CLEMSON UNIVERSITY, SCHOOL OF COMPUTING
CPSC 3500 FOUNDATIONS OF COMPUTER SCIENCE

# Assignment 4: Finite Automata

Return by 11:59pm 10/1/2019

**Problem 1 10%**

Let L be the language $AnBn = \{a^n b^n | n \geq 0\}$.

(a) Find two distinct strings $x, y \in \{a, b\}^*$ that are not L-distinguishable.

(b) Find an infinite set of pairwise L-distinguishable strings.

(c) Can you draw FA for $AnBn$?

**Problem 2 10%**

Let $n$ be a positive integer and $L = \{x \in \{a, b\}^* | \ |x| = n$ and $n_a(x) = n_b(x)\}$. What is the minimum number of states in any FA that accepts $L$? Give reasons for your answer.

**Problem 3 20%**

Choose any 4 items out of (a)-(h). For each of the following languages $L \subseteq \{a, b\}^*$, show that the elements of the infinite set $\{a^n | n \geq 0\}$ are pairwise L-distinguishable.

(a) $L = \{a^n ba^{2n} | n \geq 0\}$

(b) $L = \{a^i b^j a^k | k > i + j\}$

(c) $L = \{a^i b^j | j = i$ or $j = 2i\}$

(d) $L = \{a^i b^j | j$ is a multiple of $i\}$

(e) $L = \{x \in \{a, b\}^* | n_a(x) < 2n_b(x)\}$

(f) $L = \{x \in \{a, b\}^* |$ no prefix of x has more b's than a's$\}$

(g) $L = \{a^{n^3} | n \geq 1\}$

(h) $L = \{ww | w \in \{a, b\}^*\}$

### Problem 4 10%

For each of the languages in Problems 3c, and 3e, use the pumping lemma to show that it cannot be accepted by an FA. (I recommend to show this for all languages in the previous problem but don't submit all of them.)

### Problem 5 10%

Let $n$ be a positive integer, and let $L$ be the set of all strings in $Pal$ of length $2n$. In other words,

$$L = \{xr(x) | x \in \{a, b\}^n\},$$

where $r(x)$ is the string reverse function. What is the minimum number of states in any FA that accepts L? Give reasons for your answer.

### Problem 6 10%

Suppose $L$ is a language over $\{a, b\}$, and there is a fixed integer $k$ such that for every $x \in \Sigma^*$, $xz \in L$ for some string $z$ with $|z| \leq k$. Does it follow that there is an FA accepting L? Why or why not?

### Problem 7 20%

Choose any 4 items out of (a)-(j). For each statement below, decide whether it is true or false. If it is true, prove it. If it is not true, give a counterexample. If you prove it, you can use any theorem we saw in class. All parts refer to languages over the alphabet $\{a, b\}$.

(a) If $L_1 \subseteq L_2$, and $L_1$ cannot be accepted by an FA, then $L_2$ cannot.

(b) If $L_1 \subseteq L_2$, and $L_2$ cannot be accepted by an FA, then $L_1$ cannot.

(c) If neither $L_1$ nor $L_2$ can be accepted by an FA, then $L_1 \cup L_2$ cannot.

(d) If neither $L_1$ nor $L_2$ can be accepted by an FA, then $L_1 \cap L_2$ cannot.

(e) If $L$ cannot be accepted by an FA, then $L'$ cannot (Reminder: $L'$ is the complement to $L$).

(f) If $L_1$ can be accepted by an FA and $L_2$ cannot, then $L_1 \cup L_2$ cannot.

(g) If $L_1$ can be accepted by an FA, $L_2$ cannot, and $L_1 \cap L_2$ can, then $L_1 \cup L_2$ cannot.

(h) If $L_1$ can be accepted by an FA and neither $L_2$ nor $L_1 \cap L_2$ can, then $L_1 \cup L_2$ cannot.

(i) If each of the languages $L_1, L_2, \ldots$ can be accepted by an FA, then $\cup_{n=1}^{\infty} L_n$ can.

(j) If none of the languages $L_1, L_2, \ldots$ can be accepted by an FA, and $L_i \subseteq L_{i+1}$ for each $i$, then $\cup_{n=1}^{\infty} L_n$ cannot be accepted by an FA.

### Problem 8 10%

A set $S$ of nonnegative integers is an arithmetic progression if for some integers $n$ and $p$,

$$S = \{n + ip | i \geq 0\}.$$

Let $A$ be a subset of $\{a\}^*$, and let $S = \{|x| \mid x \in A\}$. Show that if $S$ is an arithmetic progression, then $A$ can be accepted by an FA.