Zewail City for Science, Technology, and Innovation

University of Science and Technology

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Renewable Engineering Program

Advanced Control System Project

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Model of a car suspension system

Components of Suspension System and Their Practical Correlations:

- m1 = 2500 kg: Mass of body of vehicle; in actual suspension it will be the weight of car chassis and body minus wheels and axle assemblies.

- m2 = 320 kg: It includes non-extruded mass consisting of wheels, tires, and axle assemblies in usual configuration. It represents the mass that shall directly take road irregularities.

- let k1 = 80,000 N/m and k2 = 500,000 N/m, respectively, be the vehicle and wheel suspensions' spring stiffness coefficients. So, k1 would, therefore be the main springs supporting the car body thus absorbing larger movements between the body and chassis, while k2 springs in a wheel assembly-represent resilience to immediate impacts from the road surfaces.

- Damping coefficients: B1 = 350 N.s/m, B2 = 15,020 N.s/m. B1 represents the dampers-the shock absorbers-which prevent the car body from oscillating to much by dissipating the kinetic energy in such a way as not to make it bounce. The far more important value, B2 = 15,020 N.s/m, gives evidence for significant damping at the wheels to absorb the jolts coming from closer shocks, such as potholes or bumps.

Control System Implementation for Comfort:

The model control system is tuned in a manner to have a minimum relative displacement between the car body and the wheel assembly, x1 - x2. The relative displacement here implies the motion of the body w.r.t the wheel and thus has a direct relation to passenger comfort.

Practical Implementation:

Sensors and Actuators: On today's modern vehicle, sensors take the measure of travel from both the body and the wheels. These sensors measure the displacements and speeds of these motions, supplying in real time a knowledge of how the car interacts with road surfaces.

Control Force: The magnitude of damping force to be applied is calculated by a control algorithm from sensor signals. In other words, it is the actual force realized via actuators integrated in the suspension system to actively manipulate the damping characteristics. While this may increase on poor roads to reduce oscillations, on smooth roads, damping may be relaxed to result in a softer ride.

Feedback Loops: It is feed-forward in that continuous sensor data feeds into information from the control unit on how best to control the actuators with a view to optimizing damping force hence keeping at a minimum any impact of disturbance W via efficient and fast damping out of oscillations.

That is to say, suspension control systems adjust road conditions to keep the vehicle body as stable as possible at any time for the comfort and riding feel of passengers.

Write the modeling equations of the system:

Using Newton's second law, for m_1:

$$m_1 \frac{d^2x_1}{dt} = -k_1(x_1 - x_2) - B_1\left(\frac{dx}{dt} - \frac{dx}{dt}\right) + u$$

Similarly, for m_2:

$$m_2 \frac{d^2x_2}{dt^2} = k_1(x_1 - x_2) - k_2(x_2 - W) + B_1\left(\frac{dx}{dt} - \frac{dx_2}{dt}\right) - B_2\left(\frac{dx_2}{dt} - \frac{dW}{dt}\right) - u$$

Explanation:

On m_1:

Spring force from k_1:

$$k_1(x_1 - x_2)$$

Damping force from B_1:

$$B_1\left(\frac{dx}{dt} - \frac{dx_2}{dt}\right)$$

Control force u

On m_2:

Spring force from k_1 and k_2:

$$k_1(x_1 - x_2) - k_2(x_2 - W)$$

Damping force from B_1 and B_2:

$$B_1\left(\frac{dx}{dt} - \frac{dx_2}{dt}\right) - B_2\left(\frac{dx_2}{dt} - \frac{dW}{dt}\right)$$

Control force -u

Laplace transfrom for m_1 and m_2 from last question equations and converting into matrix form, we get

$$\begin{bmatrix} m_1 s^2 + B_1 s + k_1 & -(B_1 s + k_1) \\ -(B_1 s + k_1) & m_2 s^2 + B_1 s + B_2 s + k_1 + k_2 \end{bmatrix}\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} U(s) \\ k_2 W(s) + B_2 s W(s) - U(s) \end{bmatrix}$$

3- Verify that the input-output transfer matrix

$$m_1 \frac{d^2x_1}{dt} = -k_1(x_1 - x_2) - B_1\left(\frac{dx}{dt} - \frac{dx}{dt}\right) + u$$

$$m_2 \frac{d^2x_2}{dt^2} = k_1(x_1 - x_2) - k_2(x_2 - W) + B_1\left(\frac{dx}{dt} - \frac{dx_2}{dt}\right) - B_2\left(\frac{dx_2}{dt} - \frac{dW}{dt}\right) - u$$

Formulations above is:

$$(M_1S^2 + b_1S + K_1)X_1(S) - (b_1S + K_1)X_2(S) = U(S)$$

$$M_2\ddot{X}_2 = b_1(\dot{X}_1 - \dot{X}_2) + K_1(X_1 - X_2) + b_2(W - \dot{X}_2) + K_2(W - X_2) - U$$

By compensating $U(s)$ into equation (2), we obtaining:

$$(M_1S^2X_1(S) + (M_2S^2 + b_2S + K_2)X_2(S) = (b_2S + K_2)W(S)$$

Now, we will use matrices to solve the two equations as below:

$$\begin{bmatrix} (M_1S^2 + b_1S + K_1) & -(b_1S + K_1) \\ M_1S^2 & M_2S^2 + b_2 S + K_2 \end{bmatrix} \times \begin{bmatrix} X_1(S) \\ X_2(S) \end{bmatrix} \equiv \begin{bmatrix} U(S) \\ (b_2 S + K_2)W(S) \end{bmatrix}$$

Obtaining determinant:

$$A = \begin{bmatrix} (M_1S^2 + b_1S + K_1) & -(b_1S + K_1) \\ M_1S^2 & M_2S^2 + b_2S + K_2 \end{bmatrix}$$

$$\Delta = \left((M_1S^2 + b_1S + K_1) \times (M_2S^2 + b_2 S + K_2) - (M_1S^2) \times -(b_1S + K_1)\right)$$

$$\Delta = \begin{bmatrix} (M_1M_2S^4 + M_1 b_2S^3 + M_1 K_2S^2) + (M_2 b_1S^3 + b_1 b_2 S^2 + b_1 K_2S) \\ +(M_2 K_1S^2 + b_2 K_1 S + K_1 K_2) + (M_1 b_1S^3 + M_1 K_1S^2) \end{bmatrix}$$

Calculate the inverse of matrix A:

$$A^{-1} = \begin{bmatrix} M_2S^2 + b_2S + K_2 & (b_1S + K_1) \\ -(M_1S^2) & (M_1S^2 + b_1S + K_1) \end{bmatrix}$$

Simplifying it by multiplying with both side of equation:

$$\begin{bmatrix} X_1(S) \\ X_2(S) \end{bmatrix} \equiv \frac{1}{\Delta}\begin{bmatrix} (M_2S^2 + b_2 S + K_2) \times U(S) + (b_1S + K_1) \times (b_2 S + K_2)W(S) \\ -(M_1S^2) \times U(S) + (M_1S^2 + b_1S + K_1) \times (b_2 S + K_2)W(S) \end{bmatrix}$$

4- Plot the step responses of the transfer functions:

Using the system parameters and let W =0.1m.

For transfer function

$$G_1\left(\frac{x_1-x_2}{W}\right)$$

Settling time = 21.89s

Maximum Overshoot = 94.63%

Damped Frequency = 0.837 Hz

For transfer function

$$G_2\left(\frac{x_1-x_2}{W}\right)$$

Settling time = 53.83s

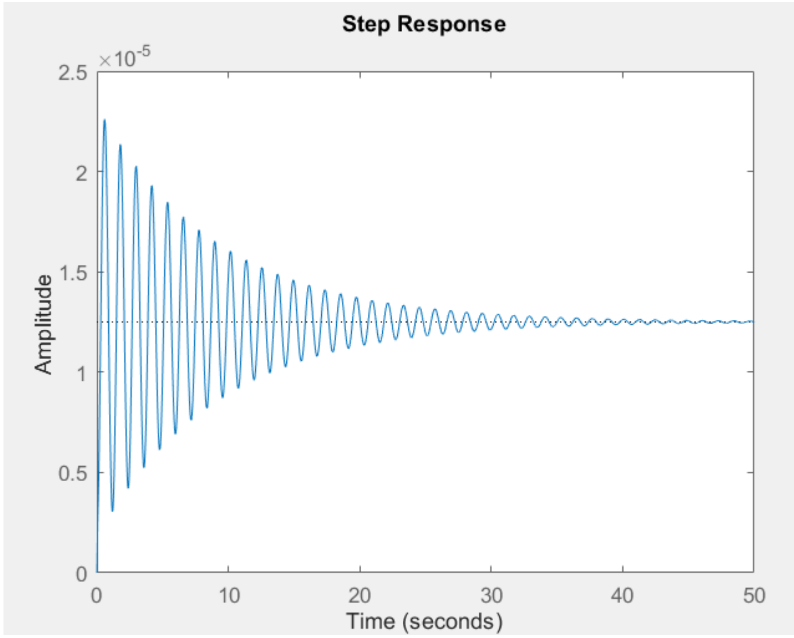
Maximum Overshoot = 4,540.47%

(indicating a significant overshoot due to high disturbance sensitivity)

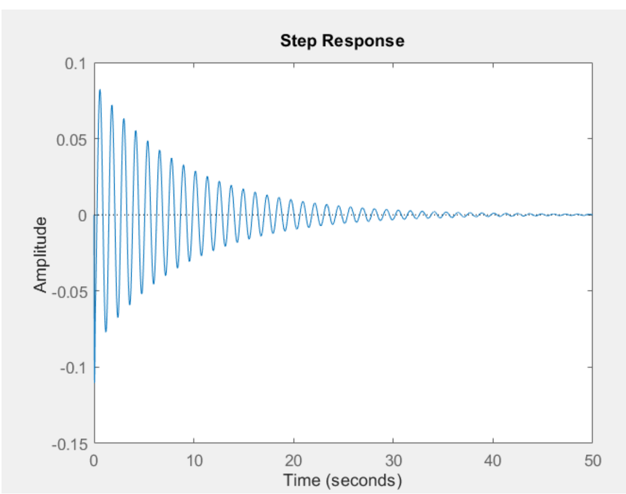
Damped Frequency = 0.837 Hz

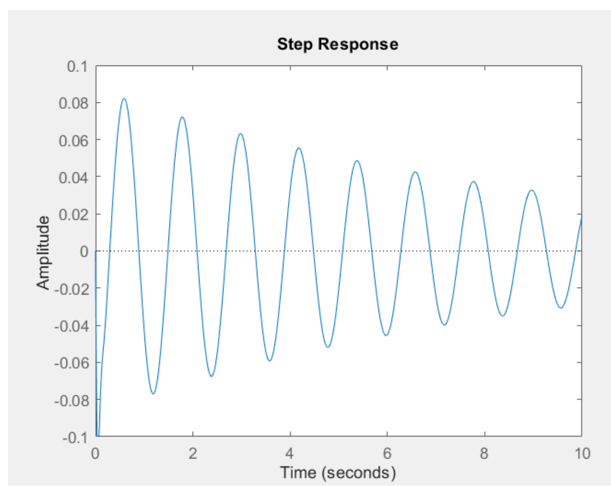
Note: System has high damping and takes longer to settle in response to disturbance (W). Overshoot for G2 is particularly large due to disturbance amplification.

For the open loop response to the unit step actuated force:



For the open loop response to 0.1 m step disturbance:





To see some details, we changed the axis to

[0 10 -0.1 0.1]

For the code:

Let

G1(s) = nump/denp

G2(s) = num1/den1

```
m1=2500;
m2=320;
k1=80000;
k2=500000;
b1 = 350;
b2 = 15020;

nump=[(m1+m2) b2 k2];
denp=[(m1*m2) (m1*(b1+b2))+(m2*b1) (m1*(k1+k2))+(m2*k1)+(b1*b2) (b1*k2)+(b2*k1)
k1*k2];
'G(s)1'
printsys(nump,denp)

num1=[-(m1*b2) -(m1*k2) 0 0];
den1=[(m1*m2) (m1*(b1+b2))+(m2*b1) (m1*(k1+k2))+(m2*k1)+(b1*b2) (b1*k2)+(b2*k1)
k1*k2];
'G(s)2'
printsys(0.1*num1,den1)
step(nump,denp)
step(0.1*num1,den1)
axis([0 10 -.1 .1])
```

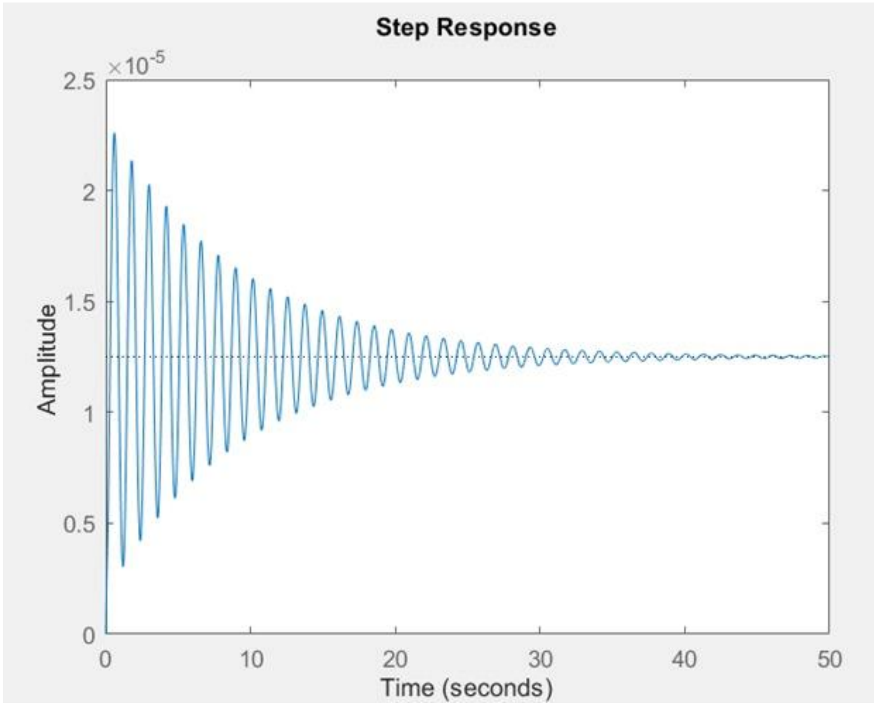
5- Write a state space model:

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{d^2x_1}{dt^2} \\ \frac{dx_2}{dt} \\ \frac{d^2x_2}{dt^2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{B_1}{m_1} & \frac{k_1}{m_1} & \frac{B_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{B_1}{m_2} & -\frac{k_1+k_2}{m_2} & -\frac{B_1+B_2}{m_2} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \frac{dx_1}{dt} \\ x_2 \\ \frac{dx_2}{dt} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{m_1} & 0 \\ 0 & 0 \\ -\frac{1}{m_2} & \frac{k_2}{m_2} + \frac{B_2 s}{m_2} \end{bmatrix} \begin{bmatrix} U \\ W \end{bmatrix}$$

Output is $y = x_1 - x_2$

$$y = \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \frac{dx_1}{dt} \\ x_2 \\ \frac{dx_2}{dt} \end{bmatrix}$$

6- Is the open-loop system stable?



This is the step response graph of a system that settles to some constant value after the initial major oscillations. Major observations to be noted are as follows:

Oscillatory Response: The response is oscillatory in nature with time-decaying amplitudes, which show that energy gets dissipated over time.

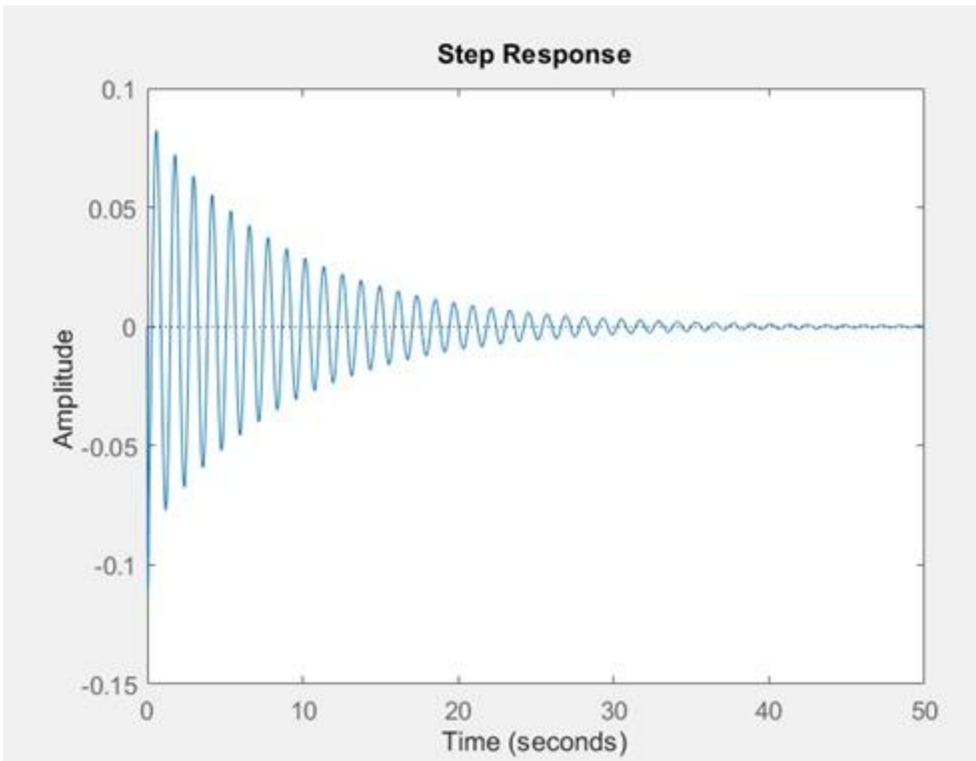
Approaching Steady State: It approaches a steady state where the oscillations die down, hinting at some stability since it does not diverge indefinitely.

Damped Oscillations: Such an oscillation amplitude reduces to indicate that there does exist a presence of damping forces within the system in order to bring it to its stable position.

Conclusion on Stability:

Open-Loop Stability: Indeed, one would have been able to deduce from this graph that it was an open-loop stable since there were no indications of unbounded behavior. The nature of underdamped, with the oscillations which start and settle down, suggests the presence of complex poles with negative real parts giving stability and non-zero imaginary parts causing the oscillation.

Practical Implication: Though technically this is a stable system, the amount of its initial oscillatory response may be problematic, as in practical application areas, fast settling or even small amounts of oscillation could result in system performance degradation and/or introduce mechanical wear. For these, feedback control approaches can receive the performance/stability enhancement which, among the reduction in overshoot, hastens the process of attaining settling time.



Based on the responses, some of the salient features which could be garnered are as below:

Step Response Graph: This graph represents the step response of the system. This apparently is a form of "damped" response that decays to zero with time.

The above graph further shows that the system contains some sort of damping because the oscillations get settled around a particular value. Moreover, during the approach, oscillation approaches a fixed and stable value that's an indication of the friction or resistance in a system.

System Dynamic Understanding: To understand the system dynamics, the magnitude needs to be known-for example, parameters of a system such as the damping coefficient or spring constant. Of that, it is an indication of the important traits, such as initial overshoot and asymptotic approach of response to zero in case of changes which may hint at a second-order or higher order system response.

Further Analysis Would Want More Information: More information would, therefore, be required to understand it as such. It would contain the trend which the system's response had taken with respect to an input over time through which constituent parts make up the system for getting further insight into the nature and dynamic characteristics of the same.

7- Check the system controllability and observability

For the controllability:

We use the next formula:

The system is controllable if the controllability matrix W_c *has rank* n, *where*
$W_c = [B \quad AB \quad \cdots \quad A^{n-1}B]$ *and* $\dim(x) = n$.

$$\text{Wc} = \begin{pmatrix} 0 & 0 & 0.0004 & 0 & -0.0005 & 225.3213 \\ 0.0004 & 0 & -0.0005 & 225.3213 & -0.0910 & 40647.9937 \\ 0 & 0 & -0.0031 & 1609.4375 & 0.1493 & -77303.2949 \\ -0.0031 & 1609.4375 & 0.1493 & -77303.2949 & -1.4545 & 796114.8718 \end{pmatrix}$$

From using matrix rank calculator: matrix rank is 4

Then the system is controllable

For the observability:

is completely observable if and only if the observability matrix

$$W_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

has rank n.

$$\text{Wo} = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -282 & -1.2338 & 1844.5 & 45.1713 \end{pmatrix}$$

From using matrix rank calculator: matrix rank is 3

Then the system is observable

8- Design a state feedback controller such that if the car experiences a 0.1-meter step disturbance, the car body will oscillate within +/- 0.1 meter and returns to a smooth ride (oscillations vanish) within 5 seconds.

To design feedback with given

- Settling time $T_s = 5$ s
- Damping ratio $\zeta = 0.7$

$$T_s = \frac{4}{\zeta\omega_n},\ \omega_n = \frac{4}{0.7{*}5} = 1.143\text{rad/s}$$

Therefore: $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -0.8 \pm j0.8$

$$s_{1,2} = -0.8 \pm j0.8, s_3 = -5, s_4 = -6$$

Then the desired characteristic equation equation is:

$$(s+0.8-j0.8)(s+0.8+j0.8)(s+5)(s+6)$$

By multiplication: $= s^4 + 12.6s^3 + 47.68s^2 + 96.08s + 38\ ^2$
Desired characteristic polynomial:

$$\lambda_{\text{desired}}\ (s) = s^4 + a_3s^3 + a_2s^2 + a_1s + a_0$$

The open loop characteristic polynomial is found by solving the determinant of (sI-A):

$$= \lambda_{open}(s) = s^4 + a_3s^3 + a_2s^2 + a_1s + a_0$$

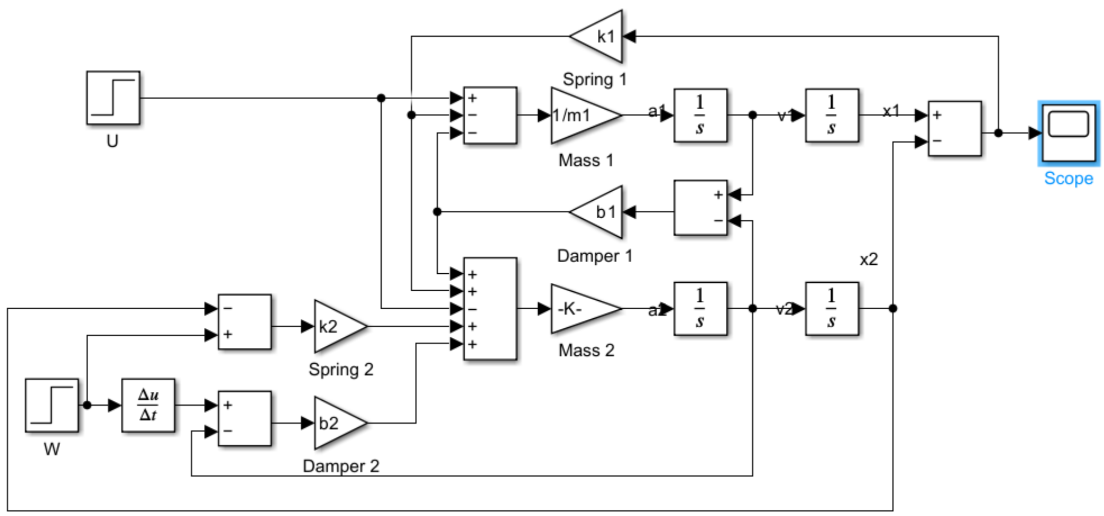
By using Ackermann's formula:

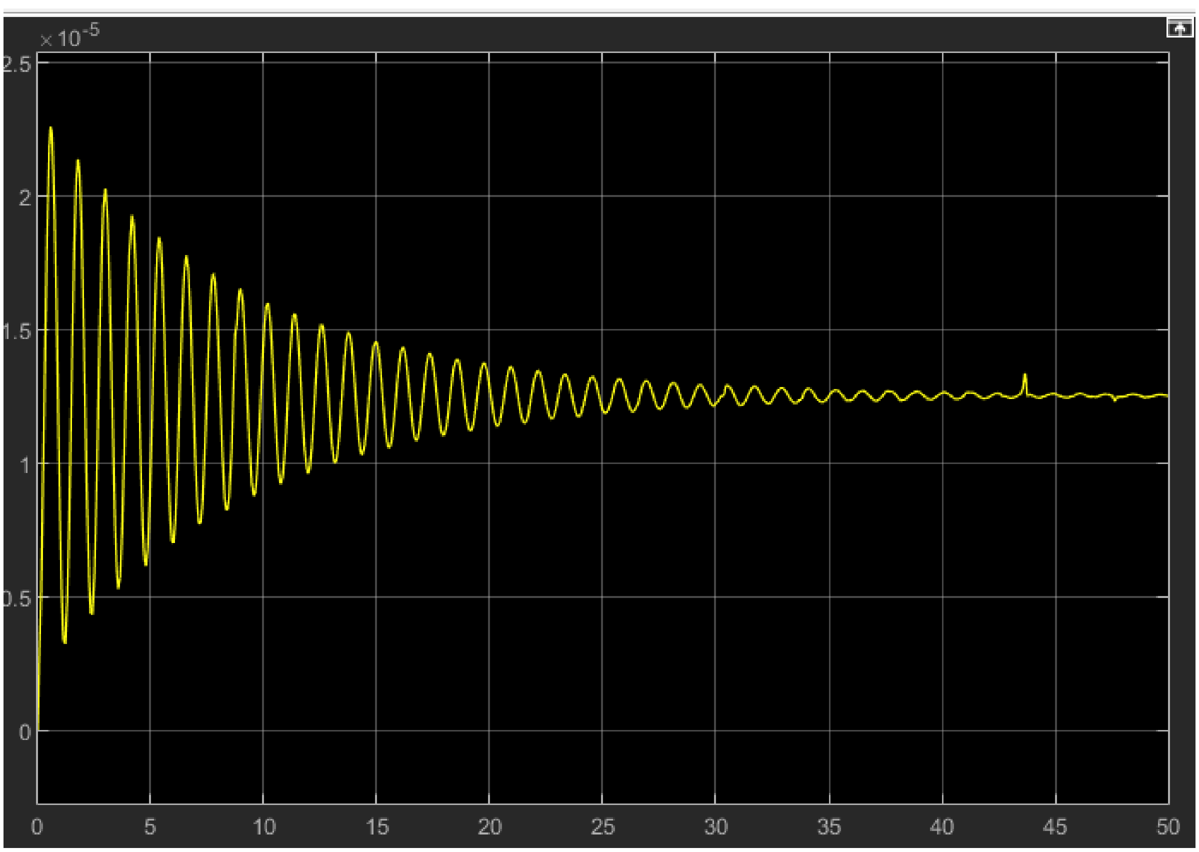
$$\mathrm{K} = [0 \quad 0 \quad 0 \quad 1]W_c^{-1}\Phi(A)$$

$$\phi(A) = \lambda_{\text{desired}}\ (A) - \lambda_{\text{open}}\ (A)$$

By using Numerical Calculations from getting Controllability Matrix and desired polynomial we can compute $K = [k_1, k_2, k_3, k_4] = [4.6, 0.25, 1730, 43]$

9- Use Simulink to simulate the system based on your design in part (8) to verify the design requirements. Plot the time responses of y1 and the control signal u.





10- Repeat parts (8) and (9) using an LQR design. Explain how to select the weighting matrices. Simulate the system. Plot the time responses of y1 and the control signal u.

Some basic but relevant steps and observations will include the selection of weighting matrices in effective ways to analyze the system.

1- The Diagonal elements of Q chosen considering the relative importance of the states, Greater values penalize large deviations in the corresponding state more. In this case, x1-x2 has been given higher priority in weighting factor -it means higher error penalties in displacement between the vehicle body and wheel in case of displacement difference.

2-Control Effort Weight (R): The scalar R weights the magnitude of the control effort. A smaller value of R would result in more aggressive control action and may be at the cost of higher energy or effort expended.

Simulating the System:

The simulation is performed under two conditions-open-loop and closed-loop:

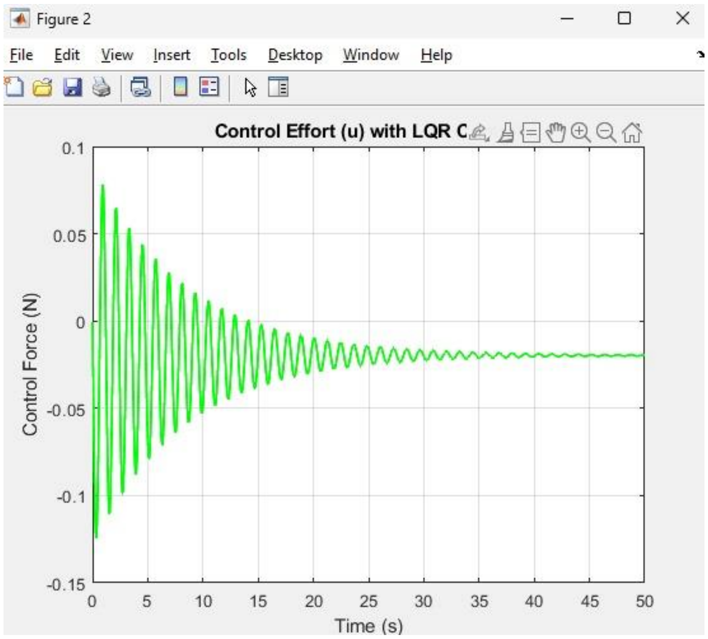
-Open-loop: The response of the system to road disturbances alone is observed with no active control intervention.

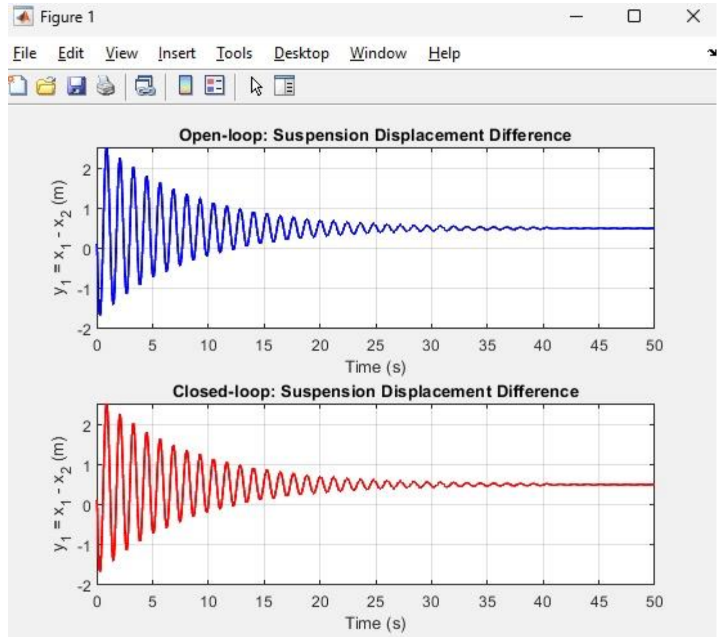
- Closed-loop: The system is working with the applied LQR controller and is continuously correcting itself against a cost function introduced by Q and R.

Time Response:

-y1: Suspension displacement difference - plots both open and closed loops; closed loop shall yield lesser oscillations and also better settling time as opposed to open loop and manifests the control.

-Control Input 'u' Control effort: Graph representing how the input to an actuator has varied over a time interval as necessary to enforce desired system response.





Comment on the Results:

- -Open-loop Response: It is observed from the graph that the suspensiondisplacement difference keeps oscillating without control. This is because of the response of the system under continued disturbances without correction.

- -Closed-loop Response: Clearly, the addition of LQR control significantly reduces oscillations and is able to stabilize the system much faster. It reflects on the effectiveness of the controller in minimizing the effect of disturbances and maintaining stability.

- -Control Effort: Initially large, the control effort starts to decrease as the system reaches stability. The peaks correspond to the controller acting against disturbances in order to maintain the system at the desired state.

All these steps and analyses, in effect, substantiate the performance of the LQR controller, but all the while illustrate how weighting in the matrices enters vitally to strike a balance between performance against control effort.

Code:

```
clear; clc; close all;
%% 1) Define System Parameters
m1 = 2500;   % Sprung mass (vehicle body)
m2 = 320;    % Unsprung mass (wheel/tire)
k1 = 80000;  % Spring constant of the suspension
k2 = 500000; % Spring constant of the tire
b1 = 350;    % Damping coefficient of the suspension
b2 = 15020;  % Damping coefficient of the tire
% Set D = 0 for all outputs in this example
D = 0;
%% 2) Define State-Space Matrices (Two-Input Version)
% States: x = [ x1, x1_dot, x2, x2_dot ]'
% x1 = displacement of the body, x2 = displacement of the wheel
%
% B has two columns:
%   - Column 1 = Force from the actuator (u1)
%   - Column 2 = Force from road disturbances (u2)
A = [  0,      1,        0,       0;
     -32, -0.14,      32,    0.14;
      0,      0,        0,       1;
    250,      0,  -1812.5, -48.03125 ];
B = [ 0,           0;
     1/2500,     0;
     0,          0;
    -1/320, 1609.4375 ];
C = [1, -1, 0, 0];  % Output = difference in displacement (x1 - x2)
%% 3) Design LQR Controller
% Weighting matrices for state and control
Q = diag([5000, 500, 500, 100]);  % State weights
R = 10;                           % Control effort weight
% Calculate LQR gain for the actuator force channel = B(:,1)
K = lqr(A, B(:,1), Q, R);
%% 4) Simulation Parameters
T = 0:0.01:50;  % Time vector for simulation (50 seconds)
W = 0.5;        % Amplitude of step disturbance
% Initial state with a slight displacement
x0 = [0.1; 0; 0; 0];
%% 5) Open-loop Simulation
% Open-loop to observe the effect of road disturbance alone
Ac_open_loop = A;
Bc_open_loop = B(:,2);  % Input from road disturbance
Cc_open_loop = C;
Dc_open_loop = D;
% Input for step disturbance matches Bc_open_loop
U_open_loop = W * ones(length(T), 1);
% Define the open-loop state-space system
sys_open_loop = ss(Ac_open_loop, Bc_open_loop, Cc_open_loop, Dc_open_loop);
% Simulate the open-loop response
[y_open_loop, t_open_loop, x_open_loop] = lsim(sys_open_loop, U_open_loop, T,
x0);
%% 6) Closed-loop Simulation
% Apply internal actuator force as a control input
Ac_closed_loop = (A - B(:,1)*K); % Modify A matrix by subtracting the control
contribution
Bc_closed_loop = B(:,2);         % Keep the road disturbance input
Cc_closed_loop = C;
```

```
Dc_closed_loop = D;
% Input for step disturbance remains the same
U_closed_loop = W * ones(length(T), 1);
% Define the closed-loop state-space system
sys_cl = ss(Ac_closed_loop, Bc_closed_loop, Cc_closed_loop, Dc_closed_loop);
% Simulate the closed-loop response
[y_closed_loop, t_closed_loop, x_closed_loop] = lsim(sys_cl, U_closed_loop,
T, x0);
%% 7) Calculate Control Effort
% Compute the control force at each time step
u_closed_loop = -K * x_closed_loop';
%% 8) Plotting Results
% Open-loop Response
figure;
subplot(2,1,1);
plot(t_open_loop, y_open_loop, 'b', 'LineWidth', 1.5); % Changed color to
blue
xlabel('Time (s)');
ylabel('y_1 = x_1 - x_2 (m)');
title('Open-loop: Suspension Displacement Difference');
grid on;
% Closed-loop Response
subplot(2,1,2);
plot(t_closed_loop, y_closed_loop, 'r', 'LineWidth', 1.5); % Changed color to
red
xlabel('Time (s)');
ylabel('y_1 = x_1 - x_2 (m)');
title('Closed-loop: Suspension Displacement Difference');
grid on;
% Control Effort
figure;
plot(t_closed_loop, u_closed_loop, 'g', 'LineWidth', 1.5); % Changed color to
green
xlabel('Time (s)');
ylabel('Control Force (N)');
title('Control Effort (u) with LQR Control');
grid on;
```