in L$ because each string in L^k is a result of k concatenations of strings from L, i.e., $\Lambda = \Lambda\Lambda\cdots\Lambda$.

Let us take any $x \in L^k$ and prove that $x \in L^{k+1}$. If $x = \Lambda$ then $x \in L^{k+1}$ as a result of concatenation of Λ from L^k, and Λ from L. If $x \neq \Lambda$ then $x = x\Lambda$ is in L^{k+1} because x is in L^k, and Λ is in L.

Solution to Problem 6

(a) If $L^k = L^*$, then L^k contains Λ, which implies that $L^k \subseteq L^{k+1}$ (see problem above) and therefore that $L^k = L^{k+1}$ because $L^{k+1} \subseteq L^*$, i.e., we have both $L^k \subseteq L^{k+1}$, and $L^{k+1} \subseteq L^* = L^k$.

On the other hand, suppose that $L^k = L^{k+1}$. Let m be the length of the shortest element of L. Then the shortest elements in L^k and L^{k+1} have lengths km and $(k+1)m$, respectively, which implies that m must be 0. Therefore, $\Lambda \in L$. It follows that $L^i \subseteq L^{i+1}$ for every i, and therefore that $L^* = \cup_{i=0}^{\infty} L^i \subseteq \cup_{i=0}^{k} L^i$ (because L^i are all equal if $i \geq k$). But in addition, $L^{k+i} \subseteq L^k$ for every i, so that $\cup_{i=k}^{\infty} L^i \subset L^k$.

We conclude that $L^k = L^{k+1}$ if and only if $L^k = L^*$.

(b) The order is 3, because L^2 contains no string of length 6, and L^3 contains strings of all lengths ≥ 4.

(c) ∞, because the language does not contain Λ

(d) It is not hard to see that this language contains every string in which the number of a's is either a multiple of 3, or 1 plus a multiple of 3. It follows from this that the order is 2.

Solution to Problem 7

yes, no, yes

Solution to Problem 8

$aa^*b^*(a+b)(a+ba^*b^*(a+b))^*$

Solution to Problem 9

(a) $a(ab+baa)^*(aba)^*(aa+bab)a$

(b) $(ab^*a+baaba)^*$

(c) $((a^*b+ab^*)ab)^+$