Assignment 2: Finite Automata

Return by 11:59pm 9/18/2018

Problem 1 20%

Choose any 4 items out of (a)-(k). In each part below: (a) draw an FA accepting the indicated language over $\{a, b\}$; (b) explain why your FA accepts the indicated language.

(a) The language of all strings containing exactly two a's.

(b) The language of all strings containing at least two a's.

(c) The language of all strings including Λ that do not end with ab.

(d) The language of all strings that begin or end with aa or bb.

(e) The language of all strings including Λ not containing the substring aa.

(f) The language of all strings in which the number of a's is even.

(g) The language of all strings in which both the number of a's and the number of b's are even.

(h) The language of all strings containing no more than one occurrence of the string aa. (The string aaa contains two occurrences of aa.)

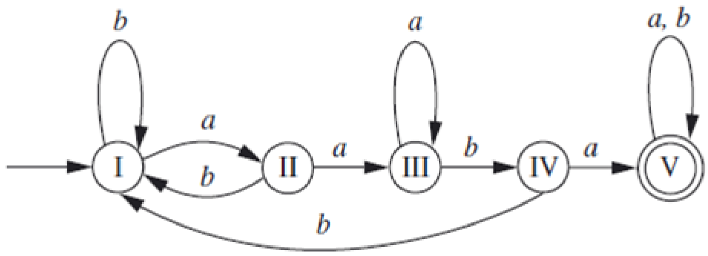
(i) The language of all strings in which every a (if there are any) is followed immediately by bb.

(j) The language of all strings containing both bb and aba as substrings.

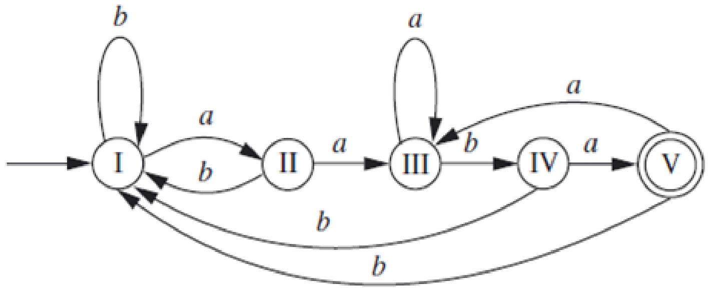
(k) The language of all strings containing both aba and bab as substrings.

Problem 2 20%

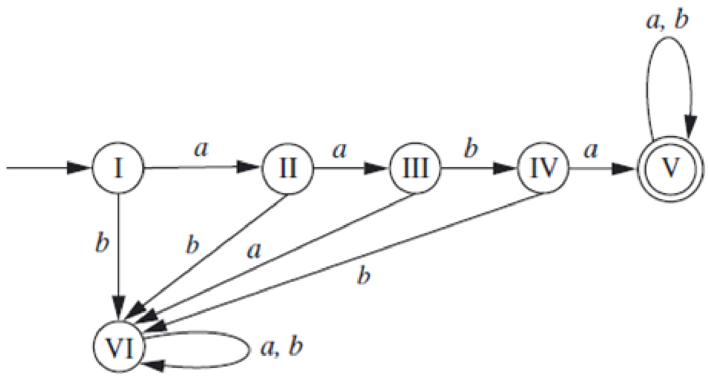
For each of the FAs pictured bellow, give a simple verbal description of the language it accepts.



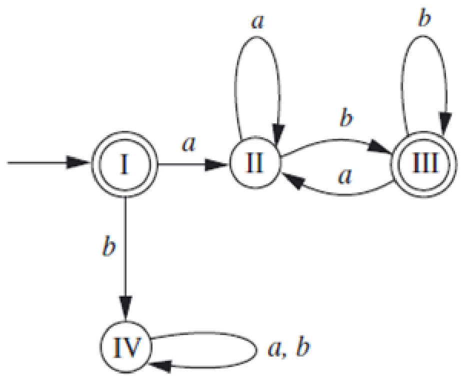
(*a*)



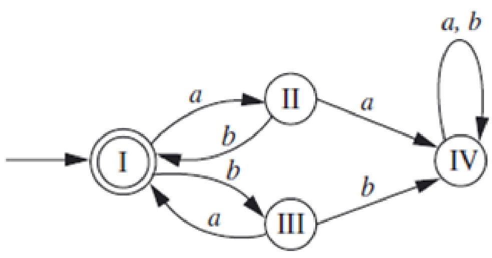
(*b*)



(*c*)



(*d*)



(*e*)

Problem 3 10%

(a) Draw a transition diagram for an FA that accepts the string *abaa* and no other strings.

(b) For a string $x \in \{a, b\}^*$ with $|x| = n$, how many states are required for an FA accepting x and no other strings? For each of these states, describe the strings that cause the FA to be in that state.

(c) For a string $x \in \{a, b\}^*$ with $|x| = n$, how many states are required for an FA accepting the language of all strings in $\{a, b\}$ that begin with x? For each of these states, describe the strings that cause the FA to be in that state.

Problem 4 20%

Draw and describe FA that accepts language L_5, the set of strings in $\{0, 1\}^*$ that are binary representations of integers divisible by 5.

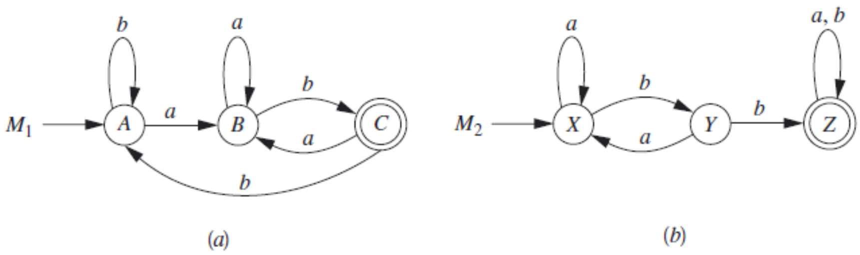
Problem 5 15%

Suppose $M = (Q, \Sigma, q_0, A, \delta)$ is an FA, $q \in Q$, and $x, y \in \Sigma^*$. Using structural induction on y, prove the formula

$$\delta^*(q, xy) = \delta^*(\delta^*(q, x), y).$$

Problem 6 15%

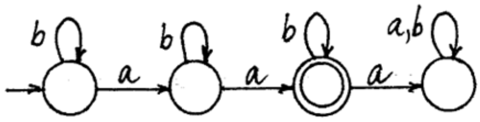
Let M_1, and M_2 be the FA pictured below. They accept languages L_1, and L_2, respectively. Draw and explain FAs accepting the following languages: (a) $L_1 \cup L_2$; (b) $L_1 \cap L_2$; and (c) $L_1 - L_2$.



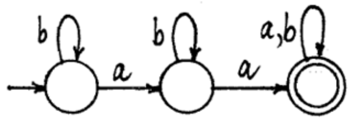
Solutions to Assignment 2

Solution to Problem 1

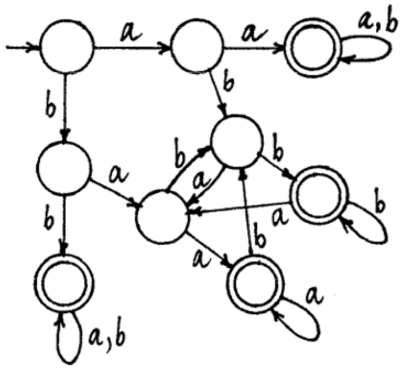
(a)



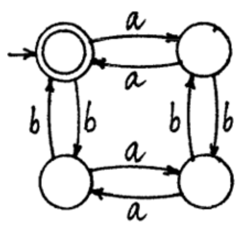
(b)



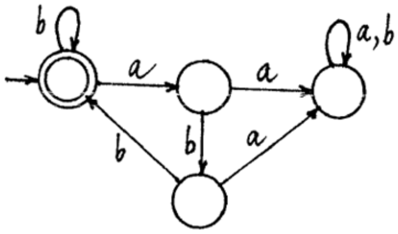
(d)



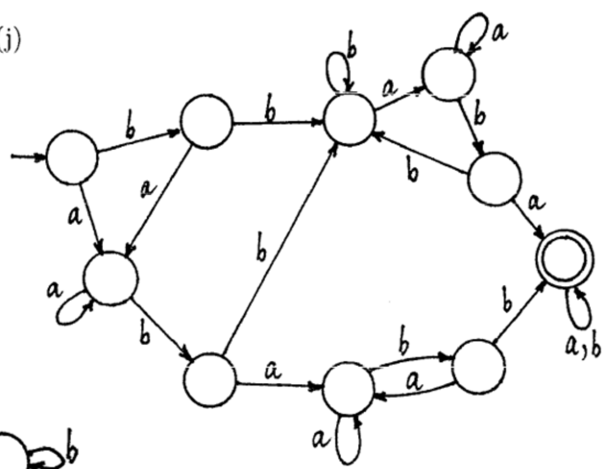
(g)



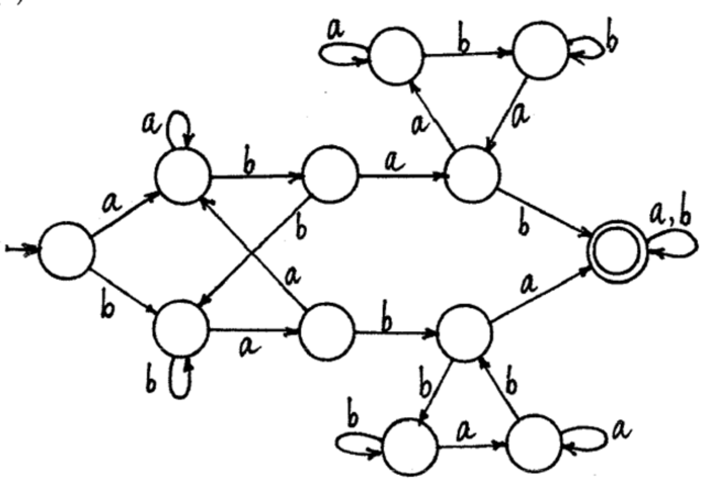
(i)

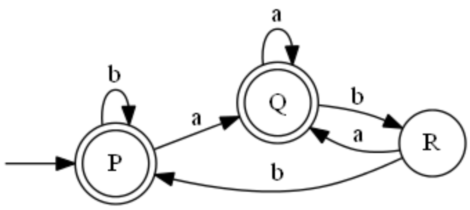


(j)

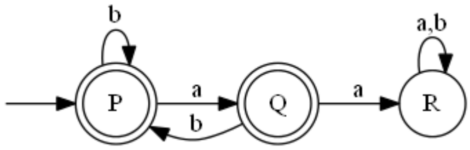


(k)

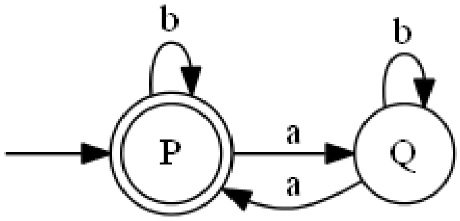




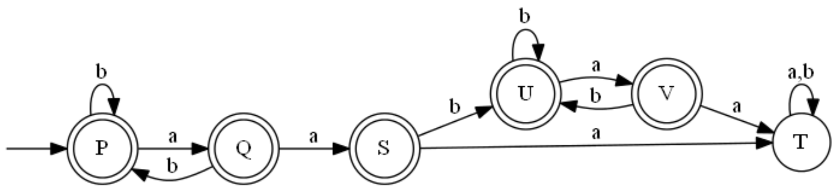
c)



e)



f)



h)

Solution to Problem 2

(a) All strings containing $aaba$.

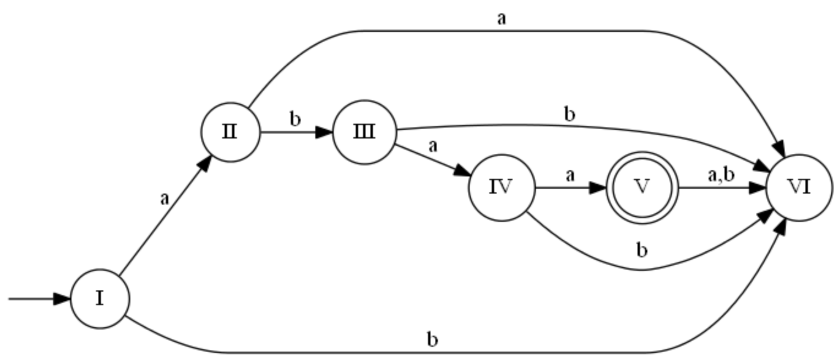
(b) All strings ending with $aaba$.

(c) All strings beginning with $aaba$.

(d) All strings that start with a and end with b including Λ.

(e) All possible concatenations of ab, and ba, including Λ.

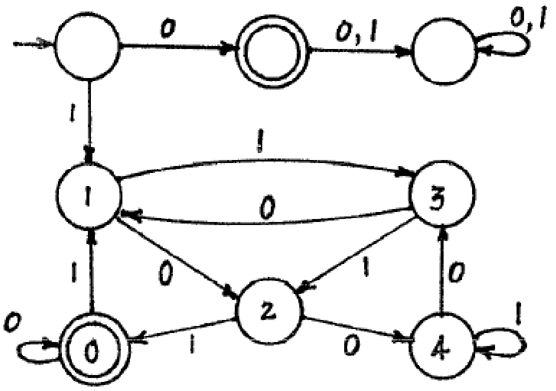
Solution to Problem 3

(a)


(b) There is an FA with $n+2$ states accepting $\{x\}$. It has one state for each of the $n+1$ prefixes of x, and one state N representing all the strings that are nonprefixes. For each state representing a prefix of x other than x itself, there is one transition to the next longer prefix and one to N. From the state corresponding to x and from N, all transitions go to N.

(c) There is an FA with $n+2$ states accepting such strings. It is the same as the one in (b) except that the transitions from the state that accept x both go back to this state.

Solution to Problem 4



Solution to Problem 5

See hint in the textbook. Exercise 2.5

Solution to Problem 6

The diagram below works for all three parts, except that in part (b) the only accepting state is (C,Y), and in (c) (C,Z) is the only one.