

Lesson 1 : DC Motor

1 Background

DC motor is a mechatronic product that consist of two parts: the mechanical part and the electrical part. A typical dc motor used by a robot is constructed by: a dc motor, a wheel encoder (for measuring rotation pulse of motor), and a gear box (for reducing the speed of motor).

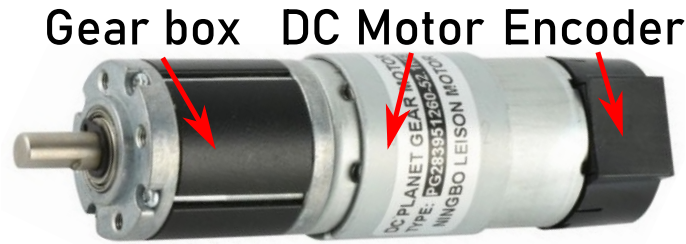


Figure 1: Typical dc motor

2 Modelling

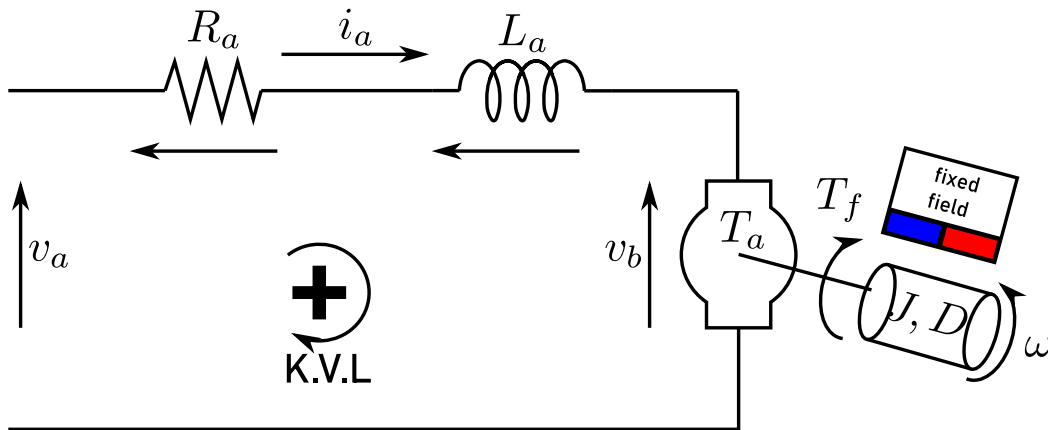


Figure 2: dc motor model

2.1 Electrical Part

$$v_b(t) = K_b \dot{\theta}(t) = K_b \omega(t) \quad (1)$$

Where:

- $v_b(t)$ is voltage at terminal conductor of motor
- K_b is back emf constant
- $\dot{\theta} = \omega$ angular velocity of motor

$$T_a = K_t i_a(t) \quad (2)$$

Where:

- T_a is rotor torque
- K_t is motor torque
- i_a is the current draw by motor

By applying Kirchoff Voltage Law to the circuit loop in Figure 2

- $v_a(t)$ is input voltage from power source
- $v_{resistance} = R_a i_a(t)$ is voltage across resistance
- $v_{inductor} = L_a \frac{di_a(t)}{dt}$ is voltage across inductor

$$\begin{aligned} v_a(t) - v_b(t) - R_a i_a(t) - L_a \frac{di_a(t)}{dt} &= 0 \\ \Rightarrow v_a(t) &= v_b(t) + R_a i_a(t) + L_a \frac{di_a(t)}{dt} \end{aligned} \quad (3)$$

Substitute Equation 1 into Equation 3, we get:

$$v_a(t) = K_b \omega(t) + R_a i_a(t) + L_a \frac{di_a(t)}{dt} \quad (4)$$

In practical dc motor the L_a is very small ($L_a \approx 0$) and neglectable.

$$\begin{aligned} v_a(t) &= K_b \omega(t) + R_a i_a(t) \\ \Rightarrow i_a(t) &= \frac{v_a(t) - K_b \omega(t)}{R_a} \end{aligned} \quad (5)$$

2.2 Mechanical Part

$$T_a = T_f + J\dot{\omega}(t) \quad (6)$$

Where:

- T_f is torque of coulomb friction and viscous friction
- J is moment of inertia

We know that:

$$T_f = T_c \text{sign}[\omega(t)] + D\omega(t) \quad (7)$$

Where:

- T_c is coulomb friction torque
- D is coefficient viscous friction

Substitute Equation 7 to Equation 6, we get:

$$T_a = T_c \text{sign}[\omega(t)] + D\omega(t) + J\dot{\omega}(t)$$

2.2.1 Approximation of coulomb friction to zero $T_c \approx 0$

$$T_a = D\omega(t) + J\dot{\omega}(t) \quad (8)$$

From Equation 2: $T_a = K_t i_a(t)$ substitute to Equation 8:

$$\begin{aligned} K_t i_a(t) &= D\omega(t) + J\dot{\omega}(t) \\ \Rightarrow i_a(t) &= \frac{D\omega(t) + J\dot{\omega}(t)}{K_t} \end{aligned} \quad (9)$$

2.2.2 Keep coulomb friction T_c

$$\Rightarrow i_a(t) = \frac{T_c \text{sign}[\omega(t)] + D\omega(t) + J\dot{\omega}(t)}{K_t} \quad (10)$$

3 Electrical and Mechanical combine

3.1 Approximation of coulomb friction to zero $T_c \approx 0$

From Equation 5 and Equation 9: Put it side by side:

$$\begin{aligned} i_a(t) &= i_a(t) \\ \frac{v_a(t) - K_b\omega(t)}{R_a} &= \frac{D\omega(t) + J\dot{\omega}(t)}{K_t} \end{aligned}$$

Get $\dot{\omega}(t)$:

$$\begin{aligned} \dot{\omega}(t) &= \frac{\frac{(v_a(t) - K_b\omega(t))K_t}{R_a} - D\omega(t)}{J} \\ \dot{\omega}(t) &= \frac{(v_a(t) - K_b\omega(t))K_t}{R_a J} - \frac{D\omega(t)}{J} \\ \dot{\omega}(t) &= \frac{v_a(t)K_t - K_b K_t \omega(t)}{R_a J} - \frac{D\omega(t)}{J} \end{aligned}$$

Separate $\omega(t)$ and $v_a(t)$:

$$\dot{\omega}(t) = -\left(\frac{K_b K_t + D R_a}{R_a J}\right)\omega(t) + \frac{K_t}{R_a J}v_a(t) \quad (11)$$

Let:

- $a = \left(\frac{K_b K_t + D R_a}{R_a J}\right)\omega(t) \quad [1/s]$
- $b = \frac{K_t}{R_a J} \quad [rad/s^2/V]$

We get lamped Parameter in a simplified form as:

$$\Rightarrow \dot{\omega}(t) = -a\omega(t) + b v_a(t) \quad (12)$$

3.2 Keep coulomb friction T_c

$$\dot{\omega}(t) = -\left(\frac{K_b K_t + D R_a}{R_a J}\right)\omega(t) + \frac{K_t}{R_a J}v_a(t) - \frac{T_c}{J}\text{sign}(\omega(t)) \quad (13)$$

Let:

- $a = \left(\frac{K_b K_t + D R_a}{R_a J}\right)\omega(t) \quad [1/s]$
- $b = \frac{K_t}{R_a J} \quad [rad/s^2/V]$
- $c = \frac{T_c}{J} \quad [dddd]$

We get lamped Parameter in a simplified form as:

$$\Rightarrow \dot{\omega}(t) = -a\omega(t) + bv_a(t) - c\text{sign}(\omega(t)) \quad (14)$$

4 Simulation

In general, the equation $\dot{\omega}(t) = -a\omega(t) + bv_a(t) - c\text{sign}(\omega(t))$ is used to represent all the dc motor in the market. By modifying the parameters a, b, c will result in different dc motor.

From equation $\dot{\omega}(t) = -a\omega(t) + bv_a(t) - c\text{sign}(\omega(t))$

- $\dot{\omega}(t)$ is the angular acceleration of dc motor and is the output of the system
- $\omega(t)$ is the angular velocity of dc motor and is the output of the system
- $v_a(t)$ is the input voltage to dc motor and is the input of the system

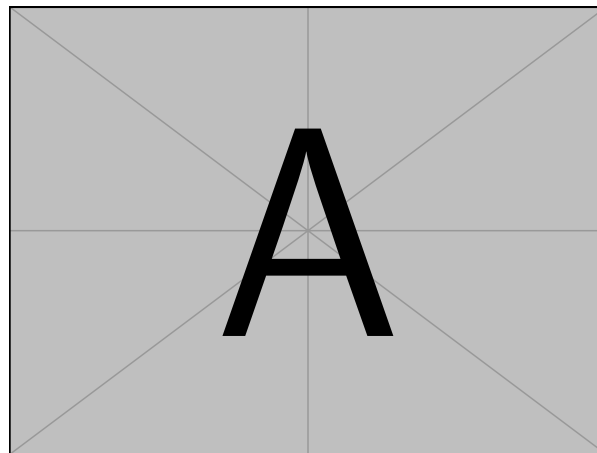


Figure 3: Simulation Flow

4.1 SIMULINK

1 `a = [1,2,3,4,5,6];`

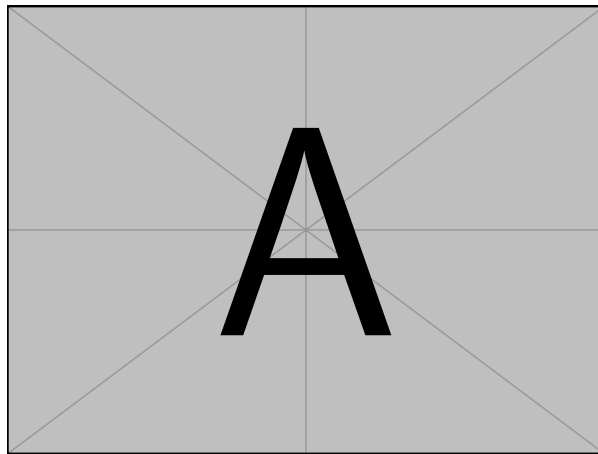


Figure 4: SIMULINK simulation