

# Mathematics Behind the Options Calculator

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## 1 European Call

$$\begin{aligned}C &= Se^{-qT}\Phi(d_1) - Ke^{-rT}\Phi(d_2) \\d_1 &= \frac{\ln\left(\frac{S}{K}\right) + (r - q + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\d_2 &= d_1 - \sigma\sqrt{T}\end{aligned}$$

$C$  Option price

$S$  Spot price

$K$  Strike price

$q$  Dividend rate

$T$  Time to maturity

$r$  Risk-free rate

$\sigma$  Volatility

$\Phi$  Standard normal CDF

$\phi$  Standard normal pdf

### 1.1 Delta

$$\Delta = \frac{\partial C}{\partial S} = e^{-qT}\Phi(d_1)$$

$$\begin{aligned}\Delta &= \frac{\partial C}{\partial S} = e^{-qT}\Phi(d_1) + Se^{-qT}\phi(d_1)\left(\frac{\partial d_1}{\partial S}\right) - Ke^{-rT}\phi(d_2)\left(\frac{\partial d_2}{\partial S}\right) \\&= e^{-qT}\Phi(d_1) + \underbrace{Se^{-qT}\phi(d_1)\frac{1}{S\sigma\sqrt{T}} - Ke^{-rT}\phi(d_2)\frac{1}{S\sigma\sqrt{T}}}_{=0} \\ \Delta &= e^{-qT}\Phi(d_1)\end{aligned}$$

## 1.2 Gamma

## 1.3 Vega

## 1.4 Theta

## 1.5 Rho

# 2 European Put

$$\begin{aligned}P &= Ke^{-rT} \Phi(-d_2) - Se^{-qT} \Phi(-d_1) \\d_1 &= \frac{\ln\left(\frac{S}{K}\right) + (r - q + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\d_2 &= d_1 - \sigma\sqrt{T}\end{aligned}$$

## 2.1 Delta

## 2.2 Gamma

## 2.3 Vega

## 2.4 Theta

## 2.5 Rho

# 3 Binary Call (Cash-or-Nothing)

$$\begin{aligned}C_{\text{bin}} &= Q e^{-rT} \Phi(d_2) \\d_1 &= \frac{\ln\left(\frac{S}{K}\right) + (r - q + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\d_2 &= d_1 - \sigma\sqrt{T}\end{aligned}$$

## 3.1 Delta

## 3.2 Gamma

## 3.3 Vega

## 3.4 Theta

## 3.5 Rho

# 4 Binary Put (Cash-or-Nothing)

$$\begin{aligned}P_{\text{bin}} &= Q e^{-rT} \Phi(-d_2) \\d_1 &= \frac{\ln\left(\frac{S}{K}\right) + (r - q + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\d_2 &= d_1 - \sigma\sqrt{T}\end{aligned}$$

#### 4.1 Delta

#### 4.2 Gamma

#### 4.3 Vega

#### 4.4 Theta

#### 4.5 Rho

### 5 American Call

$$C^A = \sup_{\tau \in \mathcal{T}[0, T]} \mathbb{E}^{\mathbb{Q}}[e^{-r\tau} (S_{\tau} - K)^+]$$

(equals the European call when  $q = 0$ , since early exercise has no value).

#### 5.1 Delta

#### 5.2 Gamma

#### 5.3 Vega

#### 5.4 Theta

## 5.5 Rho

# 6 American Put

$$P^A = \sup_{\tau \in \mathcal{T}[0, T]} \mathbb{E}^{\mathbb{Q}}[e^{-r\tau} (K - S_{\tau})^+]$$

## 6.1 Delta

## 6.2 Gamma

## 6.3 Vega

## 6.4 Theta

## 6.5 Rho