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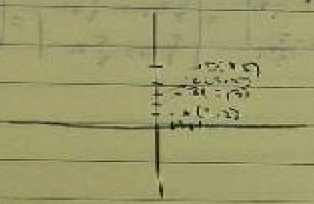
Coffee Customers Dataset

Customer	Avg spent per visit (USD)	Visits per month
A		2
B	2	3
C	3	4
D	4	5
	x_1	x_2

① 

② mean of x_1: $\frac{1+2+3+4}{4} = \frac{10}{4} = 2.5$

mean of x_2: $\frac{2+3+4+5}{4} = \frac{14}{4} = 3.5$

③

$$A_{center} = \begin{bmatrix} 1-2.5 & 2-3.5 \\ 2-2.5 & 4-3.5 \\ 3-2.5 & 5-3.5 \\ 4-2.5 & 6-3.5 \end{bmatrix}$$

④ $= A_{center} \begin{bmatrix} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{bmatrix}$

⑤ $\frac{1}{n-1} A^T A_{center}$

$$A_{center} = \begin{bmatrix} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{bmatrix}$$

(4×2)

$$\times \quad A^T_{center} = \begin{bmatrix} -1.5 & -0.5 & 0.5 & 1.5 \\ -1.5 & -0.5 & 0.5 & 1.5 \end{bmatrix}$$

(2×4)

VICTORY

DATE:

$(-1.5)^2 + (0.5)^2 + (0.5)^2 + (1.5)^2 = 5$

$\frac{1}{3}\begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{5}{3} \end{bmatrix}$

6. $\det\left(\begin{bmatrix} \frac{5}{3}-\lambda & \frac{5}{3} \\ \frac{5}{3} & \frac{5}{3}-\lambda \end{bmatrix}\right) = 0$

$\left(\frac{5}{3}-\lambda\right)\left(\frac{5}{3}-\lambda\right) - \left(\frac{5}{3}\right)\left(\frac{5}{3}\right) = 0$

$\lambda^2 - \frac{10}{3}\lambda = 0$

$\lambda\left(\lambda - \frac{10}{3}\right) = 0$

$\lambda_1 = \frac{10}{3} = 3.33$

$\lambda_2 = 0$

7. $(A - \lambda I)v = 0$

$\left(\begin{bmatrix} \frac{5}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{5}{3} \end{bmatrix} - \begin{bmatrix} \frac{10}{3} & 0 \\ 0 & \frac{10}{3} \end{bmatrix}\right)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$

$\begin{bmatrix} \frac{5}{3} - \frac{10}{3} & \frac{5}{3} \\ \frac{5}{3} & \frac{5}{3} - \frac{10}{3} \end{bmatrix} = \begin{bmatrix} -\frac{5}{3} & \frac{5}{3} \\ \frac{5}{3} & -\frac{5}{3} \end{bmatrix}$

$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -\frac{5}{3}v_1 + \frac{5}{3}v_2 \\ \frac{5}{3}v_1 - \frac{5}{3}v_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{3}v_2 = \frac{5}{3}v_1 \\ \frac{5}{3}v_1 = \frac{5}{3}v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

8. $u = \frac{v}{\|v\|}$

$v = [1, 1]$

$\sqrt{1^2 + 1^2} = \sqrt{2}$

$u_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for $v = [-1, 1]^T$:

9. $\frac{5}{3}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{5}{3}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 10/3 \\ 10/3 \end{bmatrix}$

$\lambda v = \frac{10}{3}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10/3 \\ 10/3 \end{bmatrix}$

$\frac{5}{3}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \frac{5}{3}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

10. $\lambda = \frac{10}{3}$ – ~~[illegible]~~ I chose this because it is the largest eigenvalue.

11. Projected x u_1

$\begin{bmatrix} -1.5 & -1.5 \\ -0.5 & -0.5 \\ 0.5 & 0.5 \\ 1.5 & 1.5 \end{bmatrix} \times \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\frac{-1.5 - 1.5}{\sqrt{2}} = \frac{-3}{\sqrt{2}}$

$\frac{-0.5 - 0.5}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\frac{0.5 + 0.5}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\frac{1.5 + 1.5}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

new position $\begin{bmatrix} -\frac{3}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \frac{3}{\sqrt{2}} \end{bmatrix}$