



Design a Linear Actuator for a Solar Tracking Mechanism

[Muhammad Abdelsallam 201801356](#)

[Mohamed Mansour 201-801-221](#)

[Phelopater Ramsis 202-001-171](#)

[Beshoy Zakaria 202-001-123](#)

[Aya Atya 202-000-247](#)

University of Science and Technology - Zewail City

REE 312 Machine Design II

Dr. [Mohamed Lotfi Eid Shaltout](#)

Eng. [Sherif Ezzeldin 201-600-076](#)

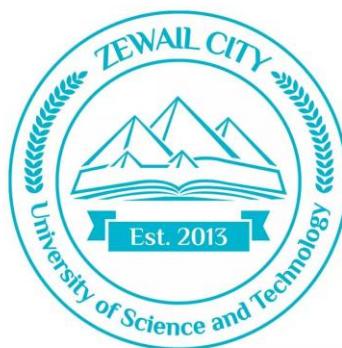




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Introduction

In PV solar tracking mechanisms, an electrically actuated mechanism is usually used to change the angle of the PV panel to track the changing Sun position during the day. The electric motor drives a set of gears. The output shaft of the gearbox is coupled to a power screw which drives a nut that slides inside a hollow cylinder and pushes the actuator rod. At a critical operating point, the linear actuator is required to provide a thrust force $F = 1500 \text{ N}$ at linear velocity $v = 4 \text{ m/s}$. At this point, the DC motor is running at $\omega_m = 1250 \text{ rpm}$. The available outer diameter D and pitch p sizes of single-square thread power screws are:

$D, \text{ mm}$	8	10	12	14	16	20	22	24	28	30	32	36	40	46	50	55	60	70	80
$p, \text{ mm}$	1.5	2	3	3	3	4	5	5	5	6	6	6	7	8	8	9	9	10	10

Power Screw Design

In the design process of a suitable power screw, the selection of material is the first choice to be made. The typical materials used in power screws are (in strength/cost order)

1. **Low-carbon steel:** Yield strength: 250-350 MPa
2. **Medium-carbon steel:** Yield strength: 400-600 MPa
3. **High-carbon steel:** Yield strength: 600-1000 MPa
4. **Stainless steel:** Yield strength: 200-2000 MPa
5. **Aluminum:** Yield strength: 70-100 MPa
6. **Brass:** Yield strength: 100-500 MPa

Starting the design process with Low-Carbon Steel ($\sigma_y = 250 \text{ MPa}$) and checking the diameter needed to achieve a Factor of Safety (FOS) of 2, if the diameters needed were extreme, Medium or High Carbon Steel will be chosen.

The coefficient of friction f for steel threaded pairs is taken to be 0.15 according to table 8.5. Choosing an outer diameter and pitch size from the available power screws $D = 10 \text{ mm}$, & $p = 2 \text{ mm}$ as an initial guess and to be increased if not satisfying the self-locking feature and minimum FOS of 2.

Self-Locking Feature

$$\begin{aligned} l &= np = 2 \text{ mm} \\ d_m &= d - p/2 = 10 - 2/2 = 9 \text{ mm} \\ d_r &= d - p = 10 - 2 = 8 \text{ mm} \end{aligned}$$

$$\pi f d_m > l \Rightarrow 0.15 * 9\pi > 2 \Rightarrow 4.24 \text{ mm} > 2 \text{ mm}$$

So, the self-locking feature is satisfied for the selected sizes.

Stress Analysis

Evaluating the raising and lowering torques by considering the torque required to overcome collar friction as the screw is a load component in the axial axis of installation.

$$T_R = \frac{Fd_m}{2} \left(\frac{l + \pi f d_m}{\pi d_m - fl} \right) + \frac{Ff_c d_c}{2}, \quad T_L = \frac{Fd_m}{2} \left(\frac{\pi f d_m - l}{\pi d_m + fl} \right) + \frac{Ff_c d_c}{2}$$

Assuming the collar diameter and friction to be $d_c = 15 \text{ mm}$, & $f_c = f = 0.15$

$$T_R = \frac{1500 * 0.009}{2} \left(\frac{0.002 + 0.15 * 0.009\pi}{0.009\pi - 0.15 * 0.002} \right) + \frac{1500 * 0.15 * 0.015}{2} = 0.585 + 1.69 \\ = 2.28 \text{ N.m}$$

$$T_L = \frac{1500 * 0.009}{2} \left(\frac{0.15 * 0.009\pi - 0.002}{0.009\pi + 0.15 * 0.002} \right) + \frac{1500 * 0.15 * 0.015}{2} = -0.37 + 1.69 \\ = -1.32 \text{ N.m}$$

The overall efficiency e in raising the load is.

$$e = \frac{Fl}{2\pi T_R} = \frac{1500 * 0.002}{2 * 2.28\pi} = 0.209 (20.9\%)$$

The axial nominal normal stress σ is.

$$\sigma = -\frac{4F}{\pi d_r^2} = -\frac{4 * 1500}{0.008^2 \pi} = -29.85 \text{ MPa}$$

The thread-root bending stress σ_b is, with one thread carrying $0.38F$

$$\sigma_b = \frac{6 * 0.38F}{\pi d_r p} = \frac{6 * 0.38 * 1500}{0.008 * 0.002\pi} = 68.04 \text{ MPa}$$

The body shear stress τ due to torsional moment T_R at the outside of the screw body is.

$$\tau = \frac{16 * T_R}{\pi d_r^3} = \frac{16 * 2.28}{0.008^3 \pi} = 22.68 \text{ MPa}$$

The tangential shear stress τ_{zx} , with one thread carrying $0.38T_R$

$$\tau_{zx} = -\frac{4 * 0.38T_R}{\pi d_r^2 p} = \frac{4 * 0.38 * 2.28}{0.008^2 * 0.002\pi} = 8.62 \text{ MPa}$$

The 3-D stresses are.

$$\sigma_x = 68.04 \text{ MPa}, \quad \tau_{xy} = 0$$

$$\sigma_y = -29.85 \text{ MPa}, \quad \tau_{yz} = 22.68 \text{ MPa}$$

$$\sigma_z = 0, \quad \tau_{xy} = 8.62 \text{ MPa}$$

The von Mises stress

$$\sigma' = \frac{1}{\sqrt{2}} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]^{1/2}$$

$$\sigma' = \frac{1}{\sqrt{2}} [(68.04 + 29.85)^2 + (-29.85)^2 + (68.04)^2 + 6(22.68^2 + 8.62^2)]^{1/2}$$

$$\sigma' = 95.53 \text{ MPa}$$

The factor of design

$$n_f = \frac{\sigma'}{\sigma_y} = \frac{250}{95.53} = 2.59 \text{ (accepted)}$$

The needed gear ratio between the motor and the power screw to deliver the required linear velocity is.

$$\omega_P = \frac{v}{l} = \frac{0.04 * 60}{0.002} = 1200 \text{ rpm}$$

$$G_R = \frac{\omega_R}{\omega_P} = \frac{1250}{1200} = \frac{25}{24}$$

The factor of design is greater than the desired factor of safety ($n_f = 2$) so smaller sizes are chosen for the power screw from the available date to check if they can satisfy the conditions. The new power screw outer diameter and pitch sizes are $D = 8 \text{ mm}$, $p = 1.5 \text{ mm}$

Self-Locking Feature

$$l = np = 1.5 \text{ mm}$$

$$d_m = d - p/2 = 8 - 1.5/2 = 7.25 \text{ mm}$$

$$d_r = d - p = 8 - 1.5 = 6.5 \text{ mm}$$

$$\pi f d_m > l \Rightarrow 0.15 * 7.25\pi > 1.5 \Rightarrow 3.42 \text{ mm} > 1.5 \text{ mm}$$

So, the self-locking feature is satisfied for the selected sizes.

Stress Analysis

Evaluating the raising and lowering torques by considering the torque required to overcome collar friction as the screw is a load component in the axial axis of installation.

$$T_R = \frac{Fd_m}{2} \left(\frac{l + \pi f d_m}{\pi d_m - fl} \right) + \frac{F f_c d_c}{2}, \quad T_L = \frac{Fd_m}{2} \left(\frac{\pi f d_m - l}{\pi d_m + fl} \right) + \frac{F f_c d_c}{2}$$

Assuming the collar diameter and friction to be $d_c = 12 \text{ mm}$, & $f_c = f = 0.15$

$$T_R = \frac{1500 * 0.00725}{2} \left(\frac{0.0015 + 0.15 * 0.00725\pi}{0.00725\pi - 0.15 * 0.0015} \right) + \frac{1500 * 0.15 * 0.012}{2} \\ = 1.19 + 1.35 = 2.54 \text{ N.m}$$

$$T_L = \frac{1500 * 0.00725}{2} \left(\frac{0.15 * 0.00725\pi - 0.0015}{0.00725\pi + 0.15 * 0.0015} \right) + \frac{1500 * 0.15 * 0.012}{2} \\ = 0.45 + 1.35 = 1.8 \text{ N.m}$$

The overall efficiency e in raising the load is.

$$e = \frac{Fl}{2\pi T_R} = \frac{1500 * 0.0015}{2 * 2.54\pi} = 0.141 (14.1\%)$$

The axial nominal normal stress σ is.

$$\sigma = -\frac{4F}{\pi d_r^2} = -\frac{4 * 1500}{0.0065^2 \pi} = -45.2 \text{ MPa}$$

The thread-root bending stress σ_b is, with one thread carrying $0.38F$

$$\sigma_b = \frac{6 * 0.38F}{\pi d_r p} = \frac{6 * 0.38 * 1500}{0.0065 * 0.0015\pi} = 111.65 \text{ MPa}$$

The body shear stress τ due to torsional moment T_R at the outside of the screw body is.

$$\tau = \frac{16 * T_R}{\pi d_r^3} = \frac{16 * 2.54}{0.0065^3 \pi} = 47.1 \text{ MPa}$$

The tangential shear stress τ_{zx} , with one thread carrying $0.38T_R$

$$\tau_{zx} = -\frac{4 * 0.38T_R}{\pi d_r^2 p} = \frac{4 * 0.38 * 2.54}{0.0065^2 * 0.0015\pi} = 19.39 \text{ MPa}$$

The 3-D stresses are.

$$\sigma_x = 111.65 \text{ MPa}, \quad \tau_{xy} = 0$$



$$\sigma_y = -45.2 \text{ MPa}, \quad \tau_{yz} = 47.1 \text{ MPa}$$

$$\sigma_z = 0, \quad \tau_{xy} = 19.39 \text{ MPa}$$

The von Mises stress

$$\sigma' = \frac{1}{\sqrt{2}} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]^{1/2}$$

$$\sigma' = \frac{1}{\sqrt{2}} [(111.65 + 45.2)^2 + (-45.2)^2 + (111.65)^2 + 6(47.1^2 + 19.39^2)]^{1/2}$$

$$\sigma' = 165.34 \text{ MPa}$$

The factor of design

$$n_d = \frac{\sigma'}{\sigma_y} = \frac{250}{165.34} = 1.51 \text{ (rejected)}$$

The factor of design is less than the desired factor of safety ($n_f = 2$) so the previous sizes are chosen. The power screw outer diameter and pitch sizes are $D = 10 \text{ mm}$, $p = 2 \text{ mm}$.

Gear Box Design

Gear properties

Teeth determination

Since the gear ratio is small enough it can be achieved with one stage of gears

$$\frac{N_1}{N_2} = \sqrt{25/24}$$

Trying $N_1 = 52$ yields $N_2 = 49.96$ and by choosing $N_2 = 50$.

For simplification the second gear set will have the same ratio thus $N_3 = N_1, N_4 = N_2$

Leading to a total relative error of 1.94% which is acceptable for power.

Transmission.

Diameter determination

In order to keep the gear box as compact as possible a small module of "1 mm/tooth" is selected

Yielding diameters of $d_1 = d_3 = 50\text{mm}$, $d_2 = d_4 = 52\text{ mm}$

Properties:

spur gear, 20°full depth, $m = 1\text{ mm}$, $\sigma_y = 250\text{ MPa}$ (Low-carbon steel)

Force analysis.

Performing force analysis for every gear set in the gear box arrangement as shown in fig 1.

To deliver a torque of 2.54 Nm. to the output shaft

Starting from gear 4, the output gear:

$$T_4 = 2.540 \text{ N.m}$$

$$w_t = \frac{2T}{d_4} = 97.7 \text{ N} \rightarrow F_{43}^t = 97.7 \text{ N}$$

$$F_{43}^r = F_{43}^t \tan(20) = 35.56 \text{ N}$$

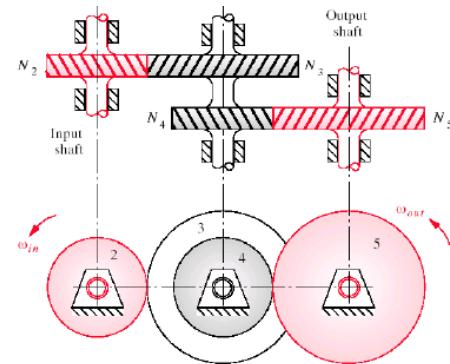
$$F_{43} = \frac{F_{43}^t}{\cos(20)} = 108.12 \text{ N}$$

$$T_3 = T_2 \rightarrow F_{43}^t d_3 = F_{21}^t d_2$$

$$\rightarrow F_{21}^t = 93.94 \text{ N}$$

$$F_{21}^r = F_{21}^t \tan(20) = 34.19 \text{ N}$$

$$F_{21} = \frac{F_{21}^t}{\cos(20)} = 110.67 \text{ N}$$



Factors of safety

Assumptions:

The Gears are made of hardened steel.

Only analysis for pinions ($G_3 \& G_4$) is required since the gears ($G_1 \& G_2$) are of the same material and equal diameters (respectively) and carry higher torques thus having higher strength and are under lower threat of breaking.

Gear specs:

Tooth system: $\Phi_n = 20^\circ$

Reliability $R = 0.95$, Cycles $N = 10^9$, $Ko = 1$, Design factor $nd = 2$, Tooth system: $\Phi_n = 20^\circ$

Quality number: $Qv = 6$, grade 1 material

Assuming $m_b \geq 1.2 \rightarrow k_b = 1$

AGMA Analysis:

$$Y_{G3} = 0.409 \rightarrow Y_{G4} = 0.412 \text{ (by interpolation)}$$

$$J_{G3} = J_{G4} = 0.443$$

$$V = \omega_4 \frac{\pi}{30} \frac{d_4}{2} = 3.26 \text{ m/s}$$

$$W_t = 97.7$$

$$B = 0.25(12 - 6)^{\frac{2}{3}} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

$$k_v = \left(\frac{59.77 + \sqrt{200 * 3.26}}{59.77} \right)^{0.8255} = 1.3413$$

For a reliability of 0.99

$$K_R = Y_Z = 0.5 - 0.109 \ln(0.001) = 1.002 \approx 1$$

From the tables 14-14 & 14-15, For cycles $N = 10^9$

$$Y_{N3} = 1.3558(10^9)^{-0.0178} = 0.9376$$

$$Y_{N4} = 1.3558 \left(10^9 * \frac{50}{52} \right)^{-0.0178} = 0.9382$$

$$Z_{N3} = 1.4488(10^9)^{-0.023} = 0.8995 \approx 0.9$$

$$Z_{N4} = 1.4488 \left(10^9 * \frac{50}{52} \right)^{-0.023} = 0.9003 \approx 0.9$$

taking $F = 10\text{mm}$ as an initial guess

Taking the necessary assumptions

$$C_{mc} = C_{pm} = C_e = 1$$

$$F = \frac{10}{25.4} = 0.3937 \text{ in}$$

$$\frac{F}{10d_p} = \frac{10}{10(50)} = 0.02 \rightarrow \frac{F}{10d_p} = 0.05$$

$$\rightarrow C_{pf} = 0.05 - 0.025 = 0.025$$

$$C_{ma} = 0.127 + 0.0158(0.3937) - 0.930 * 10^{-4}(0.3937) = 0.133206$$

$$K_H = 1 + 1(0.025 + 0.133206) = 1.15821$$

Assuming a helix angle $\psi = 20^\circ$ $K_T = Y_\theta = C_f = Z_R = m_n = 1$

$$I = \frac{\cos(20) \sin(20)}{2(1)} \frac{52/50}{52/50 + 1} = 0.082$$

Assuming the gears are made of grade 1 steel.

$$Z_E = 191\sqrt{MPa}$$

$$H_B = 200$$

$$S_t = 0.568H_B + 83.3 = 196.6 MPa$$

$$S_c = 2.22H_B + 200 = 644 MPa$$

$$Z_W = C_H = 1$$

From figure 14-6: $Y_{J3} = Y_{J4} = 0.58$

For bending

$$\sigma_{Bend} = W^t K_o K_v K_s \frac{1}{mF} \frac{K_H K_B}{Y_J}, S_F = \frac{S_t Z_N Z_W}{Y_\theta Y_Z \sigma_B}$$

$$\sigma_{G3} = \sigma_{G4} = 97.7(1.3413) \left(\frac{1}{1(10)} \right) \left(\frac{1.15821}{0.58} \right) = 26.168 MPa$$

$$S_{F3} = S_{F4} = \frac{196.6 * 0.9}{26.168} = \boxed{6.74} \rightarrow Safe##$$

For wear

$$\sigma_c = Z_E \left(W^t K_o K_v K_s \frac{K_H Z_R}{d_P F I} \right)^{0.5}, S_H = \frac{S_c Z_N Z_W}{Y_\theta Y_Z \sigma_c}$$

$$\sigma_{G3} = 191 \left(97.7 * (1.3413) * \frac{1}{50 * 10 * 0.081} \right)^{0.5} = 343.57 MPa$$

$$\sigma_{G4} = 191 \left(97.7 * (1.3413) * \frac{1}{52 * 10 * 0.081} \right)^{0.5} = 336.89 MPa$$

$$S_{H3} = \frac{644 * 0.9}{343.57} = \boxed{1.687} \rightarrow Safe##$$

$$S_{H4} = \frac{644 * 0.9}{336.89} = \boxed{1.72} \rightarrow Safe##$$

Comparing S_F with S_{H3}^2 and S_{H4}^2 shows that the threat lies in wearing of gear 3.

DC Motor Specs

$$T_1 = T_4 \left(\frac{d_2}{d_1} \right) \left(\frac{d_4}{d_3} \right) = 2.3483 \text{ Nm}$$

$$\omega_1 = \left(\frac{d_1}{d_2} \right) \left(\frac{d_3}{d_4} \right) \omega_4 = 1248.48 \text{ rpm}$$

Shaft Design

Forces on shaft (could be illustrated below in the table below, and under the figure under the table is showing the design that we are working on).

Input shaft	$F_1^r = 34.19 \text{ N}$	$F_1^T = 93.94 \text{ N}$	$D_1 = 50 \text{ mm}$
Intermediate shaft	$F_2^r = 34.19 \text{ N}$ $F_3^r = 35.56 \text{ N}$	$F_2^T = 93.94 \text{ N}$ $F_3^T = 97.7 \text{ N}$	$D_2 = 52 \text{ mm}$ $D_3 = 50 \text{ mm}$
Output	$F_4^r = 35.56 \text{ N}$	$F_4^T = 97.7 \text{ N}$	$D_4 = 52 \text{ mm}$

Then, we will calculate torque for Input, Intermediate, and output shaft. It will be calculated using the next formula:

$$T = F^T * \frac{D}{2}$$

For Input Shaft:

$$T_1 = F_1^T * \frac{D_1}{2} = 93.94 * \frac{50 * 10^{-3}}{2} = 2.3485 \text{ N} \cdot \text{m}$$

For intermediate Shaft:

$$T_2 = F_2^T * \frac{D_2}{2} = 93.94 * \frac{52 * 10^{-3}}{2} = 2.44244 \text{ N} \cdot \text{m}$$

$$T_3 = F_3^T * \frac{D_3}{2} = 97.7 * \frac{50 * 10^{-3}}{2} = 2.4425 \text{ N} \cdot \text{m}$$

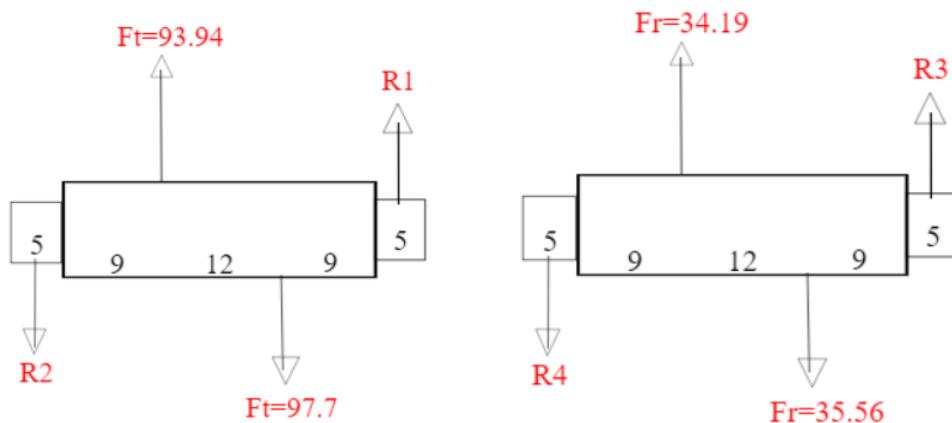
$$T_{tot} = T_3 - T_2 = 0.09776 \text{ N} \cdot \text{m}$$

For output shaft:

$$T_4 = F_4^T * \frac{D_4}{2} = 97.7 * \frac{52 * 10^{-3}}{2} = 2.5402 \text{ N} * \text{m}$$

All shafts will be assumed to be 40 mm length each.

Intermediate shaft design:



As illustrated from the FBD, we can easily calculate the reaction forces:

$$R_1 = \frac{97.7(12 + 9 + 2.5) - 93.94(9 + 2.5)}{5 + 30} = 34.7326 \text{ N}$$

Similarly:

$$R_2 = 30.9726 \text{ N}$$

$$R_3 = 12.6424 \text{ N}$$

$$R_4 = 11.2724 \text{ N}$$

Since the shaft is symmetric, it's enough to calculate the moment at a:

$$M_a (\text{from both planes}) = \sqrt{(30.9726 * 2.5)^2 + (11.2724 * 2.5)^2} = 82.4 \text{ N} \cdot \text{mm} = 0.0824 \text{ N} \cdot \text{m}$$

We will use inexpensive steel 1020CD with $S_{ut} = 469 \text{ MPa}$ $S'_e = 234.5 \text{ MPa}$

$$\therefore k_a = 3.04(469)^{-0.217} = 0.8$$

Assume $k_f = k_{fs} = 1.2$, and $k_b = k_c = k_d = k_e = 1$

$$\therefore S_e = 234.5 * 0.8 = 187.6 \text{ MPa}$$

We know that

$$M_m = T_a = 0$$

Now, calculate

$$A = 2 * 1.2 * 0.0824 = 0.19776 \quad B = 1.2 * \sqrt{3} * 0.09776 = 0.20319$$

$$\therefore d = \left(\frac{16*2}{\pi} * \left(\frac{A}{187.6*10^6} + \frac{B}{469*10^6} \right) \right)^{1/3} = 2.474mm \text{ for a factor of safety 2}$$

Since the diameter cannot be less than 7.62 for the $k_b < 1$ we will set d=8mm

Since we are not interested about notches now, we will keep $k_f = k_{fs} = 1.2$

$$\therefore k_b = \left(\frac{8}{7.62} \right)^{-0.107} = 0.994806 \quad \therefore S_e = 234.5 * 0.8 * 0.994806$$

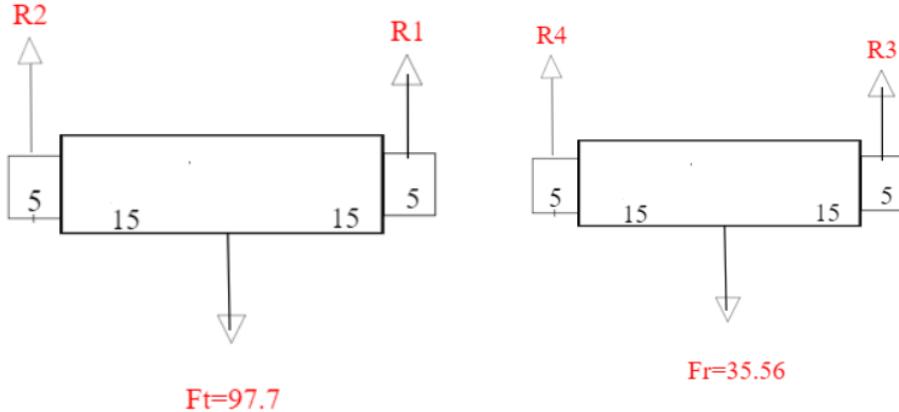
$$\therefore \sigma'_a = \frac{32 * 1.2 * 0.0824}{\pi * 0.008^3} * 10^{-6} = 1.967155 MPa$$

$$\therefore \sigma'_m = \frac{16 * \sqrt{3} * 1.2 * 0.09776}{\pi * 0.008^3} * 10^{-6} = 2.021171 MPa$$

$$\therefore n_f = \left(\frac{1.967155}{186.6256} + \frac{2.021171}{469} \right)^{-1} = 66.2$$

The intermediate diameter can be assumed a little bigger $D = 12mm$

Output shaft design:



Similar to force analysis of the intermediate shaft we can get the following:

$$R_1 = 48.85 \text{ N}$$

$$R_2 = 48.85 \text{ N}$$

$$R_3 = 17.78 \text{ N}$$

$$R_4 = 17.78 \text{ N}$$

since the shaft is symmetric, it's enough to calculate the moment at a:

$$M_a (\text{from both planes}) = \sqrt{(48.85 * 2.5)^2 + (17.78 * 2.5)^2} = 129.963 \text{ N} \cdot \text{mm} = 0.129963 \text{ N} \cdot \text{m}$$

Using the same material 1020CD $S_{ut} = 469 \text{ MPa}$ $S'_e = 234.5 \text{ MPa}$

$$\therefore k_a = 3.04(469)^{-0.217} = 0.8$$

Assume $k_f = k_{fs} = 1.2$, and $k_b = k_c = k_d = k_e = 1$

$$\therefore S_e = 234.5 * 0.8 = 187.6 \text{ MPa}$$

We know that

$$M_m = T_a = 0$$

Now, calculate

$$A = 2 * 1.2 * 0.129963 = 0.311904 \quad B = 1.2 * \sqrt{3} * 2.5402 = 5.2797$$

$$\therefore d = \left(\frac{16*2}{\pi} * \left(\frac{A}{187.6*10^6} + \frac{B}{469*10^6} \right) \right)^{1/3} = 5.087 \text{ mm} \text{ for a factor of safety 2}$$

Since the diameter cannot be less than 7.62 for the $k_b < 1$ we will set $d=8\text{mm}$

Since we are not interested about notches now, we will keep $k_f = k_{fs} = 1.2$

$$\therefore k_b = \left(\frac{8}{7.62} \right)^{-0.107} = 0.994806 \quad \therefore S_e = 234.5 * 0.8 * 0.994806$$

$$\therefore \sigma'_a = \frac{32 * 1.2 * 0.129963}{\pi * 0.008^3} * 10^{-6} = 3.102638 \text{ MPa}$$

$$\therefore \sigma'_m = \frac{16 * \sqrt{3} * 1.2 * 2.5402}{\pi * 0.008^3} * 10^{-6} = 52.518 \text{ MPa}$$

$$\therefore n_f = \left(\frac{3.102638}{186.6256} + \frac{52.518}{469} \right)^{-1} = 7.7758$$

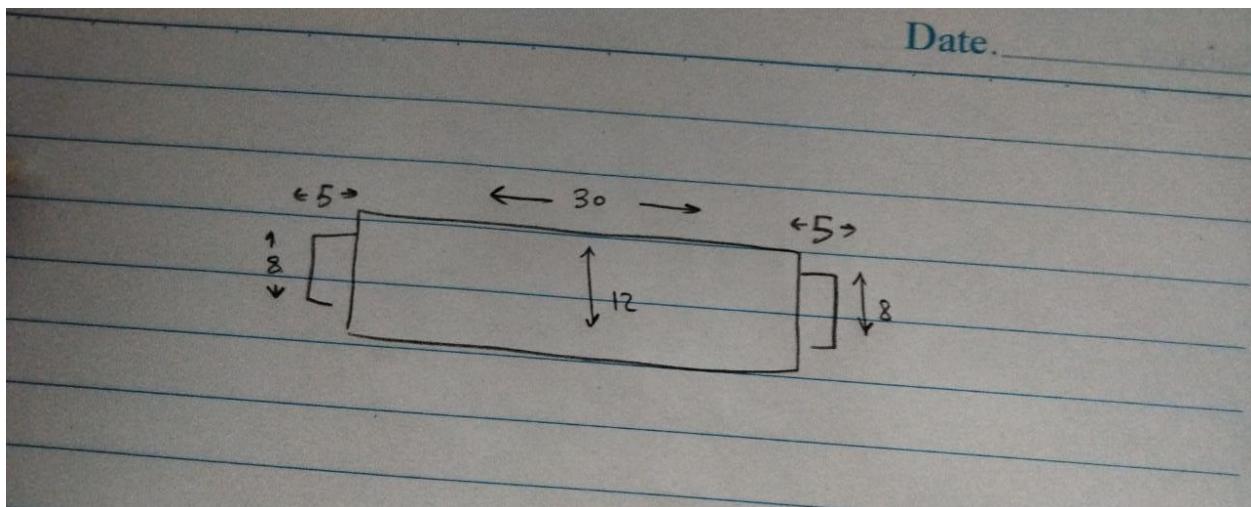
The intermediate diameter can be assumed a little bigger $D = 12mm$

Finally, there's no need to analyze the input shaft as it has less torque. So, a typical shaft to that of the output will be good enough.

Bearings Selections

Deep groove bearings selection (Single row deep groove ball bearings with a snap ring groove):

Our three shafts have a diameter shaft of 12mm, and 8mm as shown in the figure below:

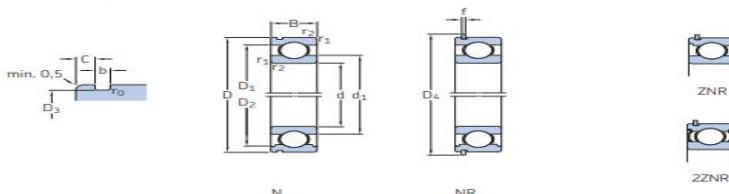


From SKF Tables: Bearings will be on the 8mm diameter so we will select the **least bearing diameter (10 mm)** of a single row deep groove ball bearing with snap ring groove (Type: 6200):



1.3 Single row deep groove ball bearings with a snap ring groove
d 10–35 mm

L.3



Principal dimensions			Basic load ratings dynamic	Basic load ratings static	Fatigue load limit	Speed ratings Reference speed	Speed ratings Limiting speed ²⁾	Mass	Designations Bearings ¹⁾	Snap ring	
d	D	B	C	C ₀	P _u	r/min	r/min	kg	—		
10	20	9	5,4	2,36	0,1	56 000	28 000	0,035	6200-ZNR	6200-ZZNR	
30	9	5,4	2,36	0,1	56 000	36 000	0,032	6200-N	6200-NR	SP 30	
12	32	10	7,28	3,1	0,132	50 000	26 000	0,037	6201-ZNR	6201-ZZNR	
	32	10	7,28	3,1	0,132	50 000	32 000	0,037	6201-N	6201-NR	SP 32
15	35	11	8,06	3,75	0,16	43 000	22 000	0,045	6202-ZNR	6202-ZZNR	
	35	11	8,06	3,75	0,16	43 000	28 000	0,045	6202-N	6202-NR	SP 35
17	40	12	9,95	4,75	0,2	38 000	19 000	0,065	6203-ZNR	6203-ZZNR	
	40	12	9,95	4,75	0,2	38 000	24 000	0,065	6203-N	6203-NR	SP 40
	47	14	14,3	6,55	0,275	34 000	17 000	0,12	6303-ZNR	6303-ZZNR	SP 47

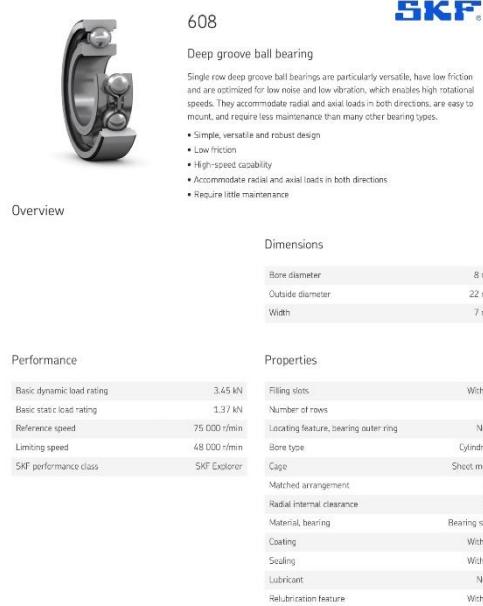
Or:

From SKF Website: A 608 deep groove ball bearing was found with bore diameter of 8 mm, and here is some specification:

From:

SKF. (n.d.). Skf.com. Retrieved May 26, 2023, from

<https://www.skf.com/sg/products/rolling-bearings/ball-bearings/deep-groove-ball-bearings/productid-608>.



Retaining rings Selections

The selected rings for gears will be 12 mm (DSHR-012) as shown below:

Item #	Shaft Dia. (mm)	Groove Size				Ring Size & Weight								Supplementary Data					
		Diameter		Width	Depth	Thickness ²		Free Diameter		Lug Ht.	Max. Sect.	Hole Dia.	Weight	Edge Margin	Thrust Load Ring	Thrust Load Groove	Allowable Radii/ Chamfers	Max. Load w/R Max. or Ch Max.	RPM Limits
		D _s	D _g	Tol.	W Min.	d	T	Tol.	D _f	Tol.	H Max.	S Max.	R Min.	kg/ 1,000	Y Min.	P _r kN	P _g kN	R/Ch Max.	P' r kN
DSHR-012	12	11.5	-0.11	1.60	0.25	1.50	-0.06	11.0		3.4	1.8	1.7	0.75	0.7	11.30	1.53	1.0	4.5	75,000
DSHR-015	15	14.3	-0.11	1.60	0.35	1.50	-0.06	13.8		4.8	2.4	2.0	1.20	0.7	15.50	3.20	1.0	4.5	50,000
DSHR-016	16	15.2	-0.11	1.60	0.40	1.50	-0.06	14.7	+0.10/ -0.36	5.0	2.5	2.0	1.20	1.2	16.70	3.26	1.0	4.5	48,000
DSHR-017	17	16.2	-0.11	1.60	0.40	1.50	-0.06	15.7		5.0	2.6	2.0	1.24	1.2	18.00	4.32	1.0	4.5	46,000
DSHR-018	18	17.0	-0.11	1.60	0.50	1.50	-0.06	16.5		5.1	2.7	2.0	1.54	1.5	26.60	5.50	1.5	5.8	43,000
DSHR-019	19	18.0	-0.11	1.60	0.50	1.50	-0.06	17.5		5.1	2.7	2.0	1.45	1.5	26.60	5.78	1.5	5.9	28,000
DSHR-020	20	19.0	-0.13	1.85	0.50	1.75	-0.06	18.5		5.5	3.0	2.0	2.25	1.5	36.30	5.60	1.5	8.2	32,000
DSHR-022	22	21.0	-0.21	1.85	0.50	1.75	-0.06	20.5		6.0	3.1	2.0	2.30	1.5	36.00	5.60	1.5	8.1	29,000
DSHR-024	24	22.9	-0.21	1.85	0.55	1.75	-0.06	22.2		6.3	3.2	2.0	2.70	1.7	34.20	7.95	1.5	7.6	29,000
DSHR-025	25	23.9	-0.21	2.15	0.55	2.00	-0.07	23.2	+0.21/ -0.42	6.4	3.4	2.0	3.35	1.7	45.00	8.30	1.5	10.3	25,000
DSHR-026	26	24.4	-0.21	2.15	0.80	2.00	-0.07	23.6		6.6	3.3	2.0	3.65	2.4	44.00	10.70	1.5	10.0	27,000
DSHR-027	27	25.5	-0.21	2.15	0.75	2.00	-0.07	24.7		6.6	3.4	2.0	3.85	2.3	45.50	10.30	1.5	10.6	25,000
DSHR-028	28	26.6	-0.21	2.15	0.70	2.00	-0.07	25.9		6.5	3.5	2.0	3.90	2.1	57.00	10.00	1.5	13.4	22,000
DSHR-029	29	27.6	-0.21	2.15	0.70	2.00	-0.07	26.9		6.5	3.8	2.0	4.30	2.1	56.50	10.40	1.5	13.3	22,000
DSHR-030	30	28.6	-0.21	2.15	0.70	2.00	-0.07	27.9		6.5	4.1	2.0	5.00	2.1	57.00	10.70	1.5	13.6	21,000
DSHR-032	32	30.3	-0.21	2.15	0.85	2.00	-0.07	29.6		6.5	4.1	2.5	5.40	2.5	57.00	12.90	1.5	13.6	20,000

The selected rings for bearings should be 8 mm, which isn't found. So, we will select the lowest one which is DSHR-012:

Item #	Shaft Dia. (mm)	Groove Size				Ring Size & Weight								Supplementary Data					
		Diameter		Width	Depth	Thickness ²		Free Diameter		Lug Ht.	Max. Sect.	Hole Dia.	Weight	Edge Margin	Thrust Load Ring	Thrust Load Groove	Allowable Radii/ Chamfers	Max. Load w/R Max. or Ch Max.	RPM Limits
		D _s	D _g	Tol.	W Min.	d	T	Tol.	D _f	Tol.	H Max.	S Max.	R Min.	kg/ 1,000	Y Min.	P _r kN	P _g kN	R/Ch Max.	P' r kN
DSHR-012	12	11.5	-0.11	1.60	0.25	1.50	-0.06	11.0		3.4	1.8	1.7	0.75	0.7	11.30	1.53	1.0	4.5	75,000
DSHR-015	15	14.3	-0.11	1.60	0.35	1.50	-0.06	13.8		4.8	2.4	2.0	1.20	0.7	15.50	3.20	1.0	4.5	50,000
DSHR-016	16	15.2	-0.11	1.60	0.40	1.50	-0.06	14.7	+0.10/ -0.36	5.0	2.5	2.0	1.20	1.2	16.70	3.26	1.0	4.5	48,000
DSHR-017	17	16.2	-0.11	1.60	0.40	1.50	-0.06	15.7		5.0	2.6	2.0	1.24	1.2	18.00	4.32	1.0	4.5	46,000
DSHR-018	18	17.0	-0.11	1.60	0.50	1.50	-0.06	16.5		5.1	2.7	2.0	1.54	1.5	26.60	5.50	1.5	5.8	43,000
DSHR-019	19	18.0	-0.11	1.60	0.50	1.50	-0.06	17.5		5.1	2.7	2.0	1.45	1.5	26.60	5.78	1.5	5.9	28,000
DSHR-020	20	19.0	-0.13	1.85	0.50	1.75	-0.06	18.5		5.5	3.0	2.0	2.25	1.5	36.30	5.60	1.5	8.2	32,000
DSHR-022	22	21.0	-0.21	1.85	0.50	1.75	-0.06	20.5		6.0	3.1	2.0	2.30	1.5	36.00	5.60	1.5	8.1	29,000
DSHR-024	24	22.9	-0.21	1.85	0.55	1.75	-0.06	22.2		6.3	3.2	2.0	2.70	1.7	34.20	7.95	1.5	7.6	29,000
DSHR-025	25	23.9	-0.21	2.15	0.55	2.00	-0.07	23.2	+0.21/ -0.42	6.4	3.4	2.0	3.35	1.7	45.00	8.30	1.5	10.3	25,000
DSHR-026	26	24.4	-0.21	2.15	0.80	2.00	-0.07	23.6		6.6	3.3	2.0	3.65	2.4	44.00	10.70	1.5	10.0	27,000
DSHR-027	27	25.5	-0.21	2.15	0.75	2.00	-0.07	24.7		6.6	3.4	2.0	3.85	2.3	45.50	10.30	1.5	10.6	25,000
DSHR-028	28	26.6	-0.21	2.15	0.70	2.00	-0.07	25.9		6.5	3.5	2.0	3.90	2.1	57.00	10.00	1.5	13.4	22,000
DSHR-029	29	27.6	-0.21	2.15	0.70	2.00	-0.07	26.9		6.5	3.8	2.0	4.30	2.1	56.50	10.40	1.5	13.3	22,000
DSHR-030	30	28.6	-0.21	2.15	0.70	2.00	-0.07	27.9		6.5	4.1	2.0	5.00	2.1	57.00	10.70	1.5	13.6	21,000
DSHR-032	32	30.3	-0.21	2.15	0.85	2.00	-0.07	29.6		6.5	4.1	2.5	5.40	2.5	57.00	12.90	1.5	13.6	20,000
DSHR-034	34	32.3	-0.25	2.65	0.85	2.50	-0.07	31.5		6.6	4.2	2.5	6.80	2.5	87.00	16.40	1.5	15.6	18,000
DSHR-035	35	33.0	-0.25	2.65	1.00	2.50	-0.07	32.2	+0.25/ -0.50	6.7	4.2	2.5	7.10	3.0	86.00	17.80	1.5	15.4	17,000
DSHR-036	36	34.0	-0.25	2.65	1.00	2.50	-0.07	33.2		6.7	4.2	2.5	7.50	3.0	101.50	20.10	2.0	18.3	16,000
DSHR-038	38	36.0	-0.25	2.65	1.00	2.50	-0.07	35.2		6.8	4.3	2.5	8.00	3.0	101.00	21.20	2.0	18.6	15,000

Gear Keys Design

Keys are used to secure moving parts by being inserted into shafts. These keys enable the transfer of torque from the shaft to the part it supports. They serve as the connecting element between the gear and shaft, or between the pulley and shaft.

All Shafts diameters are 12mm = 0.472441 incl. ($radius (r) = \frac{0.472441}{2} = 0.2362205 \text{ inch.}$)

Thus, the chosen key will have a keyway depth pf 1/16 "From Table 7_6", since the square key is better than the rectangular one.

Table 7–6 Inch Dimensions for Some Standard Square- and Rectangular-Key Applications

Shaft Diameter		Key Size		
Over	To (Incl.)	w	h	Keyway Depth
$\frac{5}{16}$	$\frac{7}{16}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{64}$
$\frac{7}{16}$	$\frac{9}{16}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{3}{64}$
		$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16}$
$\frac{9}{16}$	$\frac{7}{8}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
		$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{32}$
$\frac{7}{8}$	$1\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{3}{32}$
		$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$
$1\frac{1}{4}$	$1\frac{3}{8}$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{1}{8}$
		$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{32}$
$1\frac{3}{8}$	$1\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
		$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{16}$
$1\frac{3}{4}$	$2\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{16}$
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$
$2\frac{1}{4}$	$2\frac{3}{4}$	$\frac{5}{8}$	$\frac{7}{16}$	$\frac{7}{32}$
		$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{16}$
$2\frac{3}{4}$	$3\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
		$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{8}$

Material of the key:

Material of all Keys for all shafts will be G10200 (CD1020) since it's the cheapest (Table A-20) with a $S_y=57$ Kpsi.:.

Table A-20 Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels

[The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm ($\frac{3}{4}$ to $1\frac{1}{4}$ in). These strengths are suitable for use with the design factor defined in Sec. 1-10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.]

1 UNS No.	2 SAE and/or AISI No.	3 Process- ing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197
G10600	1060	HR	680 (98)	370 (54)	12	30	201

To get the Length of the key we will use the next formula:

$$L = \frac{2 * F * n}{t * S_y}$$

Where,

L is the length of the key

t is the key thickness

F is Force on the shaft

S_y is the shear strength

Given that from previous calculations that:

-In the inner shaft: $T_1 = 2.3485 \text{ N.m} = 20.7859757 \text{ lbf-in}$

$$\text{Then } F_1 = \frac{T_1}{r} = \frac{20.7859757}{0.2362205} = 87.99 \text{ ibf}$$

-In the Intermediate shaft:

$$T_2 = 2.44244 \text{ N.m} = 21.61741473 \text{ lbf-in}$$

$$\text{Then } F_2 = \frac{T_2}{r} = \frac{21.61741473}{0.2362205} = 91.5 \text{ ibf}$$

$$T_3 = 2.4425 \text{ N.m} = 21.61794578 \text{ lbf-in}$$

$$\text{Then } F_3 = \frac{T_3}{r} = \frac{21.61794578}{0.2362205} = 91.5 \text{ ibf}$$

-in the output shaft:

$$T_4 = 2.5402 \text{ N.m} = 22.4826636 \text{ lbf-in}$$

$$\text{Then } F_4 = \frac{T_4}{r} = \frac{22.4826636}{0.2362205} = 95.2 \text{ ibf}$$

Then the length of the key for every shaft will be (let the factor of safety (n) is 7 for all shafts since it's the lowest one among all), $t=1/16$, and $S_y=57 \text{ ksi}$ (from above tables):

$$\text{For input shaft: } L_1 = \frac{2*F_1*n}{t*S_y} = \frac{2*87.99*7}{\frac{1}{16}*57000} = 0.3458 \text{ inch} = 8.78332 \text{ mm}$$

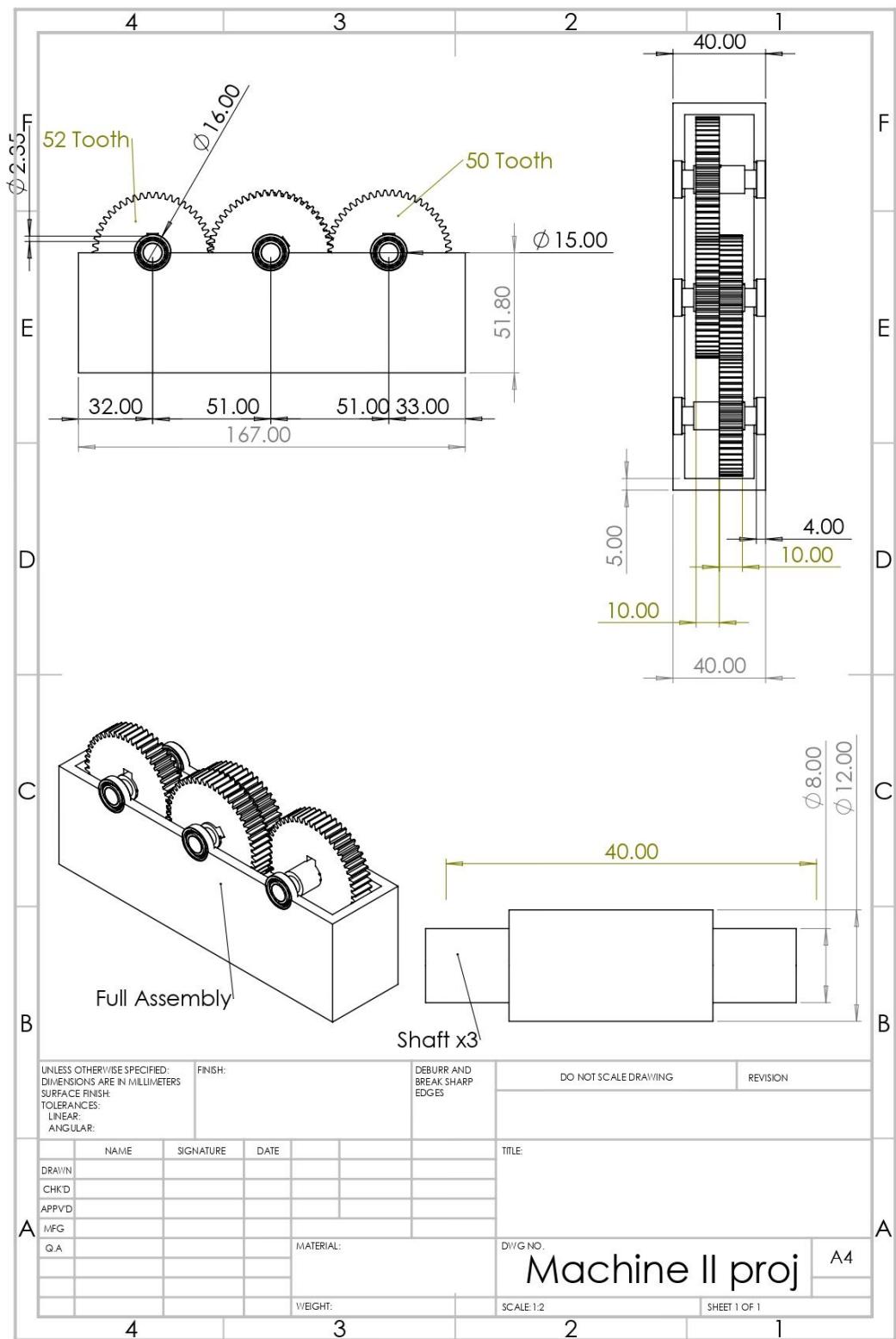
$$\text{For intermediate shaft: } L_2 = \frac{2*F_2*n}{t*S_y} = \frac{2*91.5*7}{\frac{1}{16}*57000} = 0.359 \text{ inch} = 9.1186 \text{ mm}$$

$$L_3 = \frac{2*F_3*n}{t*S_y} = \frac{2*91.5*7}{\frac{1}{16}*57000} = 0.359 \text{ inch} = 9.1186 \text{ mm}$$

For output shaft:

$$L_4 = \frac{2*F_4*n}{t*S_y} = \frac{2*95.2*7}{\frac{1}{16}*57000} = 0.374 \text{ inch} = 9.4996 \text{ mm}$$

Gearbox Assembly Drawing (Solid works)



Appendix

Table 8.5 Coefficient of friction for threaded Pairs

Table 8–5 Coefficients of Friction f for Threaded Pairs

Screw Material	Nut Material			
	Steel	Bronze	Brass	Cast Iron
Steel, dry	0.15–0.25	0.15–0.23	0.15–0.19	0.15–0.25
Steel, machine oil	0.11–0.17	0.10–0.16	0.10–0.15	0.11–0.17
Bronze	0.08–0.12	0.04–0.06	—	0.06–0.09

Source: Data from H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook*, 2nd ed., McGraw-Hill, New York, 2006.

Figure 14–14

Repeatedly applied bending strength stress-cycle factor Y_N . (ANSI/AGMA 2001-D04.)

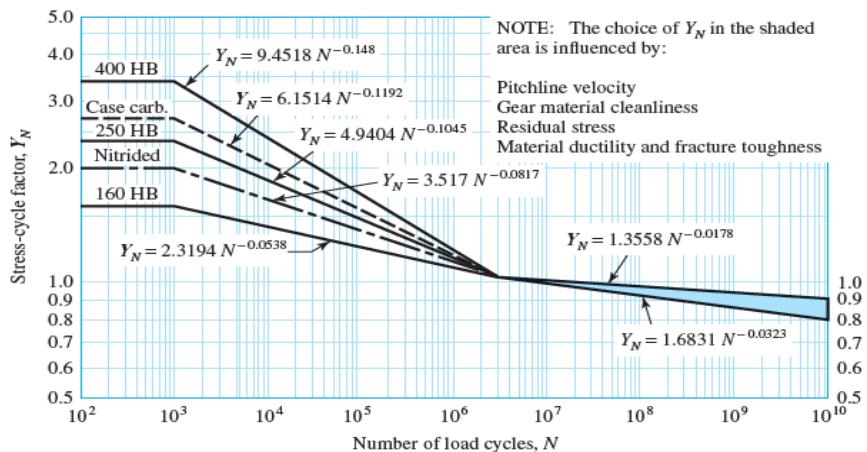
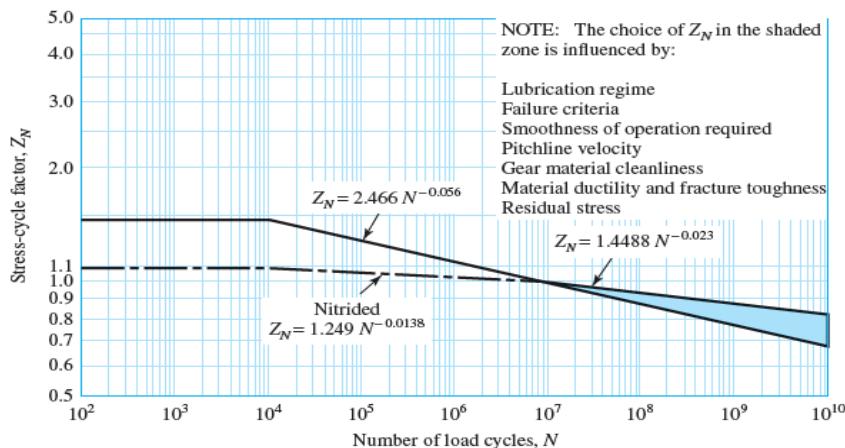
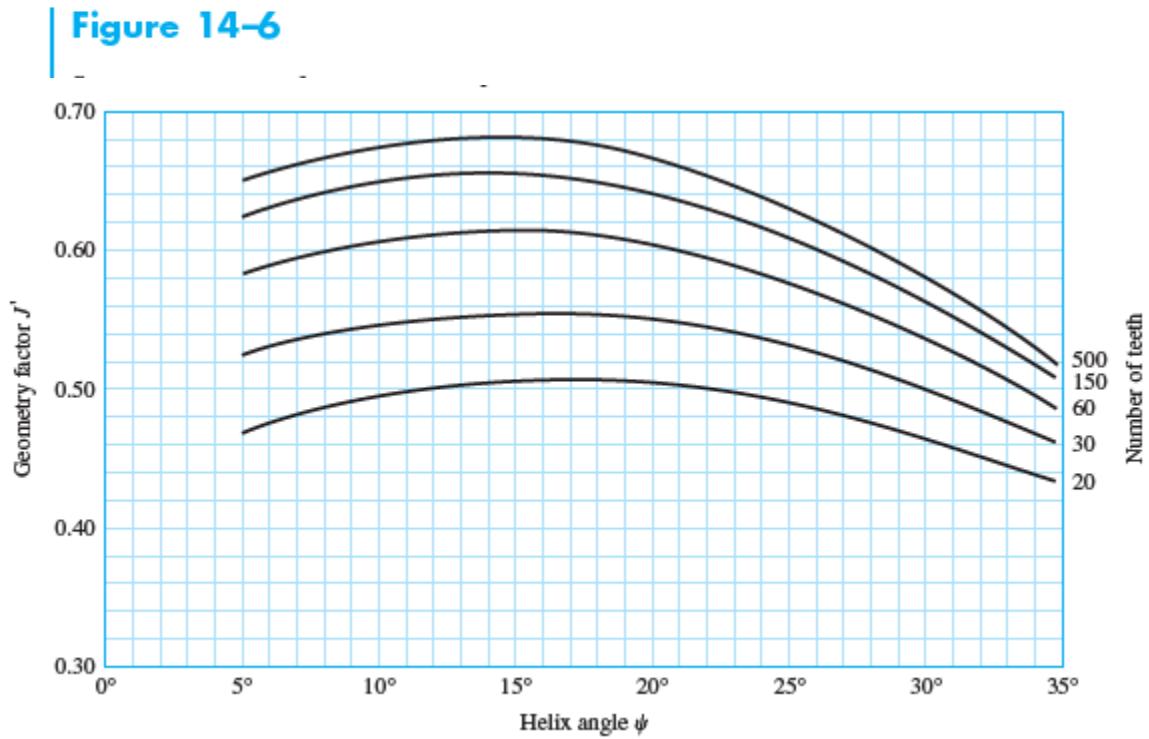


Figure 14–15

Pitting resistance stress-cycle factor Z_N . (ANSI/AGMA 2001-D04.)



Source: Data from H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook*, 2nd ed., McGraw-Hill, New York, 2006.



References

Budynas, R. G., & Nisbett, J. K. (2011). *Shigley's mechanical engineering design* (Vol. 9) New York: McGraw-Hill.