CLEMSON UNIVERSITY, SCHOOL OF COMPUTING
CPSC 3500 FOUNDATIONS OF COMPUTER SCIENCE

Assignment 7: Nondeterminism, and Kleene's Theorem

Return by 10/9/2018

Problem 1 5%

Given the transition table for an NFA with states 1-5 and input alphabet $\{a, b\}$. There are no Λ-transitions.

(a) Draw a transition diagram.

(b) Calculate $\delta^*(1, ab)$.

(c) Calculate $\delta^*(1, abaab)$.

q	$\delta(q, a)$	$\delta(q, b)$
1	$\{1, 2\}$	$\{1\}$
2	$\{3\}$	$\{3\}$
3	$\{4\}$	$\{4\}$
4	$\{5\}$	$\emptyset$
5	$\emptyset$	$\{5\}$

Problem 2 10%

A transition table is given for an NFA with seven states. Find

(a) $\Lambda(\{2, 3\})$

(b) $\Lambda(\{1\})$

(c) $\Lambda(\{3, 4\})$

(d) $\delta^*(1, ba)$

(e) $\delta^*(1, ab)$

(f) $\delta^*(1, ababa)$

q	$\delta(q, a)$	$\delta(q, b)$	$\delta(q, \Lambda)$
1	$\emptyset$	$\emptyset$	$\{2\}$
2	$\{3\}$	$\emptyset$	$\{5\}$
3	$\emptyset$	$\{4\}$	$\emptyset$
4	$\{4\}$	$\emptyset$	$\{1\}$
5	$\emptyset$	$\{6, 7\}$	$\emptyset$
6	$\{5\}$	$\emptyset$	$\emptyset$
7	$\emptyset$	$\emptyset$	$\{1\}$

Problem 3 5%

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an NFA with no Λ-transitions. Show that for every $q \in Q$ and every $\sigma \in \Sigma$, $\delta^*(q, \sigma) = \delta(q, \sigma)$.

Problem 4 10%

It is easy to see that if $M = (Q, \Sigma, q_0, A, \delta)$ is an FA accepting L, then the FA $\overline{M} = (Q, \Sigma, q_0, Q - A, \delta)$ accepts $\overline{L}$ (the FA obtained from $\overline{L} = \Sigma^* - L$). Does this still work if M is an NFA? If so, prove it. If not, find a counterexample.

Problem 5 20%

Let $M = (Q, \Sigma, q_0, A, \delta)$ be an NFA. This exercise involves properties of the Λ-closure of a set S. Since $\Lambda(S)$ is defined recursively, structural induction can be used to show that $\Lambda(S)$ is a subset of some other set.

(a) Show that if S and T are subsets of Q for which $S \subseteq T$, then $\Lambda(S) \subseteq \Lambda(T)$.

(b) Show that for any $S \subseteq Q$, $\Lambda(\Lambda(S)) = \Lambda(S)$.

(c) Show that if $S, T \subseteq Q$, then $\Lambda(S \cup T) = \Lambda(S) \cup \Lambda(T)$.

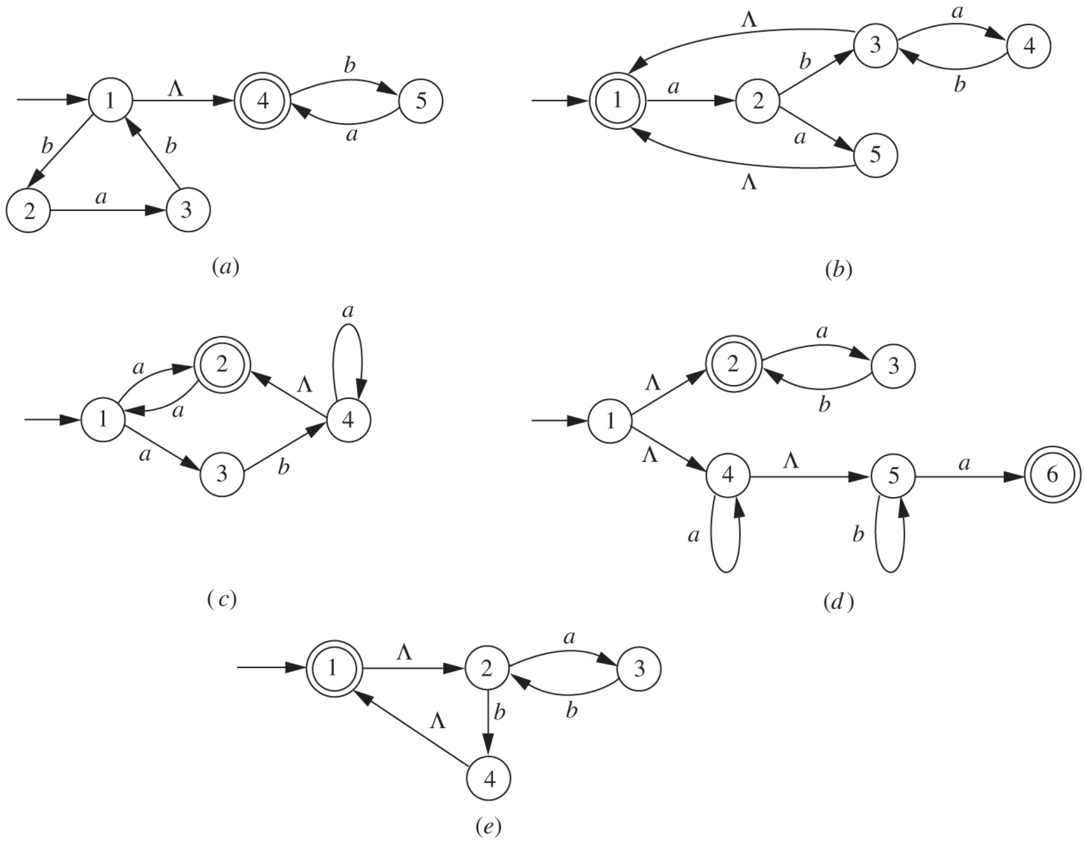
(d) Show that if $S \subseteq Q$, then $\Lambda(S) = \cup\{\Lambda(\{p\}) | p \in S\}$.

(e) Draw a transition diagram to illustrate the fact that $\Lambda(S \cap T)$ and $\Lambda(S) \cap \Lambda(T)$ are not always the same. Which is always a subset of the other?

(f) Draw a transition diagram illustrating the fact that $\Lambda(S')$ and $\Lambda(S)'$ are not always the same. Which is always a subset of the other? Under what circumstances are they equal?

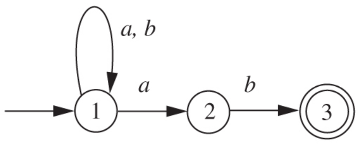
Problem 6 20%

In each part of Figure below is pictured an NFA. Use the algorithm described in the proof of Theorem 3.17 to draw an NFA with no Λ-transitions accepting the same language.

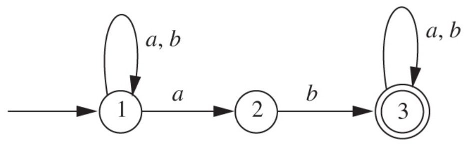


Problem 7 30%

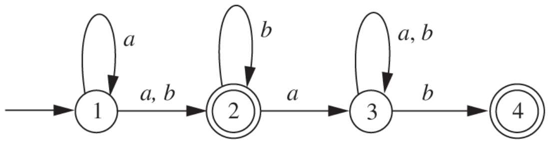
Each part of the figure below pictures an NFA. Using the subset construction, draw an FA accepting the same language. Label the final picture so as to make it clear how it was obtained from the subset construction.


a, b
a
b
1
2
3

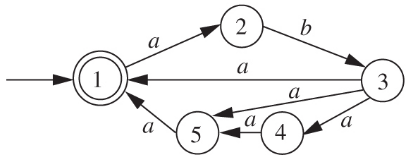
(a)


a, b
a
b
a, b
1
2
3

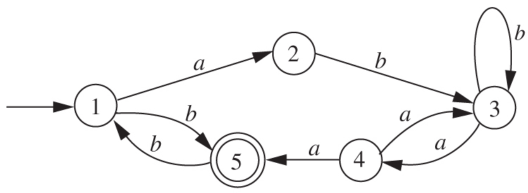
(b)


a
b
a, b
a, b
a
b
1
2
3
4

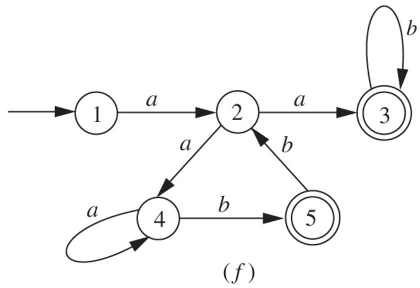
(c)


2
a
b
1
a
3
a
a
a
a
5
4

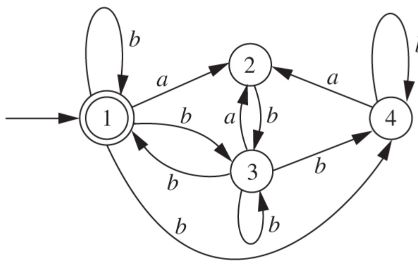
(d)


b
2
a
b
1
3
b
a
b
a
a
5
4

(e)


b
1
a
2
a
3
a
b
a
b
4
5

(f)


b
b
2
a
a
1
b
a
b
4
b
3
b
b
b

(g)

Solutions to Assignment 6

Solution to Problem 1

(a) $\delta^*(1,a) = \cup_{p\in\delta^*(1,\Lambda)}\delta(p,a) = \delta(1,a) = \{1,2\}$. $\delta^*(1,ab) = \delta(1,b)\cup\delta(2,b) = \{1,3\}$.
(b) $\delta^*(1,aba) = \delta(1,a)\cup\delta(3,a) = \{1,2,4\}$. $\delta^*(1,abaa) = \cup_{p\in\{1,2,4\}}\delta(p,a) = \{1,2,3,5\}$.
(c) $\delta^*(1,abaab) = \delta(1,b)\cup\delta(2,b)\cup\delta(3,b)\cup\delta(5,b) = \{1,3,4,5\}$.

Solution to Problem 2

(a) $\{2,3,5\}$; (b) $\{1,2,5\}$; (c) $\{1,2,3,4,5\}$; (d) $\{3,5\}$; (e) $\{1,2,4,5\}$; (f) $\{1,2,3,4,5\}$

Solution to Problem 3

$\delta^*(q,a) = \cup\{\delta(p,a) | p\in\delta^*(q,\Lambda)\} = \cup\{\delta(p,a) | p\in\{q\}\} = \delta(q,a)$.

Solution to Problem 4

No, because there may be a string x so that in M, one sequence of transitions corresponding to x ends at an accepting state and another sequence doesn't. Then $\overline{M}$ has this property also, which means that $x\in L(M)$ and $x\in L(\overline{M})$. An example can easily be constructed with just two states.

Solution to Problem 5

(a) see proof in the textbook

(b) We know from the definition of Λ-closure that $\Lambda(S)\subseteq\Lambda(\Lambda(S))$. To show the opposite inclusion using structural induction, we must show that for every $s\in\Lambda(S)$, and for every $t\in\Lambda(S)$, $\Lambda(\{t\})\subseteq\Lambda(S)$. The first is trivial, and the second is part of the definition of $\Lambda(S)$.

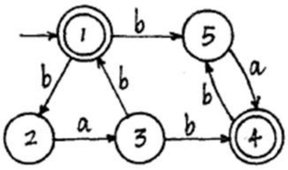
(c) The statement $\Lambda(S\cup T)\subseteq\Lambda(S)\cup\Lambda(T)$ is easily shown by structural induction. The opposite inclusion follows from the two statements $\Lambda(S)\subseteq\Lambda(S\cup T)$ and $\Lambda(T)\subseteq\Lambda(S\cup T)$, both of which are true by part (a).

(d) From (c), we have the result when $|S| = 2$, and it is easy to extend the result to arbitrary n by induction.

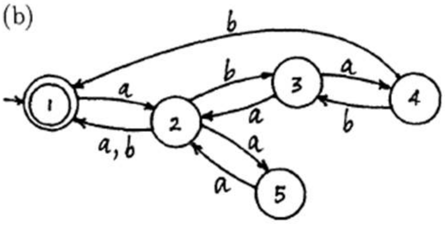
(e) $\Lambda(S\cap T)$ is a subset both of $\Lambda(S)$ and of $\Lambda(T)$, by part (a), and therefore $\Lambda(S\cap T)\subseteq\Lambda(S)\cap\Lambda(T)$.

Solution to Problem 6

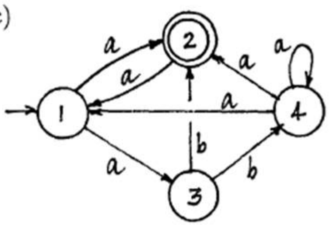
(a)



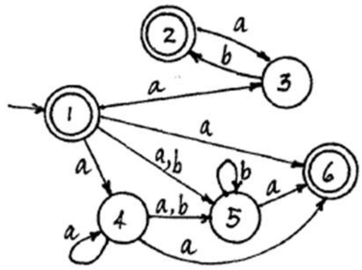
(b)



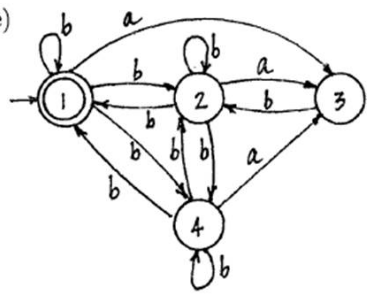
(c)



(d)



(e)



Solution to Problem 7