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Assignment - 5

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ASSIGNMENT-5

PI CONTROLLER

Actuating signal is proportional to error signal plus integral of error signal

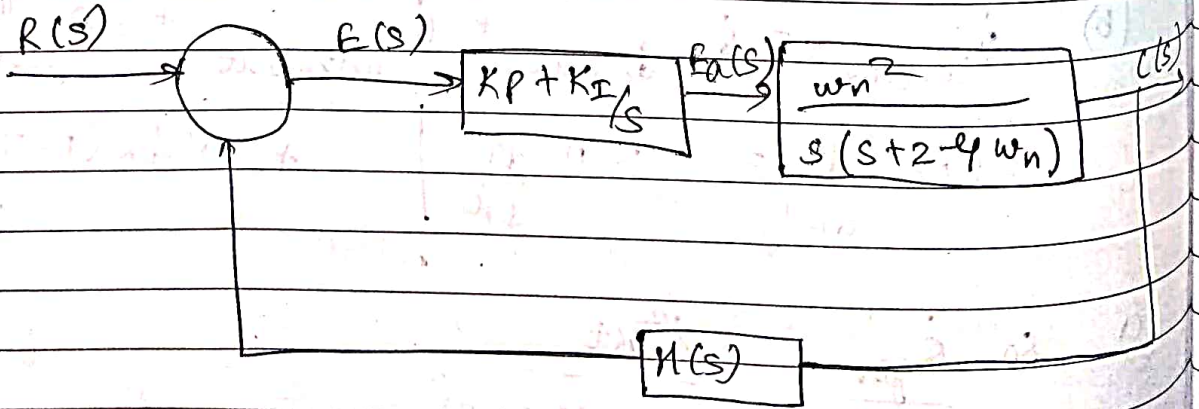
$$e_a(t) \propto e(t)$$
$$\propto \int e(t)\,dt$$

$$e_a(t) = K_p\, e(t) + K_I \int e(t)\,dt$$

K_p: proportional gain, K_I: integral gain

$$E_a(s) = K_p\, E(s) + K_I \left(\frac{E(s)}{s}\right)$$

$$\boxed{\frac{E_a(s)}{E(s)} = K_p + \frac{K_I}{s}}$$



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overall transfer function

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$\frac{C(s)}{R(s)} = \frac{\left(K_p + \frac{K_I}{s}\right)\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s}\right)}{1 + \left(K_p + \frac{K_I}{s}\right)\left(\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s}\right)}$$

$$\frac{C(s)}{R(s)} = \frac{(K_p s + K_I)\omega_n^2}{s^3 + 2\zeta\omega_n s^2 + K_p\omega_n^2 s + K_I\omega_n^2}$$

characteristic equation

$$s^3 + 2\zeta\omega_n s^2 + K_p\omega_n^2 s + K_I\omega_n^2 = 0$$

STEADY STATE ERROR

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + \frac{(K_p s + K_I)\omega_n^2}{s^2(s + 2\zeta\omega_n)}}$$

$$E(s) = \frac{s^2(s + 2\zeta\omega_n) \cdot R(s)}{s^2 + (2\zeta\omega_n + K_p\omega_n^2)s + K_I\omega_n^2}$$

$$e_{ss} = \lim_{s \to 0} s \cdot E(s)$$

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$$e_{ss} = \lim_{s \to 0} \frac{s^3 (s + 2\zeta\omega_n) R(s)}{s^3 + (2\zeta\omega_n + K_P \omega_n^2) s + K_I \omega_n^2}$$

(a) Step Input $R(s) = \frac{1}{s}$

$e_{ss} = 0$

(b) Ramp Input $R(s) = \frac{1}{s^2}$

$e_{ss} = 0$, $e_{ss}|_{PD} = \frac{2\zeta}{K_P \omega_n}$

(c) Parabolic input $R(s) = \frac{1}{s^3}$

$e_{ss} = \frac{2\zeta\omega_n}{K_I \omega_n} = \frac{2\zeta}{K_I \omega_n}$, $e_{ss}|_{PD} = \infty$