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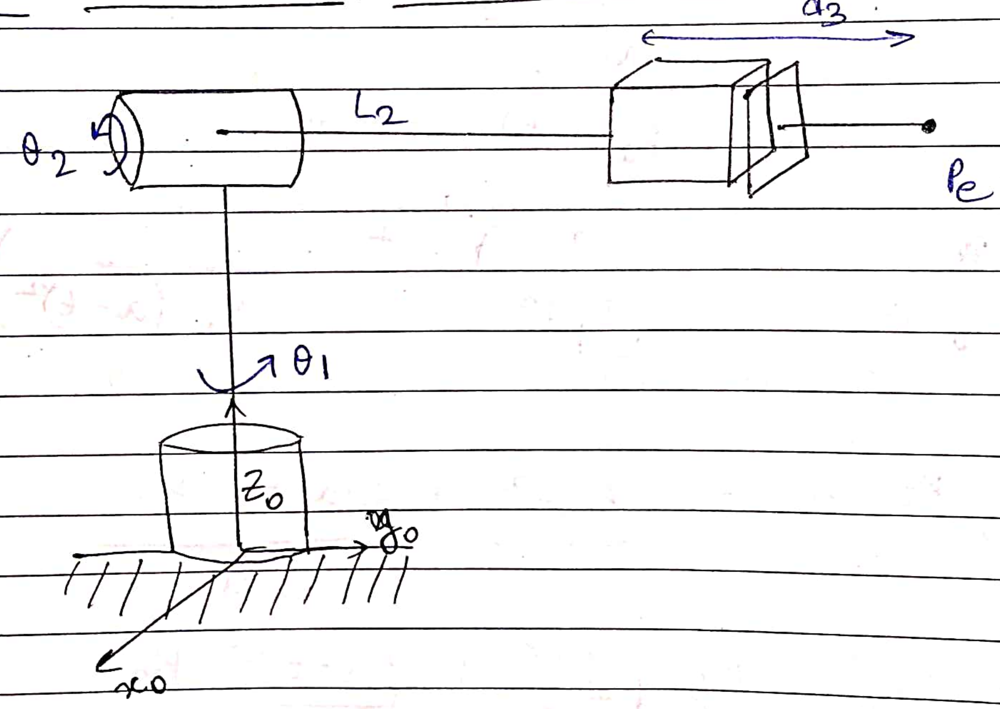
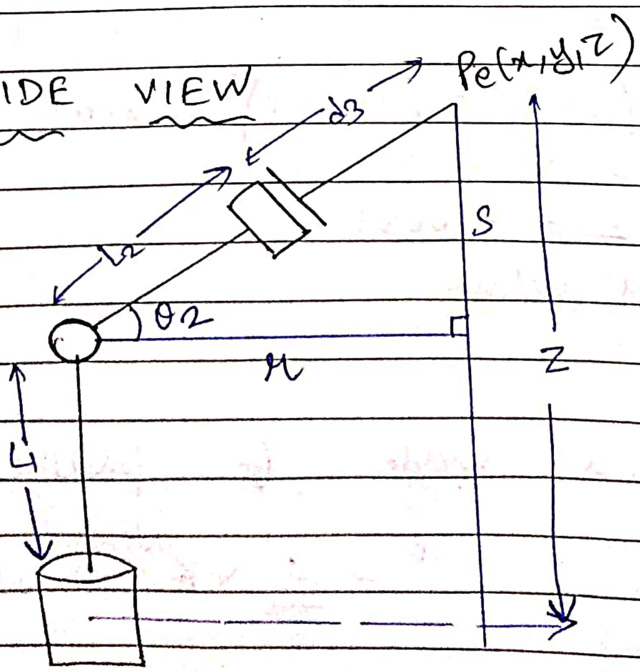
PhD Mechanical Engg.

ME639
ASSIGNMENT-4

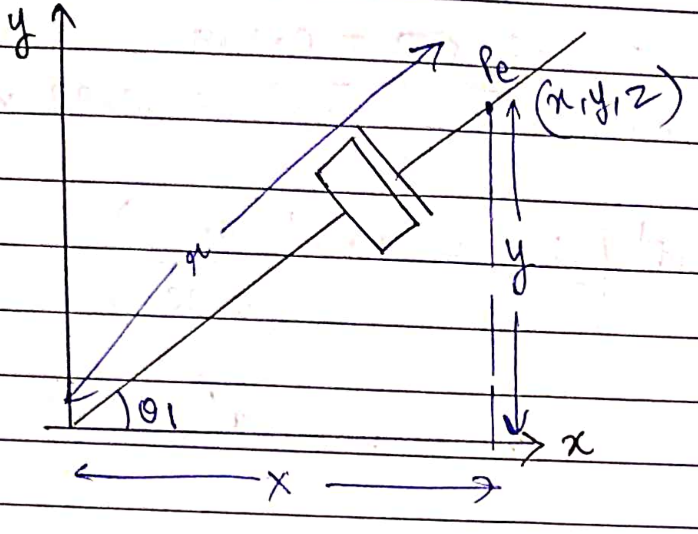
(1) RRP Stanford manipulator



SIDE VIEW



TOP VIEW



For Inverse Kinematics (referring Section 4.3.2 of textbook)

From Side view

$\theta_2 = \tan^{-1}\left(\frac{s}{r}\right)$

$s = z - L_1$

$r^2 = x^2 + y^2$

Also for d_3 $\quad (L_2 + d_3)^2 = s^2 + r^2$

$\Rightarrow L_2 + d_3 = \sqrt{s^2 + r^2}$

$\Rightarrow d_3 = \sqrt{s^2 + r^2} - L_2$

Given : end effector position (Pe)

$P_e = x, y, z, L_1, L_2$

let $x = 0.20$ m

$y = 0.25$ m

$z = 0.30$ m

$L_1 = 0.20$ m

$L_2 = 0.20$ m

From top view

$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right)$

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Substituting the values of x, y, z, L_1, L_2

$s = z - L_1 = 0.3 - 0.2 = 0.1\ m$

$r = \sqrt{x^2 + y^2} = \sqrt{0.2^2 + 0.25^2} = 0.32\ m$

$\theta_2 = \tan^{-1}\left(\frac{s}{r}\right) = \tan^{-1}\left(\frac{0.1}{0.32}\right)$

$\theta_2 = 0.3029^R$

$\boxed{\theta_2 = 17.35^\circ}$

~~for~~ , $\theta_1 = \tan^{-1}\left(\frac{y}{x}\right)$

$\theta_1 = \tan^{-1}\left(\frac{0.25}{0.20}\right)$

$\theta_1 = 0.896^R$

$\boxed{\theta_1 = 51.34^\circ}$

$d_3 = \sqrt{s^2 + r^2} - L_2$

$= \sqrt{0.32^2 + 0.1^2} - 0.2$

$= 0.335 - 0.2$

$= 0.135\ m$

$\boxed{d_3 = 0.135\ m}$

Forward Kinematics

let $\theta_1 = 51.34^\circ$ $\quad L_1 = 0.2m$

$\theta_2 = 17.35^\circ$ $\quad L_2 = 0.2m$

$d_3 = 0.135m$

Now, $\cos\theta_2 = \dfrac{r}{L_2 + d_3}$

$\Rightarrow (\cos 17.35^\circ) \times (0.2 + 0.135) = r$

$\Rightarrow \boxed{r = 0.32\ m}$

Now $\cos\theta_1 = \dfrac{x}{r}$

$\Rightarrow x = r\cos\theta_1 = 0.32 \cos 51.34^\circ$

$\boxed{x \approx 0.2\ m}$

$\sin\theta_1 = \dfrac{y}{r}$

$\Rightarrow y = r\sin\theta_1 = 0.32 \sin 51.34^\circ$

$\boxed{y \approx 0.25m}$

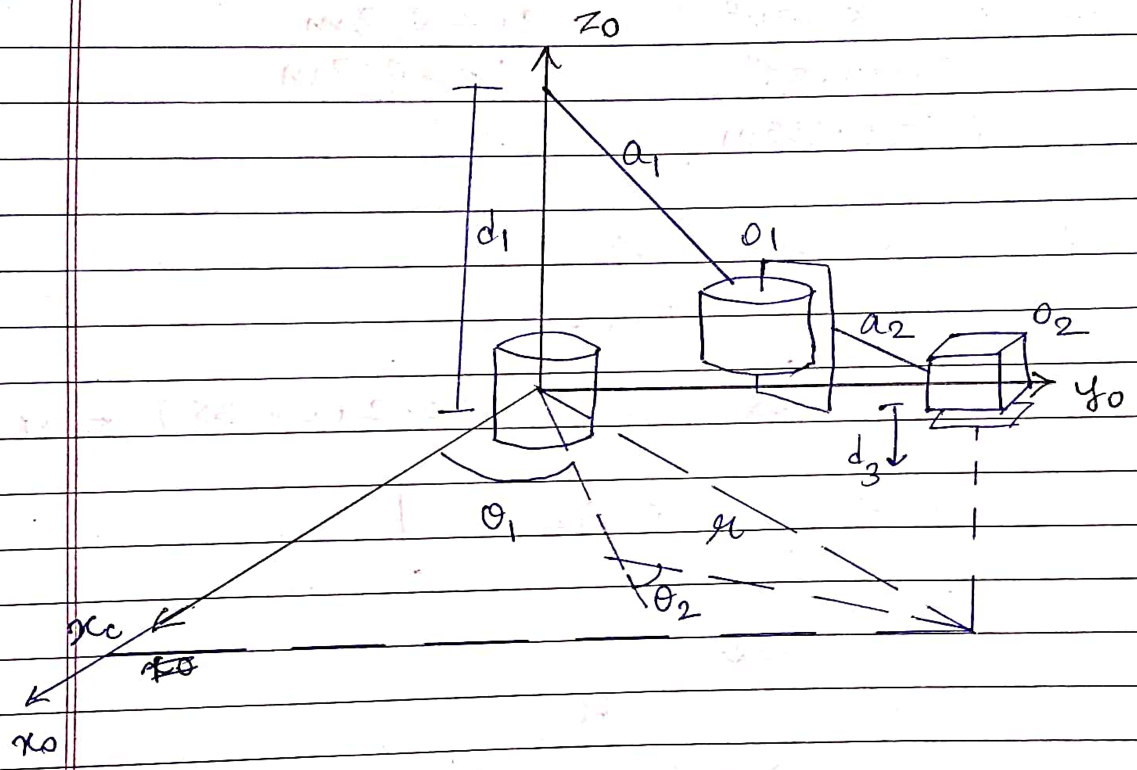
$z = S + L_1$

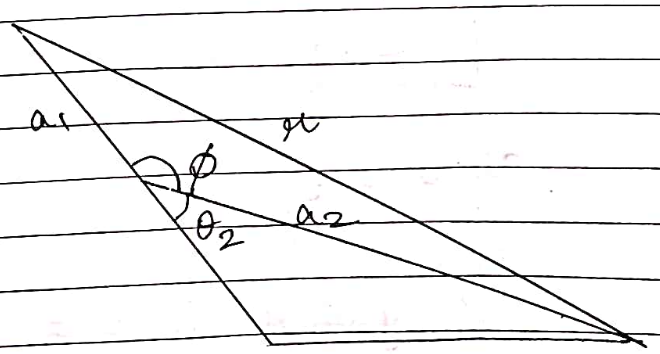
$z = \cancel{L_1} (L_2 + d_3)\sin\theta_2 + L_1$

$z = (0.2 + 0.135)\sin 17.35 + 0.2$

$\boxed{z \approx 0.30m}$

$\boxed{(2)}$ RRP SCARA MANIPULATOR





$\phi + \theta_2 = 180°$

$r^2 = a_1^2 + a_2^2 - 2a_1a_2\cos\phi$

$$\cos\phi = \frac{a_1^2 + a_2^2 - r^2}{2a_1a_2}$$

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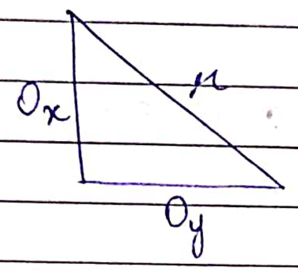
$$\therefore \cos(180-\theta_2) = -\cos\theta_2 = \cos\phi$$

$$\Rightarrow \cos\theta_2 = -\left(\frac{a_1^2+a_2^2-r^2}{2a_1a_2}\right) \quad \text{--- (1)}$$



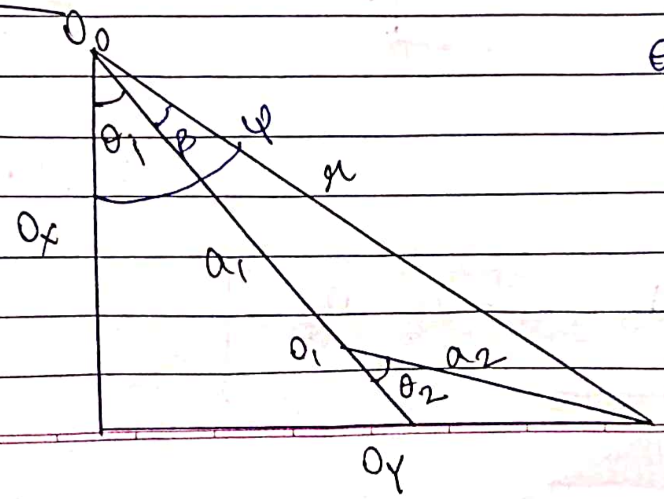
$$r^2 = O_x^2 + O_y^2$$

From eq^n (1)

$$\tan\theta_2 = \frac{\sin\theta_2}{\cos\theta_2} = \sqrt{\frac{1}{\cos^2\theta_2}-1}$$

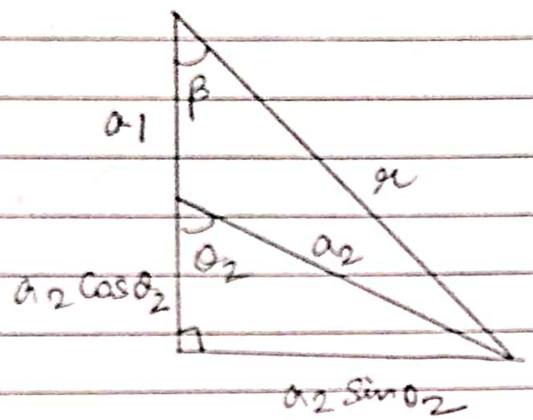
$$\boxed{\theta_2 = \tan^{-1}\left(\sqrt{\frac{1}{\cos^2\theta_2}-1}\right)}$$

For θ_1



$$\theta_1 + \beta = \psi$$
$$\theta_1 = \psi - \beta$$

$$\psi = \tan^{-1}\left(\frac{O_y}{O_x}\right)$$



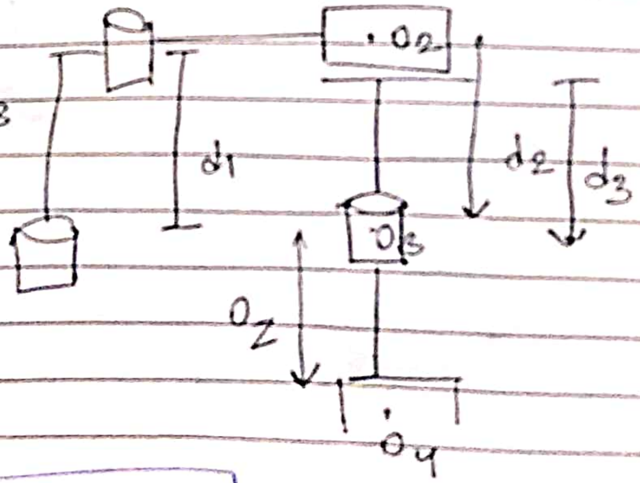
$$\beta = \tan^{-1}\left(\frac{a_2 \sin\theta_2}{a_1 + a_2\cos\theta_2}\right)$$

$$\Rightarrow \theta_1 = \psi - \beta$$

$$\boxed{\theta_1 = \tan^{-1}\left(\frac{O_y}{O_x}\right) - \tan^{-1}\left(\frac{a_2 \sin\theta_2}{a_1 + a_2\cos\theta_2}\right)}$$

For d_3

$$O_z = d_1 - d_2 - d_3$$



$$\boxed{d_3 = d_1 - d_2 - O_z}$$

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Finding θ_4

$$T_0^4 = \begin{bmatrix} C_2C_4 + S_{12}S_4 & S_{12}C_4 - C_{12}S_4 & 0 & a_1C_1 + a_2C_{12} \\ S_{12}C_4 - C_{12}S_4 & -C_{12}C_4 - S_{12}S_4 & 0 & a_1S_1 + a_2S_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\theta_4 = \theta_1 + \theta_2 - \alpha \quad \text{where} \quad \alpha = \tan^{-1}\left(\frac{r_{12}}{r_{11}}\right)$$

of $R = \begin{bmatrix} C\alpha & +S\alpha & 0 \\ S\alpha & -C\alpha & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Final equations are

$$\theta_2 = \tan^{-1}\left(\frac{\sqrt{1-B^2}}{B}\right) \quad \text{where} \quad B = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{a_2 \sin\theta_2}{a_1 + a_2\cos\theta_2}\right)$$

$$d_3 = d_1 - d_2 - O_z$$

$$\theta_4 = \theta_1 + \theta_2 - \alpha \quad \text{where} \quad \alpha = \tan^{-1}\left(\frac{r_{12}}{r_{11}}\right) \text{ of}$$

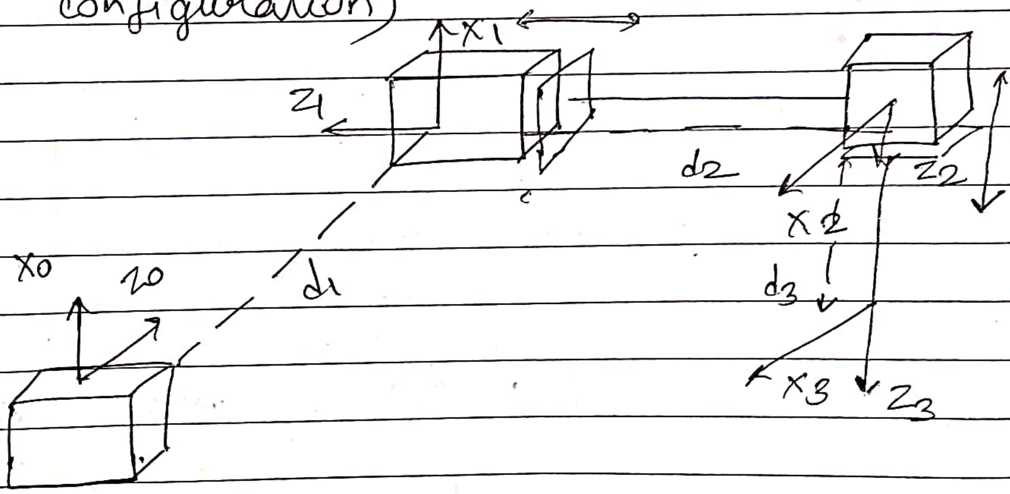
$$R = \begin{bmatrix} C\alpha & S\alpha & 0 \\ S\alpha & -C\alpha & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(3) joint velocity using end effector cartesian velocities.

$$\dot{x} = J(q)\dot{q}$$

$$\boxed{\dot{q} = (J^{-1}) \cdot (\dot{x})}$$

(7) DH parameter for a 3D printer (PPP configuration)



DH parameters

	a_i	α_i	d_i	θ_i
0-1	0	90	d1	0
1-2	0	90	-d2	-90
2-3	0	0	d3	0

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$$T_0^3 = T_0^1\, T_1^2\, T_2^3$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{T_0^3 = \begin{bmatrix} 0 & 0 & -1 & -d_3 \\ 0 & -1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

End effector position $x = -d_3$

$y = d_2$

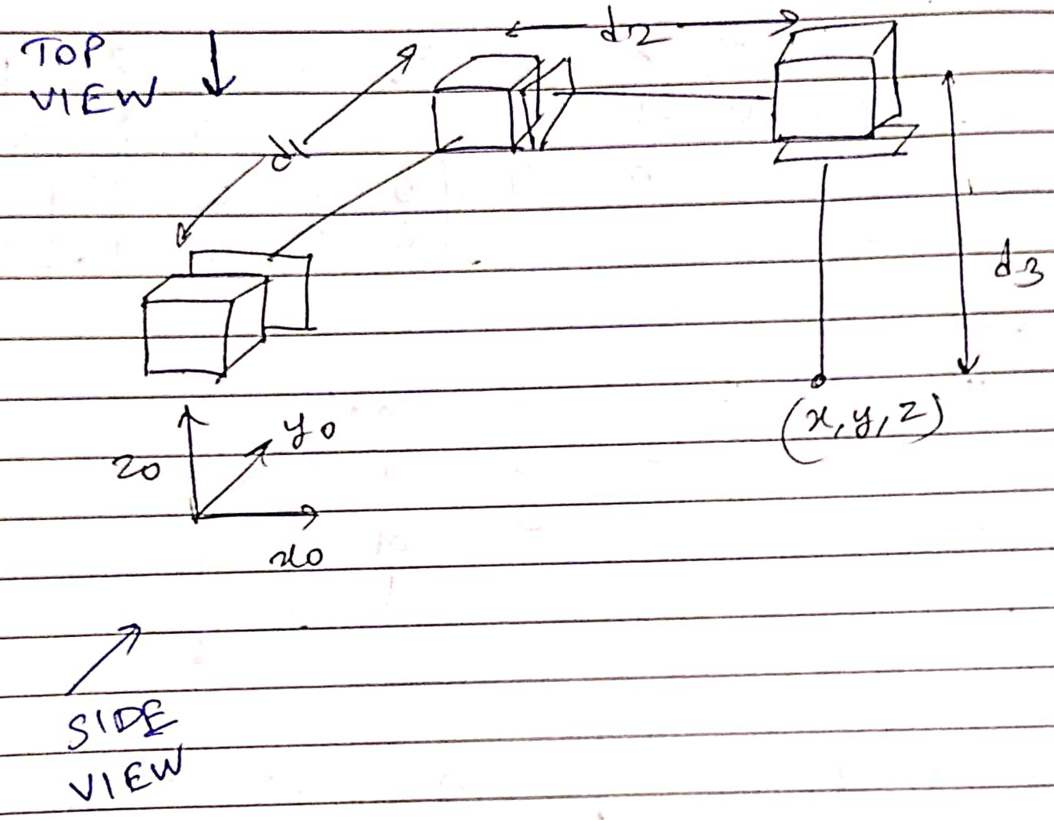
$z = d_1$

end effector velocity

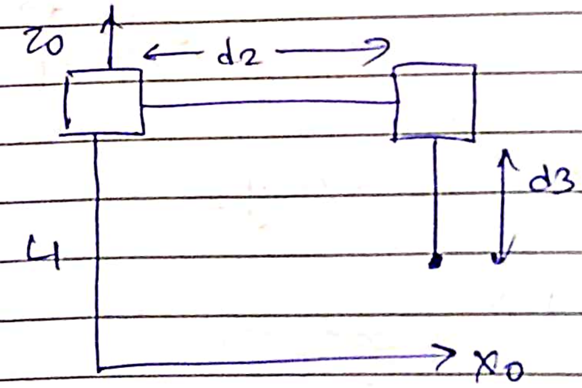
$$V = J\dot{X}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$

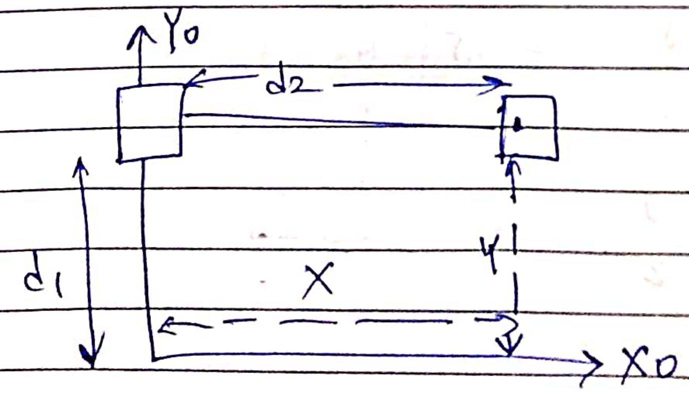
(8) INVERSE KINEMATICS FOR 3D PRINTER (PPP)



Side view



Top view



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Ans

$d_1 = Y$

$d_2 = X$

$d_3 = L_1 - Z$