# AERSP 458 Project 1

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Integrity Statements:

Aditya Singhal

☆ I have completed this work with integrity

Aditya

Han-Yu To

☆ I have completed this work with integrity

3/5/21

# AERSP 458 Project I.

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#1.

Case I & Case 2

Known: $r_0$, $v_0$, $t_0$, $t_1$, $\mu$

$\vec{h} = \vec{r_0} \times \vec{v_0}$ → constant everywhere

$P = \frac{h^2}{\mu}$

Rearrange: $\boxed{a = -\frac{\mu}{2\left(\frac{V_0^2}{2} - \frac{\mu}{r_0}\right)}}$

$\mathcal{E} = -\frac{\mu}{2a} = \frac{V_0^2}{2} - \frac{\mu}{r_0}$ → since energy is constant everywhere, we evaluate at $t_0$ to find a.

So... we know a & p

use

$p = a(1 - e^2)$ to find e.

$e = \sqrt{1 - \frac{p}{a}}$

Use chart ... to find orbit type

| | orbit shape | e |
|---|---|---|
| | circle | 0 |
| Case I → | ellipse | $0 < e < 1$ |
| | parabolic | 1 |
| Case II → | hyperbolic | $> 1$ |

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## Approximation of $U_0$ $U_1$ $U_2$

Top - down algorithm

$$f = \cfrac{a_0}{b_0 - \cfrac{a_1}{b_1 - \cfrac{a_2}{b_2 - \cfrac{a_3}{b_3 - a_4 \ddots}}}} \qquad u = \cfrac{\frac{1}{2}x}{1 - \cfrac{\alpha(\frac{1}{2}x)^2}{3 - \cfrac{\alpha(\frac{1}{2}x)^2}{5 - \cfrac{\alpha(\frac{1}{2}x)^2}{\ddots}}}}$$

So...

set $a_0 = \frac{1}{2}x$ & $a_1, a_2, a_3 \cdots a_n = \alpha(\frac{1}{2}x)^2$

$b_0 = 1$ & $b_1 = 3$ $b_2 = 5$ .....

Use this to find $u$ to $\Delta = 10^{-6}$

then....

$$U_0 = \frac{1 - \alpha u^2}{1 + \alpha u^2} \qquad U_1 = \frac{2u}{1 + \alpha u^2} \qquad U_2 = \frac{2u^2}{1 + \alpha u^2}$$

to approx $U_0$, $U_1$, $U_2$

all written as a MATLAB function

called... TopDown

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Approx $x$

Using Newton-Raphson iteration...

$$x_{new} = x_{old} - \frac{f(x_{old})}{f'(x_{old})}$$

$f(x) = 0$ to $\delta = 10^{-6}$

$$\boxed{f(x) = x - \sigma_0 - \sigma_0 U_0 - (1 - \alpha r_0) U_1 - \alpha\sqrt{\mu}(t_1 - t_0) = 0}$$

↳ where $\sigma_0 = \frac{\vec{r} \cdot \vec{v}}{\sqrt{\mu}}$

$\alpha = \frac{1}{a}$

Need...

$$\frac{df}{dx} = 1 - \sigma_0 \frac{dU_0}{dx} - (1 - \alpha r_0) \frac{dU_1}{dx} = 0$$

↳ $\frac{dU_1}{dx} = U_0$

↳ $\frac{dU_0}{dx} = -\alpha U_1$

$$\therefore \boxed{f'(x) = 1 + \sigma_0 \alpha U_1 - (1 - \alpha r_0) U_0 = 0}$$

Initial Guess for $x$ ⇒ $\boxed{x_{initial} = \alpha\sqrt{\mu}(t_1 - t_0)}$

Use this initial $x$ to find initial $U_0, U_1, U_2$

Using top down function

Repeat the process until...

$|x_{new} - x_{old}| < \delta = 10^{-6}$.

↳ Function UniversalEqn output newest $x, U_0, U_1, U_2$

cont'd

4. Calculate $F,\ G,\ G_t,\ F_t$

$$F = 1 - \frac{1}{r_0} U_2$$

$$G = \frac{r_0}{\sqrt{\mu}} U_1 + \frac{\sigma_0}{\sqrt{\mu}} U_2$$

$$\longrightarrow \vec{r}_1 = F\,\vec{r}_0 + G\,\vec{v}_0$$

$$F_t = \frac{-\sqrt{\mu}}{r r_0} U_1$$

$$G_t = 1 - \frac{U_2}{r}$$

$$\longrightarrow \vec{v}_1 = F_t\,\vec{r}_0 + G_t\,\vec{v}_0$$

5. Energy $= \mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$

@ $t_0$ $\quad \mathcal{E}_0 = \frac{v_0^2}{2} - \frac{\mu}{r_0}$

@ $t_1$ $\quad t_1 = \frac{u^2}{2} - \frac{\mu}{r_1}$

6. $\vec{h} = \vec{r} \times \vec{v}$

@ $t_0$ $\quad \vec{h}_0 = \vec{r}_0 \times \vec{v}_0$

@ $t_1$ $\quad \vec{h}_1 = \vec{r}_1 \times \vec{v}_1$

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Case 1: ------------------------------------
Orbit shape is ELLIPSE
Eccentricity e = 0.520710
Semi-major Axis a = 2.532193
r1 =
  -0.258289895863091
  -0.848613464518243
   1.044258226757839

v1 =
   6.451452954455124
   0.571675296717327
  -0.295644674662010

Energy at t0 = -7.795302 ; Energy at t1 = -7.795302
h at t0 =
  -0.346088580000000
   6.660620789999999
   5.327131890000000

h at t1 =
  -0.346088580000000
   6.660620789999999
   5.327131889999999
```

```
Case 2: ----------------------------------
Orbit shape is HYPERBOLIC
Eccentricity e = 2.462939
Semi-major Axis a = -1.003078
r1 =
   9.313762920308964
  -3.727434393887336
   5.066586002833935

v1 =
   5.052629466935396
  -3.377876065110318
   3.073036362378408

Energy at t0 = 19.678637 ; Energy at t1 = 19.678637
h at t0 =
   5.659758160000001
  -3.021950390000000
 -12.627391990000000

h at t1 =
   5.659758160000001
  -3.021950389999997
 -12.627391989999996
```