CLEMSON UNIVERSITY, SCHOOL OF COMPUTING
CPSC 3500 FOUNDATIONS OF COMPUTER SCIENCE

# Assignment 0: Mathematical Induction

Return by 11:59pm 8/27/2019

This assignment is not directly related to the topics of our course but it will help you to refresh on how to use mathematical induction.
Prove by induction the following claims

(a) $\forall n \in \mathbb{N} \quad \sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

(b) $\forall n \in \mathbb{N} \quad \sum_{i=1}^{n} \dfrac{1}{i(i+1)} = \dfrac{n}{n+1}$

(c) Let $r \in \mathbb{R}_{\neq 1}$ (r is real, not equal 1). Prove that $\forall n \in \mathbb{N} \quad \sum_{i=0}^{n} r^i = \dfrac{1-r^{n+1}}{1-r}$

(d) $\forall n \in \mathbb{N} \quad 1 + \sum_{i=1}^{n} i \cdot i! = (n+1)!$

(e) $\forall n \in \mathbb{N}_{\geq 4} \quad n! > 2^n$

(f) Let $x \in \mathbb{R}_{>-1}$. Prove that $\forall n \in \mathbb{N} \quad (1+x)^n \geq 1 + nx$

(g) The Fibonacci function $f$ is usually defined as follows.

$$f(0) = 0;\ f(1) = 1;\ \text{for every } n \in \mathbb{N}_{>1},\ f(n) = f(n-1) + f(n-2).$$

Here we need to give both the values $f(0)$ and $f(1)$ in the first part of the definition, and for each larger $n$, $f(n)$ is defined using both $f(n-1)$ and $f(n-2)$. Use induction to show that for every $n \in \mathbb{N}$, $f(n) \leq (5/3)^n$. (Note that in the induction step, you can use the recursive formula only if $n > 1$; checking the case $n = 1$ separately is comparable to performing a second basis step.)