University of Science and Technology in Zewail City

REE 308

FINAL LABORATORY EXPERIMENT REPORT

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Abstract

The aim of this experiment is to visualize what we have learned through the course learning outcomes and apply it in real life and see how a real vibrating system act. The experiment consisted of 2 parts, one in which we analyze the response of a free vibration. and in the other one, we analyze the response of a forced vibration using a shaker to generate a base excitation.

1. Introduction:

Vibrating systems have a variety of applications in real life such as detecting the health of machinery, Vibration isolation to make machines safer, and they're also implemented in machines that require rotation like washers, helicopters, and many others. As we can see, vibrating systems are everywhere in our lives which shows the importance of their study. In this experiment, we studied a simple vibrating system consisting of a cantilever beam supported on an electromagnetic shaker. We were to measure the response of the cantilever beam in 2 situations: The first one is at free vibration, and the second at a forced vibration.

In the free vibration experiment, we gave the beam an initial velocity and displacement, then we measured its response with time using an Arduino connected to the laptop to show us the readings. Moreover, In the second one, we used the shaker to create base excitation, then we measured the response of the beam accordingly. In the following pages, you'll be able to see the procedures and the analysis we made in both experiments.

2. Main body:

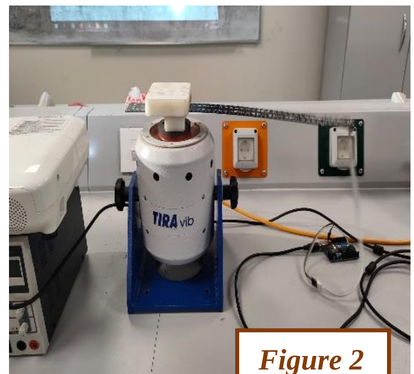
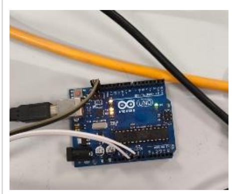
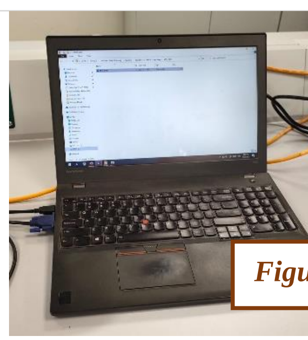
2.1. Free vibration Experiment:

2.1.1 The Experimental setup and its main components:

The Figure (1) below shows picture of the whole Experimental setup:

Figure 1

The table below contains the main components of the Experiment:

Name	Function	Photo
Shaker (TIRA Vib) which is connected to Cantilever beam with a sensor	-Shaker is hanging the cantilever beam while shaking -the sensor's main function is to read the waves and transmitted it into the Arduino	 ***Figure 2***
Arduino	Connected with the sensor to take the readings into the MATLAB (on the laptop)	 ***Figure 3***
Laptop	It was connected with the Arduino to take the readings and It was displayed in graphs on MAT LAB program	 ***Figure 4***

2.1.2 Describe the mathematical model of the system and its time response:

Mathematical model of free vibration with no forces rather than the initial one:

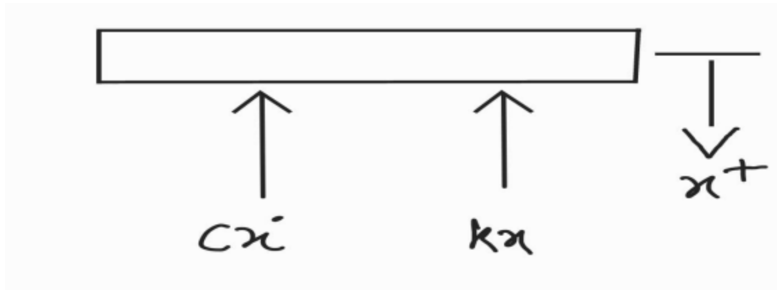


Figure 5

$$\sum F = m\ddot{x}$$
$$\sum F = -k(x-y) - c(\dot{x}-\dot{y}) = m\ddot{x}$$
$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$
$$m\ddot{x} + c\dot{x} + kx = f(t) \text{ —- eq . 1}$$

Where:
m is the mass of the beam
c is the damping coefficient
K is the stiffness
Y is the displacement done by base
$f(t)$ is the excitation force
X is the displacement
$\dot{X}$ velocity
$\ddot{X}$ is the acceleration

Since in the first experiment, then the excitation is set to equal to zero the equation will be:

$$m\ddot{x} + c\dot{x} + kx = 0$$

The solution for this differential equation at underdamped conditions will be:

$$x(t) = e - (\zeta\omega n)t\,(C1sin\,(\sqrt{1-\zeta 2\omega nt}) + C2cos\,(\sqrt{1-\zeta 2\omega nt})$$

Where:
$C1 = x(\dot{0}) + \frac{\zeta\omega nx(0)}{\omega n\sqrt{1-\zeta 2}}$ **, and** $C2 = x(0)$
And from eq 1 we can damping coefficient is equal to**:** $\zeta = c/(2\sqrt{km})$
And natural frequency and damped natural frequency are equal to:
ω**n**$= \sqrt{k/m}, Wd = 2\pi/Td = \omega n\sqrt{1-\zeta}$

2.1.3 Beam's equivalent mass and stiffness:

-Equivalent mass of cantilever beam of mass m is equal to:

$$m_{eq} = 0.23\, m$$

By knowing that:

$$mass\ (m) = Density * Volume$$

$$\textbf{Volume} = \textbf{Length} * \textbf{Width} * \textbf{height}$$

Figure 6

Data Sheet.

Equivalent Masses:

Cantilever beam of mass m.	$m_{eq} = 0.23m$
Cantilever beam of mass m carrying an end mass M.	$m_{eq} = M + 0.23m$
Simply supported beam of mass m.	$m_{eq} = 0.5m$
Simply supported beam of mass m carrying a mass M in the middle.	$m_{eq} = M + 0.5m$
Moment of Inertia, radius of gyration.	$I = mk^2$
Uniform rod pivoted about one end.	$I = \frac{mL^2}{3}$
Uniform rod pivoted about centre of gravity.	$I = \frac{mL^2}{12}$
Equivalent stiffness:	
Free end of cantilever beam.	$k_e = \frac{3EI}{L^3}$
In the middle of Simply supported beam .	$k_e = \frac{48EI}{L^3}$
Axial Stiffness of rod.	$k_e = \frac{EA}{L}$

$E_{steel} = 200 \times 10^9\ N/m^2$ $G_{steel} = 80 \times 10^9\ N/m^2$ $\rho_{steel} = 7800\ kg/m^3$ $\rho_{air} = 1.23\ kg/m^3$

$\rho_{water} = 1000\ kg/m^3$

C drag coefficient for cylinder = 1.0, Strouhal number for circular section = 0.2

-Then the total mass equivalent mass of cantilever beam is equal to:

$$m_{eq} = (0.23) * (Density) * (\textbf{Length} * \textbf{Width} * \textbf{height})$$

-Giving that the measured Length, Width, and height are **29.5 cm**, **39 mm**, and **1.5 mm** respectively. Also, giving that the density of steel ranges between **7,750** and **8,050** Kg/m^3.

-Converting all measured data to meter and take the average of maximum and minimum density to assume the nearest accurate number, then:

Length = **29.5*10^-2 m,** Width **= 39 *10^-3 m**, and height **= 1.5*10^-2 m**

Density = (7750+8050)/2 = **7900** Kg/m^3

-Plugging all data to equivalent mass equation to get it:

$$m_{eq} = (0.23) * (7900\ ) * (29.5 * 10\text{^} - 2\ \textbf{m} * 39\ \ * 10\text{^} - 3\ \textbf{m} * 1.5 * 10\text{^} - 2\ \textbf{m})$$

$$m_{eq} = 0.031357\ kg$$

-beam's stiffness:

Stiffness of cantilever beam is equal to:

$$\boldsymbol{K} = \frac{\mathbf{3}\,\boldsymbol{E}\,\boldsymbol{I}}{\boldsymbol{L}\text{^}\mathbf{3}}$$

Where Moment of inertia is equal to:

$$\boldsymbol{I} = \frac{\boldsymbol{base*height}}{\mathbf{12}} = \frac{\boldsymbol{Width*height}}{\mathbf{12}}$$

Plugging moment equation into beam's stiffness equation, then plugging all data to get beam's stiffness (giving that young modulus of steel (E)= **2*10^11 Nm-2**):

$$\boldsymbol{K} = \frac{\mathbf{3}*\boldsymbol{E}*\boldsymbol{Width}*\boldsymbol{height}}{\mathbf{12}*\boldsymbol{L}^{\mathbf{3}}}$$

$$\mathbf{K} = \frac{\mathbf{3*2*10^{11}*39*10^{-3}*1.5*10^{-2}}}{\mathbf{12*(29.5*10^{-2})^{3}}}$$

$$\mathbf{K} = \mathbf{256.355}\ \boldsymbol{N/mm2}$$

2.1.4 using the measured time response to determine the experimental system's logarithmic decrement, damping ratio, natural frequency, damping natural frequency:

i. The system's logarithmic decrement and get the damping ratio:

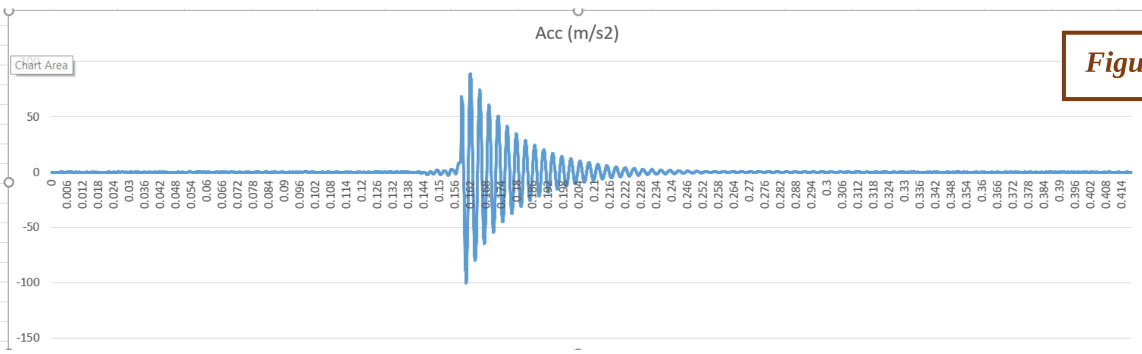


Figure 7

In order to get the system's logarithmic decrement, 2 points will be chosen from the graph:

a_1= (4.866,89.1056)

a_2= (4.98,71.369)

Logarithmic decrement method:

$$\ln \Delta = \frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}$$

Where:

- Δ is the ratio between the 2 points.
- ζ is the damping ratio.

Now we could get damping ratio by substituting the values in the above equation as shown below:

$$\ln \frac{71.369}{89.1056} = \frac{-2\pi\zeta}{\sqrt{1-\zeta^2}} \longrightarrow 1 - \zeta^2 = \frac{(-2\pi\zeta)^2}{(\ln\frac{71.369}{89.1056})^2}$$

$$\therefore \zeta = 0.0353$$

ii. *Damped Natural Frequency (ω_d):*

Is the inverse of the damped time period τ_d multiplied by 2π i.e

$$\boldsymbol{\omega_d = \frac{2\pi}{\tau_d}}$$

Where:

- τ_d is the difference between the time taken at the first acceleration and the 2nd acceleration respectively.

Thus, in this case:

τ_d is 0.114s (4.98-4.866)

$$\boldsymbol{\therefore\ \omega_d = \frac{2\pi}{0.114} = 55.116 Hz}$$

iii. *Natural frequency (ω_n):*

Could be determined by the following formula:

$$\boldsymbol{\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}}$$

$$\boldsymbol{\omega_n = \frac{55.116}{\sqrt{1-(0.02899)^2}} = 55.15 Hz}$$

2.1.5 Calculating the theoretical natural frequency, damping natural frequency, and equivalent damping coefficient of the system:

i. ***Natural Frequency (ω_d) :***

Is the inverse of the damped time period τ_d multiplied by 2π i.e

$$\omega_n = \sqrt{\frac{K}{m}}$$

Where:

- K is the stiffness of the beam which could be determined by the following Equation:

$$K = \frac{3EI}{L^3}$$

$$K = \frac{3(200*10^9)(1.0969*10^{-11})}{(0.295)^3} = 256.36N/mm2$$

the equivalent mass of the beam which is determined by:

$$m = 0.23 * \rho_{steel} * V$$

$$m = 0.23 * 7800 * (0.039 * 0.0015 * 0.295) = 0.031\,kg$$

Thus, in this case:

$$\therefore \omega_n = \sqrt{\frac{256.36}{0.031}} = 90.94Hz$$

ii. *Damped Natural frequency (ω_d):*

Could be determined by the following formula:

$$\boldsymbol{\omega_d = \omega_n * \sqrt{1-\zeta^2}}$$

Where:

- ζ is the damping ratio found in 4(i) which is 0.02899

$$\boldsymbol{\omega_d = 90.94 * \sqrt{1-0.0353^2}}$$

$$\boldsymbol{\omega_d = 90.88Hz}$$

iii. *Equivalent damping coefficient of the system (c):*

Could be determined by the following formula:

$$\boldsymbol{\zeta = \frac{c}{c_c}}$$

Rearranged to:

$$\boldsymbol{c = c_c * \zeta}$$

Where:

- Cc is the critical damping coefficient of the system which could be determined by the following Equation:

$$\boldsymbol{c_c = 2\sqrt{km}}$$

Thus

$$\boldsymbol{c_c = 2\sqrt{(256.36)(0.031)} = 5.638}$$

$$\boldsymbol{c = c_c * \zeta}$$

$$\boldsymbol{c = 5.638 * 0.0353}$$

$$\boldsymbol{c = 0.199}$$

2.1.6 plotting the measured time response and the corresponding theoretical response:

In Order to plot the corresponding theoretical graph, the initial conditions to be considered will be:

$$\dot{x}(0) = 0$$
$$x(0) = 3.0cm = 0.030m$$

And the following time response equation ($\ddot{u}(t)$) is going to be used to plot the graph:

$$u(t) = e^{-(\zeta\omega_n)t}(C_1 \sin\left(\sqrt{1-\zeta^2}\omega_n t\right) + (C_2 \cos\left(\sqrt{1-\zeta^2}\omega_n t\right)$$

$$\therefore\ u(t) = e^{-(\zeta\omega_n)t}(C_1 \sin(\omega_d t) + (C_2 \cos(\omega_d t)$$

$$\therefore\ \ddot{u}(t) = e^{-(\zeta\omega_n)t}((C_1\zeta^2{\omega_n}^2 - 2C_2\omega_d\zeta\omega_n - C_1{\omega_d}^2)\sin(\omega_d t)) + ((C_2\zeta^2{\omega_n}^2 + 2C_1\omega_d\zeta\omega_n - C_2{\omega_d}^2)\cos(\omega_d t))$$

For the initial conditions x (0) and ẋ (0)

$$C_1 = \frac{\dot{x}(0)+\zeta\omega_n x(0)}{\omega_n\sqrt{1-\zeta^2}} \qquad ; \qquad C_2 x(0)$$

Thus, for the theoretical plot:

$\zeta = 0.0353$

$\omega_d = 90.88Hz$

$\omega_n = 90.94Hz$

C1= $\frac{0+(0.0353)*(90.94)*(0.030)}{(90.94)*\sqrt{1-(0.0353)^2}} = 0.0106$

C2= 0.030

2.1.7 Compare between the experimental and theoretical results:

Measured Time response Graph:

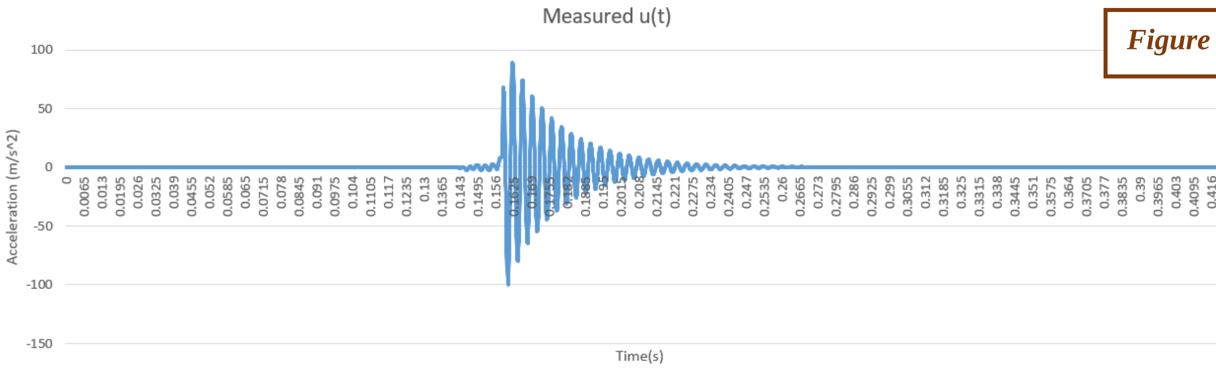


Figure 8

Theoretical Time Response Graph (with non-vibrating time period)

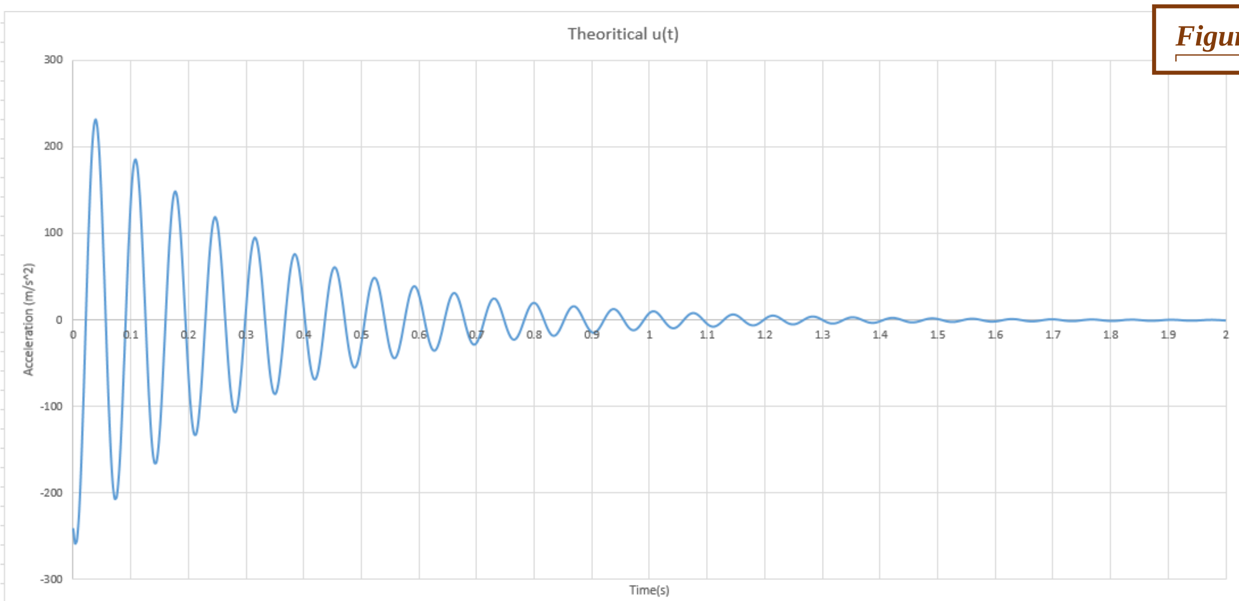


Figure 9

2.1.7 Comment on the result and sources of error:

Result comment:

As it can be observed the damped time period of the theoretical time response graph is larger than the measured time response thus ωd (damped natural frequency) is greater respectively. This leads to a significant discrepancy in the maximum acceleration between the two graphs. where the maximum acceleration in the theoretical time response graph is larger than the measured time response graph

Sources of Error:

Errors in the measured and theoretical time response u(t) are caused by:

- Constantly neglecting the mass of the accelerometer.
- unwanted system vibrations
- human errors
- Beam fixture

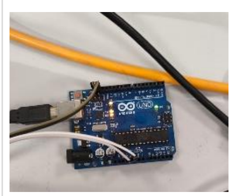
2.2 Forced Vibration Experiment:

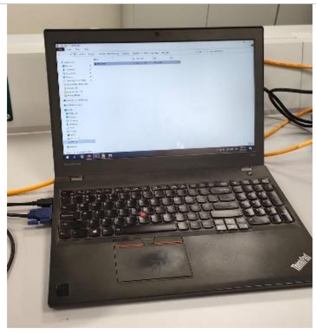
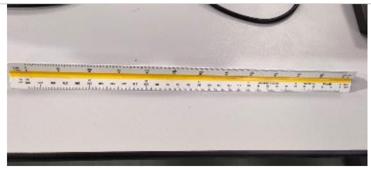
2.2.1 The Experimental setup and its main components:

The Figure (1) below shows picture of the whole Experimental setup:

Figure 10

The table below contains the main components of the Experiment:

Name	Function	Photo
Signal processor	Generate signals	
Amplifier	To modify signals and make it bigger to be transmitted to shaker to give it the needed frequencies	
Shaker (TIRA Vib) which is connected to Cantilever beam with a sensor	-Shaker is hanging the cantilever beam while shaking -the sensor's main function is to read the waves and transmitted it into the Arduino	
Arduino	Connected with the sensor to take the readings into the MATLAB (on the laptop)	

Laptop	It was connected with the Arduino to take the readings and It was displayed in graphs on MAT LAB program	
Ruler	To measure beam's dimensions	

2.2.2 Describe the mathematical model of the system and its time response:

We have the same system of a cantilever beam and there is a electrodynamic shaker and it can be considered as a spring-mass-damper system, the electrodynamic shaker is controlled by a variable frequency signal generator and a power amplifier to produce different displacements, it's also connected to an accelerometer to measure the vibration of the system and collect the needed data.

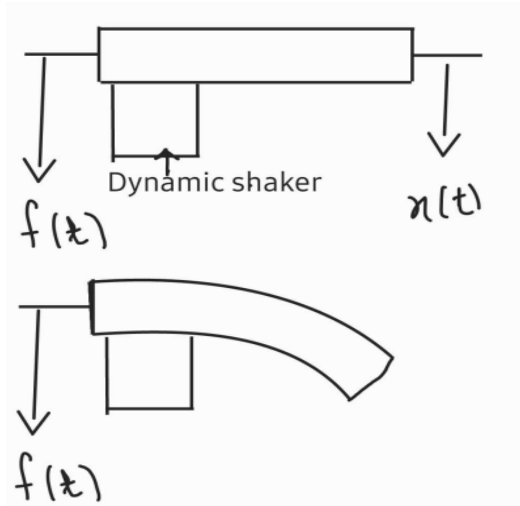


Figure 11

Free body diagram of the system:

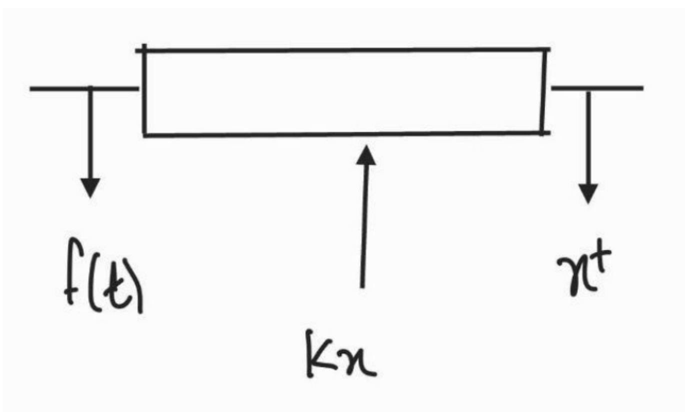


Figure 12

The equation of the system will be described with:

$$m\ddot{z} + c\dot{z} + kz = m\ddot{y}$$

Where is Z is $x - y$

M is the mass

c is the damping coefficient

K is the stiffness

The Govern equation of motion is showed below:

$$Z = Y\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}},$$

where r is ω / ωn and by knowing that Z is x-y, then the relation between x/y can be written as:

$$\frac{X}{Y} = \sqrt{\frac{1+(2zr)^2}{(1-(\mathrm{r})^2)^2+(2zr)^2}}$$

The non- homogeneous part can be expressed as:

$$x(t) = Xsin\,(\omega t - \psi)$$

$$x(t) = Y * \sqrt{\frac{1+(2zr)^2}{(1-(\mathrm{r})^2)^2+(2zr)^2}} * sin\,(\omega t - \emptyset)$$

Time response:

The following formula can be used to represent displacement as a function of time:

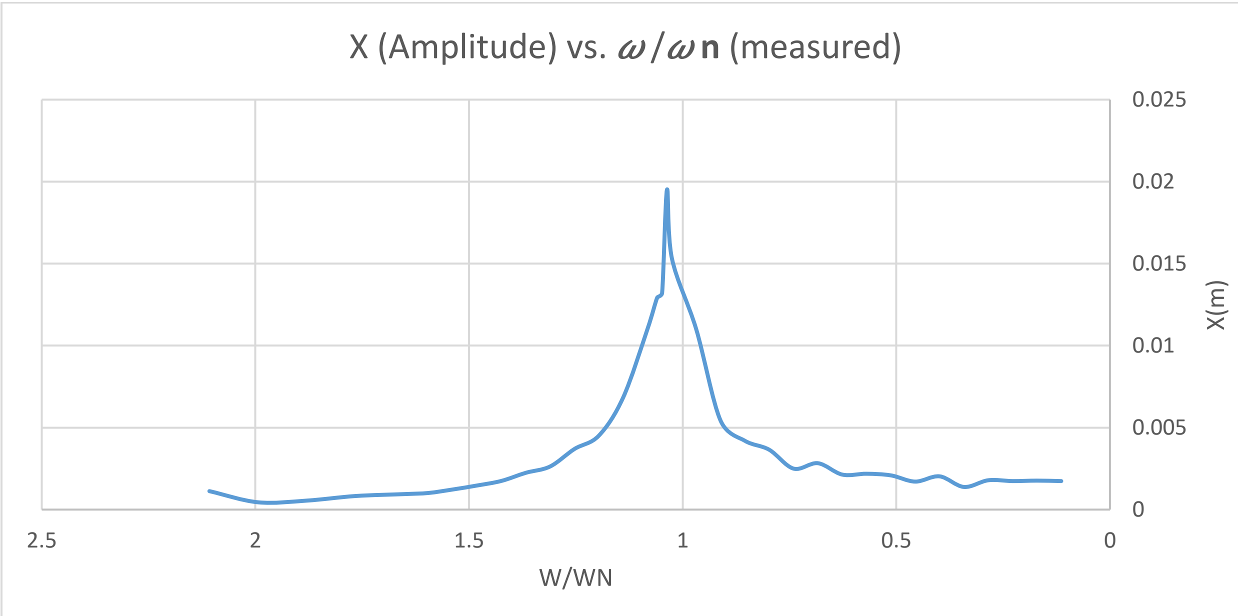
$$tan\psi = \frac{2r^3\zeta}{(1-r\hat{}2)+(2\zeta r)\hat{}2}$$

Using Transmissibility concept, x(t) equation could be written in another form:

$$Ft = F0 * \sqrt{\frac{1+(2zr)^2}{(1-(\mathrm{r})^2)^2+(2zr)^2}} * sin\,(\omega t - \emptyset)$$

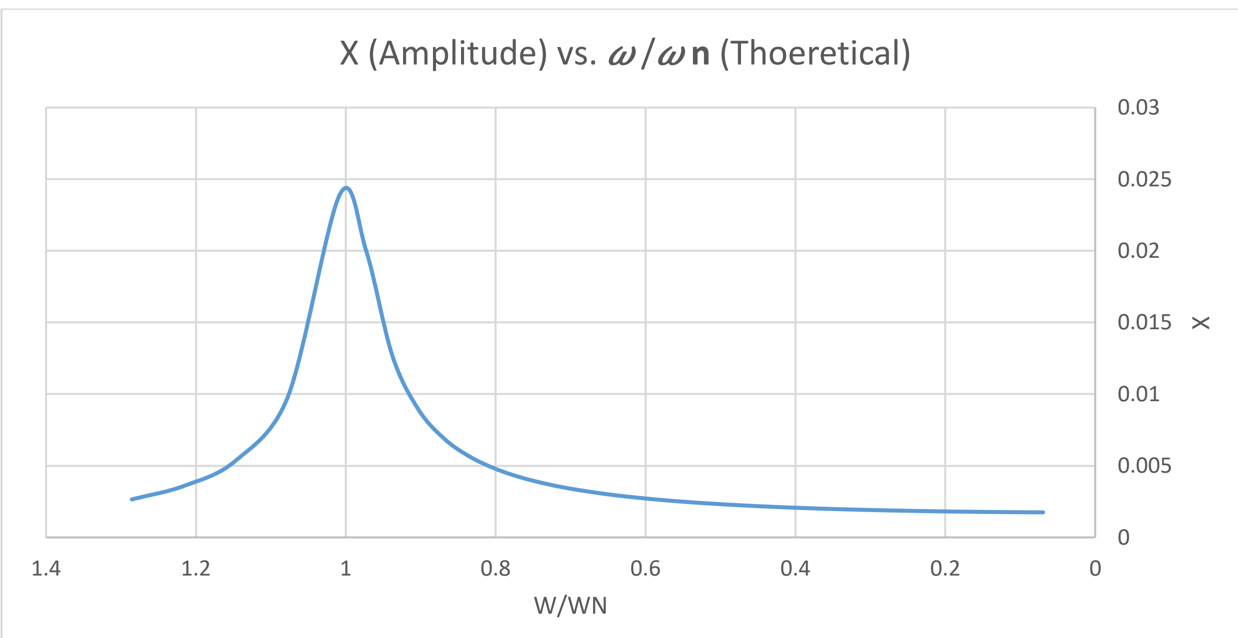
2.2.3 Plotting the measured displacement amplitudes versus the frequency ratio $\omega/\omega n$.:

Figure 13



2.2.4 Plotting the measured displacement amplitudes versus the frequency ratio $\omega/\omega n$.:

Figure 14



2.2.4 Comparing between the experimental and theoretical results:

(1)As we can see, the 2 graphs are similar to each other except that the graph plotted by the theoretical values was smother.

(2)The measured value of the $\boldsymbol{\omega}$**n** obtained from the first experiment was used to calculate the theoretical values of the amplitude.

(3) To calculate the theoretical value of $\boldsymbol{X}$ we assumed that $\boldsymbol{X} = \boldsymbol{Y}$ at low frequencies and took the value of $\boldsymbol{X}$ at one of these frequencies to be that of $\boldsymbol{Y}$.

(4) The values of the theoretical and measured amplitudes are so close to each other at low frequencies with an error percentage that can be calculated using any 2 values as following: $\boldsymbol{error} = \frac{|\mathbf{0.001739431 - 0.001747872}|}{\mathbf{0.001739431}} = \mathbf{0.4}\%$. However, at high frequencies, the error percentage increases. The differences in values can be explained by the sources of errors described below.

2.2.5 Comparing between the experimental and theoretical results:

Possible sources of error:

(1) Noises created by frequencies of the surroundings and captured by the sensor.

(2) The system isn't supposed to be a single degree of freedom system; however, we made this approximation.

(3) The measure values aren't 100% precise since the values was oscillating between a range of numbers.

(4) We were supposed to place an anchor sensor to write down a reference value for the data to ensure that the values entered manually at the beginning are correct.

3. Conclusion:

There are two experiments that have been done, and the main aim is to observe the difference between them. The two experiments were about free and forced vibrations, and we observed the behavior of the cantilever beam in many different conditions with different frequencies, then determined the theoretical and mathematical values of the natural frequency of a beam with a specific equivalent mass and stiffness. Next, determine the error percentage and guess the sources of error that caused this divergence. After observing errors, we did a comment and compared between the measured and theoretical results. Finally, we may take advantage of these sources of mistake to prevent major divergence and securely implement the motion in practical life applications.

4. References

[1] Experimental Data (Labview)

[2] 1- Rao, S. S., & Griffin, P. (2018). Mechanical vibrations.
Pearson.

[3] Lina, Design Engineer. (2018, September 17). What is structural steel? Purpose, use and most popular sections. London Structural Steel Fabricators - Steel Beams, RSJs, Steel Bars, Steel RSJ Suppliers, Fabrication, Design & Erectors - Steelo Ltd. https://www.steelo.co.uk/blog/2018/09/17/2125/

[4}What is the modulus of elasticity of steel? (2022, July 4). Byjus.com; BYJU'S. https://byjus.com/question-answer/what-is-modulus-of-elasticity-of-steel/