Mathematics Behind the Options Calculator

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1 European Call

$$
\begin{aligned}
C &= Se^{-qT}\Phi(d_1) - Ke^{-rT}\Phi(d_2) \\
d_1 &= \frac{\ln\left(\frac{S}{K}\right) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\
d_2 &= d_1 - \sigma\sqrt{T}
\end{aligned}
$$

C Option price

S Spot price

K Strike price

q Dividend rate

T Time to maturity

r Risk free rate

σ Volatility

Φ Standard normal CDF

ϕ Standard normal pdf

1.1 Delta

$$
\boxed{\Delta = \frac{\partial C}{\partial S} = e^{-qT}\Phi(d_1)}
$$

$$
\begin{aligned}
\Delta = \frac{\partial C}{\partial S} &= e^{-qT}\Phi(d_1) + Se^{-qT}\phi(d_1)\left(\frac{\partial d_1}{\partial S}\right) - Ke^{-rT}\phi(d_2)\left(\frac{\partial d_2}{\partial S}\right) \\
&= e^{-qT}\Phi(d_1) + \underbrace{Se^{-qT}\phi(d_1)\frac{1}{S\sigma\sqrt{T}} - Ke^{-rT}\phi(d_2)\frac{1}{S\sigma\sqrt{T}}}_{=0} \\
\Delta &= e^{-qT}\Phi(d_1)
\end{aligned}
$$

1.2 Gamma

1.3 Vega

1.4 Theta

1.5 Rho

2 European Put

2.1 Delta

2.2 Gamma

2.3 Vega

2.4 Theta

2.5 Rho

3 Binary Call

3.1 Delta

3.2 Gamma

3.3 Vega

3.4 Theta

3.5 Rho

4 Binary Put

4.1 Delta

4.2 Gamma

4.3 Vega

4.4 Theta

4.5 Rho

5 American Call

5.1 Delta

5.2 Gamma

5.3 Vega

5.4 Theta

5.5 Rho

6 American Put

6.1 Delta

6.2 Gamma

6.3 Vega

6.4 Theta

6.5 Rho