MATH 201 - SPRG2022

ON TIME!

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Abstract

Traffic jams are one of the most critical problems, which the whole world is facing, and especially for countries that have high population density. Traffic congestion has an impact on productivity, mobility, accessibility, mobility expenses, and environmental effects such as air pollution and global warming. This problem has been rising since a long time ago, and one of the most efficient ways is to guide riders where to go by monitoring how many entering and exiting cars at a certain node – of 2 or more road intersections and then recommend a certain road to the rider. Linear algebra has a variety of applications and contributions to facilitate solving a lot of complex problems, and sparingly! Linear Algebra will be used to solve traffic jams problems by solving linear or system equations to get the numbers of cars in each road. The software is mainly about a traffic recommendation system that takes the number of cars entering and getting out of a certain node to get the number of cars in every street and thus recommend the driver for the least traffic street. The real project will detect the cars using either radar or google maps and then recommend to the driver which street to take. The system simply consists of monitoring tools like radar or ultrasonic sensors, then software to solve the linear equations then recommend the best choice to the rider.

1. Historical Overview

Row reduction algorism or Gaussian elimination, is an algorithm for solving systems of linear equations. It includes a series of operations achieved at the corresponding matrix of coefficients. This technique also can be used to compute the rank of a matrix, the determinant of a square matrix, and the inverse of an invertible matrix. The technique is known as after *Carl Friedrich Gauss (1777– 1855).*

To carry out row reduction on a matrix, one uses a series of elementary row operations to adjust the matrix until the lower left-hand nook of the matrix is packed with zeros, as lots as possible. *There are 3 kinds of essential row operations:*

- Swapping rows,
- Multiplying a row through a nonzero number,
- Adding a more than one of 1 row to any other row.

2. Introduction

The general approach is based on the concept of "reduced echelon form," which solves a system of X-variable linear equations by augmenting the final column as the absolute term or solution. For instance, consider the linear equations in below:

- **System of linear equations:**

  Eqn. 1: 3a + b =7

  Eqn. 2: 6a -2b = 9

- **Coefficient matrix:** $\begin{bmatrix} \mathbf{3} & \mathbf{1} \\ \mathbf{6} & \mathbf{-2} \end{bmatrix}$

The coefficients of each variable are represented as ‘’a’’ and ‘’b’’ in the coefficient matrix.

- **Augmented matrix:** $\begin{bmatrix} \mathbf{3} & \mathbf{1} & \mathbf{7} \\ \mathbf{6} & \mathbf{-2} & \mathbf{9} \end{bmatrix}$

The coefficient matrix + the solution for each equation is represented by the augmented matrix.

Row reduction algorithm is shown below:

$$\begin{bmatrix} \mathbf{3} & \mathbf{1} & \mathbf{7} \\ \mathbf{6} & \mathbf{-2} & \mathbf{9} \end{bmatrix}$$

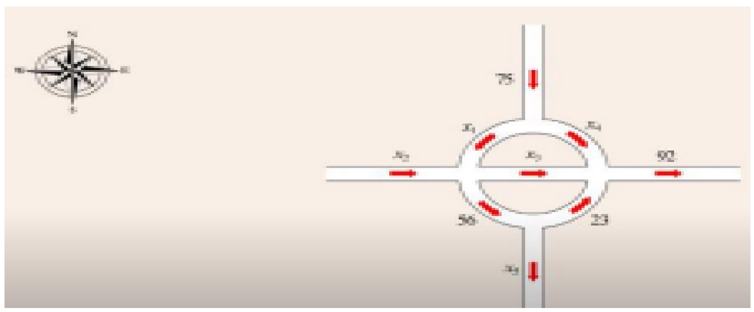
First row operations is (-2)R_1 + R_2 →R_2, when R_1 is the first row and R_2 is the second row, to get the following matrix:

$$\begin{bmatrix} \mathbf{3} & \mathbf{1} & \mathbf{7} \\ \mathbf{0} & \mathbf{-4} & \mathbf{-5} \end{bmatrix}$$

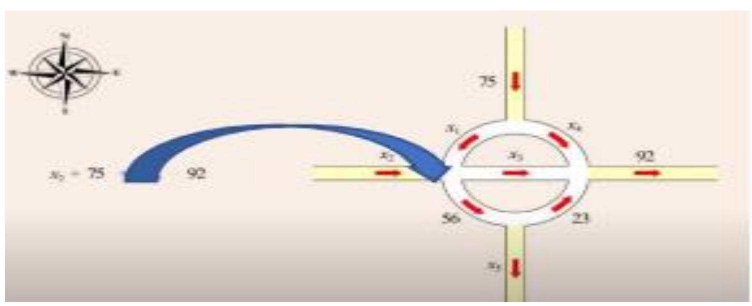
So, ‘’b’’ equals to 5/4 while ‘’b’’ equals to 23/12 [(7-5/4)/3]. The purpose of that algorism to find the exact value of each variable and determine if they are free or basic “pivot” variable.

3. How Does Linear Algebra Solve Traffic Jam?

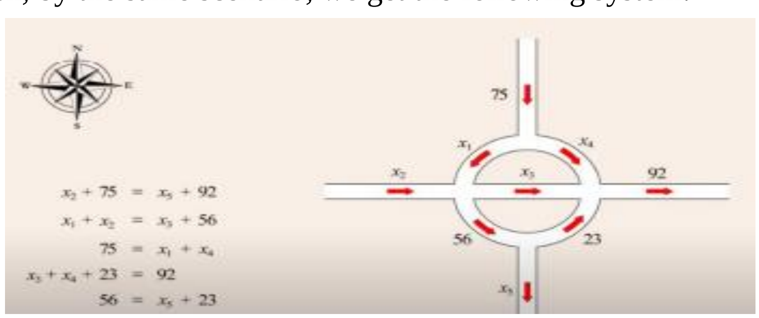
After illustrating math basics needed in the project, then the further step to demonstrate how to implement them in solving traffic jam problems. For example, the following square in figure 1, the arrows represent the direction of movements, and the number indicates how much cars pass per unit time. To describe the following system with linear algebra, absolutely we need representative equations, so the key factor is to change the arrows with numbers to variables with coefficients.



For each node, road intersection, we can construct a single equation include the entering cares and equating them with exiting cars as shown in the figure 2 below:



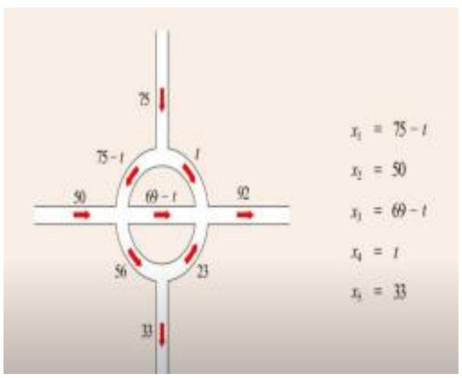
The indicated node has a unique equation for entering and exiting cars which by equating form a unique equation. Then, by the same scenario, we get the following system:



Then construct our augmented matrix as shown below:

$$
\begin{aligned}
x_2 - x_5 &= 17\\
x_1 + x_2 - x_3 &= 56\\
x_1 + x_4 &= 75\\
x_3 + x_4 &= 69\\
x_5 &= 33
\end{aligned}
\qquad
\begin{array}{c}
\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{array}\\
\left[\begin{array}{ccccc|c}
0 & 1 & 0 & 0 & -1 & 17\\
1 & 1 & -1 & 0 & 0 & 56\\
1 & 0 & 0 & 1 & 0 & 75\\
0 & 0 & 1 & 1 & 0 & 69\\
0 & 0 & 0 & 0 & 1 & 33
\end{array}\right]
\end{array}
$$

After row reduce the matrix, the parametric is made as shown below:

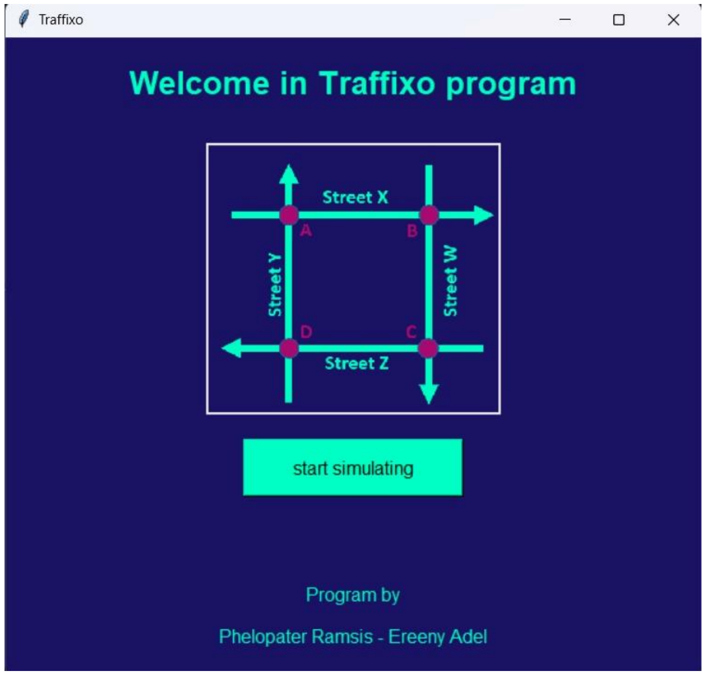


$$
\left[\begin{array}{ccccc|c}
\boxed{1} & 0 & 0 & 1 & 0 & 75\\
0 & \boxed{1} & 0 & 0 & 0 & 50\\
0 & 0 & \boxed{1} & 1 & 0 & 69\\
0 & 0 & 0 & 0 & \boxed{1} & 33\\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

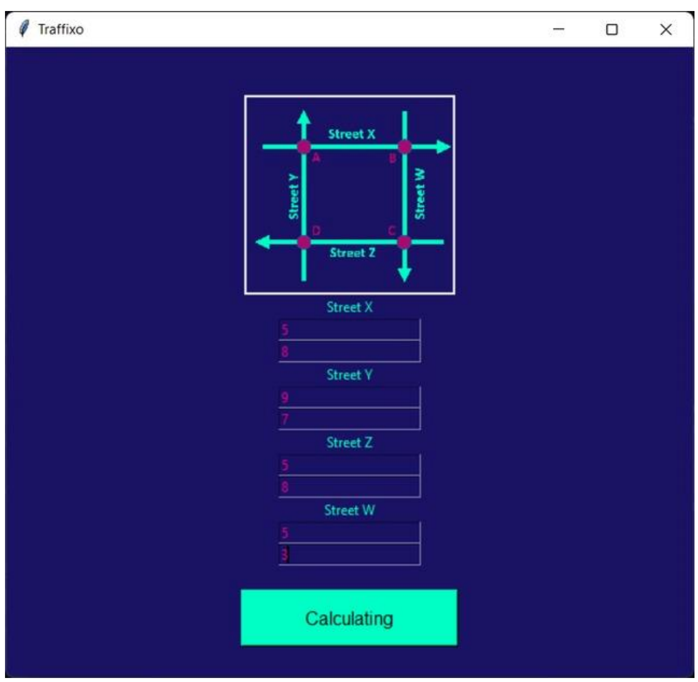
4 pivot columns and 1 free variable. So, the system has infinite number of solutions. As a result, by simple calculations and by implement the value of ''t'', and according to the direction desired for the rider, he can choose the most suitable one with less traffic jam.

4. Real-Life Implementation Using Software Application

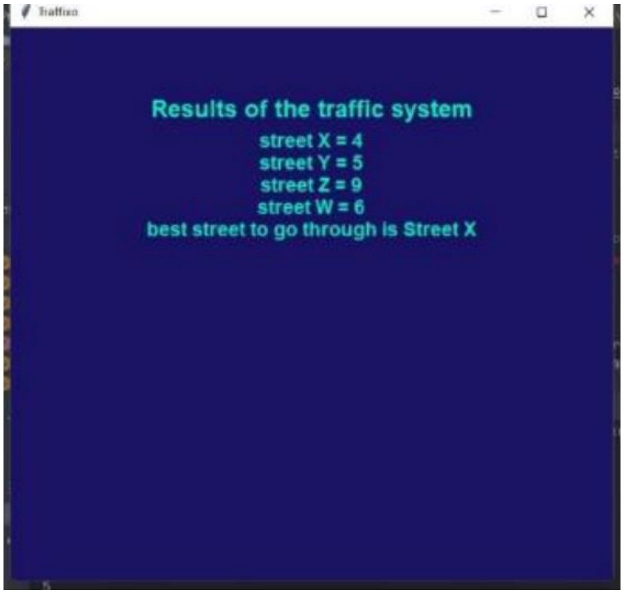
First, user will choose the layout of the area and enter how streets are linked together.



Second, user starts assigning the value of flow rate for each road.



Third, the code then starts constructing the matrix and solving the linear system equations to recommend the best road to execute.



5. Advantages and Limitations

Advantages: Traffic movements of certain place can be monitored without human engagements. The chance to stuck in urban congestion or traffic jam will decrease and time consumption will minimize. Also, Control huge amount of data and process fast.

limitations: The code designed for just single direction roads. Also, software application designed for limited road designs.

6. Conclusion

Finally, Traffic jam is considered one of Egypt's grand challenges as it can elongate the time of reaching certain place and cause road accidents and catastrophic damage if it kept uncontrolled. We can control its effects and limit the consequences of its impact without the human interaction to minimize human injuries by simple software application using python which recommends the best road to execute.

7. References

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