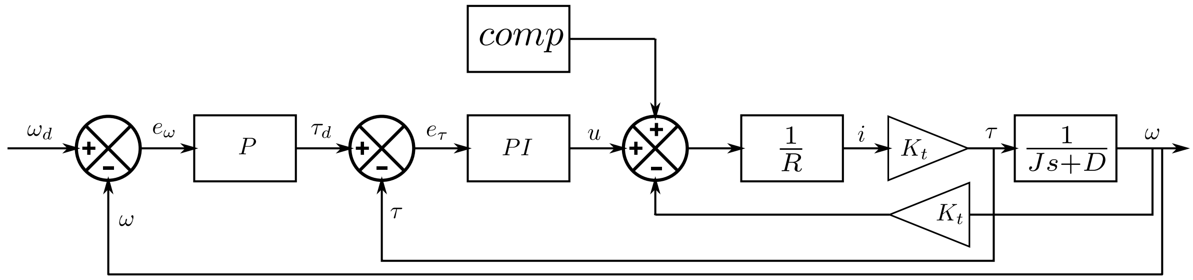
Lesson 6 : DC Motor Cascade Control

1 Outer Propositional Velocity and Inner Propositional Integral Torque Control Design



Assumption: $L = 0, K_b = K_t, Tc \neq 0$
From the Architecture, We have:

- $u = K_{pi} e_\tau + K_{ii} \int e_\tau dt + comp$
- $e_\omega = \omega_d - \omega => \dot{e}_\omega = \dot{\omega}_d - \dot{\omega}$
- $e_\tau = \tau_d - \tau => \dot{e}_\tau = \dot{\tau}_d - \dot{\tau}$
- $\tau_d = K_{po} e_\omega => \dot{\tau}_d = K_{po} \dot{e}_\omega$

From the Model of DC Model, We have:

$$u = K_t \omega + Ri$$
$$=> i = \frac{u - K_t \omega}{R}$$

We have:

$$\tau = K_t i = Tc + D\omega + J\dot{\omega}$$

By Substitute i in, We get:

$$K_t \frac{u - K_t \omega}{R} = Tc + D\omega + J\dot{\omega}$$
$$RT_c + RD\omega + RJ\dot{\omega} = K_t(u - K_t\omega)$$
$$\frac{RJ}{K_t}\dot{\omega} + \frac{RD + K_t^2}{K_t}\omega + \frac{RT_c}{K_t} = u$$

Substitute u in, We get:

$$\frac{RJ}{K_t}\dot{\omega} + \frac{RD + K_t^2}{K_t}\omega + \frac{RT_c}{K_t} = K_{pi} e_\tau + K_{ii} \int e_\tau dt + comp$$

Take Derivative to eliminate integral:

$$\frac{RJ}{K_t}\ddot{\omega} + \frac{RD + K_t^2}{K_t}\dot{\omega} = K_{pi}\dot{e}_\tau + K_{ii} e_\tau + \dot{comp}$$
$$\frac{RJ}{K_t}\ddot{\omega} + \frac{RD + K_t^2}{K_t}\dot{\omega} = K_{pi}(\dot{\tau}_d - \dot{\tau}) + K_{ii}(\tau_d - \tau) + \dot{comp}$$

From the model, We have:

$$\tau = T_c + D\omega + J\dot{\omega} => \dot{\tau} = D\dot{\omega} + J\ddot{\omega}$$

We get:

$$\frac{RJ}{K_t}\ddot{\omega} + \frac{RD + K_t^2}{K_t}\dot{\omega} = K_{pi}(K_{po}\dot{e}_\omega - (D\dot{\omega} + J\ddot{\omega})) + K_{ii}(K_{po}e_\omega - (T_c + D\omega + J\dot{\omega})) + \dot{comp}$$
$$\frac{RJ}{K_t}\ddot{\omega} + \frac{RD + K_t^2}{K_t}\dot{\omega} = K_{pi}(K_{po}\dot{e}_\omega - D\dot{\omega} - J\ddot{\omega}) + K_{ii}(K_{po}e_\omega - T_c - D\omega - J\dot{\omega}) + \dot{comp}$$
$$\frac{RJ}{K_t}\ddot{\omega} + \frac{RD + K_t^2}{K_t}\dot{\omega} = K_{pi}K_{po}\dot{e}_\omega - K_{pi}D\dot{\omega} - K_{pi}J\ddot{\omega} + K_{ii}K_{po}e_\omega - K_{ii}T_c - K_{ii}D\omega - K_{ii}J\dot{\omega} + \dot{comp}$$

$$\frac{RJ}{K_t}\ddot{\omega} + \frac{RD + K_t^2}{K_t}\dot{\omega} + K_{pi}D\dot{\omega} + K_{pi}J\ddot{\omega} + K_{ii}D\omega + K_{ii}J\dot{\omega} = K_{pi}K_{po}\dot{e}_\omega + K_{ii}K_{po}e_\omega - K_{ii}T_c + \dot{comp}$$
$$(\frac{RJ}{K_t} + K_{pi}J)\ddot{\omega} + (\frac{RD + K_t^2}{K_t} + K_{pi}D + K_{ii}J)\dot{\omega} + K_{ii}D\omega = K_{pi}K_{po}\dot{e}_\omega + K_{ii}K_{po}e_\omega - K_{ii}T_c + \dot{comp}$$

Multiply both side by -1 to reverse the sign:

$$-(\frac{RJ}{K_t} + K_{pi}J)\ddot{\omega} - (\frac{RD + K_t^2}{K_t} + K_{pi}D + K_{ii}J)\dot{\omega} - K_{ii}D\omega = -K_{pi}K_{po}\dot{e}_\omega - K_{ii}K_{po}e_\omega + K_{ii}T_c - \dot{comp}$$

Adding both of the equation for compensation with :

$$\begin{aligned} &+ (\frac{RJ}{K_t} + K_{pi}J)\ddot{\omega}_d \\ &+ (\frac{RD + K_t^2}{K_t} + K_{pi}D + K_{ii}J)\dot{\omega}_d \\ &+ K_{ii}D\omega_d \end{aligned}$$

We get:
On LHS:

$$(\frac{RJ}{K_t} + K_{pi}J)(\ddot{\omega}_d - \ddot{\omega}) + (\frac{RD + K_t^2}{K_t} + K_{pi}D + K_{ii}J)(\dot{\omega}_d - \dot{\omega}) + K_{ii}D(\omega_d - \omega) =$$

On RHS:

$$= -K_{pi}K_{po}\dot{e}_\omega - K_{ii}K_{po}e_\omega + K_{ii}T_c - \dot{comp} + (\frac{RJ}{K_t} + K_{pi}J)\ddot{\omega}_d + (\frac{RD + K_t^2}{K_t} + K_{pi}D + K_{ii}J)\dot{\omega}_d + K_{ii}D\omega_d$$

Then:
On LHS:

$$(\frac{RJ}{K_t} + K_{pi}J)\ddot{e}_\omega + (\frac{RD + K_t^2}{K_t} + K_{pi}D + K_{ii}J)\dot{e}_\omega + K_{ii}De_\omega + K_{pi}K_{po}\dot{e}_\omega + K_{ii}K_{po}e_\omega =$$

On RHS:

$$= K_{ii}T_c - \dot{comp} + (\frac{RJ}{K_t} + K_{pi}J)\ddot{\omega}_d + (\frac{RD + K_t^2}{K_t} + K_{pi}D + K_{ii}J)\dot{\omega}_d + K_{ii}D\omega_d$$

On LHS:

$$(\frac{RJ}{K_t} + K_{pi}J)\ddot{e}_\omega + (\frac{RD + K_t^2}{K_t} + K_{pi}D + K_{ii}J + K_{pi}K_{po})\dot{e}_\omega + (K_{ii}D + K_{ii}K_{po})e_\omega =$$

On RHS:

$$= K_{ii}T_c - \dot{comp} + (\frac{RJ}{K_t} + K_{pi}J)\ddot{\omega}_d + (\frac{RD + K_t^2}{K_t} + K_{pi}D + K_{ii}J)\dot{\omega}_d + K_{ii}D\omega_d$$

On the LHS, Using the 2nd Order Differential Standard Form : $\ddot{X} + 2\zeta\omega_n\dot{X} + \omega_n^2X = 0$, We get:

$$2\zeta\omega_n = \frac{(\frac{RD+K_t^2}{K_t} + K_{pi}D + K_{ii}J + K_{pi}K_{po})}{(\frac{RJ}{K_t} + K_{pi}J)}$$

$$\omega_n^2 = \frac{(K_{ii}D + K_{ii}K_{po})}{(\frac{RJ}{K_t} + K_{pi}J)}$$

We solve above equation for K_{pi} and K_{ii} in terms of K_{po}, ζ, ω_n, We get:

$$K_{pi} = -\frac{D^2R + J^2R\omega_n^2 + k_{po}(K_t^2 - 2JR\omega_n\zeta) + D(K_t^2 + R(k_{po} - 2J\omega_n\zeta))}{K_t(D^2 + k_{po}^2 + J^2\omega_n^2 - 2Jk_{po}\omega_n\zeta + 2D(k_{po} - J\omega_n\zeta))}$$

$$K_{ii} = \frac{J(-K_t^2 + k_{po}R)\omega_n^2}{K_t(D^2 + k_{po}^2 + J^2\omega_n^2 - 2Jk_{po}\omega_n\zeta + 2D(k_{po} - J\omega_n\zeta))}$$

On the RHS, We have our compensation:

$$\dot{comp} = K_{ii}T_c + (\frac{RJ}{K_t} + K_{pi}J)\ddot{\omega}_d + (\frac{RD + K_t^2}{K_t} + K_{pi}D + K_{ii}J)\dot{\omega}_d + K_{ii}D\omega_d$$

$$comp = \int \left[K_{ii}T_c + (\frac{RJ}{K_t} + K_{pi}J)\ddot{\omega}_d + (\frac{RD + K_t^2}{K_t} + K_{pi}D + K_{ii}J)\dot{\omega}_d + K_{ii}D\omega_d\right] dt$$